On the Calculation of Sextupole Correctors Strength in Main Injector using TPOT

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1 Introduction

This paper reports calculations done about two years ago on the sextupole correctors strength in the Main Injector using the thin lens code TPOT. The goal at that time was to verify whether the TPOT results and Rod Gerig's results using the code TEVLAT [1] agree. Calculations were done using the lattice MI15. Lately questions have been raised as to the validity of TPOT's sextupole corrector strengths. The following calculations demonstrate that TPOT and TEVLAT results agree quite well.

2 First: Units

In [1] the units for the sextupole strengths are in $Kg/m^2 - m$. It has the unit of an integrated sextupole field and not just sextupole field. One has to be careful when working with TPOT which uses MKS units but also uses many confusing notations. One such confusion comes from the multipole errors in magnets through the input parameters $SIG(A,B)(0-9)$. SIG here has nothing to do with the σ used to refer to an rms value. The fact is that SIG stands here for a fractional multipole of the corresponding main field evaluated at 1 m. To further confuse the issue the word "unit" is widely used at Fermi to refer to units of multipole in 10^4 of the corresponding main field B_m at 1 in instead of 1 m. Therefore to convert a "unit" to a TPOT unit we need to do the following conversion:

$$TPOT\ Units\ for\ SIG(A,B)(n) = \frac{``Units'' \times 10^{-4}}{(2.54 \times 10^{-2})^{n-m}}$$

where n is the multipole number that varies from 0 to 9 and m is the order of the main field (for a dipole $m = 0$, for a quadrupole $m = 1$, etc..).

Another source of confusion comes from TPOT reported values of sextupoles in correcting for chromaticity. These values represent geometric factors $\tilde{a}_n$ and $\tilde{b}_n$ as described in [2] and are not the normal b_n and skew a_n although the same notation is used. The relation between the two is as follows:

$$(\tilde{a}_n, \tilde{b}_n) = \frac{LB_m(a_n, b_n)}{P_o/e}$$

where L is the length of the magnet, B_m the main field, and P_o/e the momentum all in MKS units. This latter is also equal to $B\rho$ where B is the magnet field and ρ the bending radius. In order to convert the TPOT sextupole units to units in $Kg/m^2 - m$ we need to do the following conversion:

$$Sextupole\ (Kg/m^2 - m) = TPOT\ Sextupole \times Energy\ in\ eV/3\ 10^8 \times 10$$

where it is assumed that the main field is that of a dipole ($m = 0$). One has to be careful when dealing with quadrupole as now m becomes equal to 1 and all units are defined accordingly.

3 Sextupole Correctors Strength

Calulations are made at the four machine states 8.9, 19.1, 120 and 150 $[GeV/c]$. The sextupole component in the dipole is given as the sum of three sources:

- due to remanent field
- due to eddy currents
- due to saturation

To enable a comparison with Rod's results the remanent field is assumed to be $0.027T/m^2$ at all energies. For the same purpose the saturation sextupole is set to zero for $B < 7.82Kg$ and $\frac{(B_o - 7.82)}{974}$ "units" for higher values. The sextupole due to eddy current equals $0.24T/m^2$ with a ramp rate of 240 GeV/s. These numbers are converted to units of TPOT and compared with some results obtained by Ostiguy [3]. At 19.1 GeV for which the dipole field $B_o = 0.23707\ T$ the sextupole due to eddy current from the above ramp, and following Figure A-2 revised in [1] is $\sim 0.115T/m^2$. In TPOT units this number becomes $0.115/0.23707 = 0.485$ and in units of "units" it is 3.13. The number obtained by Ostiguy at this energy level is 3.40 . Next I looked at the saturation sextupole at 120 GeV for which the dipole field is 1.375 T. The above formula gives 1.269 "units". At 150 GeV where $B_o = 1.718\ T$ this number equals 7.88 "units". The numbers obtained by Ostiguy are respectively -1.34 and -8.29 for a dipole field of 1.38T and 1.73T. As seen the level of agreement between the numbers used by Rod and the numbers obtained by Ostiguy is quite close except for the minus sign. It is not clear from the equations in [1] what sign is used for each source of sextupole. But based on my understanding and the dipole sextupole plot in Figure A-2 of [1] the sextupole due to eddy current is positive, due to saturation is negative at high fields and due to remanent is negative. Table 1 lists the sextupole strengths in both units.

Energy (GeV)	Dipole (T)	Used Sextupole ("units")	TPOT units
8.9	0.1	-1.75	-0.27
19.1	0.237	2.80	0.44
120	1.375	-0.50	-0.077
150	1.718	-8	-1.24

Table 1: Used sextupole strengths in dipoles

The numbers used in TPOT are the ones that correspond to Fig. A-2. In units of TPOT this translates according to the first equation of section 2. Table 2 compares the sextupole corrector strength in units of $kg - m/m^2$ as obtained by TPOT to the numbers obtained by Rod using TEVLAT and read from Figure A-2 revised. The horizontal and vertical chromaticities are corrected to be respectivley $+20$ and -20.

Energy (GeV)	SF/SD TPOT results ($Kg/m^2 - m$)	SF/SD TEVLAT results ($Kg/m^2 - m$)
8.9	3.856/-3.23	4/-3
19.1	-1.55/-23.23	-2/-22
120	236/-396	230/-394
150	447.5/-245	425/-230

Table 2: Required sextupole strengths

4 Chromatic Sensitivity

Change in chromaticity depends on the focussing and defocussing sextupoles strengths S_F, S_D and on the sextupole content of the dipoles S_{dip}. This can be modeled by the equations

$$\xi_H = \xi_{nat_H} + \frac{299.8}{P}(aS_F + bS_D + cS_{dip})$$

$$\xi_V = \xi_{nat_v} + \frac{299.8}{P}(dS_F + eS_d + fS_{dip})$$

Again one has to be careful with units. Here P is in [Gev/c], sextupole strengths are still in $Kg/m^2 - m$ and the coefficients should be in m^2. These coefficients are calculated in two steps. First by setting $S_{dip} = 0$ and solving for the two equations with two unknowns. Next by setting $S_F = S_D = 0$ the dipole sensitivity coefficients are calculated. The natural chromaticities are given by TPOT as $\xi_{nat_H} = -27.387$ and $\xi_{nat_V} = -28.315$. The results of these calculations are given in Table 3.

	TPOT (m^2)	TEVLAT (m^2)
a	0.1141	0.1104
b	0.01716	0.0173
c	0.1667	0.1686
d	-0.0279	-0.0282
e	-0.0624	-0.0646
f	-0.1598	-0.1621

Table 3: Sensitivity Coefficients

These sensitivity coefficients are energy independent. Recent calculations by Ostiguy [3] have shown that the saturation sextupole strengths in the main injector design dipole can be a factor of two bigger. This factor comes from using a different method of extrapolating the B-H curve beyond the measured point. The effect this factor might have on the sextupole strengths can be deduced from the calculated sensitivity coefficients.

5 Conclusions

These calculations have demonstrated the reliability of using TPOT to calculate sextupole corrector strengths. Results obtained are in good agreement with results obtained using TEVLAT, a completely different code. The sensitivity coefficients allows one to predict the change in corrector strengths due to a change in sextupole dipole strength.

References

[1] Rod Gerig, *MI-0026*, July 1990.

[2] L. Schachinger and R. talman, "Teapot: A thin element accelerator program for optics and tracking", PA-22, pp. 35-56, 1987.

[3] Jean-François Ostiguy, *MI-0036, MI-0037*, October 1990.