Notes on tune drift measurements at 150 Gev

Mike Martens, Jerry Annala

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Introduction

During Collider Run II operations it has been noticed that the tunes in the Tevatron drift with time by as much as 0.015 units while the Tevatron is at its front porch energy of 150 Gev. The time scale of the tune drifts and the b2 chromaticity correction algorithm are the same suggesting that the drift might be caused by orbit offsets and sextupole feeddown effects in the Tevatron. In order to investigate this possibility the tunes in the Tevatron were measured as a function of time at 150 Gev, RF frequency, and current in the chromaticity correction sextupoles T:SF and T:SD. The data were collected on 5/15/02 while the Tevatron was at 150 Gev after at 30 minutes store at 980 Gev. The data from these measurements and analysis of the sextupole feeddown hypothesis are presented in this note.

In the Tevatron the main sources of time varying sextupole fields are the b2 component of the Tevatron dipoles and the chromaticity correction sextupole circuits T:SF and T:SD. If the observed tune drifts are caused by time varying sextupole fields then there must be an average horizontal orbit offset in these magnets (since a vertical orbit offset creates a coupling effect.) To determine the average orbit offset in the T:SF and T:SD magnets the horizontal orbit positions were changed by varying the RF frequency and then measurements were made of the tune changes as a function of current in the T:SF and T:SD circuits. For instance we determined when the horizontal orbit, averaged over all the T:SF magnets, was centered in the T:SF magnets by finding the RF frequency that produced no tune change when the current in the T:SF circuit was varied. Once the average orbit offset was determined then the magnitude of the tune drift could be calculated.

An analysis of these data shows that the time varying strength of the chromaticity sextupoles does not explain the observed tune drift. We also conclude that an average horizontal orbit offset of about 1 mm in the dipoles is required to explain the tune drift if the sextupole feeddown hypothesis is correct. During this particular study we did not measure coupling changes. However, we note that previous studies also show a time varying coupling change that, if left uncorrected, results in a minimum tune split of about 0.02 tune units after about 2 hours. If this coupling change is due to feeddown effects then a vertical orbit offset in the sextupole fields is required but this was not examined in this study.

Another possible explanation for the time varying coupling and tune shifts is time varying quadrupole fields from persistent currents in the Tevatron dipoles or quadrupoles. This possibility is not examined in detail in this note.

Analysis

The tune change from horizontal orbit offsets in sextupole magnetic fields is given by the formula

$$\Delta n_x = \frac{1}{4p}\sum(b_x\ \Delta \mathrm{K}_2\ \mathrm{L}\ \mathrm{x})$$

where β_x is the horizontal beta function, ΔK_2 is the change in sextupole field, L is the length of the magnet, and x is the horizontal orbit position. The sum is over all sextupole fields which, in the case of the Tevatron, consists of the main dipoles and the magnets in the T:SF and T:SD circuits.

For convenience we express the change in the T:SF and T:SD sextupole fields in two parts. The first part, ΔK_2, is the time varying part which is used to compensate for the time varying b2 component in the dipoles and keeps the chromaticity at a constant value. The second part, δK_2, is the amount of sextupole field added to T:SF and T:SD as part of the tune drift measurement study. We can also express the orbit offset, x, in two parts. The first, x_0, is the offset in the magnet with the nominal orbit and nominal RF frequency and the second, D_x $(\Delta p/p)$, is the orbit offset due to the dispersion and a beam momentum offset. If we include these effects then the tune change is given by

$$\Delta n_x = \frac{1}{4p}\sum\left(b_x\ \mathrm{L}\ \left(\Delta \mathrm{K}_2 + dK_2\right)\left(\left(\Delta p/p\right)\mathrm{D}_x + \mathrm{x}_0\right)\right).$$

The strength of the T:SF and T:SD circuits are adjusted as a function of time to keep the chromaticity constant while the Tevatron is at 150 Gev. Since we know the current in the T:SF and T:SD circuits and we know that the measured chromaticity does not change, $\Delta\xi_x = 0$, we can calculate the average value of the sextupole field in the dipoles from the expression for the change in chromaticity

$$\Delta x_x = \frac{1}{4p}\sum_{\text{Dipole}}\left(b_{\text{x}}\Delta \text{K}_2\text{LD}_{\text{x}}\right) + \frac{1}{4p}\sum_{\text{T:SF}}\left(b_{\text{x}}\Delta \text{K}_2\text{LD}_{\text{x}}\right) + \frac{1}{4p}\sum_{\text{T:SD}}\left(b_{\text{x}}\Delta \text{K}_2\text{LD}_{\text{x}}\right) = 0$$

With this expression for the change in chromaticity we can express the change in tune as

$$\Delta n_x = \frac{1}{4p}\sum\left(b_{\text{x}}\ \Delta \text{K}_2\ \text{L}\ \text{x}_0\right) + \left(x_x + \Delta x_x\right)\left(\Delta p/p\right) + \frac{1}{4p}\sum\left(b_{\text{x}}\ \text{L}\ dK_2\ \left(\left(\Delta p/p\right)\text{D}_{\text{x}} + \text{x}_0\right)\right)$$

where ξ_x is the horizontal chromaticity and $\Delta\xi_x$ is the change in horizontal chromaticity as a function of time.

The first term in this equation is the tune drift at 150 Gev due to the sextupole feeddown effects. This is the term that we are interested in calculating to determine if the time varying sextupoles are responsible for the drifting tune. We assume we know the lattice functions, that the changing sextupole fields in the T:SF and T:SD circuits are known, and that we can calculate the average time varying sextupole field in the dipoles from the expression $\Delta\xi_x = 0$. This leaves the orbit offsets, x_0, in the dipoles and T:SF and T:SD magnets as the unknown variables in the first term of the equation. The second term is the change in tune due to the change in momentum and the chromaticity and does not change as a function of time since the chromaticity is kept constant.

The third term is useful because we can adjust the sextupole field, δK_2, in the T:SF or T:SD magnets and then measure the change in tune as a function of $\Delta p/p$. This allows us to determine the average value of the horizontal orbit offset in T:SF and T:SD circuit. Using the relationship between the RF frequency and the beam momentum, $(\Delta f/f) = -\eta(\Delta p/p)$ where η is the slip factor, we can write the third term as

$$\Delta n_x = \left(dK_2\text{L}\right)\frac{N}{4p}\left(-\frac{1}{h}\left\langle b_{\text{x}}\text{D}_{\text{x}}\right\rangle\left(\Delta f/f\right) + \left\langle b_{\text{x}}\text{x}_0\right\rangle\right)$$

where N is the number of magnets in the circuit and $\langle\rangle$ denotes the average value.

The value $\langle \beta_x x_0 \rangle$ is then determined from

$$\langle b_{\mathrm{x}} \mathrm{x}_0 \rangle = \frac{1}{h} \langle b_{\mathrm{x}} \mathrm{D}_{\mathrm{x}} \rangle * (\Delta f / f)_{\Delta n_x = 0}$$

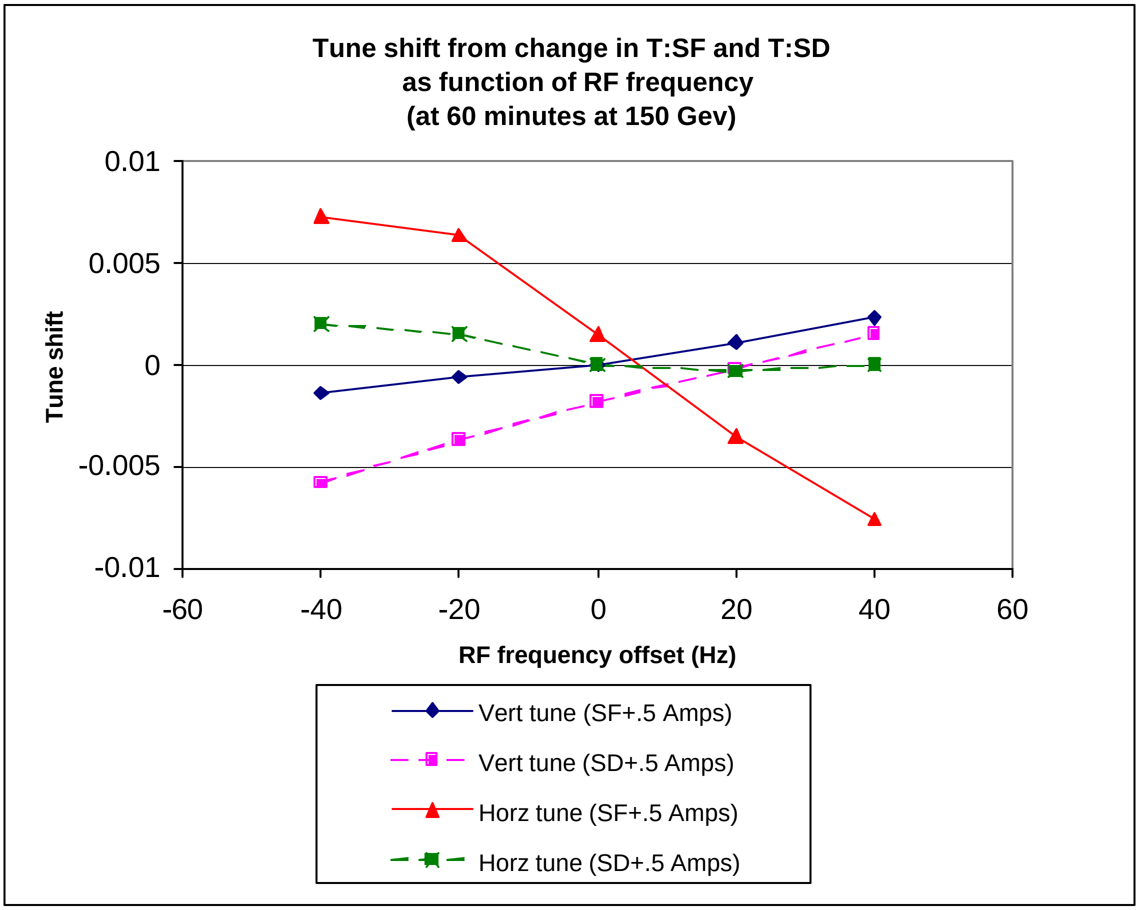
where $(\Delta f / f)_{\Delta n_x = 0}$ is the value of the RF frequency shift that gives zero tune shift when the current is changed in the T:SF or T:SD circuits.

Tune measurements

The tunes of the Tevatron were measured as a function of time at 150 Gev, a function of RF frequency, and current in the T:SF and T:SD sextupoles. The data were collected on 5/15/02 while the Tevatron was at 150 Gev after a 30 minutes store at 980 Gev. At each RF frequency offset (-40 Hz, -20 Hz, 0 Hz, +20 Hz, +40 Hz) the tune was measured with 1) no change in T:SF and T:SD, 2) +0.5 Amps added to T:SD, and 3) +0.5 Amps added to T:SF. The complete set of data is plotted in Appendix A.

For this section of the note we focus on the measurements of the tune drifts as a function of time at 150 Gev and the measurements of the tune difference resulting from a 0.5 Amp change in the chromaticity sextupole circuits. For each RF frequency offset the change in tune was measured and the results are shown in the table and figure below. These data were extracted from the tune measurements taken when the Tevatron was at 150 Gev for 60 minutes.

RF frequency Offset (Hz)	Vert tune (SF+.5 Amps)	Vert tune (SD+.5 Amps)	Horz tune (SF+.5 Amps)	Horz tune (SD+.5 Amps)
-40	-0.0014	-0.0058	0.0073	0.002
-20	-0.0006	-0.0037	0.0064	0.0015
0	0	-0.0018	0.0015	0
20	0.0011	-0.0002	-0.0035	-3E-04
40	0.0023	0.0015	-0.0076	0



Linear fits to this data are used to determine the RF frequency that results in no net change in the tune when the currents in T:SF and T:SD are varied. The results of these fits are shown in the table below.

