Low-Dispersion γ_t Jump Scheme for the Main Injector

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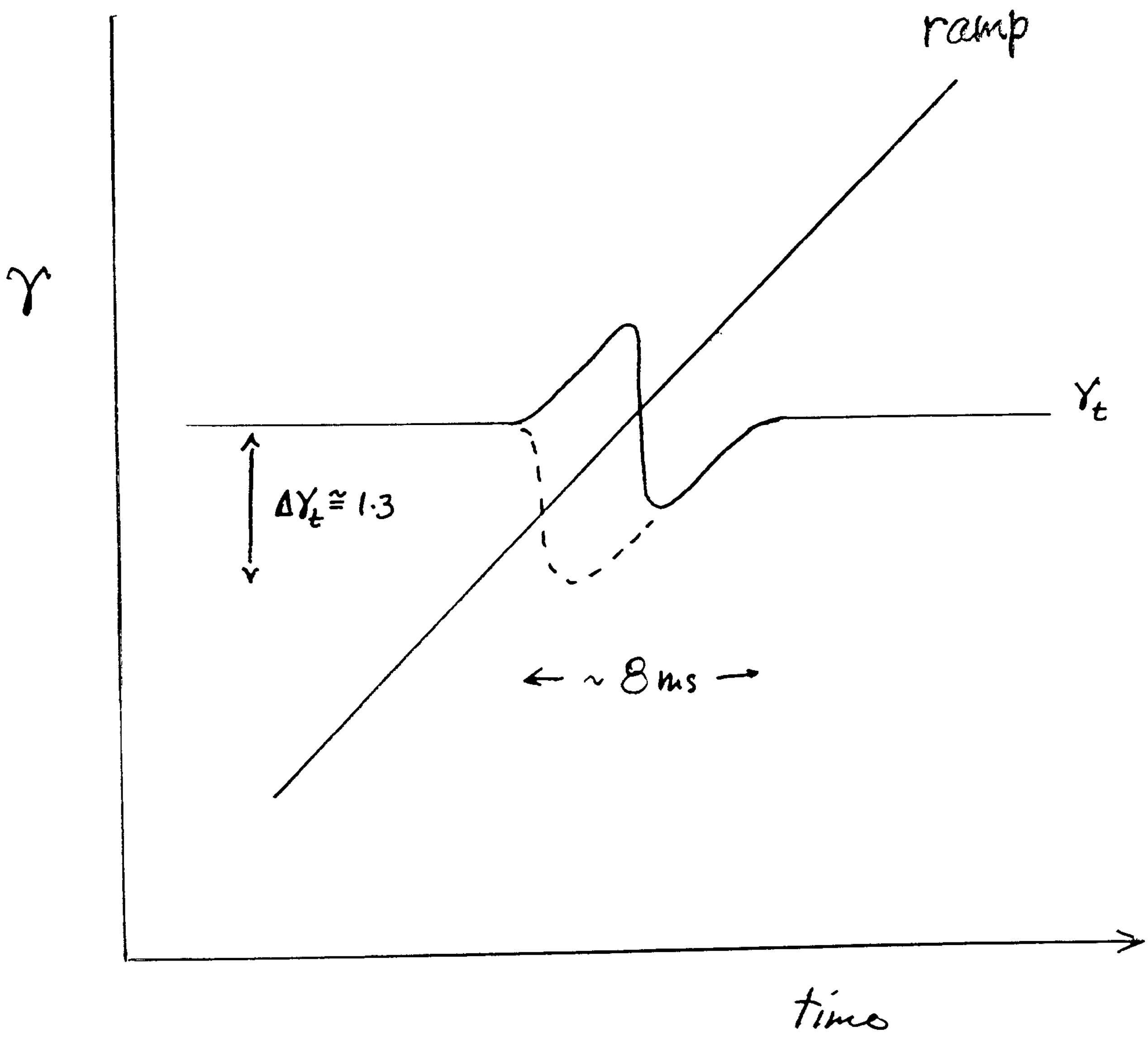

ramp
γ
γ_t
$\Delta\gamma_t \cong 1.3$
← ~ 8 ms →
time

A bipolar γ_t jump scheme was worked out for MI-17:

