MI–0061

TRANSITION CROSSING WITH POSITIVE CHROMATICITY – A PROPOSED MAIN RING MEASUREMENT OF α_1 VS CHROMATICITY

S.A. Bogacz and I. Kourbanis

Accelerator Division,
Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, IL 60510

June 1991

Here, we propose a direct measurement of α_1 (dispersion of the momentum compaction factor α), which itself is a crucial parameter governing kinematic nonlinearities of transition crossing. Ability to measure and control α_1 is essential to systematic handling of kinematic effects at transition. Furthermore, the proposed simple study is aimed at verifying both analytic and tracking predictions of *strong variation of α_1 with chromaticity*. According to the prediction, going through transition with large positive chromaticity (roughly twice the natural chromaticity) would virtually suppress the Johnsen effect, known from the tracking studies of transition crossing in the Main Injector to be the dominant cause of beam loss and emittance blow up. To make it a feasible scheme of transition crossing in the Main Injector one has to assess the slow head-tail instability below transition for large positive chromaticities. Qualitative comparison of the slow head-tail instability for both machines shows that the Main Injector constitutes more stable configuration. If one were to show experimentally that the Main Ring instability was not fatal it would validate the proposed control of α_1 with chromaticity as an extremely simple scheme of transition crossing in the Main Injector.

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*Operated by the Universities Research Association, Inc., under a contract with the U.S. Department of Energy

INTRODUCTION – JOHNSEN EFFECT

Simulation of transition crossing in the Main Injector, using ESME, have shown that the Johnsen effect is the dominant cause of beam loss and emittance blow-up[1, 2]. This effect is rooted in the variation of the transition gamma, with the momentum offset. A useful parameter characterizing the strength of this effect[3] is the rms average of transition crossing time delay (with respect to the synchronous particle) taken over the entire bunch. The so called Johnsen time is directly related to the lattice parameters α_0 and α_1, which are defined by the following equation

$$\frac{\Delta C}{C_0} = \alpha_0\,\delta + \alpha_1\,\delta^2 + \dots, \qquad \delta = \frac{\Delta p}{p_0}, \tag{1}$$

where C_0 is the nominal closed orbit path length, and ΔC is the increase in path length for an off momentum particle. The coefficients α_0 and α_1 are geometrical properties of the lattice, given by

$$\alpha_0 = \frac{2\pi}{C_0}\langle\,\eta_0\,\rangle\,, \quad \alpha_1 = \frac{2\pi}{C_0}\langle\,\eta_1\,\rangle\,, \tag{2}$$

where angle brackets $\langle \dots \rangle$ denote averaging weighted by bend angle. The quantities being averaged are component dispersions in a momentum expansion of the total dispersion.

If it is possible to measure and control α_1, then it should be possible to reduce the Johnsen time to zero, and therefore ameliorate the damage done by the Johnsen effect[4]. This can be accomplished by setting[5,6]

$$\alpha_1 = -\frac{3}{2}\alpha_0 + \frac{1}{2}\alpha_0{}^2\,. \tag{3}$$

The last term in the above equation is very small and therefore may be neglected.

α_1 – FODO FORMULA

Consider an accelerator made up of identical FODO cells. The quadrupoles are thin, and there are no drift spaces. All of the half cell of length L is filled with two identical dipoles of bend radius R. There are two thin chromatic correction sextupoles (f and d) per half cell, immediately adjacent to the focusing and defocusing quadrupoles. We have previously shown[5] that for the above lattice α_o and α_1 coefficients reduce to the following simple expressions

$$\alpha_o = \frac{L^2}{R^2}\frac{1}{s^2}[\, 1 - \frac{s^2}{12}\,] \, , \tag{4}$$

$$\alpha_1 = \frac{L^2}{R^2}\frac{1}{s^2}[\, 1 - f + \frac{s^2}{12}\,] \, . \tag{5}$$

Here, $s = \sin(\phi_{1/2})$, where $\phi_{1/2}$ is the half cell phase advance. The chromatic sextupoles are turned on at a strength to correct for f times the natural chromaticity (f = 1 sets the net chromaticity in both planes to zero).

One can see from Eq.(5) that α_1 may be easily controlled with chromaticity, while α_o is not sensitive to it. The desired condition to eliminate the Johnsen effect, Eq.(3), can be accomplished by adjusting the chromaticity as follows

$$f = 2.5 - \frac{s^2}{24} \, . \tag{6}$$

The above result substantiate our claim that keeping large positive chromaticity throughout the transition would ameliorate the damage done by the Johnsen effect.

α_1 – MAD SIMULATION

Here we consider the mi_17 Main Injector lattice[7]. Two families of sextupole magnets are used to assign the chromaticity in both planes to some desired value. The MAD code simulates elongation of the closed orbit path-length for off momentum particles (in a realistic momentum offset range, $\Delta p/p \pm 0.003$). The results in terms of

$$\alpha_p = \frac{p}{C}\frac{dC}{dp} \tag{7}$$

are summarized in Figure 1, where curves of α_p vs $\delta = \Delta p/p$ are plotted for different values of net chromaticity x. Values of α_1, extracted from the slope of each curve are also displayed. The bold curve corresponds to the desired situation, when the Johnson time is reduced to zero, $\alpha_1 = -\frac{3}{2}\alpha_0$. Indeed this curve is labeled with large positive chromaticity, x = 26.3.

