Fermilab

Doc Number: MI-0207
Version: 0.57 DRAFT

Analytic Results on Requirements for Recycler Gradient Magnet Longitudinal Uniformity

Bruce C. Brown
Main Injector Department, Beams Division
Fermi National Accelerator Laboratory *
P.O. Box 500
Batavia, Illinois 60510

22-Jan-1998

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* Operated by the Universities Research Association under contract with the U. S. Department of Energy

Contents

Abstract

As final designs are prepared for the Fermilab Recycler Ring, specifications for longitudinal uniformity of the gradient magnets are required. We will use a simple analytic model of the design field shapes, orbit shape, and the lattice β function to explore the interaction of various specifications with the nominal design features of the lattice and magnets.

1 Introduction

The nominal design of the Recycler Ring[1] uses gradient magnets for bending and most of the focusing in the ring. They are specified as straight (rectangular) magnets with a uniform dipole, quadrupole and sextupole component along the length. Steve Holmes has explored[2][3] issues concerned with the interaction between the specifications for straight magnets and the curved particle orbit which will be experienced in the Recycler. Measurements of the longitudinal uniformity of PDD 8 GeV line dipoles have been reported[4] and indicate that the non-uniformity approaches 25% for the strength tuning procedure used for producing those dipoles. It was shown previously[5] that the bend center changes expected for a 1% non-uniformity might not be unimportant. This larger non-uniformity raises additional issues. In this document we will extend these considerations to very non-uniform longitudinal fields, taking into account the curved orbit in a straight dipole.

The design features of the gradient magnet and the orbit can be quite precisely represented by polynomial expressions. If we adopt a polynomial expansion for the longitudinal non-uniformity, an analytic solution is readily available and can be manipulated to provide insight into appropriate design specifications for the magnets. Our strategy is to examine the resulting expressions, applying numerical constraints from the current design so that both qualitative and quantitative understanding of the important limits can be obtained.

Since the effects are sufficiently important that measurements are required, we will also derive forms for suitably presenting the measured results. Moments of the z profile (weighted integrals) will take full advantage of the information measured while minimizing potential limitations of the measurement. We will define moments and the most suitable variables for presenting the moment results. Effects from the moments on machine properties will be evaluated.

2 Mathematical Description

2.1 A Polynomial Description of Fields and Focusing

We obtain an explicit polynomial description of the lattice and magnets in the following way.

- Since the magnet design is for a straight hybrid permanet gradient magnet, the normalized harmonics of the design field are uniform along the length (neglecting small end effects).

$$B_y(x) = B_0(1 + b_2\frac{x}{a} + b_3(\frac{x}{a})^2) \tag{1}$$

  where a is the reference (normalization) radius for the harmonic representation of the fields.

- The pole potential and thereby the fields may experience a longitudinal modulation due to non-uniform placement of ferrite drive material such as reported for the PDD dipoles[4]. The realistic field shape for this construction technique decomposes naturally (and to good accuracy) into terms which independently depend upon x and z. To illustrate, let us use a quadratic dependence on z.

$$B_y(x, z) = B_0(1 + b_2\frac{x}{a} + b_3(\frac{x}{a})^2)(1 + \alpha_0 + \alpha_1(\frac{2z}{L}) + \alpha_2(\frac{2z}{L})^2) \tag{2}$$

  where α_0 is selected so that B_0 has the value for the uniform-in-z design (*i.e.* the same for Equations 1 and 2).

- The β function of the ring is quite smooth and over the length of a magnet can be very adequately represented by a polynomial.

$$\beta(z) = \beta_{mid}(1 + \frac{\beta'}{\beta_{mid}}(\frac{2z}{L}) + \frac{\beta''}{2\beta_{mid}}(\frac{2z}{L})^2) \tag{3}$$

$$\beta(z) = \beta_{mid}(1 + \beta_{sl}(\frac{2z}{L}) + \beta_{curv}(\frac{2z}{L})^2) \tag{4}$$