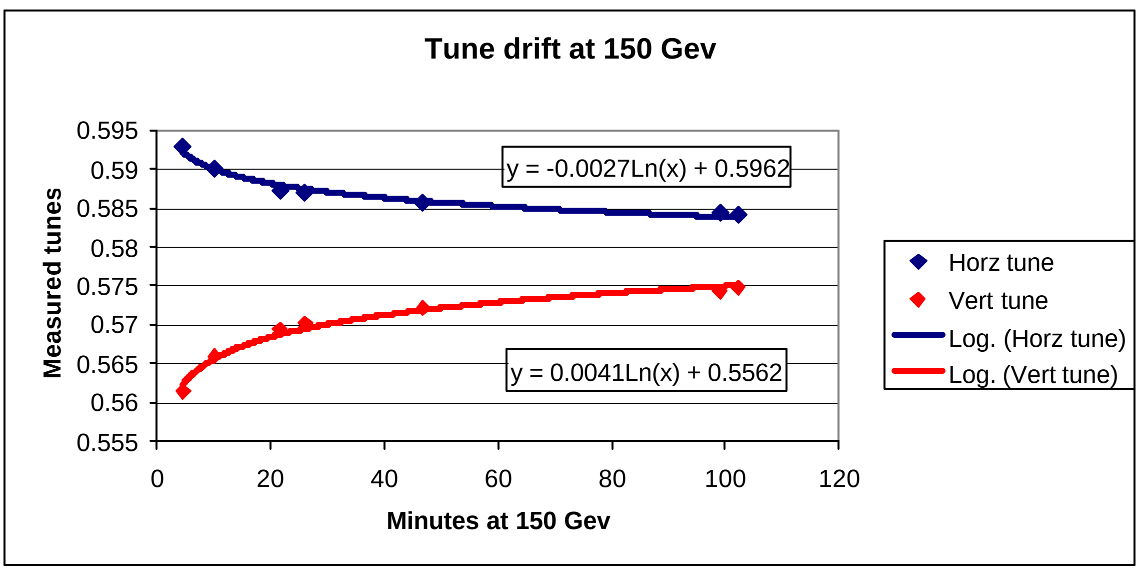
	Slope, $\Delta\nu/\Delta RF$ (Hz^{-1})	Intercept (tune units)	$(\Delta f)_{\Delta n_x=0}$ (Hz)	Momentum offset $\Delta p/p$	$\langle\beta x_0\rangle$ (m^2)	$\langle x_0\rangle$ (mm)
Vert tune (SF+.5 Amps)	4.55E-05	0.00028	-6.15	4.16E-05	-0.485E-2	-0.158
Horz tune (SF+.5 Amps)	-19.9E-05	0.00082	4.13	-2.8E-05	3.20E-2	+0.106
Vert tune (SD+.5 Amps)	9.05E-05	-0.002	22.1	-15.0E-5	1.00E-2	+0.343
Horz tune (SD+.5 Amps)	-2.90E-05	0.00064	22.1	-15.0E-5	1.06E-2	+0.343

From the results of the fit we determine the intercept on the frequency axis, $(\Delta f)_{\Delta n_x=0}$, and the value of $\langle\beta_x x_0\rangle$. Using the approximation $\langle x_0\rangle = \langle\beta_x x_0\rangle/\langle\beta_x\rangle$ we calculate the average horizontal offset in the T:SF and T:SD circuits. The data suggests that the average orbit offset in the T:SD sextupoles is +0.34 mm horizontally and the average orbit offset in the T:SF sextupoles is -.03 mm although the vertical tune and horizontal tune measurements for T:SF give different average orbit offsets. (No attempt to estimate the errors on these measurements has been made.)

We also show, in the table below, the measured slope and the theoretical slope of $\Delta\nu/\Delta f$ for a 0.5 Amp change assuming the design values for the lattice functions and the known gradient fields in the T:SF and T:SD magnets. There is good agreement in the calculated and measured values for the T:SD circuit, but the T:SF circuit shows a disagreement of about 20-30%

	Calculated Slope $\Delta\nu/\Delta f$ (Hz^{-1})	Measured Slope, $\Delta\nu/\Delta f$ (Hz^{-1})
Vert tune (SF+.5 Amps)	5.85E-5	4.55E-05
Horz tune (SF+.5 Amps)	-15.0E-5	-19.9E-05
Vert tune (SD+.5 Amps)	9.08E-5	9.05E-05
Horz tune (SD+.5 Amps)	-2.93E-5	-2.90E-05

Another part of the data is the measurement of the horizontal and vertical tune as a function of time at 150 Gev with the nominal RF frequency and nominal current in T:SF and T:SD. This is shown in the plot below.



A fit to the horizontal and vertical tune drift data using a logarithmic function gives

$$\Delta n_x = -2.7\text{x}10^{-3} * \ln(t) + 0.5962$$
$$\Delta n_y = +4.1\text{x}10^{-3} * \ln(t) + 0.5562$$

where t is the time at 150 Gev in minutes.

We also have the fits to the time varying sextupole gradients in the Tevatron dipoles and T:SF and T:SD circuits that were calculated in Appendix B,

$$\Delta(\text{K}_2\text{L})_{\text{SF}}\ (\text{m}^{-2}) = -3.12\text{x}10^{-3} * \ln(t)\ -1.13\text{x}10^{-3}$$
$$\Delta(\text{K}_2\text{L})_{\text{SD}}\ (\text{m}^{-2}) = -4.79\text{x}10^{-3} * \ln(t)\ -4.79\text{x}10^{-3}$$
$$\Delta(\text{K}_2\text{L})_{\text{Dipole}}(\text{m}^{-2}) = 9.51\text{x}10^{-4} * \ln(t) + 2.94\text{x}10^{-3}$$

Combining the measured orbit offsets, the tune drift measurement, and the time varying sextupole fields we can calculate the average orbit offset in the Tevatron dipoles, $\langle\beta_x x_0\rangle_{\text{Dipole}}$. We substitute the results of the measurements into the equation for the tune drift due to the orbit offsets in the sextupole fields,

$$\Delta n_x = \frac{1}{4p}\sum\left(b_x\ \Delta\text{K}_2\ \text{L}\ \text{x}_0\right)$$

which can be written as

$$\Delta n_x = \frac{1}{4p}\begin{pmatrix} \text{N}_{\text{Dipole}}\Delta(\text{K}_2\text{L})_{\text{Dipole}}\langle b_x \text{x}_0\rangle_{\text{Dipole}} + \\ \text{N}_{\text{T:SF}}\Delta(\text{K}_2\text{L})_{\text{T:SF}}\langle b_x \text{x}_0\rangle_{\text{T:SF}} + \\ \text{N}_{\text{T:SD}}\Delta(\text{K}_2\text{L})_{\text{T:SD}}\langle b_x \text{x}_0\rangle_{\text{T:SD}} \end{pmatrix}$$

where all the terms are known except the average orbit offset in the dipoles, $\langle\beta_x x_0\rangle_{\text{Dipole}}$.

Concentrating on the logarithmic variation we get

$$\Delta\nu_x = -2.7\text{x}10^{-3} * \ln(t) = \frac{1}{4\pi}\begin{pmatrix} 774 * (9.74\text{x}10^{-4}) * \langle \beta_x \text{x}_0 \rangle_{\text{Dipole}} + \\ 88 * \left(-3.12\text{x}10^{-3}\right) * \left(1.00\text{x}10^{-2}\right) + \\ 88 * \left(-4.79\text{x}10^{-3}\right) * \left(1.06\text{x}10^{-2}\right) \end{pmatrix} * \ln(t)$$

$$\Delta\nu_x = -2.7\text{x}10^{-3} * \ln(t) = \left(0.586\text{x}10^{-3} * \langle \beta_x \text{x}_0 \rangle_{\text{Dipole}} - 0.218\text{x}10^{-3} - 0.356\text{x}10^{-3}\right) * \ln(t)$$

which gives

$$\langle \beta_x \text{x}_0 \rangle_{\text{Dipole}} = -3.54\text{x}10^{-2}\ \text{m}^2$$

or

$$\langle \text{x}_0 \rangle_{\text{Dipole}} = \langle \beta_x \text{x}_0 \rangle_{\text{Dipole}} / \langle \beta_x \rangle_{\text{Dipole}} = -0.632\ \text{mm}$$

This means that there must be an average horizontal offset of –0.632 mm in the Tevatron dipoles if the time varying sextupole fields are to explain the horizontal tune drift at 150 Gev. If the feeddown hypothesis explaining the tune drift is correct then the T:SF and T:SD circuits contribute 8% and 13% to the horizontal tune drift respectively while the dipoles contribute 79% to the horizontal tune drift.

The same analysis is done using the vertical tune drift measurement and the equation for the vertical tune drift

$$\Delta\nu_y = -\frac{1}{4\pi}\sum\left(\beta_y\ \Delta\text{K}_2\ \text{L}\ \text{x}_0\right)$$

which can be written as

$$\Delta\nu_y = -\frac{1}{4\pi}\begin{pmatrix} N_{Dipole}\Delta(K_2L)_{Dipole}\langle\beta_y x_0\rangle_{Dipole} + \\ N_{T:SF}\Delta(K_2L)_{T:SF}\langle\beta_y x_0\rangle_{T:SF} + \\ N_{T:SD}\Delta(K_2L)_{T:SD}\langle\beta_y x_0\rangle_{T:SD} \end{pmatrix}$$

where all the terms are known except the average orbit offset in the dipoles, $\langle\beta_y x_0\rangle_{Dipole}$.

Concentrating on the logarithmic variation we get

$$\Delta\nu_y = 4.1\text{x}10^{-3} * \ln(t) = -\frac{1}{4\pi}\begin{pmatrix} 774 * 9.51\text{x}10^{-4} * \langle\beta_y x_0\rangle_{Dipole} + \\ 88 * (-3.12\text{x}10^{-3}) * (-4.85\text{x}10^{-3}) + \\ 88 * (-4.79\text{x}10^{-3}) * (3.20\text{x}10^{-2}) \end{pmatrix} * \ln(t)$$

$$\Delta\nu_y = 4.1\text{x}10^{-3} * \ln(t) = -\left(0.586\text{x}10^{-3} * \langle\beta_y x_0\rangle_{Dipole} + 0.106\text{x}10^{-3} - 1.07\text{x}10^{-3}\right) * \ln(t)$$

which gives

$$\langle\beta_y x_0\rangle_{Dipole} = -5.35\text{x}10^{-2}\ \text{m}^2$$

or

$$\langle x_0\rangle_{Dipole} = \langle\beta_y x_0\rangle_{Dipole} / \langle\beta_y\rangle_{Dipole} = -0.923\ \text{mm}$$

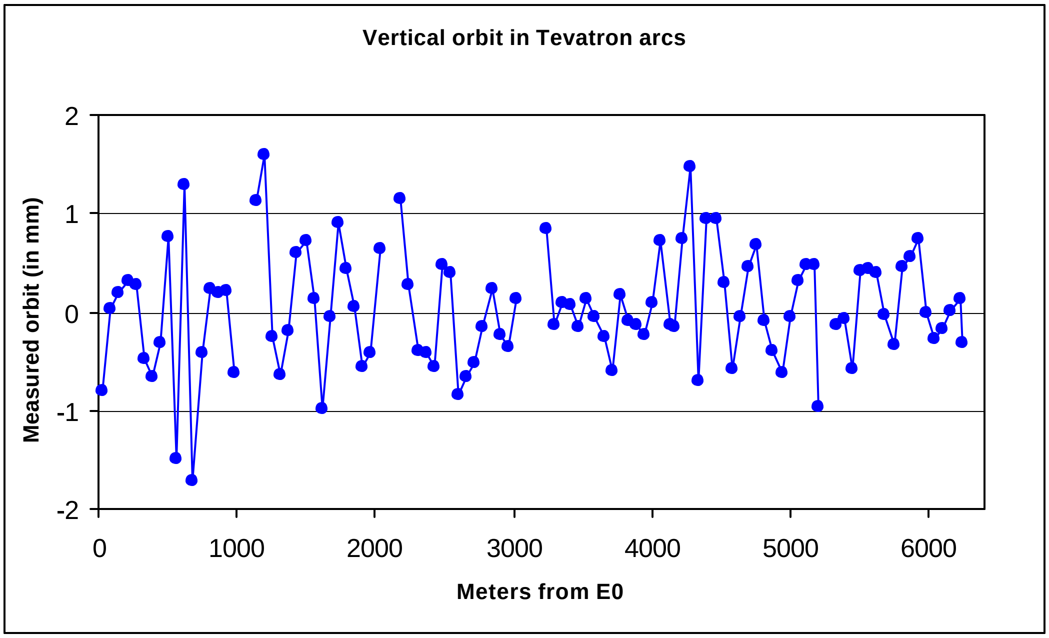
This means that there must be an average horizontal offset of –0.923 mm in the Tevatron dipoles if the time varying sextupole fields are to explain the vertical tune drift at 150 Gev. If the feeddown hypothesis explaining the vertical tune drift is correct then the T:SF and T:SD circuits contribute –2.6% and 26% to the vertical tune drift respectively while the dipoles contribute 76% to the vertical tune drift.