Now the question of other consequences of having positive chromaticity below transition is in order. The natural reservation is the stability of various modes of the transverse slow head-tail instability. This problem will be addressed in the next section.

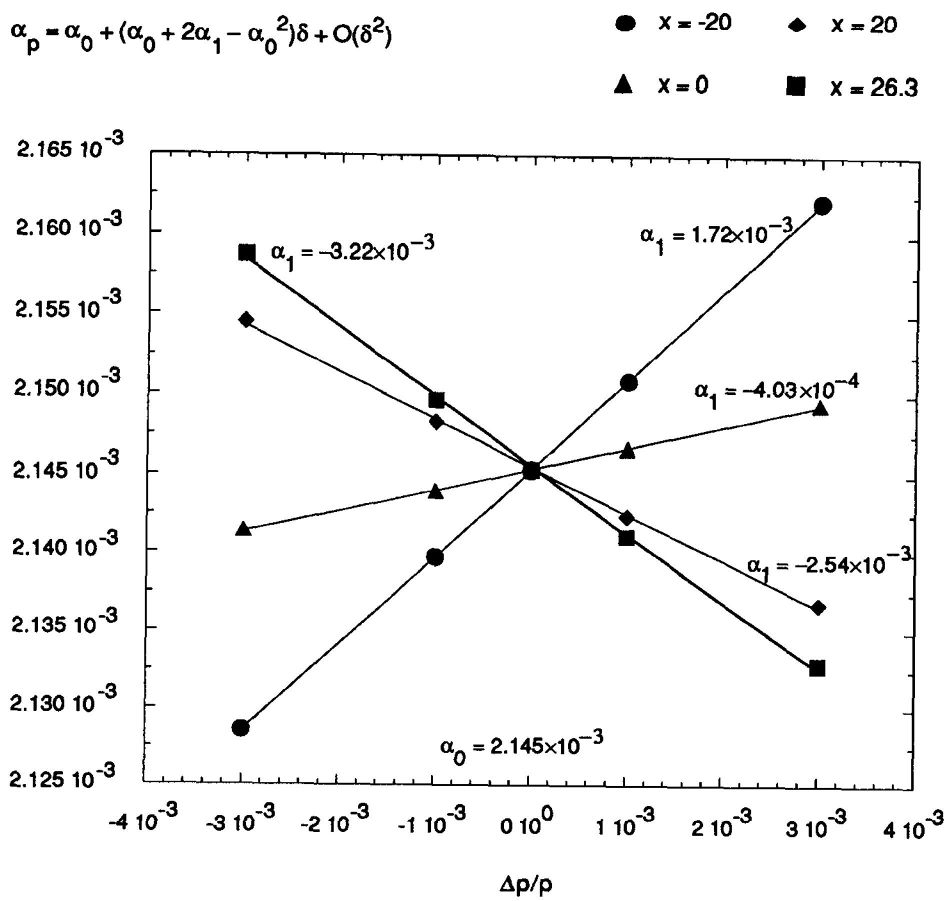

Figure 1 A family of curves (α_p vs $\delta = \Delta p/p$) plotted for different values of net chromaticity x. The bold curve corresponds to the reduction of the Johnson time to zero.

SLOW HEAD-TAIL INSTABILITY

Following Sacherer's model[9] one can generalize a simple equation of motion describing a wake field driven coherent betatron motion of a coasting beam to model the head-tail instability of the bunched beam. Assuming only one dominant contribution to the transverse coupling impedance (due to the kicker magnets), the inverse growth-times were calculated numerically[10]. Figure 2 illustrates variation of the characteristic growth-rates with chromaticity for various modes.

Our model overestimates the instability since it does not include any Landau damping mechanism (all particles have the same value of the betatron tune). The model does not include nonlinearities leading to saturation (especially higher modes). From that point of view it may serve as the worst possible case of the instability.

The Main Ring is dominated by the head-tail mode with the characteristic growth-time of 3×10^{-3} sec., with the higher modes also displaying significant instability. Our study shows that careful adjustment of chromaticity (avoiding peak values of the growth-rates) may serve as a possible way of suppressing this instability in the Main Ring. More systematic approach is to explore variation of the betatron tune with amplitude induced by the presence of small octupole field. This way of introducing Landau damping may be used as an effective scheme to control the instability.

The Main Injector already performs much better (the lowest mode stable and the growth-time of the dominant unstable mode of about 5×10^{-3}sec.) due to larger chromatic frequency shift, which governs 'overlap' of the beam spectrum and the driving transverse impedance.

Severity of the slow head-tail instability in the Main Ring is not conclusively determined at this time. If one can show experimentally that the Main Ring slow head-tail instability is not catastrophic for large positive chromaticities, than, on the basis of the above study, one can also extend the same conclusion to the case of the Main Injector.

