X-Y Coupling of Betatron Motion from MI Dipole Bus Current

James A. MacLachlan

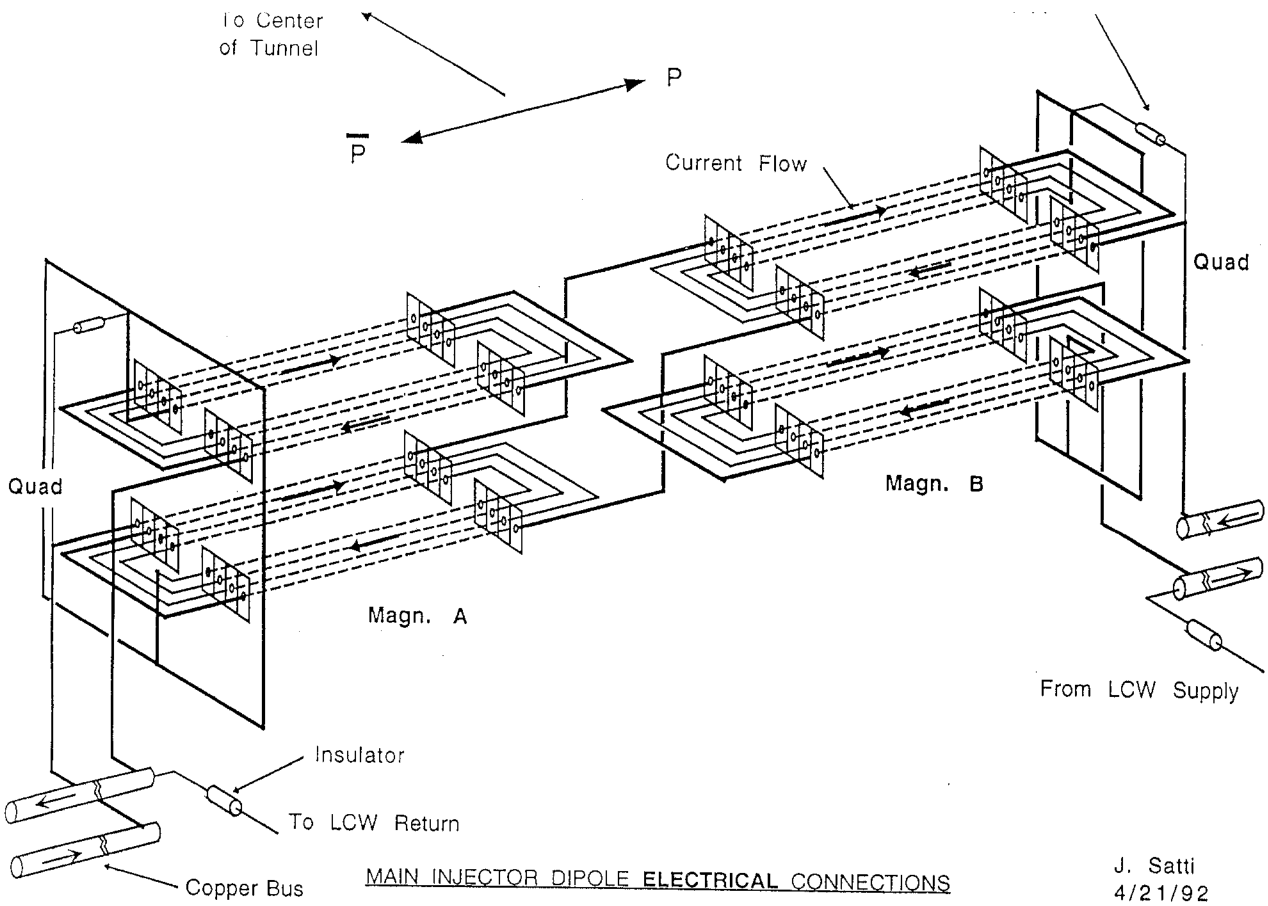
25 October 1994

Introduction

The coils of the MI dipoles have been designed to eliminate an external return bus between magnets. One half turn of a coil is replaced by a straight through bus in each magnet. In type A and C magnets, this is the left side of the lower coil, and for type B and D magnets it is the left side of the upper coil, where "left" is in each case as viewed from the feed end. Each pair of magnets is then bussed together by a short pair of conductors that pass from the bottom of A and C magnets to the top of B and D magnets. These connections are shown in the figure. One also sees from the figure that the lower coil of A and C magnets and the top coil of B and D magnets have only three turns on the end where they are joined. The net effect of the unmatched fourth turn ends and the bottom-to-top connection from type A to type B is a single-turn rectangular loop approximately 20 in $\times$ 7.5 in. This loop carries the full dipole current and will produce a longitudinal magnetic field on the central axis. The longitudinal field will couple the nominally independent modes of betatron oscillation. The strength of this coupling has been evaluated by introducing solenoid elements into the lattice between adjacent dipoles. The off-axis field generated by the bus is not, of course, purely longitudinal, so some focusing effect is also to be expected. The solenoid insertions approximately model the detuning also.

The feed end of the dipoles also exhibit some azimuthal current paths which may be expected to produce longitudinal field. The details of the flow are a little complicated, but qualitatively there is local cancelation and the effect should be smaller than that from the opposite end. Even if the feed-end fields were independently comparable, their combined effect must nearly vanish because they occur in pairs of opposite sense with only about five degrees of betatron phase separating them. Therefore, the feed ends have been ignored in this calculation which has as its principal object determining whether the order of magnitude of the coupling is large enough to warrant concern or a more detailed calculation.