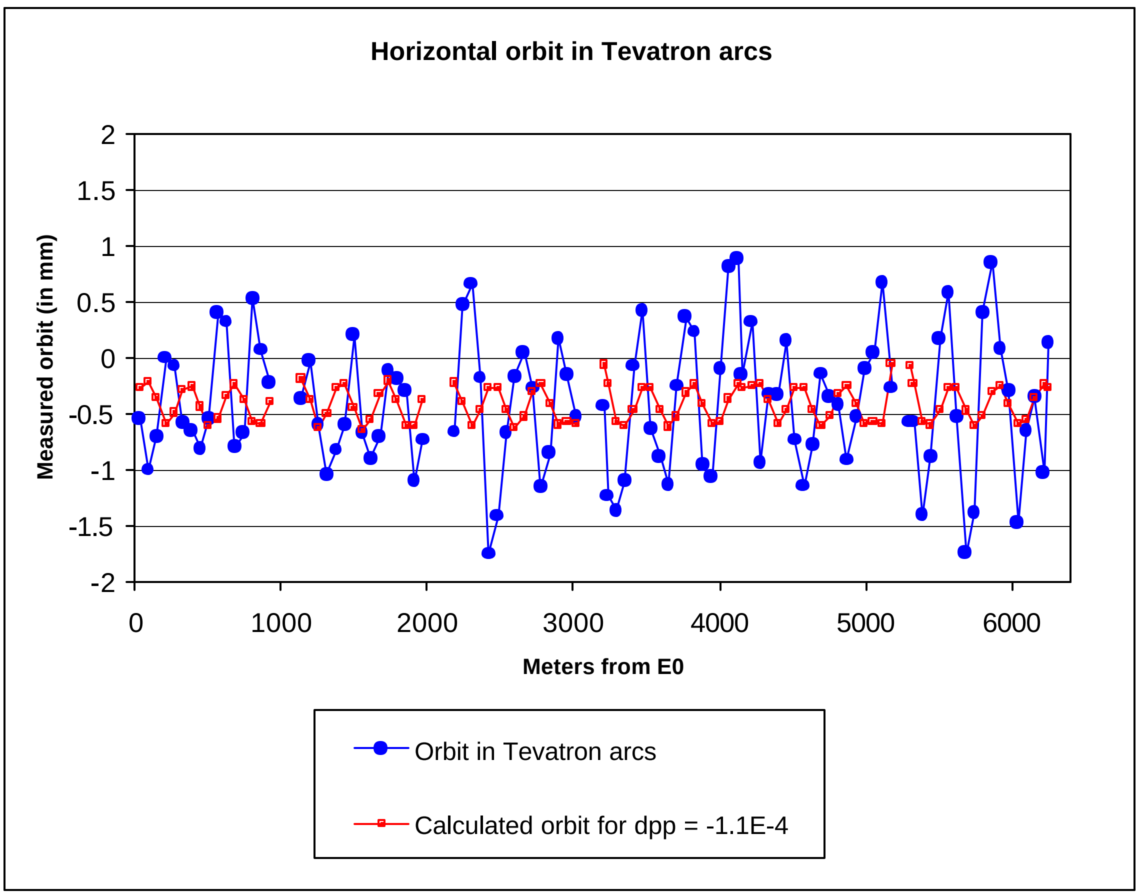
Orbits during tune drift

The horizontal orbit in the Tevatron arcs is shown in the plot below. The BPMs in the straight sections, from the 48 locations through the 11 locations, have been removed from this plot. Also shown is the calculated orbit for a beam momentum offset of $\Delta p/p = -1.1E-4$ which is equivalent to a frequency offset of $\Delta f = 16$ Hz. The measured orbit and the calculated orbit with the beam momentum offset both have an average horizontal BPM reading of -0.41 mm in the BPMs in the Tevatron arcs. As is clear from the plot the orbit distortions in the horizontal plan cannot be explained by the dispersion function and a momentum offset.

Because the magnets in the T:SF circuit are very close to the horizontal BPMs we can determine that the horizontal position in the T:SF magnets is -0.45 mm according to the BPM measurements. This data is not consistent with the measurement of the average orbit offset determined from the tune change measurements made by varying the current in T:SF. (The tune versus T:SF current measurements gave results suggesting that the average horizontal orbit offset was -0.158 mm if the vertical tune was used to determine the orbit offset and +0.106 mm if the horizontal tune was used to determine the orbit offset in the T:SF sextupoles.)

The vertical orbits in the arcs of the Tevatron are shown in the plot below. The BPMs in the straight sections, from the 49 locations to the 11 locations, have been removed from the plot. With these BPMs removed the average vertical offset is 0.03 mm which is small relative to the average horizontal orbit offset.

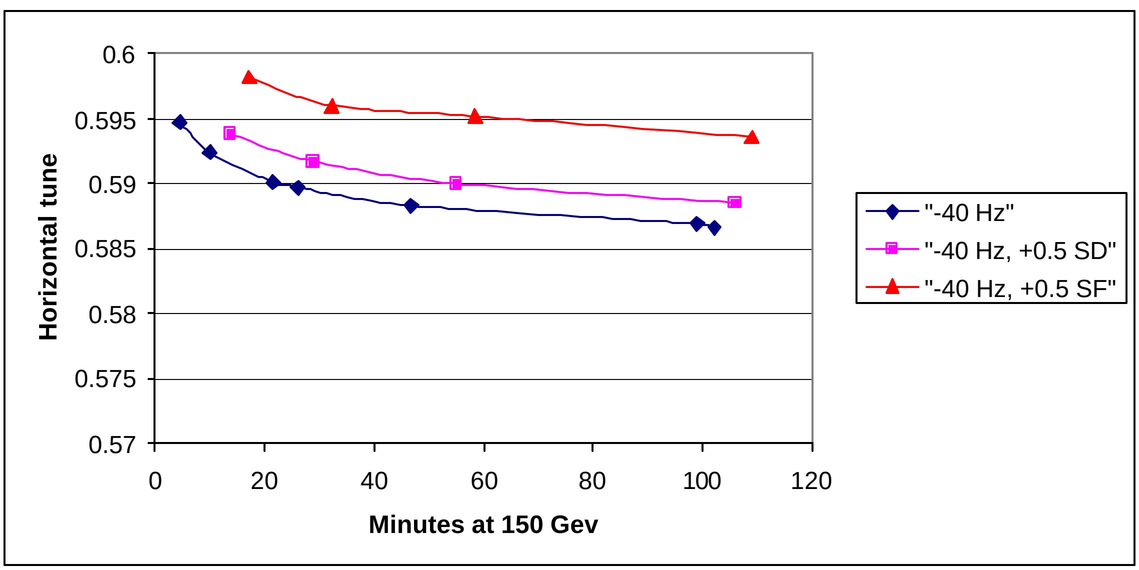

Vertical orbit in Tevatron arcs
Measured orbit (in mm)
2
1
0
-1
-2
0
1000
2000
3000
4000
5000
6000
Meters from E0

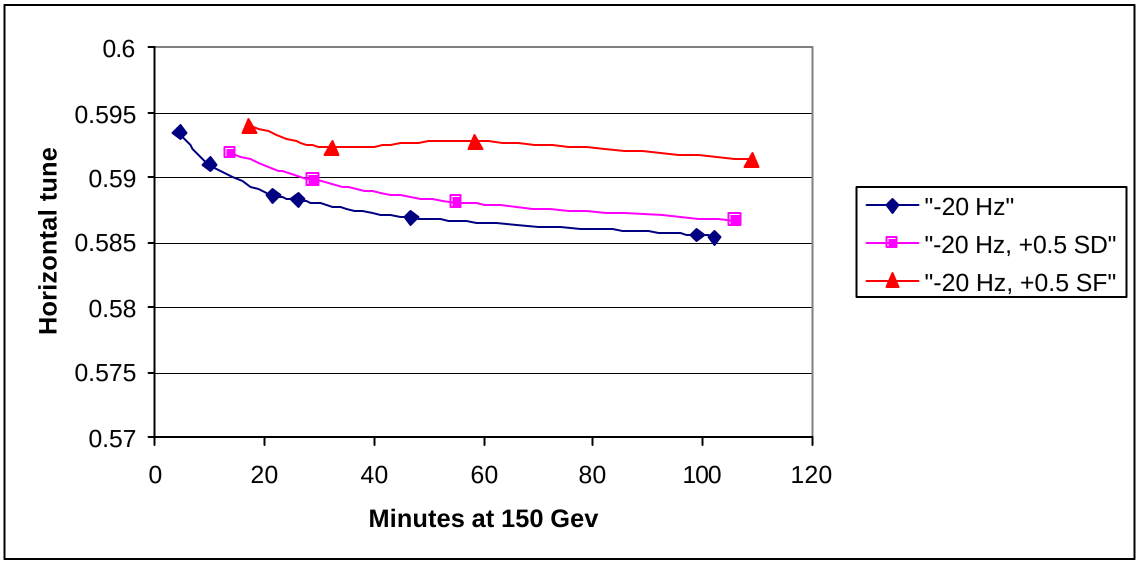

Horizontal orbit in Tevatron arcs
Measured orbit (in mm)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
1000
2000
3000
4000
5000
6000
Meters from E0
Orbit in Tevatron arcs
Calculated orbit for dpp = -1.1E-4

Appendix A: Tune measurement as function of RF frequency, time at 150 Gev, and current in T:SF and T:SD.

