The Recycler Ring will be constructed from 344 combined function and 88 discrete quadrupole magnets. Absolute field strength, magnet-to-magnet variations, and field non-uniformities within magnets must be controlled in order to assure Recycler performance. The purpose of this note is to define a magnetic uniformity specification for the Recycler combined function and quadrupole magnets. This note defines requirements in magnet-to-magnet and field uniformity relative to the absolute field strength requirements defined in MI-0215.

I. Systematic Strength Variations

The absolute multipole strengths listed in MI-0215 are regarded as the target values for the absolute strengths of the combined function and quadrupole magnets. It is assumed that the magnets are both aligned and measured with respect to a well defined fiducial, or mechanical center, thereby eliminating ambiguities in interpretation of the dipole and quadrupole components. This alignment should be achieved to better than 0.25 mm. Once an average bending field for the 8 GeV line magnets has been established, it is assumed that the absolute values listed in MI-0215 will be rescaled appropriately to reflect the expected central orbit momentum of the Recycler.

Tolerances in systematic variations around the target strengths are listed below. Tolerances are based on the criteria listed, and the underlying calculation supporting the performance specification is referenced.

Absolute strength of dipole field at the center of the long combined function magnet

<table>
<thead>
<tr>
<th>Specification: ( (&lt;BL&gt;-BL_{\text{nominal}})/BL_{\text{nominal}} &lt; 5 \times 10^{-4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion: Peak orbit offset due to momentum mismatch with 8 GeV line &lt; 1mm</td>
</tr>
<tr>
<td>Notes: ( BL_{\text{nominal}} ) is defined in MI-0215. This value may be modified following 8 GeV line magnet production. This specification applies to the average of all long combined function magnets. Specification is based on a maximum dispersion of 2 m in the Recycler.</td>
</tr>
</tbody>
</table>
Strength of dipole field at the center of short, relative to long, combined function magnets
Specification: \((<B'_L>-B'_L\text{nominal})/B_L\text{nominal} < 5\times 10^{-4}\)
Criterion: Peak closed orbit distortion < 1.5 mm
Notes: \(B\text{nominal}\) is 2/3 the strength of the average dipole component in all long combined function magnets. This specification applies to the average of all short combined function magnets. (Reference: TDR 1.2, page 2.44)

Ratio of gradient-to-dipole strength as measured at the center of all combined function magnets
Specification: \((<B'_L'B'_L>-B'_L'B'_L\text{nominal})/B_L\text{nominal} < 1\times 10^{-4}/\text{inch}\)
Criterion: rms beta function distortion < 1%
Notes: \(B'_L/B_L\text{nominal}\) is defined in MI-0215. This specification applies to the average of all combined function magnets. (Reference: TDR 1.2, page 2.44)

Ratio of sextupole-to-dipole strength as measured at the center of long combined function magnets
Specification: \(0.5(<B''_L>-B''_L\text{nominal})/B_L\text{nominal} < 0.5\times 10^{-4}/\text{inch}^2\)
Criterion: Chromaticity within range of correction system.
Notes: \(B''_L/B_L\text{nominal}\) is as defined in MI-0215. This specification applies to the average of all long combined function magnets. (Reference: TDR 1.2, page 2.56)

Integrated gradient strength in quadrupole magnets
Specification: \((<B'_L'B'_L>-B'_L'B'_L\text{nominal})/B'_L\text{nominal} < 5\times 10^{-4}\)
Criterion: Tune shift due to systematic gradient offset is less than .003. Beta function distortion < 0.2% (rms)
Notes: \(B'_L\text{nominal}\) is defined in MI-0215. This value may be modified following 8 GeV line magnet production. This specification applies to the average of all quadrupole magnets.
II. Magnet-to-magnet Strength Variations

In addition to systematic variations of the magnetic strength of the combined function and quadrupole magnets, random variations around the mean are anticipated. We assume that a Gaussian distribution of strengths will be observed relative to the values listed in MI-0215 during the magnet production run. Tolerances on the rms width of these distributions around the average values are listed below. Tolerances are based on the criteria listed, and the underlying calculation supporting the performance specification is referenced.