	MI-17	up	down
ν_x	26.40748	26.40606	26.40940
ν_y	25.40998	25.40836	24.41182
β_x Max (m)	59.86643	78.04456	68.88534
β_y Max (m)	63.17612	64.12093	65.25474
D Max (m)	1.97820	2.04025	2.20696
γ_t	21.58789	+0.6826	−0.6254
$\int B' d\ell$ (1)		+0.1822 T	−0.1586 T
$\int B' d\ell$ (2)		−0.4305 T	+0.3724 T

Gradient of normal quad at transition

$\sim$ 2.8 T/m

Simple Formula for γ_t Jump (Risselada)

- For a horizontal orbit θ_i at location s_i, the closed orbit changes by

$$\Delta x(s) = \sum_{i=1}^{N} \theta_i m(s_i, s) \ ,$$

where

$$m(s_i, s) = \frac{\sqrt{\beta(s_i)\beta(s)}}{2 \sin \pi \nu_x} \cos(\pi \nu_x - |\phi(s_i) - \phi(s)|) \ .$$

- The change in orbit length is, to first order,

$$\Delta C = \int_0^C \frac{\Delta x(s)}{\rho(s)} ds$$

or

$$\Delta C = \sum_{i=1}^{N} \theta_i \int_0^C \frac{m(s_i, s)}{\rho(s)} ds = \sum_{i=1}^{N} \theta_i D_i \ .$$

- Now, the horizontal kick θ_i can be a result of pulsing a special quadrupole at s_i to strength K_i. The particle of off-momentum δ will pass through this quad at

$$x = D_i^* \delta$$

where D_i^* is the *perturbed* dispersion at the quad. This off-momentum particle will therefore be bent by the quad by the angle

$$\theta_i = -x(s_i)K_i = -D_i^* K_i \delta$$

- The change in orbit length is therefore

$$\Delta C = -\delta \sum_{i=1}^{N} K_i D_i^* D_i$$

- The change in momentum compaction is given by

$$C_0 \Delta(\gamma_t^{-2}) \equiv \Delta \left(\frac{dC}{d\delta} \right) = - \sum_{i=1}^{N} K_i D_i^* D_i \ .$$

- We need the relation between D_i and D_j^*.

- Due to the extra quadrupoles, the closed orbit of the off-momentum particle at s_j is

$$x(s_j) \equiv \delta D_j^* = \delta D_j - \sum_{i=0}^{N} \overbrace{\delta D_i^* K_i}^{-\theta_i} m(s_i, s_j)$$

or in vector form

$$\vec{D^*} = \vec{D} + \boldsymbol{M} \vec{D^*}$$

where

$$\boldsymbol{M}_{ij} \equiv -K_j m_{ij} \equiv -K_j m(s_i, s_j)$$

- We can solve for $\vec{D^*}$

$$\vec{D^*} = (1 - \boldsymbol{M})^{-1} \vec{D} = (1 + \boldsymbol{M} + \boldsymbol{M}^2 + \cdots) \vec{D}$$

- Then the change in momentum compaction is

$$C_0\Delta(\gamma_t^{-2}) = -\sum_{i=1}^{N} K_i(1+\boldsymbol{M}+\boldsymbol{M}^2+\ \cdots\ )D_i^2\ .$$

- Note that

$$\boldsymbol{M}_{ij} \equiv -K_i m_{ij} = -\frac{K_i\sqrt{\beta_i\beta_j}}{2\sin\pi\nu_x}\cos(\pi\nu_x - |\phi_i-\phi_j|)\ .$$

Therefore the expansion is in terms of $\mathcal{O}(K\beta)$.

Requirements for γ_t Jump

1. Large γ_t jump.

2. No changes in betatron tunes ν_x and ν_y.

3. $(\beta_x)_{\max}$, $(\beta_y)_{\max}$ below reasonable values.

4. $D_{\max}$ below reasonable value.

The Amount of Jump

- For the Main Injector crossing transition $\dot{\gamma}_t = 163.1/$sec.