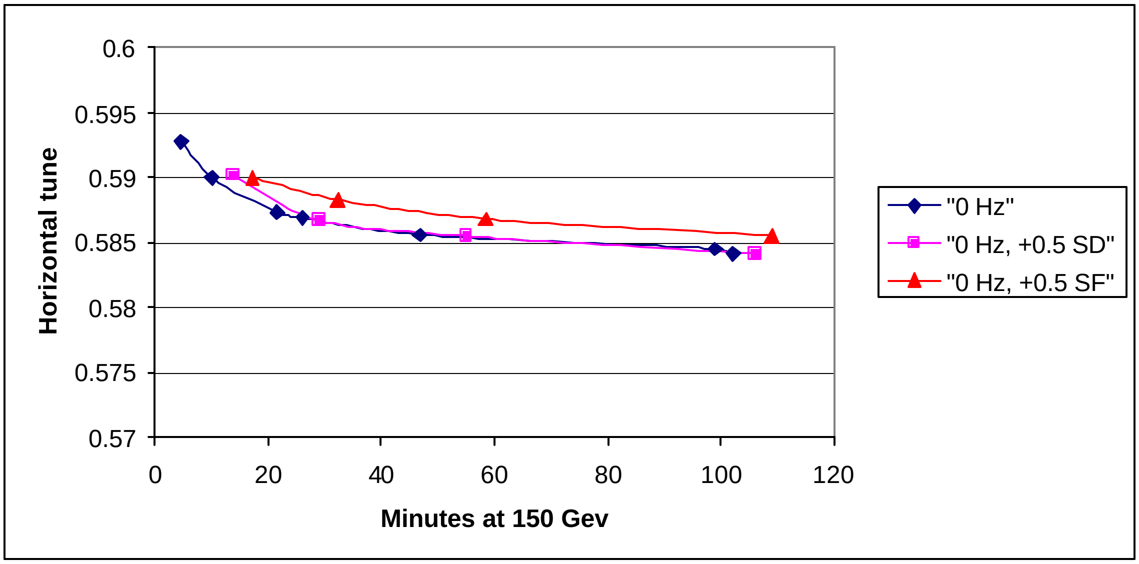
Tune measurement

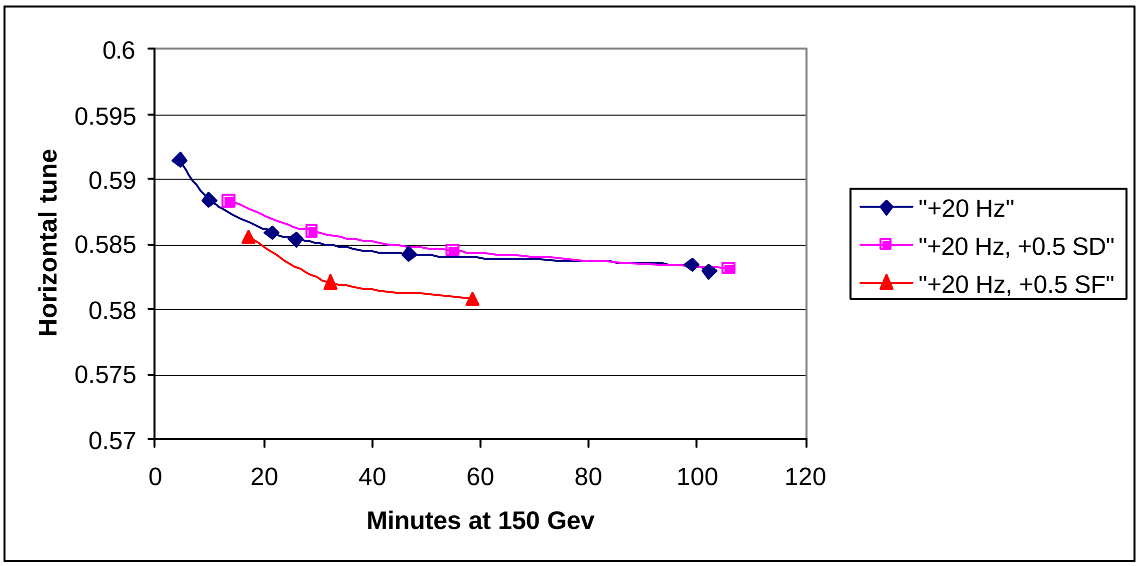
These are plots of the horizontal and vertical tunes measured as a function of time while the Tevatron was at 150 Gev after at 30 minutes store at 980 Gev. These data were collected on 5/15/02 and are recorded in the Tevatron E-log. In these plots the horizontal axis is the time at 150 Gev in minutes and the vertical axis is the measured tune. At each RF frequency (-40 Hz, -20 Hz, 0 Hz, +20 Hz, +40 Hz) the tune was measured with 1) no change in T:SF and T:SD, 2) +0.5 Amps added to T:SD, and 3) +0.5 Amps added to T:SF.

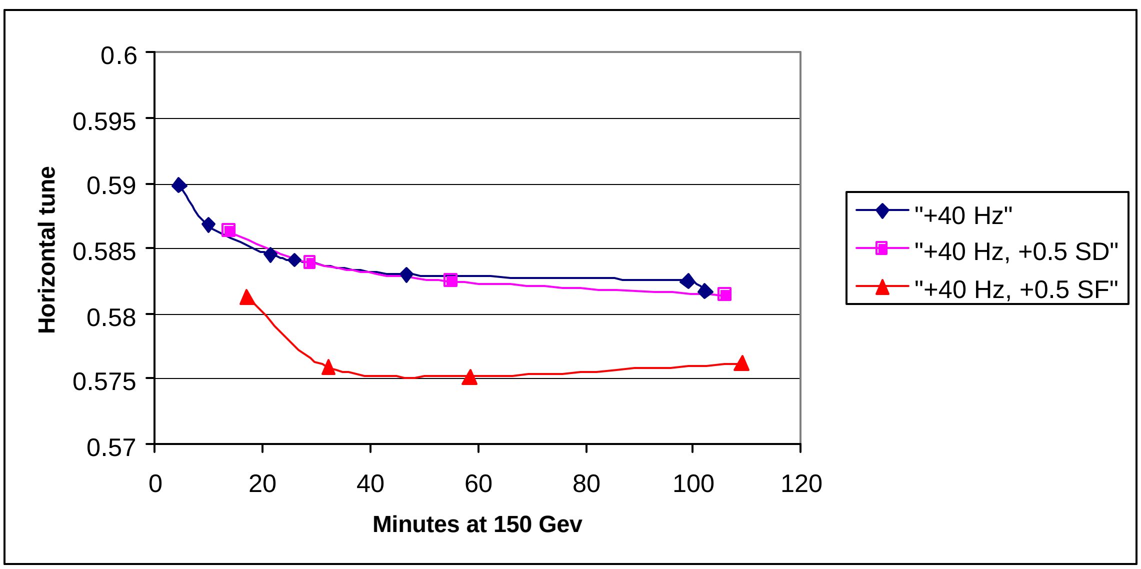


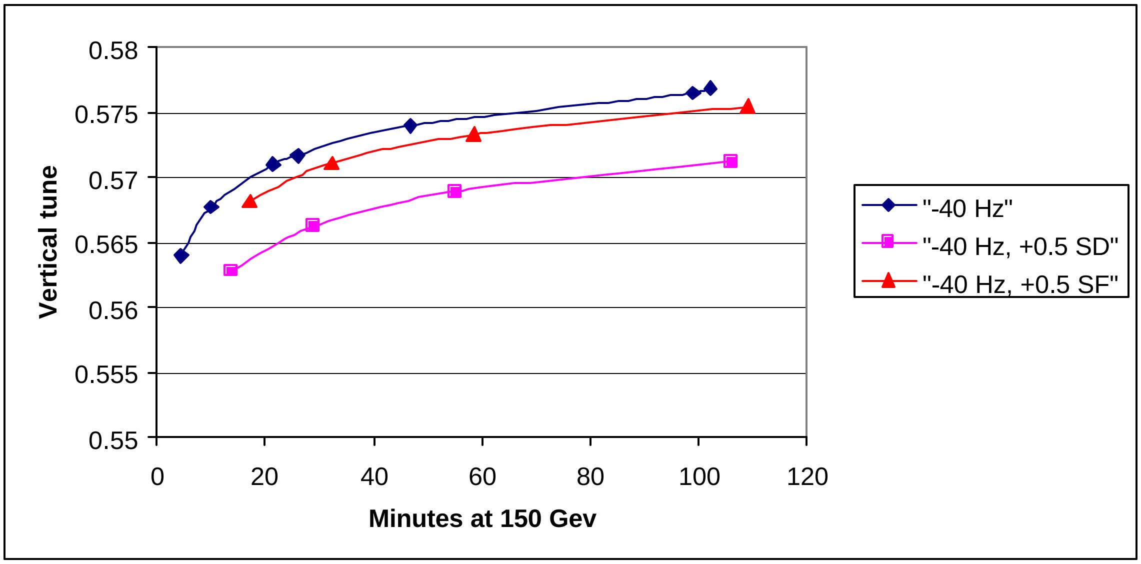


0.6
0.595
0.59
0.585
0.58
0.575
0.57
Horizontal tune
0
20
40
60
80
100
120
Minutes at 150 Gev
"-20 Hz"
"-20 Hz, +0.5 SD"
"-20 Hz, +0.5 SF"

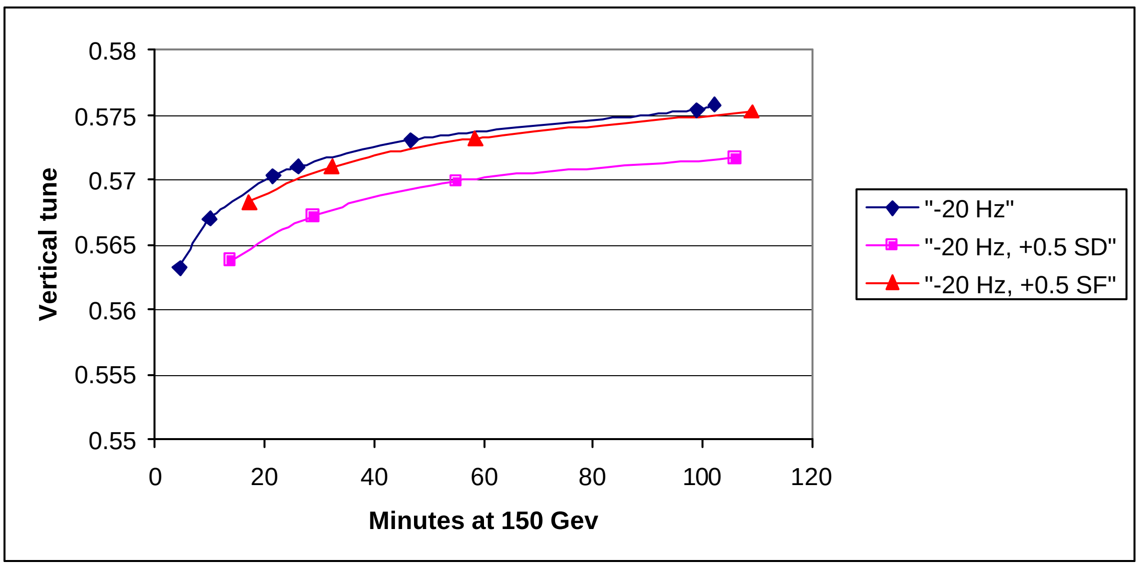

Horizontal tune
0.6
0.595
0.59
0.585
0.58
0.575
0.57
0
20
40
60
80
100
120
Minutes at 150 Gev
"0 Hz"
"0 Hz, +0.5 SD"
"0 Hz, +0.5 SF"


Horizontal tune
0.6
0.595
0.59
0.585
0.58
0.575
0.57
0
20
40
60
80
100
120
Minutes at 150 Gev
"+20 Hz"
"+20 Hz, +0.5 SD"
"+20 Hz, +0.5 SF"

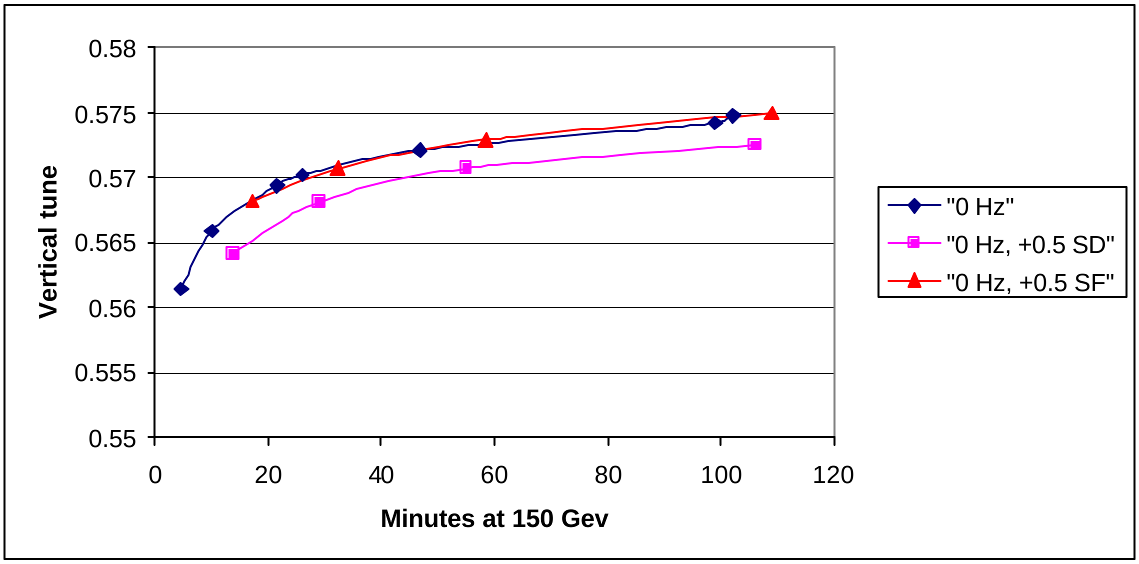

Horizontal tune
0.6
0.595
0.59
0.585
0.58
0.575
0.57
0
20
40
60
80
100
120
Minutes at 150 Gev
"+40 Hz"
"+40 Hz, +0.5 SD"
"+40 Hz, +0.5 SF"

