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Some Estimation Concerning Crossing Transition of the Main Injector

King-Yuen Ng
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, Illinois 60510

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I. INTRODUCTION

We estimate some parameters pertaining to the transition crossing of the Main Injector. These include the nonadiabatic time, bunch length and bunch height at transition, the microwave growth across transition driven by a longitudinal impedance, and the parameters that govern the Umstätter's and the Johnsen's effects.

II. BUNCH LENGTH AND HEIGHT AT TRANSITION

At time T_c before and after transition, the bucket changes so rapidly that the bunch is not able to follow it. We call this region the nonadiabatic region. This characteristic time is given by[1,2]

$$T_c = \left[\left(\frac{\pi\beta_t^2\gamma_t^4}{h\omega_0^2\dot{\gamma}_t}\right)\left(\frac{E_0/e}{V_{\rm rf}\cos\phi_0}\right)\right]^{\frac{1}{3}} , \tag{2.1}$$

where

$$\begin{aligned} E_0 &= 0.93827 \text{ GeV, rest energy of proton,} \\ \gamma_t &= 20.4 = \left(1-\beta_t^2\right)^{-1/2}, \text{ transition gamma,} \\ \omega_0 &= 566.78 \text{ kHz, angular revolution frequency,} \\ h &= 588, \text{ rf harmonic number,} \\ V_{\rm rf} &= 2.78 \text{ MV, rf voltage,} \\ \phi_0 &= 37.6^\circ \text{ rf phase,} \end{aligned} \tag{2.2}$$

and

$$\dot{\gamma}_t = \frac{\omega_0 e V_{\rm rf}\sin\phi_0}{2\pi E_0} = 163.1 \text{ sec}^{-1} \tag{2.3}$$

is the rate of change of γ at transition. We find $T_c = 1.96$ ms. If every particle crosses transition at exactly the same time, the evolution of the bunch can be computed easily. At transition, the bunch ellipse in the longitudinal phase space is tilted. The *maximum* rms bunch length (not at zero momentum offset) is given by

$$\sigma_\tau = \frac{2}{3^{5/6}\Gamma(1/3)}\left(\frac{AT_c^2 eV_{\rm rf}\dot{\gamma}_t}{E_0\beta_t^2\gamma_t^4}\right)^{\frac{1}{2}} = 0.371 \text{ ns} , \tag{2.4}$$

where $\Gamma(1/3) = 2.678939$ is the gamma function and the bunch area $A = 0.4$ eV-sec has been assumed. The *maximum* rms energy spread is

$$\sigma_E = \frac{\Gamma(1/3)}{3^{2/3}2\pi}\left(\frac{A\beta_t^2\gamma_t^4 E_0}{T_c^2\dot{\gamma}_t}\right)^{\frac{1}{2}} , \tag{2.5}$$

or $\sigma_E/E = 3.45\times10^{-3}$. Note that here $6\pi\sigma_E\sigma_\tau$ is not equal to the bunch area A because the ellipse is tilted.

III. MICROWAVE GROWTH

The growth of microwave amplitudes across transition is unavoidable because, for a certain time interval, the frequency-slip parameter $\eta = 1/\gamma_t^2 - 1/\gamma^2$ is too small to provided enough frequency spread for Landau damping. It has been shown by Courant and Synder[1,2] as well as by Herrera[3] that, if one assumes η/E to be a linear function of time in that interval, the invariant of the the longitudinal phase space can be solved analytically in terms of Bessel function and Neumann function of order 2/3. A dispersion relation can be set up and the growth rate can then be solved numerically.[4] If we further assume that Z/n is real and the bunch is gaussian in shape, the problem can be solved approximately resulting in a handy formula.[5] The total growth across transition is $\exp(S_b + S_a)$, where

$$S_{b,a} = \int \mathcal{I}m\ \Delta\Omega dt \Big|_{\mathcal{I}m\ \Delta\Omega>0} \tag{3.1}$$