- Nonadiabatic time $= \pm 1.96$ ms

  Nonlinear time $= \pm 2.12$ ms for $S = 0.4$ eV-sec, $\alpha_1 = \frac{1}{2}$.

- Need $\Delta\gamma_t \approx 163.1 \times 2 \times (0.002 + 0.002) = 1.3$.

- Nonlinear time may be less because S will be much smaller.

- Here, we look for a unipolar jump $\Delta\gamma_t \approx -1.3$

  or a bipolar jump $\Delta\gamma_t \approx \pm 0.65$.

Zero Tune Change

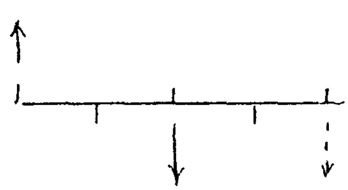
- For small quad strength K_i's, the change in tune is

$$\Delta\nu_x = \frac{1}{4\pi}\sum_{i=1}^{N}\beta_i K_i$$

- For $\Delta\nu_x = 0$, we can go with doublets, each having

$$\beta_1 K_1 + \beta_2 K_2 = 0$$

- Or we put one special quad of strength K at the F quad of one cell and another of strength $-K$ at the F quad of some later identical cell.



- We put these special quads at the F quads to maximize $\Delta\gamma_t$, since β_x and D are usually at a maximum there.

Confining Betatron Wave

- For a small quad of strength K_i, the betatron wave downstream is

$$\Delta\beta(s) = -\beta(s)\beta_i K_i \sin[2(\phi(s) - \phi_i)]$$

same sign $\cos(\phi_1 - \phi_2)$ 90°

opp sign $\sin(\phi_1 - \phi_2)$ 180°

- the dispersion wave downstream is

$$\Delta D(s) = -\sqrt{\beta(s)\beta_i} D_i K_i \sin[\phi(s) - \phi_i]$$

same sign $\cos \frac{1}{2}(\phi_1 - \phi_2)$ 180°

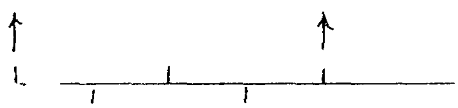
opp sign $\sin \frac{1}{2}(\phi_1 - \phi_2)$ 360°

- To confine betatron wave, in 90° cells, like MI, we can place a doublet of special quads of the *same* sign at successive identical cells.

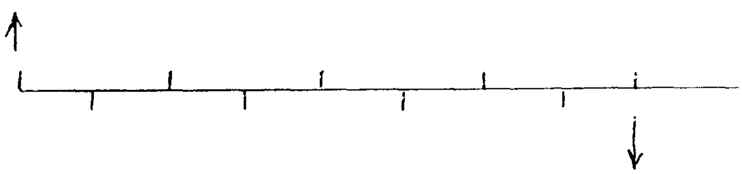
- But to accommodate $\Delta\nu_x = 0$, we need to place one at F of one cell and the other of *opposite* sign at F of the 3rd cell that is 180° downstream.

Confining Dispersion Wave

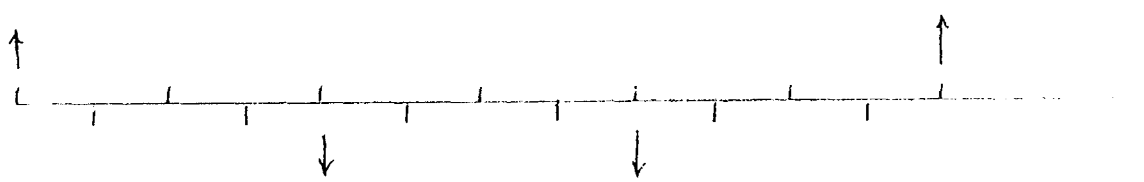
- Dispersion wave can be confined between two special quads of the *same* sign placed at 90° cells 180° apart.



- Or two quads of *opposite* sign placed 360° apart.



- One way to accommodate all these requirement is



- But these 4 quads give $\Delta\gamma_t = 0$.

- It appears that there is no way to satisfy all the 4 requirements.