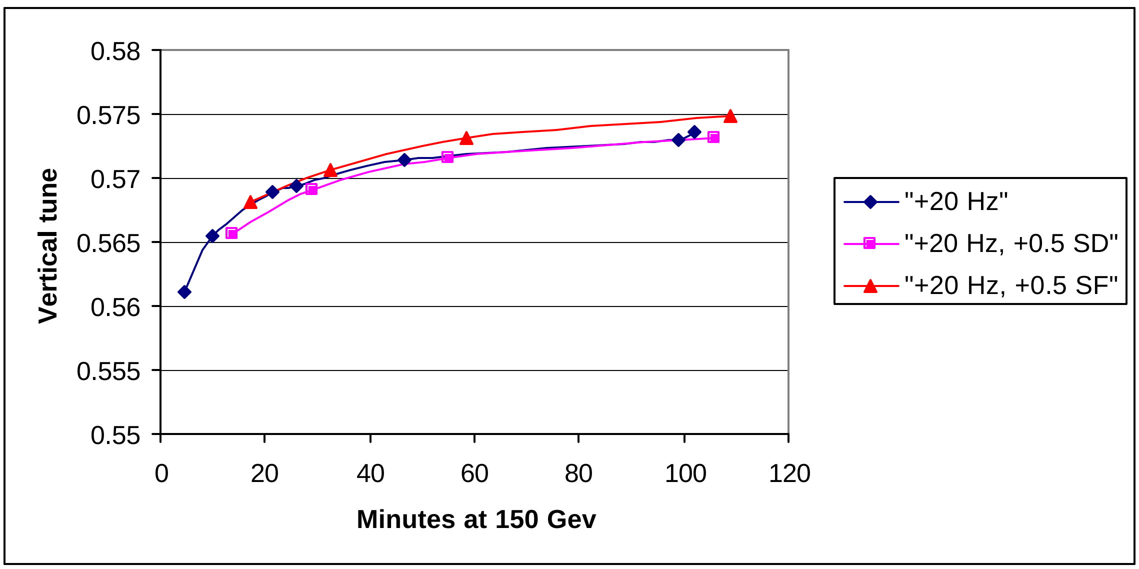


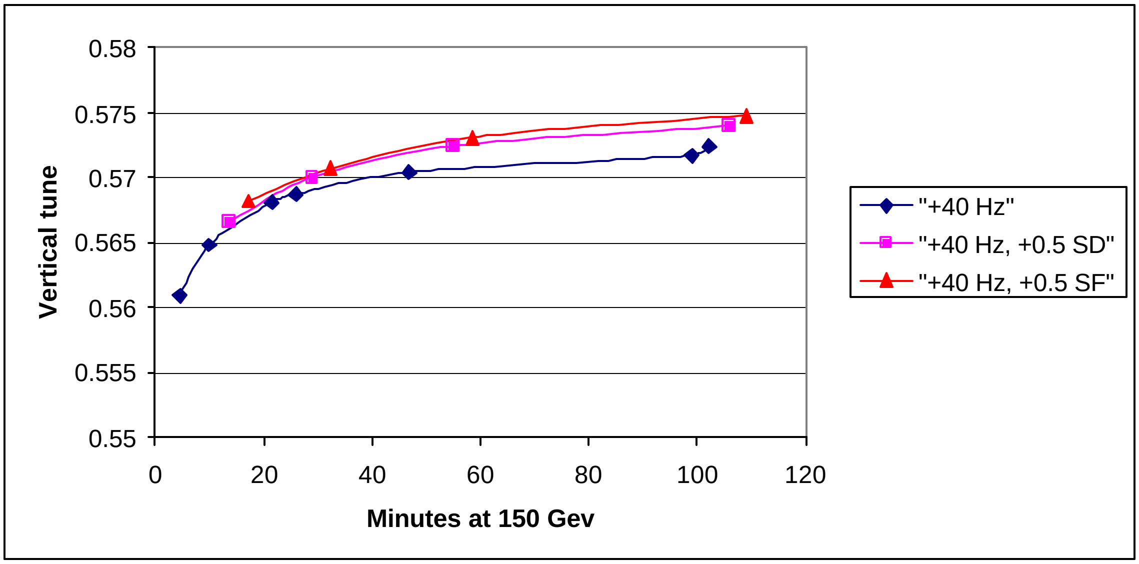
0.58
0.575
0.57
0.565
0.56
0.555
0.55
Vertical tune
0
20
40
60
80
100
120
Minutes at 150 Gev
"-40 Hz"
"-40 Hz, +0.5 SD"
"-40 Hz, +0.5 SF"



0.58
0.575
0.57
0.565
0.56
0.555
0.55
Vertical tune
0
20
40
60
80
100
120
Minutes at 150 Gev
"-20 Hz"
"-20 Hz, +0.5 SD"
"-20 Hz, +0.5 SF"


0.58
0.575
0.57
0.565
0.56
0.555
0.55
Vertical tune
0
20
40
60
80
100
120
Minutes at 150 Gev
"0 Hz"
"0 Hz, +0.5 SD"
"0 Hz, +0.5 SF"

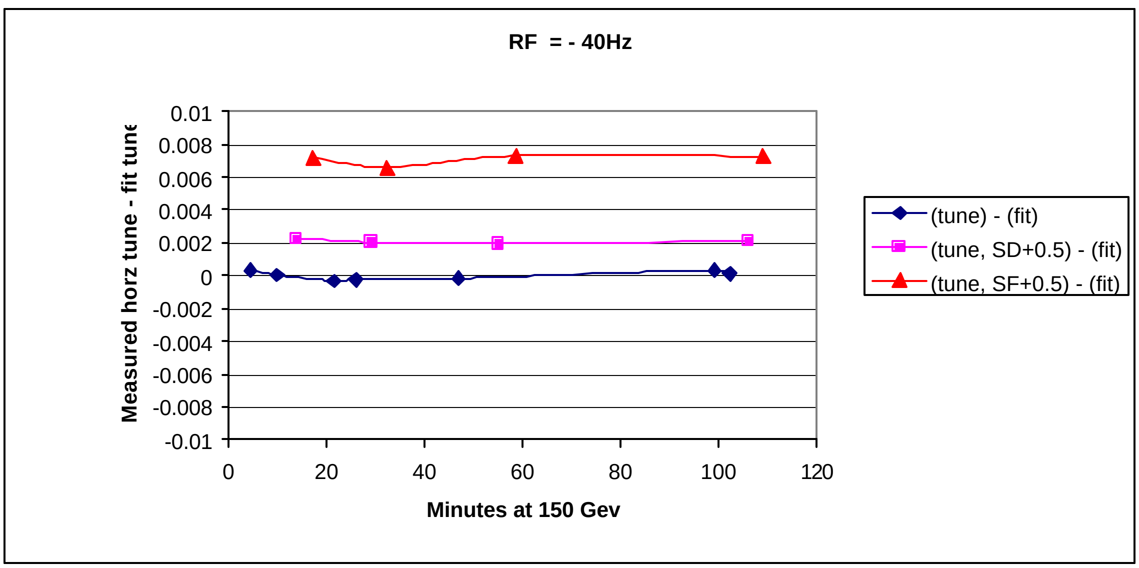

0.58
0.575
0.57
0.565
0.56
0.555
0.55
Vertical tune
0
20
40
60
80
100
120
Minutes at 150 Gev
"+20 Hz"
"+20 Hz, +0.5 SD"
"+20 Hz, +0.5 SF"

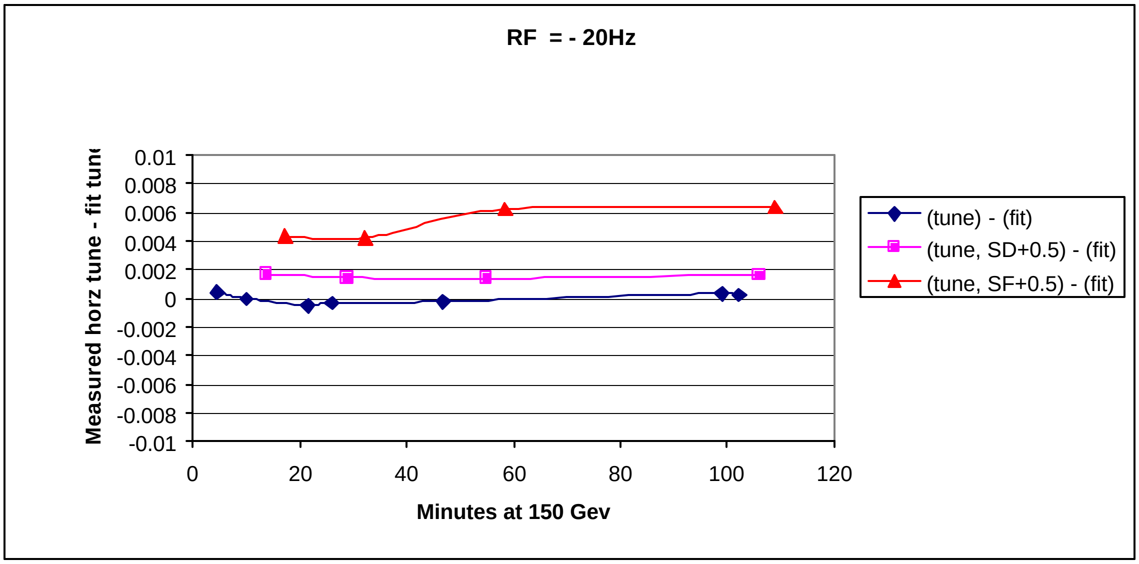


0.58
0.575
0.57
0.565
0.56
0.555
0.55
Vertical tune
0
20
40
60
80
100
120
Minutes at 150 Gev
"+40 Hz"
"+40 Hz, +0.5 SD"
"+40 Hz, +0.5 SF"

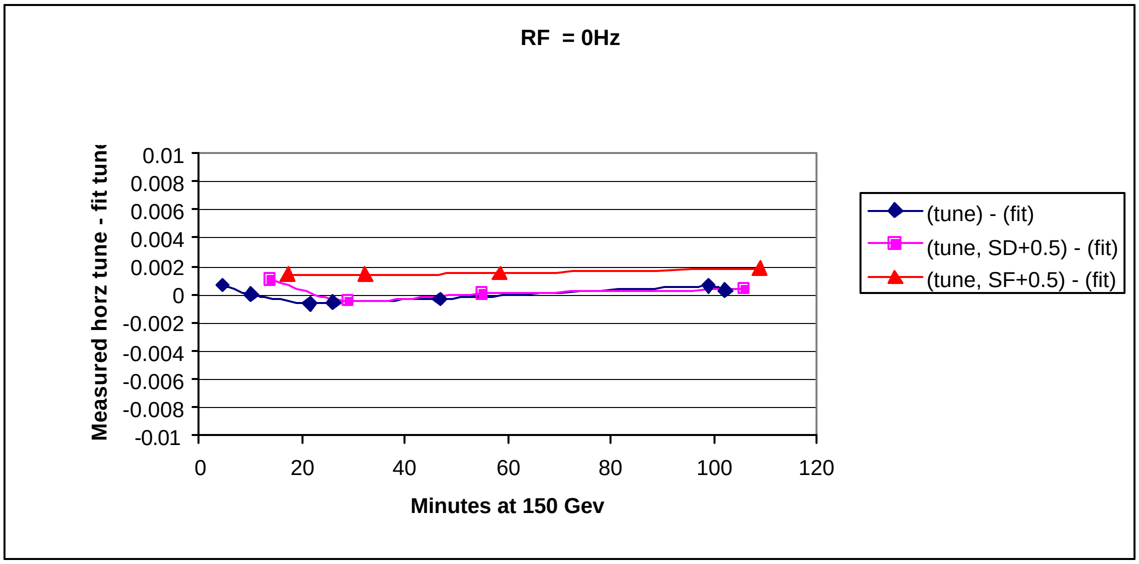
Tune difference with current in T:SF and T:SD.

These plots show the difference between the tune with and without T:SF and T:SD with an additional 0.5 Amps. In each case the data at the same RF frequency and with zero current in T:SF and T:SD were fitted to a logarithmic function $\Delta\nu(t) = a \ln(t_{150\ Gev}) + b$. This fit was then subtracted from the measured data for all three cases 1) no current in T:SF and T:SD, 2) an additional 0.5 amps in T:SD, and 3) an additional 0.5 amps in T:SF. If the average orbit in the sextupole circuits were zero then we would expect to see no change in the tune when the current in the sextupoles was changed.