represent the integrated growth rate $\mathcal{I}m\ \Delta\Omega$ before and after transition. The handy formula gives

$$\frac{S_b}{n} = \frac{S_a}{n} = \frac{F_1\left[eN(Z/n)\gamma_t^2\right]^2}{V_{\rm rf}\sin\phi_0}\,\frac{(E_0/e)^2\,\sigma_\tau}{(A/e)^3}\ , \tag{3.2}$$

where N is the number of particle of charge e per bunch, n is the harmonic of the microwave frequency, and $F_1 = 8.735$ is a numerical constant. In the above, σ_τ is the rms time spread of the bunch at time $-t_0$ when stability is lost before transition or at time t_0 when stability is regained after transition. This time t_0 and time spread σ_τ are found to be

$$t_0 = \frac{F_2 eN(Z/n)\gamma_t^4}{\omega_0 V_{\rm rf}\sin\phi_0}\,\frac{(E_0/e)^2\sigma_\tau}{(A/e)^2}\ , \tag{3.3}$$

$$\frac{\sigma_\tau}{T_c} = \begin{cases} \dfrac{2}{3^{1/3}\Gamma(1/3)}\left(\dfrac{AeV_{\rm rf}\omega_0\sin\phi_0}{6\pi E_0^2\beta_t^2\gamma_t^4}\right)^{\frac{1}{2}}\left(1+0.6859\dfrac{t_0}{T_c}\right) & \dfrac{t_0}{T_c} \ll 1\ , \\ \left(\dfrac{AeV\omega_0\sin\phi_0}{6\pi^2E_0^2\beta_t^2\gamma_t^4}\right)^{\frac{1}{2}}\left(\dfrac{t_0}{T_c}\right)^{\frac{1}{4}} & \dfrac{t_0}{T_c} \gtrsim 1\ . \end{cases} \tag{3.4}$$

Assuming that $(NZ/n) \gtrsim 10$ ohms, we obtain

$$\begin{aligned} t_0 &= 0.0705\left(N\frac{Z}{n}\right)^{4/3}\ \text{ms}\ , \\ \sigma_\tau &= 0.176\left(N\frac{Z}{n}\right)^{1/3}\ \text{ns}, \\ \frac{S_{b,a}}{n} &= 1.44\times10^{-7}\left(N\frac{Z}{n}\right)^{7/3}\ , \end{aligned} \tag{3.5}$$

where N is in 10^{10} and Z/n in ohms. An illustration is given in Table I. The growth of the microwave amplitude in the last row was computed by assuming a broad band centered at 1.5 GHz corresponding to $n = 16629$. One has to bear in mind that the actual growth is usually less than indicated because nonlinear effect may come in eventually to suppress the growth rate. The growth of the microwave amplitude will dilute the bunch area and lead to a growth of the bunch area. However, the relation between the two growths is not known.

The longitudinal space-charge force will help stability before transition but help instability after transition. It leads to the shortening of t_0 before transition and lengthening of t_0 after transition. Nevertheless, its contribution will not affect the estimation in Table I by very much.

$N\frac{Z}{n}$	10 Ω	20 Ω	30 Ω
t_0	1.52 ms	3.83 ms	6.57 ms
σ_τ	0.380 ns	0.478 ns	0.547 ns
$\frac{S_{b,a}}{n}$	3.11×10^{-5}	1.57×10^{-4}	4.04×10^{-4}
$e^{S_b+S_a}$	2.81	13.5	820