- In general, there are 2 ways to design the γ_t jump depending on the machine.

- One way is to let each doublet of special quads to null the tune change and confine betatron wave but *not* the dispersion wave.

- The simple formula for $\Delta\gamma_t$ becomes simpler.

$$(M^2)_{ij} = \sum_k K_k m_{ik} K_j m_{kj} = K_j \sqrt{\beta_i \beta_j} \sum_k K_k \beta_k \cos_{ik} \cos_{kj}$$

where

$$\cos_{ik} = \frac{1}{2 \sin \pi \nu_x} \cos(\pi \nu_x - |\phi(s_i) - \phi(s_k)|)$$

- The contribution of a 180° $\Delta\nu_x = 0$ doublet is

$$K_k \beta_k + K_{k+1} \beta_{k+1} = 0$$

$$\vec{D}^* = (1 + \boldsymbol{M})\vec{D}$$

$$C_0 \Delta\gamma_t^{-2} = -\sum_{i=1}^{N} K_i (1 + \boldsymbol{M})_{ij} D_i D_j$$

- In most cases, the first-order term vanishes, leaving

$$\Delta\gamma_t^{-2} = \frac{1}{2C_0 \sin \pi\nu_x} \sum_{i=1}^{N} D_i^2 \beta_j K_i K_j \cos(\pi\nu_x - |\phi_i - \phi_j|)$$

- So dispersion change in linear in $\boldsymbol{M}$ or $K\beta$, while the change in γ_t is second order in $\boldsymbol{M}$ or $(K\beta)^2$.

- Since the dispersion wave is not confined, it may become large. So this second-order $\Delta\gamma_t$ will not be small, even $(K\beta)^2$ may be small.

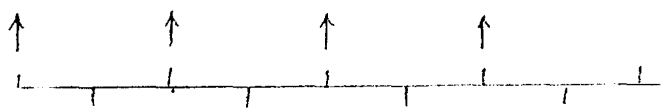
- 2 families of doublets can be placed to give a bipolar jump.

- This scheme is used in CERN PS and the Fermilab Booster.

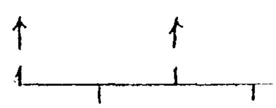
- It has also been used by Steve Holmes and J. Shan in MI.

- The 2nd scheme is to let the doublets confine both the betatron and dispersion wave, but ignore the change in tune.

- For 90°, we put special quads in group of 4:



- We place another family of special quad doublets at F's 90° apart, but in dispersion-free region.



- Here betatron wave is again confined. But no dispersion wave to worry; so we need only groups of 2, and there is no contribution to $\Delta\gamma_t$.

- We adjust the strength of this 2nd family to null out $\Delta\nu_x$.

- All orders of $\boldsymbol{M}$ or $K\beta$ contribute. But the first order dominates.

- This scheme was used by Bogacz, Harfoush, Peggs in MI (90 Workshop), Peggs, Tepikian, Trbojevic in RHIC (94 IEEE).

- This scheme is termed *matched* by Steve Peggs, and the first one *unmatched*.

- This terminology is confusing. It actually implies *dispersion confined* and *unconfined*.

- This scheme *cannot* be used in our Booster or CERN PS because there is no dispersion-free region.

Application to Main Injector

- Two-fold symmetric. So study one half of lattice only.
- There are 19 90° cells in a row in the arc.
- Put in 20 special quads (5 sets of 4's).
- There are 6 90° cells in a row.
- put in 4 special quads (1 set of 4's)
- Second family in $D \approx 0$ region: groups of 2's.
- 2 quads at MI40 (ν extraction).
- 2 quads at MI32 (opposite to kicker)
- 4 quads at MI30 (opposite of rf).
- 2 quads at MI22 (opposite to kicker).

- Results: $\Delta\gamma_t = +0.683$ and -0.625.
- Max dispersion is within 2.2 m.
- Max β_x less than 78 m.
- Both betatron tunes are matched up to < 0.002.
- The cells are not exactly $90°$.
- The dispersion-free regions are not exactly at $D = 0$.
- The cells are not well matched.
- Previous results are with MI-15.
- There, Max $D = 2.64$ m and Max $\beta_x = 97.5$ m.