Magnet-to-magnet variation in the strength of dipole field of combined function magnets
Specification: \( \text{rms}(B_L - \langle B_L \rangle)/\langle B_L \rangle < 5 \times 10^{-4} \)
Criterion: Orbit distortion < 3 mm (rms)
Notes: \( \langle B_L \rangle \) is the average dipole strength of all combined function magnets of a given class. (Reference: TDR 1.2, page 2.46)

Magnet-to-magnet variation in the strength of gradient field of combined function magnets
Specification: \( \text{rms}(B'_L - \langle B'_L \rangle)/\langle B'_L \rangle < 1 \times 10^{-4}/\text{inch} \)
Criterion: rms beta function distortion < 4%
Notes: \( \langle B'_L \rangle \) is the average gradient strength of all combined function magnets of a given class. This specification applies to the average of all combined function magnets. (Reference: TDR 1.2, page 2.46)

Magnet-to-magnet variation in the strength of sextupole field of combined function magnets
Specification: \( \text{rms}(B''_L - \langle B''_L \rangle)/\langle B''_L \rangle < 1 \times 10^{-4}/\text{inch} \)
Criterion: Chromaticity within range of correction system.
Notes: \( \langle B''_L \rangle \) is the average sextupole strength of all combined function magnets of a given class. This specification applies to the average of all combined function magnets.

Magnet-to-magnet variation in the integrated strength of quadrupole magnets
Specification: \( \text{rms}(B'L - \langle B'L \rangle)/\langle B'L \rangle < 8 \times 10^{-4} \)
Criterion: Beta function distortion < 0.4% (rms)
Notes: \( \langle B'L \rangle \) is the average dipole strength of all quadrupole magnets in the Recycler.
**III. Field Uniformity**

The presence of field components other than those specifically designed into the magnets can negatively impact the ability of the Recycler to support stored beams. Analyses based on resonance widths, detuning with amplitude, and tracking calculations are used to provide guidance on the acceptable multipole content and overall field flatness. Systematic and random variations can have different impacts and so tolerances are listed separately.

Tolerances in systematic and random variations, relative to zero, are listed below. Tolerances are based on the criteria listed, and the underlying calculation supporting the performance specification is referenced.

**Allowed systematic multipole components in the combined function magnets**

Specification: All normal and skew multipole components, both systematic and random, are required to be within the tolerances listed below. Tolerances are in "units" defined as the 10⁴ times the ratio of the multipole field to the bending field as referenced to a 1" radius.

<table>
<thead>
<tr>
<th>Multipole Component</th>
<th>Normal (Systematic)</th>
<th>Normal (Random)</th>
<th>Skew (Systematic)</th>
<th>Skew (Random)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrupole</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sextupole</td>
<td>0.5</td>
<td>1</td>
<td>--</td>
<td>0.5</td>
</tr>
<tr>
<td>Octupole</td>
<td>0.5</td>
<td>0.5</td>
<td>--</td>
<td>0.5</td>
</tr>
<tr>
<td>10-pole</td>
<td>0.2</td>
<td>0.5</td>
<td>--</td>
<td>0.5</td>
</tr>
<tr>
<td>12-pole</td>
<td>0.1</td>
<td>0.5</td>
<td>--</td>
<td>0.5</td>
</tr>
<tr>
<td>14-pole</td>
<td>0.1</td>
<td>0.5</td>
<td>--</td>
<td>0.5</td>
</tr>
<tr>
<td>16-pole</td>
<td>0.1</td>
<td>0.5</td>
<td>--</td>
<td>0.5</td>
</tr>
<tr>
<td>18-pole</td>
<td>0.1</td>
<td>0.5</td>
<td>--</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Criteria: Dynamic aperture as determined by particle tracking of 70π mm-mr over a momentum range of ±0.3%.

Notes: Specification refers to systematic multipole component as measured in Fermilab "units" referenced to the dipole field component. In the case of random, the unit is applied to the rms of the distribution.

**Allowed field non-uniformity (flatness) in the combined function magnets**

Specification: The integrated field strength shall be required to lie within ±1.5x10⁻⁴ of its nominal value, as defined by the field components listed in Table I.2, over a range of ±25 mm as measured on the vertical mid-plane of the magnet.

Criterion: Tune variation over the physical beam size + excursions <.01
Notes: Beam size for a 40π beam is ±15 mm. Specification allows for a significant orbit excursion.

Allowed systematic multipole components in the quadrupole magnets

Specification: All normal and skew multipole components, both systematic and random, are required to be within the tolerances listed below. Tolerances are in "units" defined as the 10⁴ times the ratio of the multipole field to the quadrupole field as referenced to a 1" radius.

