Design and Multiparticle Simulation of the Half Integer Slow Extraction System for the Main Injector

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Abstract
One of the roles of the new Main Injector ring, in the second phase of the Fermilab upgrade, is to deliver all year round the slow extracted 120 GeV test beams. The half-integer slow extraction system design and results from a Monte-Carlo simulation of fast spill are presented. The simulation was performed with a computer tracking program based on the TEVLAT program with a large number of particles (up to 1000). Particle tracking included the systematic errors produced by the magnetic multipoles within the dipoles and quadrupoles as well as random multipole errors.

I. Introduction
The Main Injector will replace the existing Main Ring in the Fermilab Accelerator Complex and allow significant enhancements to both the Fermilab collider and fixed target programs. Number of protons delivered to the antiproton production target or total number of protons delivered to the Tevatron will be two-three times higher than the present one. In addition the Main Injector will provide continuous 120 GeV beam to the experimental area during the collider operation, a capability which does not presently exist in the Main Ring [1]. The Main Injector will be built from newly constructed dipole magnets. In this report we present a design system for the half-integer resonant extraction for the Fermilab Main Injector. One of the major intentions of this project is to reconfirm the quality of the magnetic field in the Main Injector dipole and quadrupole magnets at 120 GeV operation. The beam size at 120 GeV is too small (for the normalized emittance of $\epsilon = 40 \times 0 \text{ mm mrad}$ the beam $\sigma_{\text{max}} \approx 1.7mm$) to cause any concerns about the aperture. The particle trajectories during the slow extraction process attain large betatron amplitudes. The field quality always deteriorates away from the axis. At high values of the magnetic field slow extraction represents the major aperture probe and sets the limits on the magnetic field quality. In the simulation process different quality of the magnetic field in the dipoles and in the quadrupoles were examined. In the first part of the report a short introduction into the half integer resonant extraction is presented with the positions and roles of the quadrupole and octupole magnets. A modeling of the half-integer resonant extraction with a computer program based on the TEVLAT [2] is shown in part III. The magnetic field errors in the dipoles and the quadrupoles were included in the simulation and the results of a Monte-Carlo simulation are presented.

II. Half Integer Extraction in the Main Injector
The Main Injector lattice used in the half integer extraction system is based on a FODO cell structure with a 90° basic cell [1]. This lattice, as reported in the conceptual design, had both the horizontal and vertical tunes close to a value of 22.4, and natural chromaticities of $\zeta_x = -26.02$ and $\zeta_y = -26.35$. There were a total of 300 hundred dipoles in the ring. The maxima and minima of the beta functions in the FODO cells were $\beta_{\text{max}} \approx 58$ meters and $\beta_{\text{min}} \approx 11$ meters. The maximum momentum is 150 GeV/c while the slow extraction is performed at 120 GeV/c. This choice is related to the dipole's magnetic field properties at the high field values-multipole content (the momentum of 150 GeV/c requires the magnetic field of 17.26 kG which is close to saturation). The slow extraction electrostatic and magnetic septa are located at the fast, one-pulse extraction straight section. The magnetic septa-Lambertson magnets will be common elements for both fast and slow extraction processes. The fast kicker magnets, used for the one pulse fast-extraction, were located in the design report upstream of the Lambertson magnets with a betatron phase difference of 90°. During the slow extraction process the beam is horizontally deflected into the field region of the Lambertson magnet where it is bent upward.

The choice of the half integer slow extraction for the Fermilab Main Injector came mostly from the experience during operations in both Fermilab accelerators-the Main Ring and the Tevatron. Although both the third and half integer resonant extraction processes were designed in detail during the Tevatron design the half integer was chosen for operation [3]. The half integer resonance is a linear resonance and the beam is either stable or unstable. The resonant extraction starts with a change of the tune toward the half integer value ($\nu_s = 22.5$). The zero-harmonic octupole introduces a tune shift $\Delta \nu_s \propto \sigma^2$.

A combination of the linear-quadrupole and nonlinear-octupole elements creates a stable region in the phase space where a particle oscillates between the "fixed" points. The amplitude of oscillation increases from turn to turn. The motion of particles is presented in figure 1 [4]. The central stable phase space is reduced in size to be too small to contain the beam. Particles then follow the trajectories of the unstable phase space increasing their amplitudes of motion. The simulation of the slow extraction process included only a pair of quadrupoles with opposite polarities and a zero-harmonic octupole at the same location as the slow extraction focussing quadrupole. The slow extraction focussing quadrupole was located upstream of the electrostatic septum with a betatron phase difference of $\approx 90°$ degrees with respect to the septum. The other, defocusing slow
The extraction quadrupole, was located at the opposite side of the ring.