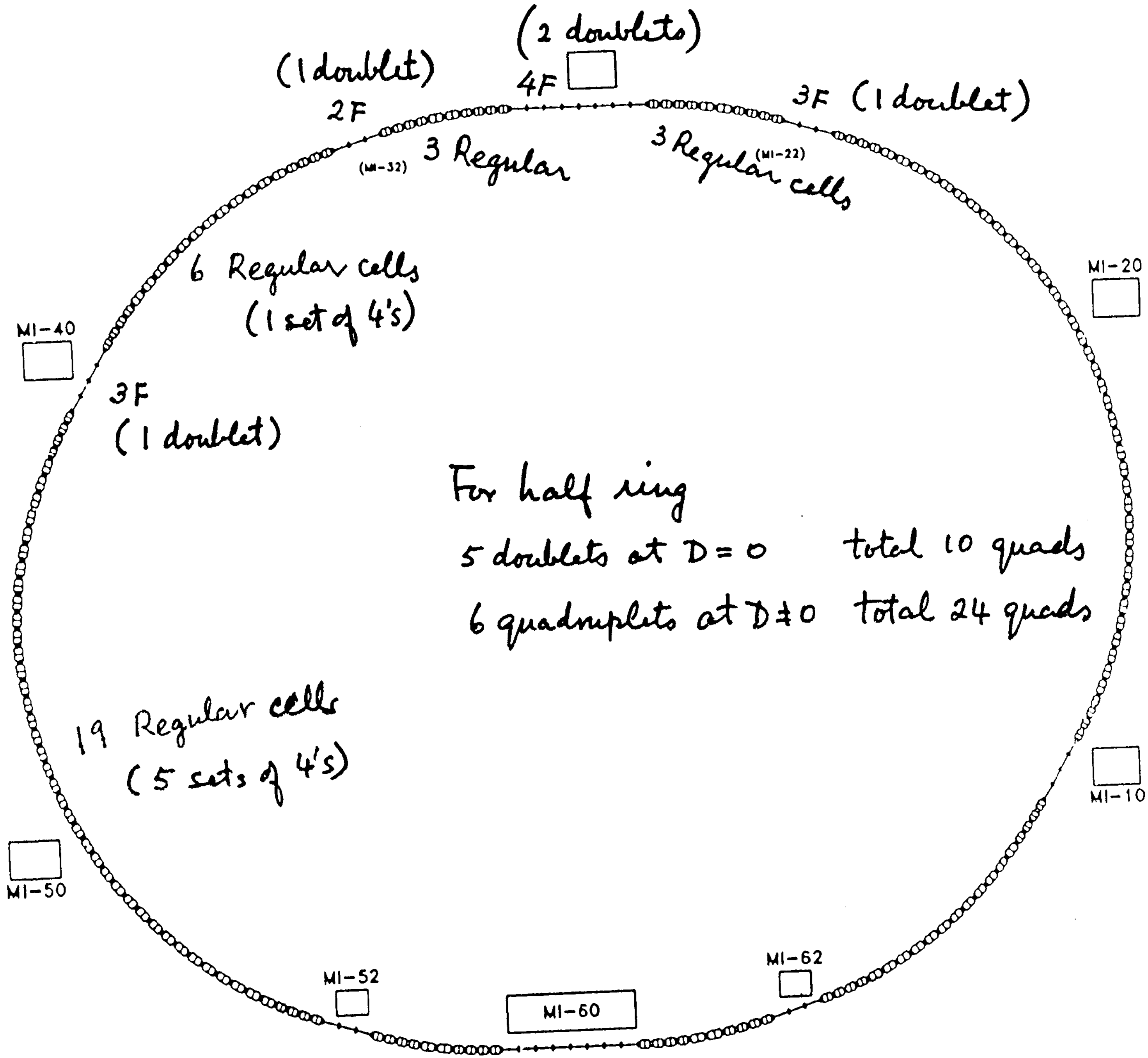


Figure 2.1-1. Main Injector Geometric Layout Showing Locations of Service Buildings and Straight Sections