- As was shown in MI-0195[2], the orbit is still adequately represented by a polynomial, since the deviation from a circlular orbit is small. The "natural" description for the closed orbit thru a curved dipole would describe x as $x(s) = x_{offset}$, where s is the parameter which identifies the coordinate along the (curved) orbit. For a straight dipole, if z is a rectilinear coordinate, the "maximal aperture" orbit might appear to be $x = d(0.5 - (\frac{2z}{L})^2)$. However, the natural orbit to consider for a gradient magnet is one offset with respect to the center of the rectangular magnet geometry such that the bend will be independent of the quadrupole term. This is the orbit[1] $x = d(\frac{1}{3} - (\frac{2z}{L})^2)$. We will consider the more general orbit

$$x = x_{offset} + x_{slope} z + d(\frac{1}{3} - (\frac{2z}{L})^2). \tag{5}$$

We expect to make $x_{slope} = 0$ by suitable demands on the bend achieved and the global orbit distortions considered. By assumption for these magnets, the sagitta is unaffected by the deviations from a non-uniform field, and is given, in the small angle approximation, by

$$d = R(1 - \cos(\frac{\theta}{2})) = \frac{R\theta^2}{8} = \frac{L\theta}{8}. \tag{6}$$

2.2 Properties to Examine

With this description of the hardware, we wish to explore a number of properties of the lattice and orbits which will be modified by symmetric and asymmetric non-uniformities of the magnets. We will calculate the following properties.

- The bend angle, θ, required in the regular cells of the Main Injector by dipoles or in the Recycler gradient magnets is $2\pi/(301\ 1/3)$ (the angle required for the dispersion suppressor cells is 2/3 of that angle). For a magnetic rigidity of $B\rho$ we have

  $$(B\rho)\ \theta = \int_{-L/2}^{L/2} B(x,z)dz \tag{7}$$

  where L is the magnet length and we have neglected the extra path length associated with the curvature. Note that z is referred to the mechanical center of the magnet. The design value for $(B\rho)$ for the Recycler is 29.6501 T-m. We will adjust parameters for each non-uniformity we consider to preserve the design value for the bend. We

[1] Observe that for a straight uniform dipole, $\int Bdz = B_0 L$. For a gradient magnet we have $\int Bdz = \int_{-L/2}^{L/2} B_0(1 + b_2 x)dz$. We demand that this also integrate to $B_0 L$ over the orbit. A particle orbit with an offset of 1/3 of the sagitta satisfies this requirement.

will define B_0 as in Equation 2 such that it has the value it would assume in a uniform curved dipole. It assumes the same value, to first order[2], in a straight dipole: $B_0 L = (B\rho)\theta$.

- As shown in MI-0162[5], the parameter which establishes orbit distortions due to bend center displacement is

$$(B\rho)\ \delta(\theta L) = \int_{-L/2}^{L/2} zB(x,z)dz \tag{8}$$

  We will calculate $\delta(\theta L)$ to understand magnet placement and/or orbit distortion implications for each non-uniformity considered.

- The tune of the machine will be modified if there are changes in the phase advance due to modification of the gradient distribution provided by each gradient magnet. We will calculate the tune change across a modified (longitudinally non-uniform) gradient magnet, assuming that the β function is unmodified. We begin with Equation 3.151 from Page 95 of Edwards and Syphers[6]

$$\delta\nu = \frac{1}{4\pi}\oint \frac{\beta(s)B'(s)}{(B\rho)}ds \tag{9}$$

  where $\beta(s)$ is the design (unmodified) focusing function, and B' is the gradient error. We will calculate the tune change for various magnet errors by examining this integral. We will then separately consider effect due to systematic and random combinations of such errors. Beginning with Equation 2, for a non-uniform gradient magnet we find

$$B'(x,z) = B_0(\frac{b_2}{a} + \frac{b_3}{a}(\frac{x}{a}))(1 + \alpha_0 + \alpha_1(\frac{2z}{L}) + \alpha_2(\frac{2z}{L})^2) \tag{10}$$

  Equation 9 has been derived for error terms, so we must be judicious in our interpretations. However, since $\delta\nu$ is linear in B', we can calculate the integrals and then consider separately the terms of interest as perturbations on the net result.