**Figure 1.** The phase space trajectories during the half integer resonant slow extraction.

The position of the electrostatic septum was selected at the available drift space upstream of the Lambertson magnet with a betatron phase difference of 75.66° degrees. It is always better to position the electrostatic septum downstream of the fast kicker magnets due to unavoidable small beam losses at the wire array of the septum. The beam will not induce radioactive damage to the kicker magnet. The length of the electrostatic septum was 3.6 meters. The septum high voltage was 54.4 kV across a 14 mm gap between the electrode and the wire array. This kick was strong enough to establish a 6 mm distance for the extracted beam at the magnetic septum. The magnetic septum - Lambertson magnet has a magnetic field-free region where the beam circulates through the center of the aperture. The beam is bent in the vertical plane within the extraction channel. The smallest separation between the field free region and the extraction channel is 2 mm. The separation wall of the vacuum chamber in the field free region has two sides with an angle of 45° degrees.

### III. Computer Simulation

The slow extraction should provide a smooth constant rate of extracted beam throughout the cycle. Particles in the beam have unavoidable tune spread which arises from the final beam momentum spread: the tune shift due to chromatic effects and space charge tune-shift which is intensity dependent. The tune spread also comes from the nonlinear elements: the guide field nonlinearities, sextupoles or other nonlinear elements which induce betatron amplitude tune shift. The spill rate is usually divided into two kinds: a slow spill - where the length of the spill is of the order of more than one second and a fast spill where the length of the spill is of the order of 200 turns (one turn \( \approx 10 \mu s \)). The time scale of the fast spill is small enough that a computer simulation of the complete process (with 1000 turns) is possible without any approximation within reasonable computer time. The computing was performed with the workstation SUN SPARK-1. The code in simulation, developed in Fermilab [2], is a kick code where the particle ensemble is propagated from element to element. The multipole field of the dipoles is presented with kicks of thin elements in the middle of the dipole. The simulation of the extraction process was performed only after the length of the ring, the natural chromaticities, the betatron functions, and the tunes obtained by one particle tracking showed complete agreement with results obtained with the SYNCH computer program. The particles (400-1000 particles) initial positions, slopes of both coordinates, and momentum offsets, were chosen randomly. When the slow extraction process was simulated the gradients of the main quadrupoles in the lattice were first set to produce the horizontal tune of 22.485 with the vertical tune at a previous value of 22.4. The chromaticity sextupoles changed the chromaticities to values \( \approx \pm 3 \). The slow extraction quadrupoles and zero-harmonic octupoles were tuned to produce a stable phase space where an ensemble of particles oscillates with well defined stable points, as presented in fig. 1. The slow extraction quadrupoles could follow either a half sinusoidal or a linear ramp pulse above a d.c. value. The extraction of particles occurred when the pulsed part of the slow extraction quadrupoles was turned on. The electrostatic wire array was placed at 17 mm away from the center of the horizontal aperture. When a particle reached this distance it would undergo a horizontal kick of 0.12 mrad. If during this motion in a horizontal space a particle hit the wire (effective thickness of 0.1 mm) it was considered lost. This was used as an efficiency measurement and at the same time this was a way to determine if the step size in process was correctly defined. During a design procedure there is always a compromise between the efficiency which can be reached and the shape of the phase space of the extracted particles. The further position of the septum array from the aperture center is the smaller losses are but the worse magnetic field in the dipoles is. The maximum horizontal offset through the dipoles, in the whole Main Injector ring and before particles reached the septum array in the ring, was 44.9 mm. Because the electrostatic septum was placed close to the extraction Lambertson septum the extracted particles travel through a short part of the ring. The maximum horizontal offset in the magnets reached by the extracted particles was 25.4 mm. At the time of the tracking studies the new Main Injector magnet had not been built yet and the magnetic field quality was not known. The magnetic multipoles in the dipole field \( B(z) \) at offset \( z \) are defined as \( B(z) = B_0[1+1/2!b_2z^2+1/4!b_4z^4+\ldots], \) and \( B_0 \) is the vertical field at the center. The Main Injector will replace the present Main Ring in Fermilab. The multipole content of the two kinds of the Main Ring dipoles (B1 and B2 dipoles) is known and has been measured. In the simulations a set of multipoles from the B2 Main Ring dipoles, which should resemble the Main Injector dipoles, were used. Because the Main Ring dipoles saturation occurs at higher current values (magnets were designed for the 400 GeV machine) the values of all multipoles were doubled. At Fermilab values of the multipoles in the magnetic field are defined as the relation ship between the field produced by the multipole and the main dipole field \( b_0 \) at the distance of one inch multiplied with \( 10^4 \). The average values of the measured (at the current of 1700 A normal multipoles in the Main Ring dipoles are presented in Table 1. The multipole expansion was cut at the eleventh pole. The simulation was performed with the twice higher multipoles presented in table 1. During the simulation process multipoles in the dipole magnetic field were chosen randomly from dipole to dipole. The limits in the values of multipoles were set by the standard deviations obtained in magnetic measurements (in the table 1 "sdv").
The distributions of an ensemble of 400 particles in the phase space at the electrostatic and magnetic septum, during the simulation, are presented in figures 2 and 3, respectively.