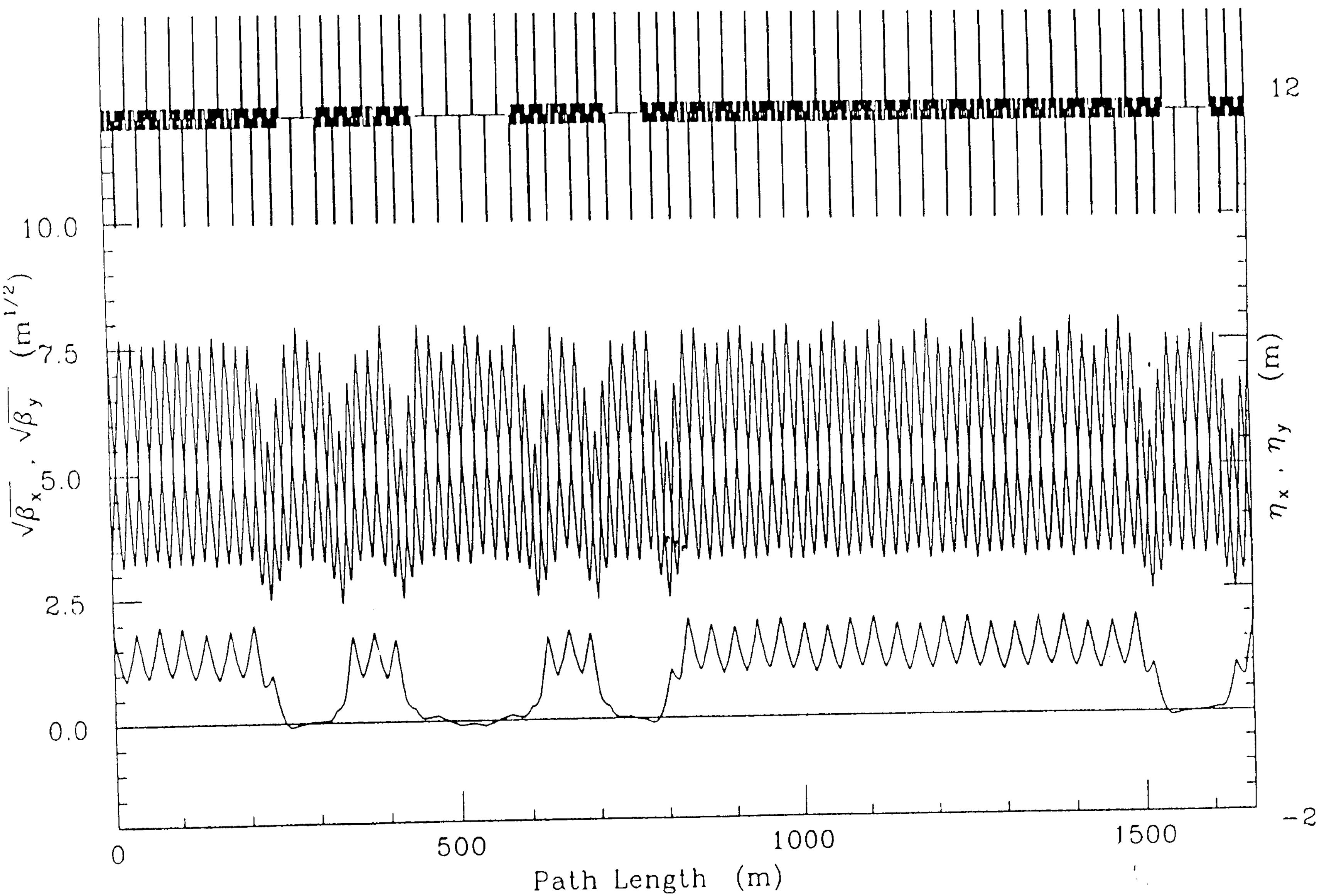
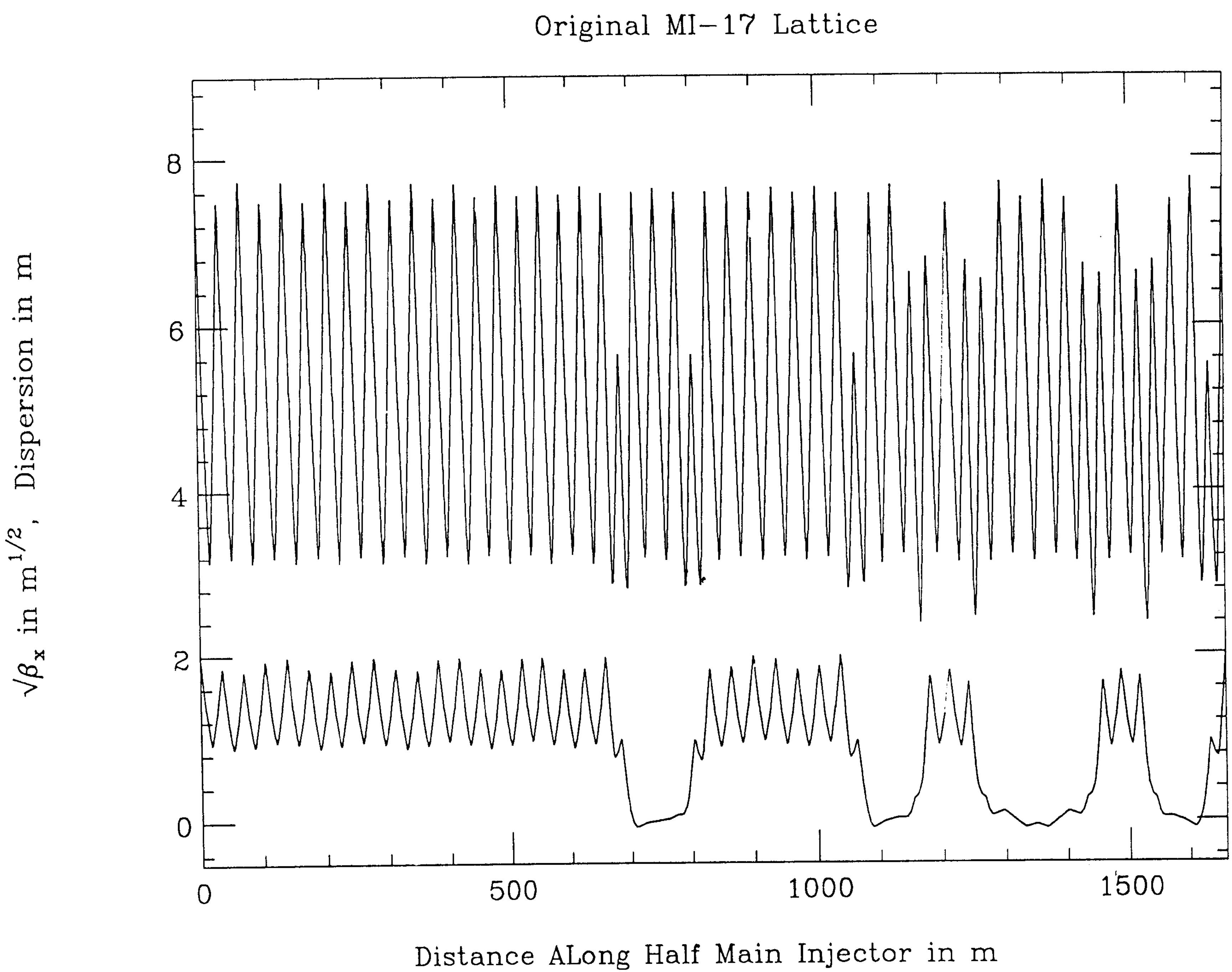


Figure 2.1-3. Main Injector Lattice Functions for One-Half the Ring


Original MI-17 Lattice
$\sqrt{\beta_x}$ in $m^{1/2}$, Dispersion in m
Distance ALong Half Main Injector in m
0
500
1000
1500
0
2
4
6
8

$J_1 = -0.00235/m$, $J_2 = +0.005518/m$, $\Delta\gamma_t = -0.625$