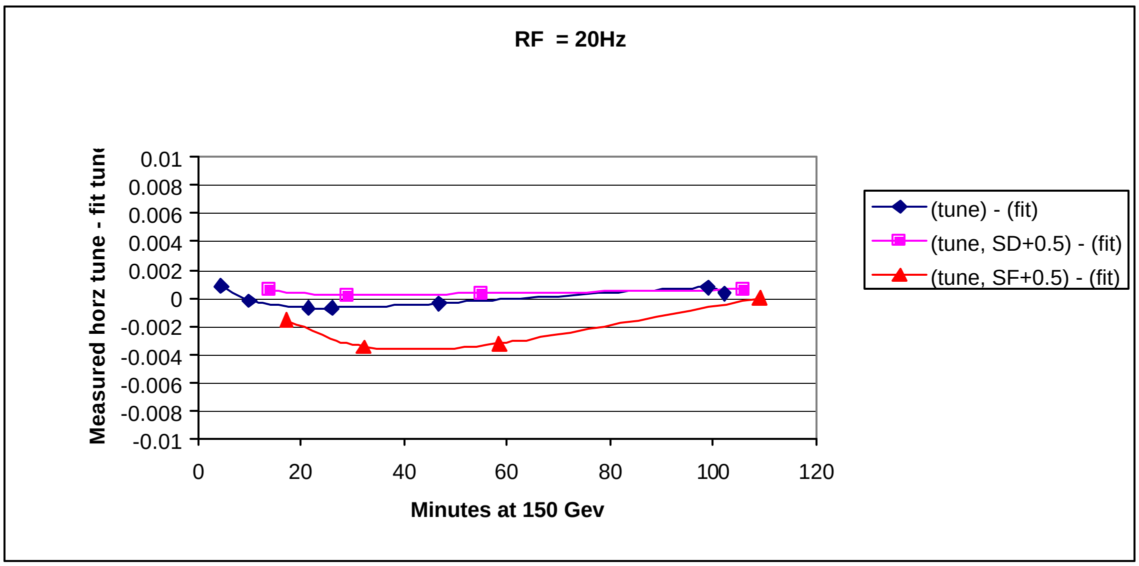




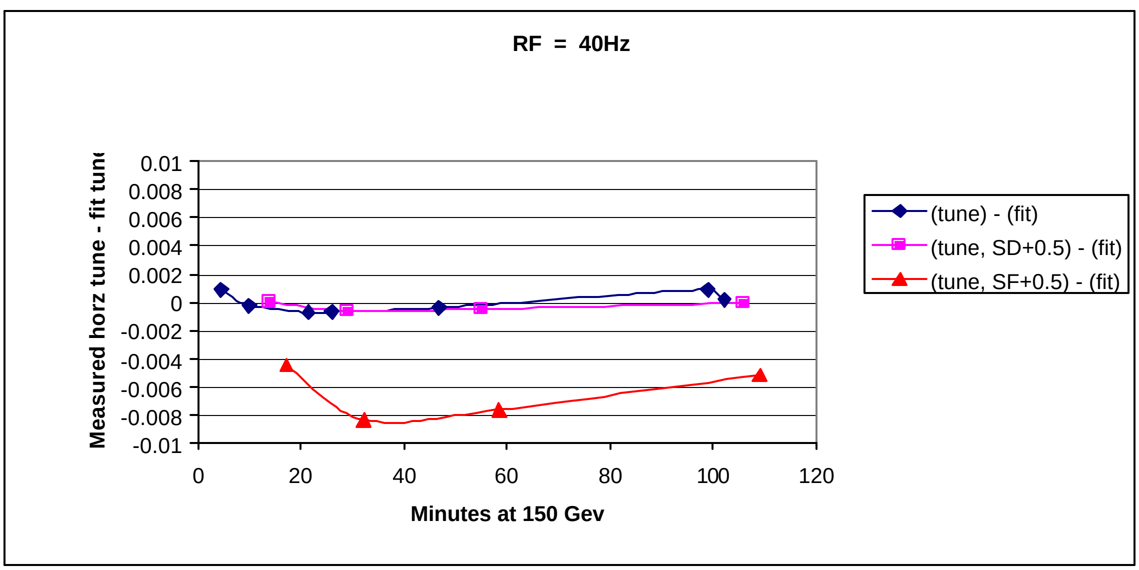
RF = - 20Hz
Measured horz tune - fit tune
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)



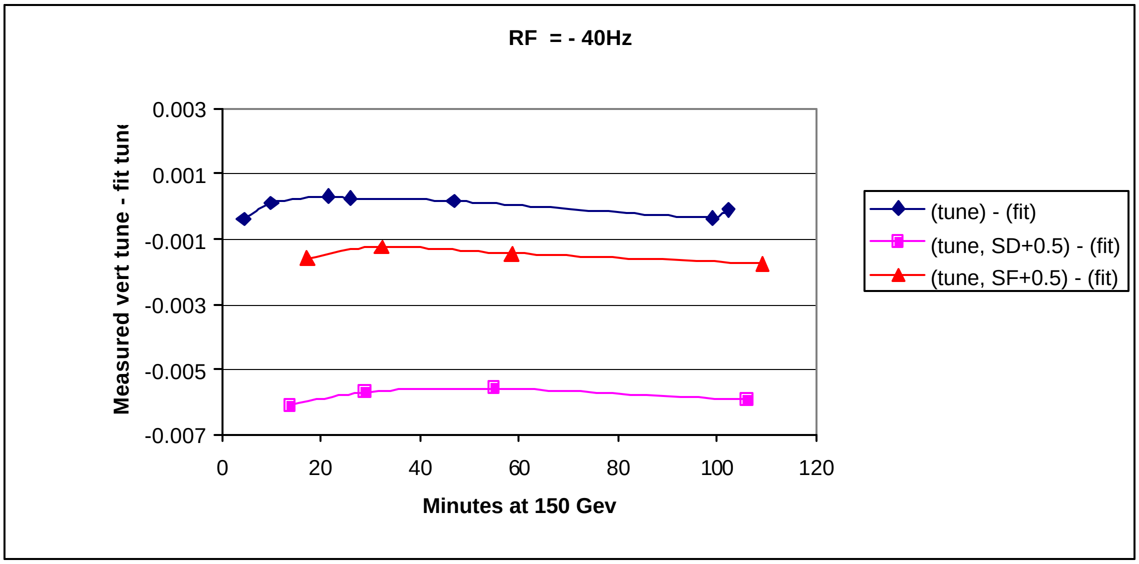
RF = 0Hz
Measured horz tune - fit tune
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)

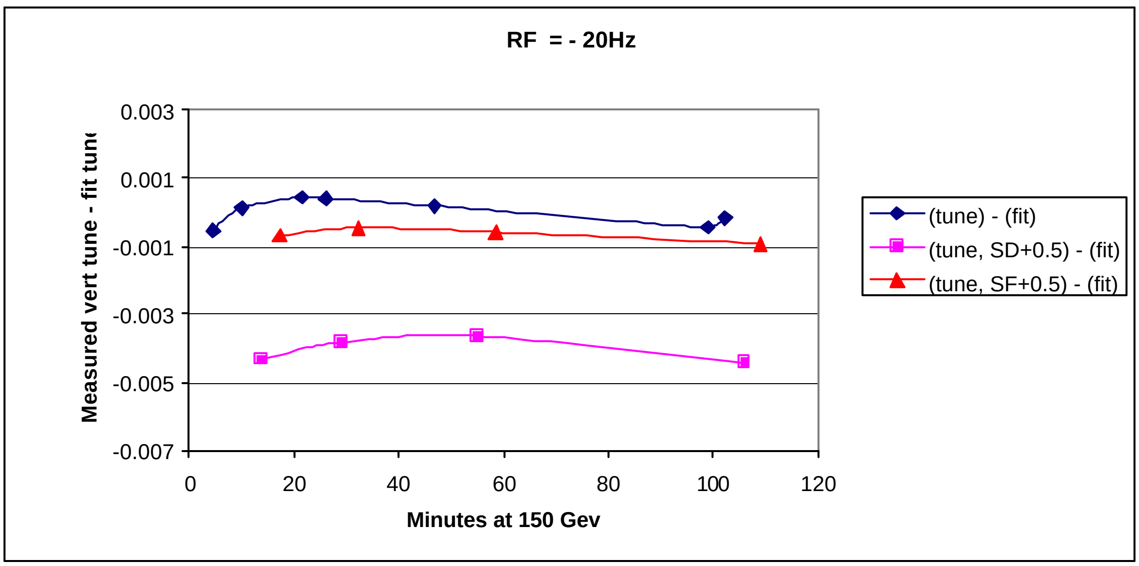


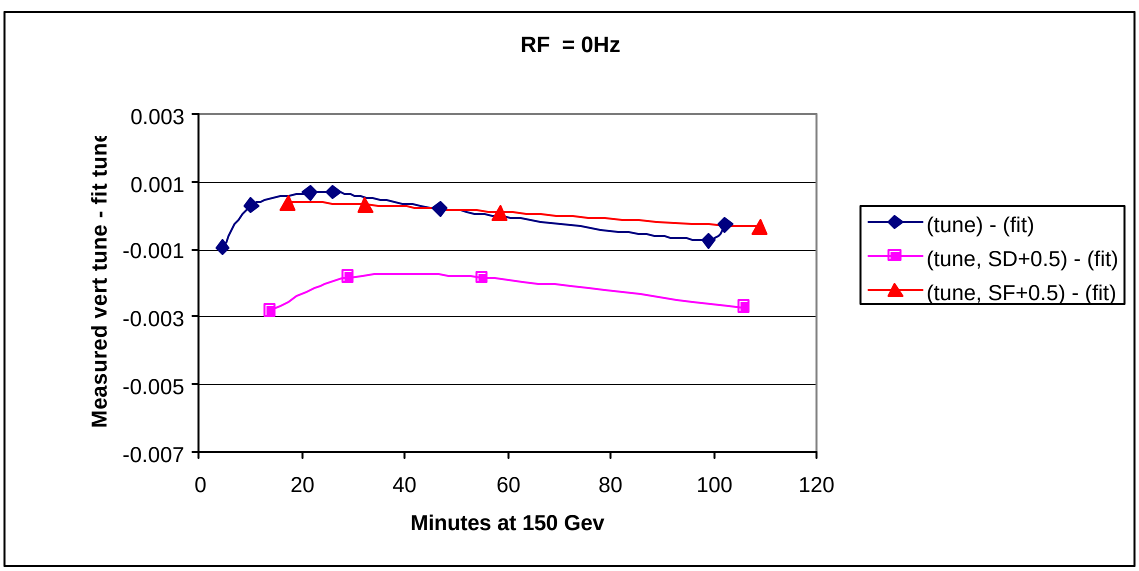
RF = 20Hz
Measured horz tune - fit tune
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)

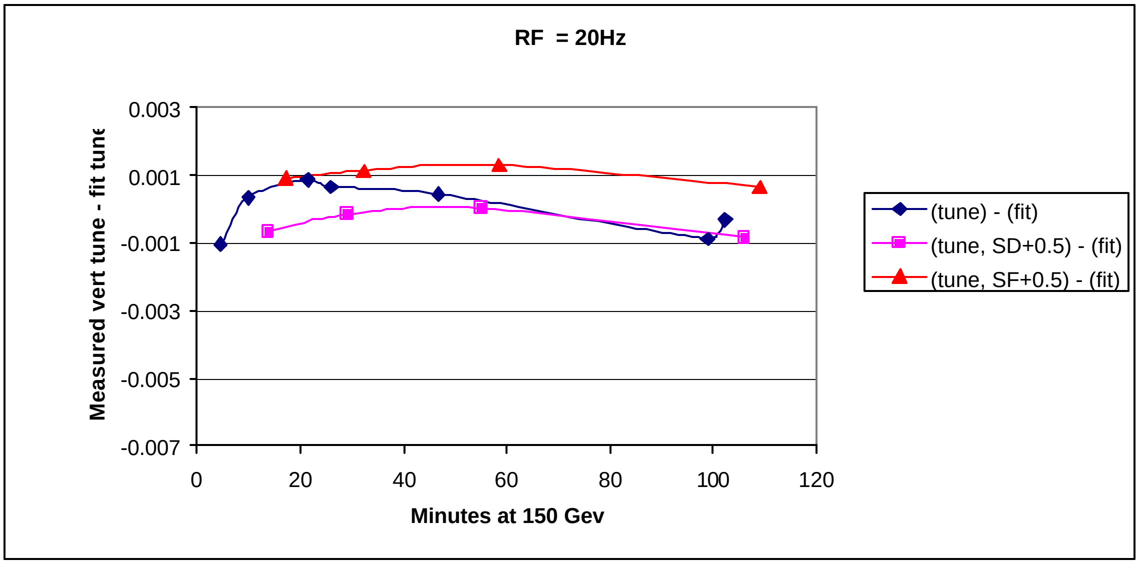


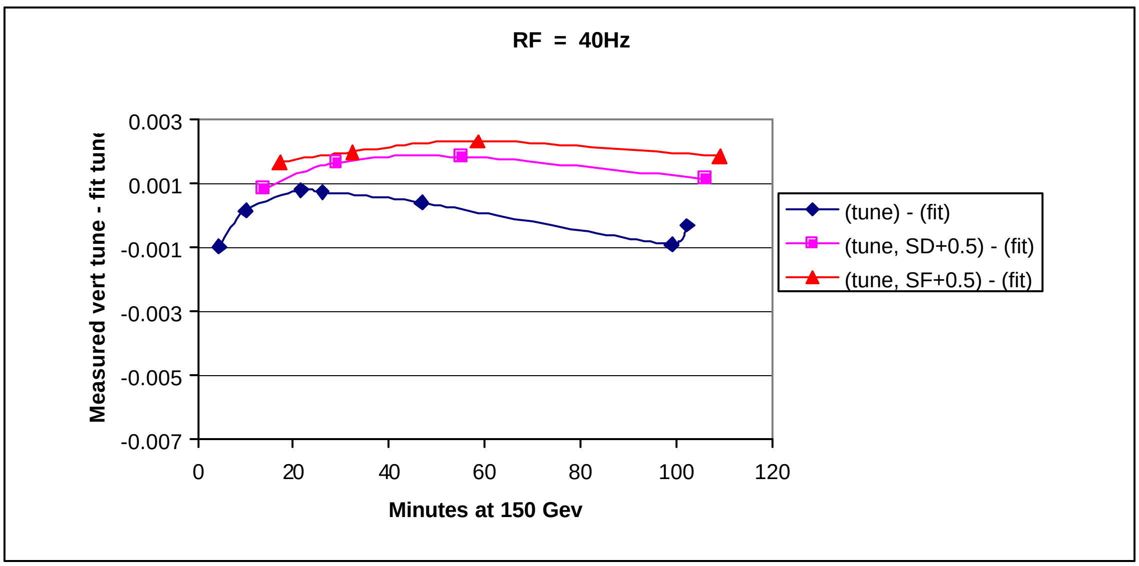
RF = 40Hz
Measured horz tune - fit tune
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)