**Table 1**

<table>
<thead>
<tr>
<th>Normal Multipoles ( \times 10^4 )</th>
<th>Skew Multipoles ( \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_2 ) = 0.037, sdev = 0.230</td>
<td>( b_2 ) = 3.910, sdev = 0.391</td>
</tr>
<tr>
<td>( b_4 ) = 0.059, sdev = 0.443</td>
<td>( b_4 ) = 0.049, sdev = 0.197</td>
</tr>
<tr>
<td>( b_6 ) = 0.008, sdev = 0.071</td>
<td>( b_6 ) = 0.321, sdev = 0.221</td>
</tr>
<tr>
<td>( b_8 ) = 0.009, sdev = 0.328</td>
<td>( b_8 ) = -0.026, sdev = 0.046</td>
</tr>
<tr>
<td>( b_{10} ) = 0.002, sdev = 0.051</td>
<td>( b_{10} ) = 0.092, sdev = 0.161</td>
</tr>
</tbody>
</table>

The distributions of an ensemble of 400 particles in the phase space at the electrostatic and magnetic septum, during the simulation, are presented in figures 2 and 3, respectively.

**Figure 2. Slow extraction simulation during the fast spill.** The distribution of 400 particles in the phase space at the electrostatic septum (540 turns).

**Figure 3. Slow extraction simulation during the fast spill.** The distribution of 400 particles in the phase space at the Lambertson magnet (540 turns).

Most of the quadrupoles in the Main Injector will be the reused Main Ring quadrupoles (according to the design report [1]). At the high field the octupole harmonic in the Main Ring quadrupoles had been measured to be 6 Fermilab units with respect to the field of the quadrupoles at 1 inch \((B_4/B_2 = 8.0 \times 10^{-4})\). Another set of simulation runs with the measured value of the octupole multiple within the quadrupole magnets was performed. The influence of the octupole harmonic from the quadrupoles on the particle distribution in the phase space showed that the strength of the zero-harmonic octupole needed to be lowered. The error in quadrupole misalignment, always present in the real machine, produces a natural half integer stop band and propagates at twice the tune (45th harmonic). The slow extraction quadrupoles will be used at the same time to correct the natural stop band of the 45th harmonic. In the simulation a presence of the natural 45th harmonic was not taken into account. The strength of the slow extraction quadrupoles at the best efficiency with a particle loss of less than 1% was \(5.4 \text{kGm/m} \) at 120 GeV/c while the strength of the octupole was \(2.7 \times 10^5 \text{kGm/m}^3\). The octupole strength is a strength of a single octupole. In the Main Injector zero-harmonic octupoles will be distributed around the ring.

**IV. Concluding Remarks**

One of the roles of the future accelerator-Main Injector will be to provide slow extracted beams up to \(3 \times 10^{13}\) protons to the experimental areas during the collider runs for use in high sensitivity K decay and neutrino experiments and for the test beams. The design of the half-integer slow extraction system and the multiparticle simulation of the extraction process has been presented. The simulation has shown that it is possible to design an efficient half-integer slow extraction system with the expected field quality of the Main Injector magnets. The results of the simulation showed that the new Main Injector magnets should have better multipole content than twice the size of the multipoles within the B2 Main Ring magnet at 120 GeV. The simulation also showed that the distortion of the extracted particles phase space is acceptable even with the maxima in the horizontal offsets through the dipoles during the last extraction turns of the order of 40 mm. The available aperture fulfills conditions required for the slow extraction beam trajectories. The elements of the slow extraction system fit well within the available drift space of the lattice presented in the design report [1]. For the slow extraction quadrupoles it is possible to use the existing Main Ring small harmonic correction quadrupoles (20 cm long with an integrated strength of 0.255 kGm/m/A [5]). The strength of the zero-harmonic octupoles would require 50 octupoles of the same strength as the Main Ring chromaticity octupoles with a current through the coil of 20 Amp (the strength of the Main Ring harmonic octupoles is \(b_2 b_{eff} = 27.92 \text{kGm/m}^3/A\)).

**References**