Table I: Microwave growth across transition

IV. UMSTÄTTER'S EFFECT

The transverse space-charge force will lower the betatron tune of those particle at the transverse edge of the bunch near the center, and therefore lower the transition γ. These particles will cross transition at a time earlier than the synchronous particle.[6] This effect is called the Umstätter's effect. Roughly, the depression of γ_t is given by

$$2\gamma_t\Delta\gamma_t = -\varepsilon\lambda(0) \ , \tag{4.1}$$

where

$$\lambda(0) = \frac{N}{\sqrt{2\pi}\sigma_\tau h\omega_0} \tag{4.2}$$

is the linear particle density at the center of the bunch and

$$\varepsilon = \frac{4hr_pR}{\beta_t^2\gamma_t^3}\left[\frac{1}{a(a+b)} - \frac{\epsilon_1}{h_v^2}\right] , \tag{4.3}$$

with a and b the half-width and half-height of the beam, $r_p = 1.535 \times 10^{-18}$ m the classical proton's radius, $R = 528.30$ m the ring radius, $\epsilon_1 = 0.172$ the electrostatic image coefficient corresponding to rectangular 2" by 4" beam pipe, and h_v the half-height of the vacuum chamber. Assuming a normalized transverse emmitance of the beam 20π mm mr and a minimum beta-function 11.6 m, we get $a = b = 6.74$ mm and therefore $\varepsilon = 2.52 \times 10^{-12}$. Using $\sigma_\tau = 0.380$ ns and $N = 5.10 \times 10^{10}$ from Section II, we get for the maximum linear density $\lambda(0) = 16.0 \times 10^{10}$. This leads to a maximum depression of $\Delta\gamma_t = 0.00995$. From Eq. (2.2), the rate of acceleration is $\dot{\gamma}_t = 163.1$. Therefore, some particles at the center of the bunch will cross transition at a time $\Delta T = \Delta\gamma_t/\dot{\gamma}_t = 0.583$ ms earlier than the synchronous particle. Since this time is much less than the adiabatic time $T_c = 1.96$ ms, Umstätter's effect should be negligible.

V. JOHNSEN'S EFFECT

Each particle inside a bunch has momentum slightly different from the synchronous momentum p_0. It travels along a different closed orbit and has a different momentum compaction factor. If the momentum deviation is Δp, its orbit length is given by

$$L = L_0\left[1 + \alpha_0\frac{\Delta p}{p_0}\left(1 + \alpha_1\frac{\Delta p}{p_0}\right)\right] , \tag{5.1}$$

where L_0 is the synchronous orbit length and α_0 is the momentum compaction factor of the synchronous particle. Obviously, this off-momentum particle will have a different γ_t and crosses transition at a time ΔT earlier. It can be shown that ΔT, which is also called the *nonlinear time*, is given by[7]

$$\Delta T \approx \frac{(\alpha_1 + 3/2)\gamma_t}{\dot{\gamma}_t}\left(\frac{\Delta E}{E}\right) , \tag{5.2}$$

where $\Delta E/E$ is approximately equal to the relative height of the bunch if every particle crosses transition at exactly the same time (or $\alpha_1 = -1.5$). From Section II, $\sigma_E/E = 3.45 \times 10^{-3}$ at transition, we have maximum $\Delta E/E \sim 8.45 \times 10^{-3}$. This gives

$$\Delta T\big|_{\max} \sim 1.057(\alpha_1 + 1.5) \text{ ms} . \tag{5.3}$$

If one assumes a perfect FODO-cell structure of phase advanced φ_c with two sets of sextupoles at, respectively, the F-quads and D-quads canceling a fraction f of the natural chromaticity, α_1 can be derived readily to give[8]

$$\alpha_1 = \frac{1 + s_c/12 - f}{1 - s_c/12} \,, \tag{5.4}$$

where $s_c = \sin\varphi_c/2$. If the sextupoles are not turned on, $f = 0$. The Main Injector consists of 90°-cells. Therefore $\alpha_1 = 25/23$. Thus, we obtain maximum $\Delta T = 2.7$ ms or maximum $\Delta T/T_c = 1.4$. the growth of bunch area due to Johnsen's effect will be appreciable.[9] On the other hand, if there is a complete cancelation of chromaticity, $f = 1$ and $\alpha_1 = 1/23$. Then, $\Delta T/T_c = 0.84$. The growth in bunch area will be much less. The details will be discussed in a future paper.

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