RF = - 40Hz
Measured vert tune - fit tune
0.003
0.001
-0.001
-0.003
-0.005
-0.007
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)


RF = - 20Hz
Measured vert tune - fit tune
0.003
0.001
-0.001
-0.003
-0.005
-0.007
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)


RF = 0Hz
Measured vert tune - fit tune
0.003
0.001
-0.001
-0.003
-0.005
-0.007
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)


RF = 20Hz
Measured vert tune - fit tune
0.003
0.001
-0.001
-0.003
-0.005
-0.007
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)


RF = 40Hz
Measured vert tune - fit tune
0.003
0.001
-0.001
-0.003
-0.005
-0.007
0
20
40
60
80
100
120
Minutes at 150 Gev
(tune) - (fit)
(tune, SD+0.5) - (fit)
(tune, SF+0.5) - (fit)

Appendix B: Sextupole fields in the Tevatron

This appendix gives formulas, calculations, and data concerning the time varying sextupole fields in the Tevatron at 150 Gev. The main sources of these fields are the b2 component of the Tevatron dipoles and the chromaticity correction sextupole circuits T:SF and T:SD.

Chromaticity sextupole magnet strength

First we define the sextupole magnetic field as

$$B_x(x,y) = B_2 xy$$
$$B_y(x,y) = \tfrac{1}{2} B_2 (x^2 - y^2)$$

and want to relate this to the magnetic field in a Tevatron sextupole magnet. From "The Tevatron Energy Doubler: A superconducting Accelerator", H. Edwards, (*Ann. Rev. Nucl. Part Sci.* 1985, 35:605-60) we have that the integrated strength of a sextupole magnet is 57 kG-in at 1 inch at 50 Amps for the chromaticity sextupoles and 44 kG-in at 1 inch at 50 Amps for the sextupole magnets in the downstream packages. The chromaticity sextupoles are located in the upstream packages so their strength is

$$B_2 L/I = 8.98 \text{ T/m/A}$$

where L is the length of the sextupole and I is the current in the magnet.

$$\int B_y(at\ 1\ inch) \bullet dl = 57\, kG - in @ 50 Amp$$

$$(0.5) B_2 L (1\ inch)^2 / I = 5.7\ T - inch / 50 A$$

$$B_2L/I = 2.0 * 5.7\ T/50A/inch * (inch/0.0254meter) = 8.98T/m/A$$

If the sextupoles are treated as a thin sextupole magnet, then in the MAD convention the thin sextupole strength is given by

$$\mathrm{K_2L} = (B_2L/I)\frac{1}{|Br|}I$$

where I is the current in the sextupole, and $|Br|$ is the magnetic rigidity and we can also relate the sextupole field, K_2, to the magnetic field

$$K_2 = \frac{1}{|Br|}\frac{\partial^2 B_y}{\partial x^2}.$$

With 1 Amp in the T:SF circuit the gradient strength at 150 Gev is,

$$\mathrm{K_2L} = (B_2L/I)\frac{1}{|Br|}I = (8.98T/m/A)/(3335.64T-m/Tev * 0.15Tev)(1.0\ A)$$

$$\mathrm{K_2L} = 0.01795\ \mathrm{m^{-2}/Amp}$$

In the Tevatron there are about 88 elements in each of the T:SF and T:SD circuits. (Some individual magnets may be jumpered out of the circuit due to magnet failures.) The average beta functions, β_x, β_y, average dispersion, D_x, and average product, β_x*D_x, are listed in the table below as calculated using MAD with the design Tevatron lattice.

Circuit	Number of elements	Average β_x (m)	Average β_y (m)	Average D_x (m)	Average β_x*D_x (m^2)	Average β_y*D_x (m)
T:SF	88	93.81	30.09	3.810	358.7	115.8
T:SD	88	30.30	93.29	2.301	70.84	214.6

Tune shift and coupling from feeddown effect

Next we look at the effective quadrupole field that the sextupole field creates for an off-center closed orbit. If the closed orbit through a thin sextupole has the coordinates x_0 and y_0, then a thin sextupole with strength K_2L and a tilt angle ψ will give a normal quadrupole field with strength

$$(\mathrm{K_1L})_{\mathrm{no}} = \mathrm{K_2L}\left(\mathrm{x}_0 \cos 3\psi + \mathrm{y}_0 \sin 3\psi\right)$$

and a skew quadrupole field with strength

$$\left(\mathrm{K_1L}\right)_{\mathrm{sq}} = \mathrm{K_2L}\left(\mathrm{x}_0 \sin 3\psi - \mathrm{y}_0 \cos 3\psi\right)$$

For the Tevatron the chromaticity sextupoles have zero tilt angle, $\psi = 0$, and then

$$(\mathrm{K_1L})_{\mathrm{no}} = \mathrm{K_2L}\,\mathrm{x}_0$$
$$\left(\mathrm{K_1L}\right)_{\mathrm{sq}} = -\mathrm{K_2L}\mathrm{y}_0$$

Once we know the strengths and feeddown effects of the sextupoles we can calculate the effect they have on the tune and coupling. The change in horizontal and vertical tune is given by the simple equations

$$\Delta\nu_x = \frac{1}{4\pi}\sum \beta_{\mathrm{x,i}}\,(\mathrm{K_1L})_{\mathrm{no,i}}$$

$$\Delta\nu_y = -\frac{1}{4\pi}\sum \beta_{y,i}\,(\mathrm{K_1L})_{\mathrm{no,i}}$$

where β_x, β_y are the beta functions at the locations of the feeddown sextupoles, and $(\mathrm{K_1L})_{\mathrm{no,i}}$ is the normal component of the quadrupole field created by the off-center closed orbit and the sextupole field.

To quantify the effect that the feeddowns have on the coupling we calculate a sine and cosine term of the coupling as

$$\Delta C_{sq} = \frac{1}{2p} \sum (K_1 L)_{sq,i} \sqrt{b_{x,i} b_{y,i}} \cos(f_y - f_x)$$

$$\Delta S_{sq} = \frac{1}{2p} \sum (K_1 L)_{sq,i} \sqrt{b_{x,i} b_{y,i}} \sin(f_y - f_x)$$

where $(K_1 L)_{sq,i}$ is the skew component of the quadrupole field and $(f_y - f_x)$ is the difference between the vertical and horizontal betatron phase advances.

To better understand the coupling constants we express the minimum tune split in terms of $\Delta C_{sq}, \Delta S_{sq}$. If the Tevatron is tuned up such that there is no coupling on the proton helix then in principle the horizontal and vertical tunes could be set equal to one another. However, if we add a small amount of coupling then the minimum tune split is

$$\Delta n_{min} = \sqrt{\Delta C_{sq}^2 + \Delta S_{sq}^2}$$

Chromaticity correction from T:SF and T:SD

The chromaticity is defined as

$$\Delta n = x \left(\frac{\Delta p}{p} \right)$$

from which we can calculate the expected change in chromaticity from the T:SF and T:SD circuits:

$$\Delta n_x = \frac{1}{4p} \sum b_{x,i} \, (K_1L)_{no,i}$$

$$\Delta n_x = \frac{1}{4p} \sum b_{x,i} (K_2Lx)_i$$

$$\Delta n_x = \frac{1}{4p} \sum b_{x,i} (K_2LD_x)_i \left(\frac{\Delta p}{p} \right)$$

$$x_x = \frac{1}{4p} \sum b_{x,i} (K_2LD_x)_i$$

$$x_y = -\frac{1}{4p} \sum b_{y,i} (K_2LD_x)_i$$

With the chromaticity sextupole strength of $K_2L = 0.01795\ m^{-2}$ and using the average beta functions and dispersion functions at the locations of the chromaticity sextupole magnets we can calculate the change in chromaticity from the T:SF and T:SD circuits at 150 Gev in the Tevatron.

There are 88 SF magnets at an average horizontal beta of about 93.8 meters, an average vertical beta of 30.1 meters, and an average dispersion of 3.81 meters. There are 88 SD magnets at an average horizontal beta of about 30.3 meters, an average vertical beta of 93.3 meters, and an average dispersion of 2.3 meters. This gives

$$x_x = \frac{88}{4p} * 93.8\ m * 0.01795\ m^{-2} * 3.81\ m = 44.9\ \text{units/(Amp in T:SF)}$$

$$x_y = -\frac{88}{4p} * 30.1\ m * 0.01795\ m^{-2} * 3.81\ m = -14.4\ \text{units/(Amp in T:SF)}$$

$$x_x = \frac{88}{4p} * 30.3\ m * 0.01795\ m^{-2} * 2.3\ m = 8.76\ \text{units/(Amp in T:SD)}$$

$$x_y = -\frac{88}{4p} * 93.3\ m * 0.01795\ m^{-2} * 2.3\ m = -27.0\ \text{units/(Amp in T:SD)}$$

A MAD calculation with K_2L increased by $0.01795\ m^{-2}$ (corresponding to 1.0 Amps at 150 Gev) gives the following changes to the chromaticity

$$\begin{pmatrix} dCh \\ dCv \end{pmatrix} = \begin{pmatrix} 45 & 8.9 \\ -14.36 & -26.96 \end{pmatrix} \begin{pmatrix} T:SF\,(amps) \\ T:SD(amps) \end{pmatrix}$$

which is consistent with the above calculations.

As a comparison between actual Tevatron and the theoretical calculations based on the design lattice we give the calculational constants from C49 that are used to relate the change in current to the change in chromaticity

$$\begin{pmatrix} dCh \\ dCv \end{pmatrix} = \begin{pmatrix} 43.14 & 6.99 \\ -12.51 & -24.25 \end{pmatrix} \begin{pmatrix} T:SF\,(amps) \\ T:SD(amps) \end{pmatrix}$$

which agrees well with the theoretical calculations.

Chromaticity from b2 component of dipoles

There are 772 dipoles in the Tevatron. The average beta functions, β_x, β_y, average dispersion, D_x, and average product, β_x*D_x, are listed in the table below as calculated using MAD with the design Tevatron lattice.