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[2] For completeness, we note that the path length difference between the straight line used for measurement and the circular orbit is $\delta s = R\theta - R(2\sin\frac{\theta}{2}) \approx L\theta^2/24 = 7.333 \times 10^{-5}m$ or 16.7 ppm for the regular cell gradient magnets in the Recycler

2.3 Numerical Values of Design Properties

To arrive at realistic values for the contributions of various effects, we will examine various results in light of the design values of the magnet and accelerator properties. For the sake of using widely available documented values, we would employ the design in the "The Fermilab Recycler Ring Technical Design Report" [1], but a $\times 2$ error exists in the specified sextupole field. We choose to instead specify an assumed design in Table 1.

Magnet Name	Length m	B_0 $Tesla$	B_2 T/m	b_2 @$1''$	B_3 T/m^2	b_3 @$1''$
RGF	4.4958	0.1375	0.3414	0.0631	0.1713	8.0353e-04
RGD	4.4958	0.1375	0-.3296	-0.0609	-0.2879	-1.3508e-03
RGS	2.9972	0.1375	0.7457	0.1377	0.0000	0.0000

Table 1: A Current Set of Design Properties of Gradient Magnets for the Recycler Ring. These are for Lattice RRV11. Normalized harmonics are quoted at a reference radius of $1''$.

3 Calculations

3.1 Offset in Longitudinally Uniform Gradient with Sextupole

We begin by re-establishing some results previously obtained by Holmes. In a gradient magnet, we will achieve the required bend by selecting a suitable offset. As discussed above, an offset of $d/3$ provides a position which makes the bend independent of the gradient. With the sextupole which has been designed into the recycler gradient magnets, this result is modified slightly. If we describe the orbit as in Equation 5, and we describe the design field by Equation 1, we can set $\int_{-L/2}^{L/2} B(z)dz = B_0 L$ and solve, as above, for a value of x_{offset} which imposes the same bend as for a uniform dipole.

$$x_{offset} = \frac{-ab_2 \pm \sqrt{(ab_2)^2 - (16/45)b_3^2 d^2}}{2b_3} \approx -\frac{4}{45}\frac{b_3 d^2}{ab_2} \tag{11}$$

which yields values of -6.1191×10^{-6} m (-10.658410^{-6} m) for RGF (RGD) magnets respectively (taking the small offset solution).

3.2 Parameters for a Longitudinal Non-Uniformity

If we consider Equation 2 to describe the magnetic field, we will wish to constrain the parameters to preserve, as far as possible, a relatively simple description of the error terms. We included the term α_0 so that the correct bend could be obtained without modifying B_0. For a non-uniform dipole, we will solve the equation

$$B_0 L = \int_{-L/2}^{L/2} dz\ B_0(1 + \alpha_0 + \alpha_1(\frac{2z}{L}) + \alpha_2(\frac{2z}{L})^2). \tag{12}$$

Since we have α_0,α_1,and α_2 which are dimensionless, we find, upon solving this equation, $\alpha_0 = -\alpha_2/3$. We interpret this to say that if the quadratic term decreases the field at the ends by 15%, the central field must be enhanced by 5% to maintain the same integral and the apparent non-uniformity which is observed for this would be 20%.

3.3 Bend Effects of Longitudinal Non-Uniformity

We consider the effects of a longitudinal non-uniformity on the bending properties of the gradient magnets by integrating the field of Equation 2 on the orbit described by Equation 5, with $x_{slope} = 0$. Our result is

$$\begin{aligned}\int_{-L/2}^{L/2} dz\ B(x,z) &= B_0 L(1 - \frac{4\alpha_2 b_2 d}{45a} + \frac{4b_3}{45}(\frac{d}{a})^2 + \frac{16\alpha_2 b_3}{945}(\frac{d}{a})^2 \\ &\quad + \frac{b_2 x_{offset}}{a} - \frac{8\alpha_2 b_3 d x_{offset}}{45a^2} + b_3(\frac{x_{offset}}{a})^2). \end{aligned} \tag{13}$$