The x-y coupling has also been examined by R. Baiod.[1] His report is not yet available, but his conclusion agrees qualitatively at least with what is reported here, *viz.*, that the coupling is entirely negligible as a practical matter.



MAIN INJECTOR DIPOLE ELECTRICAL CONNECTIONS

Strength of the Perturbation

The source of the error field is a rectangular loop of approximately $51 \times 19\ \overline{\text{cm}}^2$ centered on the beam orbit. Regardless of the shape of a loop, its central field is proportional to R^{-1} where R is the radius for an equivalent circular loop. Therefore, the rectangular loop is treated as circular loop of the same dipole moment; the equivalent radius is $R = 17.55$ cm. The axial field of a one-turn loop is

$$B_{\text{axial}} = \frac{\mu_o I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}} ,$$

where $z = 0$ is the center of the loop. The distance between adjacent dipoles, 0.3339 m, is about one foot, just about the diameter of the equivalent circular loop. Just to make the formula for the $\int B_z\, dz$ a little tidier the diameter of the loop is set to precisely the inter-dipole space so $2R = L$. The integrated effect is

$$\int B\, dz = \frac{\mu_o I}{2} \int_{-L/2}^{L/2} \frac{(L/2)^2}{[z^2 + (L/2)^2]^{3/2}} dz = \frac{\mu_o I}{\sqrt{2}} .$$

This is the same field integral that would result from a solenoid of length L and strength

$$B_z = \frac{\mu_o I}{\sqrt{2}L} \quad .$$

For beam optics one uses the momentum normalized strength

$$K_s = \frac{B_z}{B\rho} \quad .$$

Using the numbers for the MI at 120 GeV/c,

$$B_z = \frac{4\pi \cdot 10^{-7} \times 7.1 \cdot 10^3}{\sqrt{2} \times .3339} = 0.0189 \quad [\mathrm{T}] \quad ,$$

$$B\rho = \frac{10p \quad [\mathrm{GeV/c}]}{2.99793} = 400.27 \quad [\mathrm{Tm}] \quad ,$$

and

$$K_s = 4.721 \cdot 10^{-5} \quad [\mathrm{m}^{-1}] \quad .$$

The Effect of the Perturbation

The transfer matrix for a solenoid is

$$\begin{pmatrix} C^2 & SC/k & SC & S^2/k \\ -kSC & C^2 & -kS^2 & SC \\ -SC & -S^2/k & C^2 & SC/k \\ kS^2 & -SC & -kSC & C^2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad ,$$

where

$$\begin{aligned} k &= \frac{1}{2}K_s \\ S &= \sin kL \\ C &= \cos kL \quad . \end{aligned}$$