<table>
<thead>
<tr>
<th>Multipole Component</th>
<th>Normal (Systematic)</th>
<th>Normal (Random)</th>
<th>Skew (Systematic)</th>
<th>Skew (Random)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sextupole</td>
<td>0.5</td>
<td>1.5</td>
<td>--</td>
<td>1.5</td>
</tr>
<tr>
<td>Octupole</td>
<td>0.5</td>
<td>1.5</td>
<td>--</td>
<td>1.5</td>
</tr>
<tr>
<td>10-pole</td>
<td>0.5</td>
<td>1.5</td>
<td>--</td>
<td>1.5</td>
</tr>
<tr>
<td>12-pole</td>
<td>0.5</td>
<td>1.5</td>
<td>--</td>
<td>1.5</td>
</tr>
<tr>
<td>14-pole</td>
<td>0.5</td>
<td>1.5</td>
<td>--</td>
<td>1.5</td>
</tr>
<tr>
<td>16-pole</td>
<td>0.5</td>
<td>1.5</td>
<td>--</td>
<td>1.5</td>
</tr>
<tr>
<td>18-pole</td>
<td>0.5</td>
<td>1.5</td>
<td>--</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Criteria: Dynamic aperture as determined by particle tracking of 70π mm-mr over a momentum range of ±0.3%.

Notes: Specification refers to systematic multipole component as measured in Fermilab "units" referenced to the dipole field component. In the case of random, the unit is applied to the rms of the distribution.
IV. Longitudinal Uniformity

Bend Center

The nature of the permanent magnet construction creates the potential for longitudinal displacements of magnet bend centers from the nominal position. Deviations of the longitudinal bend center can create closed orbit distortions as described by the expression,

\[ x_{co}(s) = \frac{\sqrt{\beta_s}}{2 \sin \pi \nu} \sum_i L_i \theta_i \sin(\phi(s) - \phi_i - \pi \nu) \frac{\sqrt{\beta_i}}{ \sqrt{\beta_i} } . \]

where \( L_i \) is the displacement of the bend center and \( \theta_i \) is the bend angle in the magnet. For a random distribution the rms orbit distortion is related to the rms longitudinal displacement via,

\[ \sigma_{co} = \sqrt{N_D \theta_D \sigma_L} = \frac{2\pi}{\sqrt{8 N_D \sin \pi \nu}} \sigma_L \approx 0.25 \sigma_L \]

for Recycler parameters (\( N_D=344, \ \nu=.42 \)).

Magnet-to-magnet variation in the bend center location of combined function magnets

- Specification: \( \text{rms}(L_C-L_{C\text{nominal}}) < 8 \text{ mm} \)
- Criterion: Orbit distortion < 2 mm (rms)
- Notes: \( L_C \) is the longitudinal position of the combined magnet bend center.

Uniformity

Longitudinal variations of the field strength in the magnets can lead to orbit distortions, beta distortions, and tune shifts. Such effects are analyzed for a parabolic field variation in MI-Note 202.

Longitudinal uniformity in the field strength of the combined function magnet

- Specification: \( \mathcal{B}(z) - <\mathcal{B}(z)> < 2\% \) over the physical length of any magnet
- Criterion: Tune shift <.005
- Notes: Allowed peak-to-peak variation is 4%. (Reference: MI-Note 202)
V. Temperature Range

Operational Temperature Range

Specification: Specified performance will hold over the temperature range 20°C-35°C (68°F-98°F)

VI. Temporal Variations

It is assumed that long term systematic changes of the systematic dipole and gradient fields with time can probably be tolerated to a factor of four-to-five beyond the tolerances suggested in Section II by retuning of the accelerator complex. However, multipole components must remain fixed within the given specification over the operating lifetime of the Recycler.

VII. Summary

Field quality specifications for Recycler combined function and quadrupole magnets have been given. This specification is based on semi-analytic and tracking analyses incorporating the complete interaction between various field components, misalignments, etc. It is envisioned that as magnets are produced to this specification tracking evaluation will proceed utilizing the measured systematic and random field components of real production magnets. It is further assumed that once an average bending field for the 8 GeV magnets has been established, the absolute values listed in MI-0215 will be rescaled appropriately to reflect the expected central orbit momentum of the Recycler.