Circuit	Number of elements	Average β_x (m)	Average β_y (m)	Average D_x (m)	Average β_x*D_x (m^2)	Average β_y*D_x (m^2)
Dipoles	772	57.41	57.94	2.827	170.2	155.6

Thus if K2 = 1E-4 m^{-3} then

$$x_x = \frac{1}{4p} \sum b_{x,i} (K_2 L D_x)_i$$

$$x_x = \frac{772}{4p} * 57.41\,m * 1E - 4\,m^{-3} * 6.1214\,m * 2.827\,m = 6.10\ \text{units/(1E - 4 m}^3\text{of K2)}$$

$$x_y = -\frac{1}{4p} \sum b_{y,i} (K_2 L D_x)_i$$

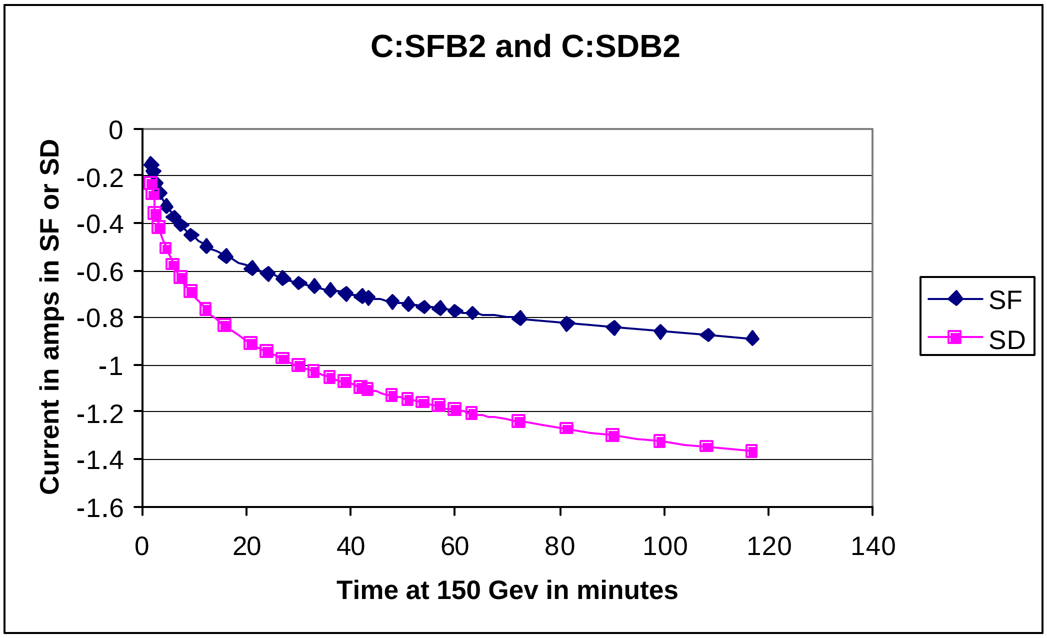
$$x_y = -\frac{772}{4p} * 57.94\,m * 1E - 4\,m^{-3} * 6.1214\,m * 2.827\,m = -6.16\ \text{units/(1E - 4 m}^3\text{of K2)}$$

Changing K2 in the BEND elements in the MAD file by 1E-4 changes the horizontal chromaticity from 10 units to 16.44 units and the vertical chromaticity from 10 units to 4.14 units which confirms the above calculations

Estimate of b2 component of dipoles

The average, over all the dipoles in the Tevatron, of the time varying sextupole field, ΔK_2, can be estimated from the time variation in the T:SF and T:SD sextupoles and the fact that these circuits are tuned up to keep the chromaticity constant.

During the studies on tune drift measurements taken on 5/15/02 the T:SF and T:SD b2 windup curves played out the following current as a function of time



and fits to a logarithmic function gives

$$\Delta I_{SF}\ (\text{amps}) = -0.174 * \ln(t) - 0.0627$$
$$\Delta I_{SD}\ (\text{amps}) = -0.267 * \ln(t) - 0.0974$$

or, with the transfer constant K_2L = 0.01795 m^{-2}/Amp, in terms of the sextupole gradient strength

$$\Delta(K_2L)_{SF}\ (m^{-2}) = -3.12\text{x}10^{-3} * \ln(t) - 1.13\text{x}10^{-3}$$
$$\Delta(K_2L)_{SD}\ (m^{-2}) = -4.79\text{x}10^{-3} * \ln(t) - 1.21\text{x}10^{-3}$$

Using this data we can calculate the change in chromaticity due to the change in the T:SF and T:SD circuits and then estimate the change in the K_2 component of the Tevatron dipoles. Recent measurements showed that the b2 windup curves were tuned up well at 150 Gev and the measured chromaticity changed by only one or two units (out of about 40 to 50 units of chromaticity change over two hours if the b2 were not compensated.) So we take $\Delta\xi_x$ = 0 and use

$$\Delta x_x = \frac{1}{4p}\sum_{\text{Dipole}}\left(b_x \Delta \mathrm{K}_2 \mathrm{LD}_x\right) + \frac{1}{4p}\sum_{\text{T:SF}}\left(b_x \Delta \mathrm{K}_2 \mathrm{LD}_x\right) + \frac{1}{4p}\sum_{\text{T:SD}}\left(b_x \Delta \mathrm{K}_2 \mathrm{LD}_x\right) = 0$$

to calculate $(\Delta K_2 L)_{Dipole}$. From a MAD calculation of the design lattice at 150 Gev

$$\sum_{\text{Dipole}}\left(b_x \mathrm{D}_x\right) = 1.32\mathrm{x}10^5 \ \mathrm{m}^2, \quad \sum_{\text{T:SF}}\left(b_x \mathrm{D}_x\right) = 3.16\mathrm{x}10^4 \ \mathrm{m}^2, \quad \sum_{\text{T:SD}}\left(b_x \mathrm{D}_x\right) = 6.23\mathrm{x}10^3 \ \mathrm{m}^2$$

which gives

$$\left(\Delta \mathrm{K}_2 \mathrm{L}\right)_{\text{Dipole}} \sum_{\text{Dipole}}\left(b_x \mathrm{D}_x\right) = -\Delta(\mathrm{K}_2\mathrm{L})_{\text{SF}} \sum_{\text{T:SF}}\left(b_x \mathrm{D}_x\right) - \Delta(\mathrm{K}_2\mathrm{L})_{\text{SD}} \sum_{\text{T:SD}}\left(b_x \mathrm{D}_x\right)$$

$$\left(\Delta \mathrm{K}_2 \mathrm{L}\right)_{\text{Dipole}} * 1.32\mathrm{x}10^5 = \begin{pmatrix} (3.12\mathrm{x}10^{-3} * \ln(t) + 1.13\mathrm{x}10^{-3}) * 3.16\mathrm{x}10^4 + \\ (4.79\mathrm{x}10^{-3} * \ln(t) + 1.21\mathrm{x}10^{-3}) * 6.23\mathrm{x}10^3 \end{pmatrix}$$

$$\left(\Delta \mathrm{K}_2 \mathrm{L}\right)_{\text{Dipole}}(\mathrm{m}^{-2}) = 9.74\mathrm{x}10^{-4} * \ln(t) + 3.28\mathrm{x}10^{-4}$$

$$\left(\Delta \mathrm{K}_2\right)_{\text{Dipole}}(\mathrm{m}^{-3}) = 1.59\mathrm{x}10^{-4} * \ln(t) + 5.35\mathrm{x}10^{-4}$$

If we use the vertical chromaticity to calculate the K_2 component of the dipole we get

$$\left(\Delta \mathrm{K}_2 \mathrm{L}\right)_{\text{Dipole}} \sum_{\text{Dipole}}\left(b_y \mathrm{D}_x\right) = -\Delta(\mathrm{K}_2\mathrm{L})_{\text{SF}} \sum_{\text{T:SF}}\left(b_y \mathrm{D}_x\right) - \Delta(\mathrm{K}_2\mathrm{L})_{\text{SD}} \sum_{\text{T:SD}}\left(b_y \mathrm{D}_x\right)$$

$$\left(\Delta \mathrm{K}_2 \mathrm{L}\right)_{\text{Dipole}} * 1.33\mathrm{x}10^5 = \begin{pmatrix} (3.12\mathrm{x}10^{-3} * \ln(t) + 1.13\mathrm{x}10^{-3}) * 1.01\mathrm{x}10^4 + \\ (4.79\mathrm{x}10^{-3} * \ln(t) + 3.28\mathrm{x}10^{-4}) * 1.92\mathrm{x}10^4 \end{pmatrix}$$

$$\left(\Delta \mathrm{K}_2 \mathrm{L}\right)_{\text{Dipole}}(\mathrm{m}^{-2}) = 9.28\mathrm{x}10^{-4} * \ln(t) + 2.60\mathrm{x}10^{-4}$$

$$\left(\Delta \mathrm{K}_2\right)_{\text{Dipole}}(\mathrm{m}^{-3}) = 1.52\mathrm{x}10^{-4} * \ln(t) + 4.26\mathrm{x}10^{-4}$$

The factors multiplying the ln(t) term when using the horizontal and vertical chromaticity agree to within 5%. The difference in the constant term is only 4% of the value of $(\Delta K_2$at time t=5 minutes and even less for later times. Therefore we quote the average value of these two determinations ΔK_2 and get

$$\left(\Delta K_2 L\right)_{Dipole}(m^{-3}) = 9.51x10^{-4} * \ln(t) + 2.94x10^{-4}$$

for the average time variation of the sextupole field in the Tevatron dipoles.

Appendix C: Momentum, circumference, and RF

For a constant bend field the change in revolution period is

$$\frac{\Delta t}{t} = \frac{\Delta L}{L} - \frac{\Delta v}{v}$$

$$\frac{\Delta L}{L} = \frac{1}{g_t^2}\left(\frac{\Delta p}{p}\right)$$

$$\frac{\Delta v}{v} = \frac{1}{g^2}\left(\frac{\Delta p}{p}\right)$$

$$h = \frac{1}{g_t^2} - \frac{1}{g^2}$$

$$\frac{\Delta t}{t} = h\left(\frac{\Delta p}{p}\right)$$

$$\frac{\Delta f}{f} = -h\left(\frac{\Delta p}{p}\right)$$

For the Tevatron at 150 Gev we have $a = \frac{1}{g_t^2}$ from a MAD calculation

$$a = \frac{1}{g_t^2} = 0.0028266 = \frac{1}{18.809^2}$$

$$h = \frac{1}{18.809^2} - \frac{1}{(150/0.938)^2} = 0.002787$$

$$\left(\frac{\Delta p}{p}\right) = -358.74\left(\frac{\Delta f}{f}\right)$$

$$\left(\frac{\Delta p}{p}\right) @ +40Hz = -358.74 * \left(\frac{40Hz}{53.103703MHz}\right) = -0.0002702$$

Appendix D: Tevatron tune quadrupoles

Quadrupole magnet strength

First we define the quadrupole magnetic field as

$$B_x(x, y) = B_1 y$$
$$B_y(x, y) = B_1 x$$

and want to relate this to the magnetic field in a Tevatron quadrupole trim magnet. From "The Tevatron Energy Doubler: A superconducting Accelerator", H. Edwards, (*Ann. Rev. Nucl. Part Sci.* 1985, 35:605-60) we have that the integrated strength of a trim quadrupole magnet is 75 kG-in at 1 inch at 50 Amps giving a strength of

$$B_1 L / I = 0.15 \text{ T/A}$$

where L is the length of the sextupole and I is the current in the magnet.

$$\int B_y(at\ 1\ inch) \bullet dl = 75\, kG - in @ 50 Amp$$

$$B_1 L(1\, inch)/I = 7.5\, T - inch / 50A$$

$$B_1 L/I = 7.5\, T / 50A = 0.15 T / A$$

If the quadrupoles are treated as a thin quadrupole magnet, then in the MAD convention the thin quadrupole strength is given by

$$\mathrm{K_1 L} = (B_1 L / I) \frac{1}{|Br|} I$$

where I is the current in the quadrupole, and $|Br|$ is the magnetic rigidity. With 1 Amp in the trim quadrupole circuit we can calculate the gradient strength at 150 Gev,

$$\mathrm{K}_1\mathrm{L} = (B_1 L / I)\frac{1}{|Br|} I = (0.15T / A)/(3335.64T - m / Tev * 0.15Tev)(1.0\ A)$$

$$\mathrm{K}_1\mathrm{L} = 0.00030\ \mathrm{m}^{-1} / \mathrm{Amp}$$

This can then be related to the change in tune that it produces

$$\Delta n_x = \frac{1}{4p} \sum b_{\mathrm{x,i}}\ (\mathrm{K}_1\mathrm{L})_{\mathrm{no,i}}$$

$$\Delta n_y = -\frac{1}{4p} \sum b_{y,i}\ (\mathrm{K}_1\mathrm{L})_{\mathrm{no,i}}$$

As an order of magnitude we use 1 Amp in the tune quad at a location where the beta function is 93 meters. Then the tune change is

$$\Delta n_x = \frac{1}{4p} \sum 93m\ \ * 0.00030\ \mathrm{m}^{-1}/\mathrm{Amp} = 0.00222/\mathrm{Amp}$$