The thin lens version of this matrix is

$$\begin{pmatrix} 1 & 0 & kL & 0 \\ -k^2L & 1 & 0 & kL \\ -kL & 0 & 1 & 0 \\ 0 & -kL & -k^2L & 1 \end{pmatrix}$$

which partitions in an obvious way into focusing diagonal blocks and coordinate rotating off-diagonal blocks. This version is more than sufficient for the needs of this calculation, but, because a matrix code (MAD) is used, the full matrix is employed anyway. Note, however, the focusing is second order in k; it can hardly be significant. The coupling terms are first order, and in the crudest estimate look almost significant.

The crudest approximation is to represent the coupling of all 172 dipole pairs by a single thin solenoid. Then, the angle of rotation of the axes is

$$\psi = \frac{N_{\text{dip}}}{2} \times kL = 172 \times 2.36 \cdot 10^{-5} \times 0.3339 = 1.36 \cdot 10^{-3} \quad .$$

This is already apparently negligible, but it will be interesting to find the coupling resulting from the distribution of solenoids because this result should agree more closely with that of Baiod who, as I understand it, treated the coupling as resulting from a continuous solenoid around the entire ring.

Matrix Formalism for X-Y Coupling

To evaluate the global coupling one wants to calculate the off-diagonal blocks of the one-turn transfer matrix. I follow here the technique described by R. Talman.[2] Denote the transfer matrix by M and denote by $\bar{M}$ its so-called symplectic conjugate

$$\bar{M} = -SM^TS = \begin{pmatrix} \bar{A} & \bar{C} \\ \bar{B} & \bar{D} \end{pmatrix} \quad ,$$

where S is the symplectic matrix

$$S = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

and

$$\bar{A} = \overline{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = A^{-1}\det|A| \quad .$$

Because $\det|M| = 1$, $\bar{M} = M^{-1}$. The matrix that is convenient for the normal mode analysis is actually

$$M + \bar{M} = \begin{pmatrix} A + \bar{A} & B + \bar{C} \\ C + \bar{B} & D + \bar{D} \end{pmatrix} = \begin{pmatrix} \text{tr}A & \bar{E} \\ E & \text{tr}D \end{pmatrix} \quad .$$

The desired off-diagonal block is

$$E = C + \bar{B} = \begin{pmatrix} c_{11} + b_{22} & c_{12} - b_{12} \\ c_{21} - b_{21} & c_{22} + b_{11} \end{pmatrix} \quad .$$

The global coupling coeficient is usually measured as the width of the difference resonance, *i.e.*, the minimum attainable separation of the normal mode tunes. The number to be calculated from M is

$$|\nu_y - \nu_x|_{\min} = \frac{\sqrt{\det E}}{\pi(\sin \mu_x + \sin \mu_y)}$$

where the x and y subscripts actually refer to the normal mode tunes.

Numerical Results

The matrix code MAD has been used with the MI-17 lattice file to generate the one-turn transfer matrix M. The space between all dipoles, $L = .3339$ m, is replaced with a solenoid of that length with the strength $K_s = 4.721 \cdot 10^{-5}$ m^{-1} calculated above. The result is

$$M = \left(\begin{array}{cc} \begin{pmatrix} -0.905591 & 4.389695 \\ -0.045903 & -0.881747 \end{pmatrix} & \begin{pmatrix} 0.000009 & 0.000168 \\ 0.000000 & -0.000167 \end{pmatrix} \\ \begin{pmatrix} 0.000167 & -0.000337 \\ 0.000001 & -0.000010 \end{pmatrix} & \begin{pmatrix} -0.882762 & 31.532495 \\ -0.008125 & -0.842567 \end{pmatrix} \end{array} \right) .$$

The tunes $\nu_x = 26.425939$ and $\nu_y = 25.415603$ are identical to the unperturbed tunes; therefore the focusing effect of the solenoids is truly negligible, as expected. A maximum vertical dispersion of $8.6 \cdot 10^{-5}$ m indicates a principal axis rotation of order $4 \cdot 10^{-5}$ radian since this must be a projection of the 2 m maximum horizontal dispersion of the unperturbed lattice. This can be compared to the extreme over-estimate quoted for a lumped solenoid, *viz.*, about one milliradian.