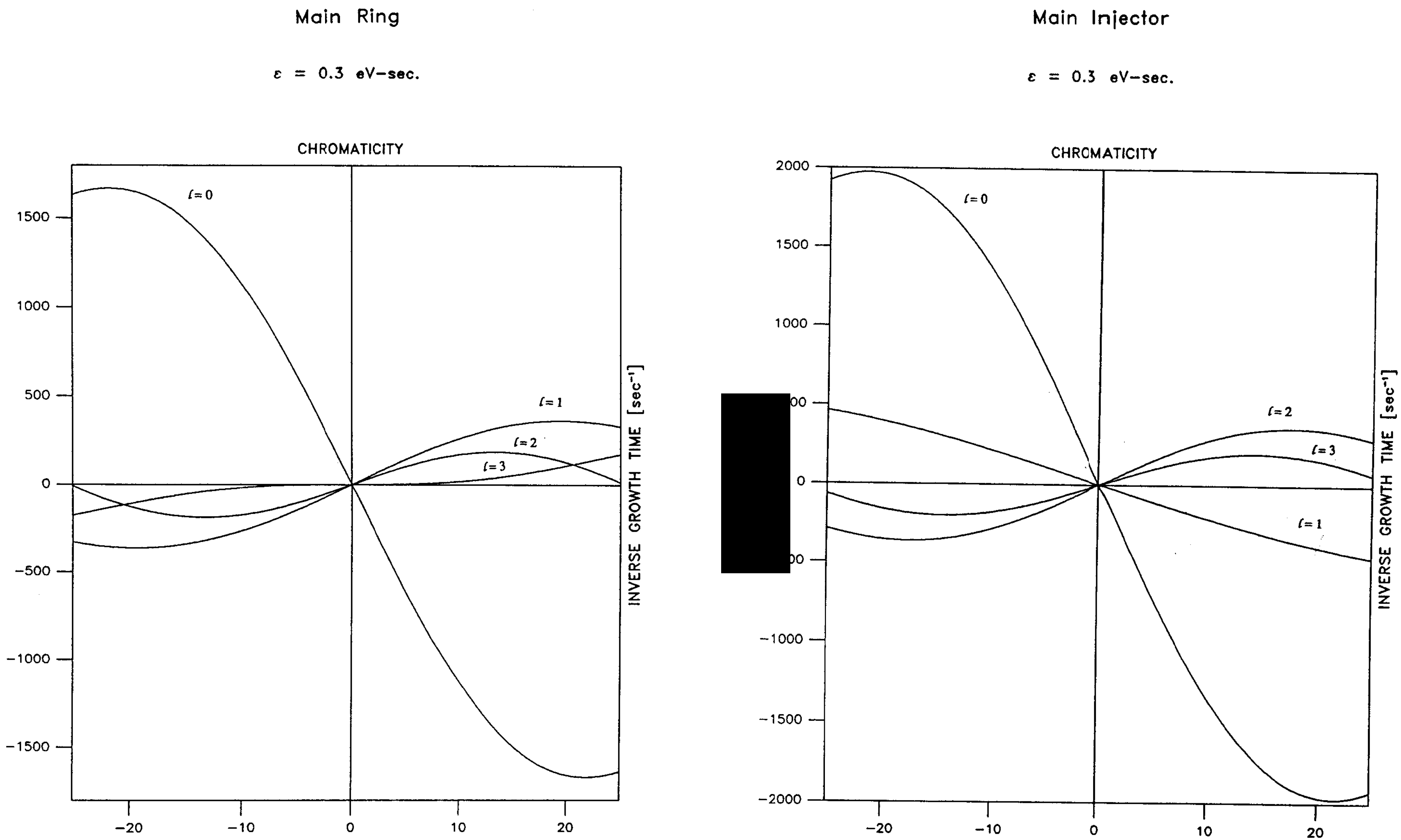

Figure 2 Comparison of the slow head-tail instability in the MR and MI – characteristic growth-rates vs chromaticity for various modes.

CONCLUSIONS

The first part of this proposal – the measurement of α_1 in the Main Ring is a very important task in itself and it should definitely be carried out. The measurement of α_1 could be done the same way as the one already performed[11], since its reported modest accuracy of 30% is quite sufficient to verify our hypothesis. Nevertheless a more precise measurement is under investigation and it, more likely, will be used in the proposed experiment. Comparison of the experimental results against the simulations would not only check our predictions but it would also increase confidence in the MAD code, frequently used to design new structures eg. the Main Injector lattice, transfer lines. Conversely, the above comparison would also indicate how well our model lattice reflects optics of the real machine.

If one were to show experimentally that the Main Ring instability was not fatal we could extend the same conclusions to the Main Injector and it would validate the proposed control of α_1 with chromaticity as an extremely simple scheme of transition crossing in this machine. Whether this is the case will further be examined by a full blown study of the slow head-tail instability at transition.

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