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## 3.13 A Lattice Structure for the MI with an Imaginary Transition Gama

With the present Main Injector design transition is one of the major restrictions for high intensity beams (of the order of $10^{14}$ protons per cycle). At transition the bunch length becomes very small while the momentum spread extends to infinity. In the Main Injector the transition gama ($\gamma_t$) is 20.56.

Deiter Mohle from CERN and Tom Collins from FNAL suggested that another Main Injector lattice might be examined with an imaginary transition gama.

The transition occurs during acceleration due to relativistic effects. The transition gama is defined by:

$$1/(\gamma_t)^2 = (\Delta L/L)/(\Delta p/p),$$

where "L" is the total length of the accelerator while $\Delta L$ is a difference in a path of an off momentum particles ($p+\Delta p$). To avoid transition the $1/(\gamma_t)^2$ should be less than zero. The source of dispersion is the dipole where the higher/lower momentum particles are bent less/more.

$$\Delta x_i = D_i * \Delta p/p.$$

The $1/(\gamma_t)^2$ will be negative if the sum of the dispersion function through the dipoles is a negative number. If the dispersion function is presented by Floquet's transformation(1) where the coordinates $\chi(s)$ (the y-axis) and $\xi(s)$ (the x-axis) are defined as:

$$\chi_i = D_i\sqrt{\beta_i} \quad \text{and} \quad \xi_i = D'_i \sqrt{\beta_i} + D_i*\alpha_i/\sqrt{\beta_i}$$

where the $\alpha$ and $\beta$ are the betatron functions while D is dispersion with the slope of the dispersion $D'=dD/ds$, then it becomes clear that the dipoles should be placed in a lattice in the third and fourth quadrant of the $\chi$ and $\xi$ coordinate system. This idea was implemented and an example of a lattice with an imaginary $\gamma_t$ is presented below where the $\gamma_t = i*37.29$.

The maximum of the dispersion function in the lattice with an imaginary $\gamma_t$ is 1.25 m, the tunes are around 33.688, while the chromaticity is $\xi_x=-63.2$ and $\xi_y=-56.05$. The maxima of the betatron functions $\beta_x$ and $\beta_y$ are 74 m, while the minima of the $\beta_x$ and $\beta_y$ in the regular FODO cell are 13.23 m. Figure 1 presents the lattice functions within a repetitive cell. Fifteen of these cells make the whole ring. The straight sections are not designed yet. The lenght of the ring without straight sections is 3366.6 meters. A rough estimate of the length with the straght sections can be done by adding the drift space in the Main Injector straigths to the length of the special low beta cells of this example. This calculation raises the cicumference up to 3700 meters. This lattice example has 270 dipoles with respect to the 300 dipoles in the Main Injector design.

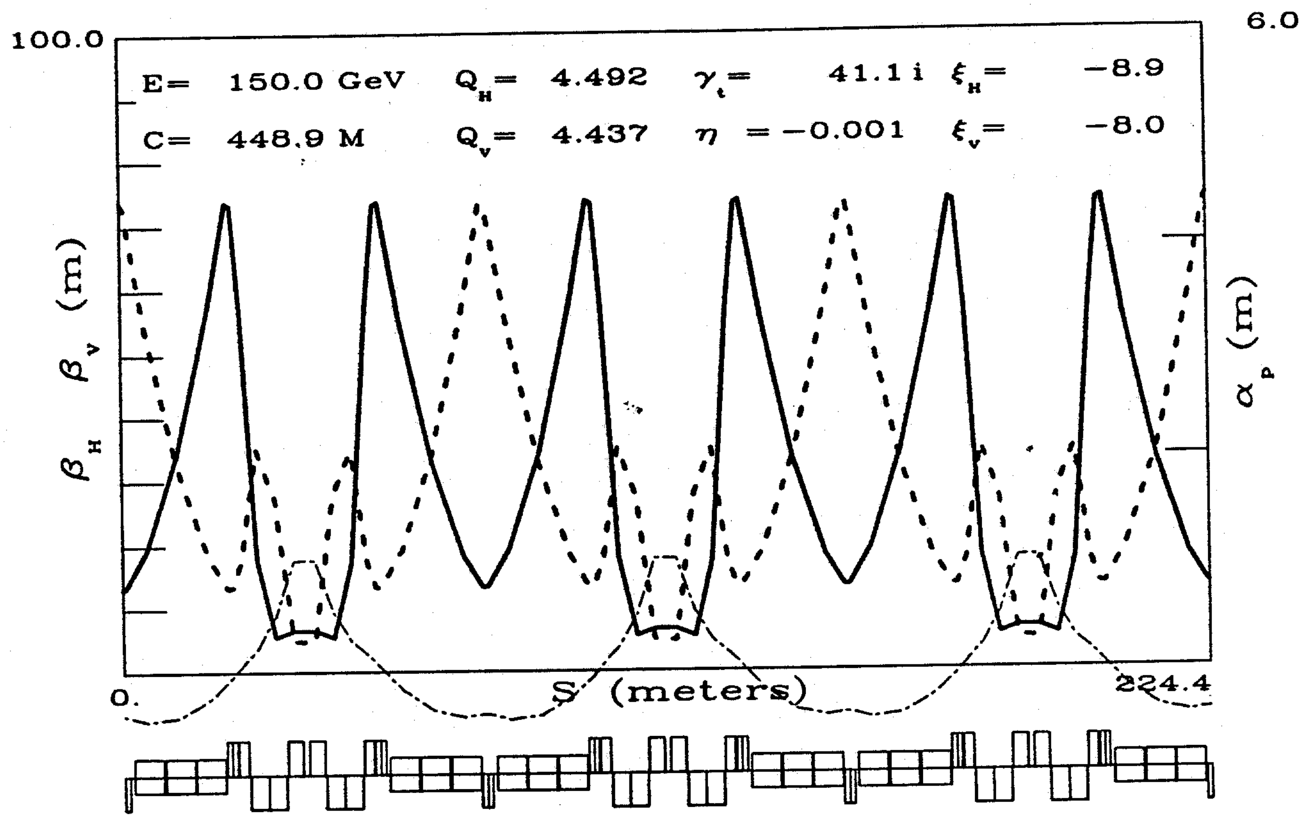


Figure 1 Lattice functions in a repetitive cell.

(1) E.D. Courant and H.S. Snyder, "Theory of the Alternating Gradient Synchrotron," Ann. Phys 3, 1(1958).