The sub-matrix E is

$$E = \begin{pmatrix} 0.0 & -0.000338 \\ 0.000001 & 0.000010 \end{pmatrix}$$

with determinant $\det|E| = 3.38 \cdot 10^{-10}$. Since the perturbation is so small, one can clearly use the x and y tunes for the eigen-tunes. Therefore, the coupling constant can be written

$$K_{xy} = |\nu_y - \nu_x|_{\min} = \frac{\sqrt{\det|E|}}{\pi(\sin\mu_x + \sin\mu_y)} = 6.13 \cdot 10^{-6}.$$

This is three orders of magnitude below practical significance.

References

[1] Rachid Baiod,"Evaluation of Beam Coupling due to Dipole Current Loop", MI-0113 (in preparation)

[2] Richard Talman, "Single Particle Motion" in Frontiers of Particle Beams, M. Month and S. Turner eds., Springer-Verlag(1989)

Addendum

The assertion that a coupling constant of 10^{-5} is neglible seems rather uncontroversial, but it is more concrete to put this result into the immediate context. So, what coupling is expected from the skew quad field of the dipoles? The Technical Design Manual indicates that this is considered rather small and easily corrected. Therefore, the same procedure has been followed to find the coupling from the skew quad error of the dipoles for comparison. In the same spirit of approximation used in the foregoing, the skew component measured at (approximately) 120 GeV/c excitation for IDA-10 is taken as a systematic error and represented as a skew quad between each A-B and C-D pair. This should represent an upper limit for the effect.

The normalized skew quad coefficient for IDA-10 is $-7 \cdot 10^{-5}$. The skew gradient is

$$B'_{\mathbf{skew}} = \frac{a_2 B_\circ}{d} \ ,$$

where d is the reference radius of one inch, and $B_\circ$ is the bend field, 1.37 T. Therefore, $B'_{\mathbf{skew}}$ is -0.004 T/m and the focusing strength is

$$k_{\mathbf{skew}} = \frac{B'_{\mathbf{skew}}}{B\rho} = -0.004/400.26 = -10^{-5}\ \mathrm{m}^{-2} \ .$$

This strength is to be associated with a skew quad filling the space between each pair of dipoles, so it must be multiplied by the ratio of twice the average dipole length of 5.34 m divided by the dipole spacing of 0.339 m. Thus, the $K1$ parameter for the skew quad in the MAD calculation is $-3.2 \cdot 10^{-4}$

The one turn transfer matrix obtained is

$$M = \begin{pmatrix} \begin{pmatrix} -0.908348 & 4.336841 \\ -0.045277 & -0.884785 \end{pmatrix} & \begin{pmatrix} 0.000452 & 0.253469 \\ 0.000205 & 0.003221 \end{pmatrix} \\ \begin{pmatrix} 0.002243 & 0.250591 \\ 0.000209 & 0.000879 \end{pmatrix} & \begin{pmatrix} -0.879560 & 31.890338 \\ -0.008223 & -0.838856 \end{pmatrix} \end{pmatrix} \ .$$

The sub-matrix E is

$$E = \begin{pmatrix} 0.005464 & -0.002878 \\ 0.000004 & 0.001331 \end{pmatrix}$$

with determinant $\det|E| = 1.78 \cdot 10^{-6}$. Because the denominator in the coupling coefficient expression is the same for both calculations, the ratio the coupling constants is just the square root of the ratio of the determinants. Thus, one finds the coupling resulting from the dipole bus current loop is one percent of the effect expected from a systematic skew quad component of the bending magnet field.