We note that the dependance on α_1 has dropped from the equation. The terms which depend upon α_2 are linear in the quadrupole, linear in the sextupole and bilinear in the sextupole times x_{offset}. Solving for x_{offset}, setting $\alpha_0 = -\alpha_2/3$, $\alpha_2 = .15$ (20% overall non-uniformity) and evaluating the result numerically, we find $x_{offset} = 0.000150$ m for RGF or $x_{offset} = 0.000145$ m for RGD. Although these effects are negligible, they are still more than an order of magnitude larger than the direct sextupole effect (see Section 3.1). Following the same procedure but setting $b_3 = 0$ provides the simple formula

$$x_{offset} = \frac{4\alpha_2 d}{45}, \tag{14}$$

for the position which results in the design bend, which, for the same values of d and α_2 as above gives $x_{offset} = 0.000156$ m. We note that this is noticeably different than the above result with sextupole considered. Although all of these results are small enough to ignor, the second order interactions among these non-uniformities are not negligible compared with direct effects.

3.4 Bend Center Effects of Longitudinal Non-Uniformity

[Calculations to be performed.]

3.5 Tune Effects of Longitudinal Non-Uniformity

The equations contain more terms when one examines the effects of non-uniformity on the focusing properties of gradient magnets. Let us simplify our considerations initially by ignoring the small design sexupole in our magnets. By applying the form of Equation 9 but integrating over only a single gradient magnet, we find the change tune contribution from that magnet.

$$\delta\nu = \int_{-L/2}^{L/2} \frac{\beta(s)B'(s)}{4\pi(B\rho)} ds = \int_{-L/2}^{L/2} \frac{\theta\beta(s)B'(s)}{4\pi(B_0 L)} ds \tag{15}$$

where we obtain the second form by applying the result $B_0 L = (B\rho)\theta$. By applying Equations 2,4 and 5 but with $x_{slope} = 0$, $b_3 = 0$ we have,

$$\delta\nu = \frac{\beta_{mid} b_2 \theta}{4\pi a}(1 + \frac{\beta_{curv}}{3} + \frac{4\alpha_2\beta_{curv}}{45} + \frac{\alpha_1\beta_{sl}}{3}) \tag{16}$$

The result for $b_3 \neq 0$ is only a bit longer:

$$\begin{aligned} \delta\nu \quad = \quad & \frac{\beta_{mid} b_2 \theta}{4\pi a}((1 + \frac{2b_3 x_{offset}}{ab_2})(1 + \frac{\beta_{curv}}{3} + \frac{4\alpha_2\beta_{curv}}{45} + \frac{\alpha_1\beta_{sl}}{3}) \\ & - \frac{8b_3 d}{45ab_2}(\alpha_2 + \beta_{curv} + \frac{11\alpha_2\beta_{curv}}{21} + \alpha_1\beta_{sl})) \end{aligned} \tag{17}$$

For the gradient magnet locations in the Recycler Ring, fits have been performed to the design β functions [only x so far] and the results used to calculate the effects described in Equation 16. Table 2 displays the results of these fits and calculations. Note that $\alpha_2 = 1\%$ might be interpreted as 1.3% non-uniformity. The systematic and random effect have only been combined for single magnet types. For detailed comparison with other calculations, further assumptions will be required.

3.6 Incomplete....

At least the following items are incomplete in this draft: Numbers for Vertical Tune Effects, Numbers for sextupole effects, and analysis of delta z effects.

	Horizonal Focus			1 magnet		Systm.	Random
Loc	β_{mid}	β_{sl}	β_{curv}	$\delta\nu_x$	$\delta\nu_x$	$\delta\nu_x$	$\delta\nu_x$
Type	m			$\alpha_2 = 1\%$	$\alpha_1 = 1\%$	$\alpha_2 = 1\%$	$\alpha_1 = 1\%$
RGF	49.335	-0.1082	-0.04962	-8.9697E-6	-7.3346E-5	-0.0009687	-0.00076224
RGD	13.522	-0.1827	0.08949	16.177E-6	3.2762E-5	0.00174711	0.000340468
RGS(f)	49.562	-0.1027	-.04487	11.855E-6	10.1749E-5	0.00075869	0.000814
RGS(d)	10.405	-.2406	0.08017	-4.4467E-6	5.0044E-5	-0.0002846	0.0004003

Table 2: Four types of lattice locations employ the three types of gradient magnets for the recycler. There are 108 locations for the RGF and RGD magnets and 64 focusing and 64 defocusing locations for RGS magnets. We tabulate here the parameters of these locations and the tune effect results for each location type in this table. Single magnet effects are shown in columns 5 and 6. In column 7 we show column 5 times the number of locations, the result if there is a systematic effect in α_2 of the size shown (all 8 Gev line magnets have the same sign of α_2). In column 8 we show columns 6 time the square root of the number of locations, the result if there is a random α_1 with an RMS of 1%.

4 Using Moments to Relate to Measured Profiles

Measured profiles of the longitudinal (z) distribution of field will be required in Recycler Gradient magnets. To take best advantage of these measurements, we will choose analysis strategies which extract the most relavant data. Consider moments of the distributions defined by

$$^kB_{zMom} = \int_{-\infty}^{\infty} z^k B_y(z)dz / \int_{-\infty}^{\infty} B(z)dz \tag{18}$$

Where we normalize to the integrated strength. The two moments of interest can be re-expressed in terms of more physically understandable entities as follows.

$$\delta z =^1 B_{zMom} \tag{19}$$

where δz is the location of the bend center (in the whatever frame z in measured). We describe $^1B_{zMom}$ as the first moment of longitudinal profile, $B_y(z)$. The second moment measures the width of the distribution of the field. A natural way to express this is to compare it to the second moment of a rectangular distribution. If $B_y(z)$ has value B_0 from $-L/2$ to $L/2$ then the value of $^2B_{zMom}$ is $L^2/12$. Let us define the second moment ratio deviation as

$$^2\Delta_{zMom} = \frac{B^2_{zmom}}{L^2/12} - 1 = \frac{12}{L^2}\frac{\int_{-\infty}^{\infty} z^2 B_y(z)dz}{\int_{-\infty}^{\infty} B(z)dz} - 1 \qquad (20)$$

and we will call this quantity the second harmonic normalized deviation (Delta_z2Mom_rel).

Let us now evaluate the effects on the tune which will be created by the deviations of the moments from the design values. We calculate the tune effects using Equation 9 or 15.

5 Analysis and Conclusions

[I would expect to discuss both results of this calculation and comparisons to the calculations by Norman Gelfand and Shekhar Mishra in this section.....]

6 Acknowledgments

This work was prompted by discussions with Gerry Jackson while measuring the large effects observed in 8 GeV line dipoles and gradient magnets. I have been greatly assisted by discussions with Norman Gelfand, Shekhar Mishra and James Holt. A discussion with Stan Pruss provided crucial insight. Don Edwards has help locate useful references.

A Analytic Results on Magnet Properties

In examining the longitudinal non-uniformity results, it is useful to compare them directly to other magnet requirements on an equivalent basis. We will compile some results here for comparison with other calculations and some requirements specifications.

A.1 Effects of Gradient Errors

The tune change due to a gradient error in a single gradient magnet can be evaluated using Equation 15 and Equation 1 (with B' calculated as for Equation 10).

$$\delta\nu = \int_{-L/2}^{L/2} \frac{\beta(s)B'(s)}{4\pi(B\rho)}ds = \int_{-L/2}^{L/2} \frac{\theta\beta(s)B'(s)}{4\pi(B_0 L)}ds = \frac{\theta}{4\pi}\int_{-L/2}^{L/2} \frac{\beta(s)B_0 b_2}{a(B_0 L)}ds \qquad (21)$$

but since this calculation is for a uniform longitudinal distribution, the integral on ds gives L and we have

$$\delta\nu = \frac{\theta}{4\pi a}\beta\ \delta b_2 \tag{22}$$

where β is the average over the magnet. The various magnet designs each appear at only one type of lattice location so we can use those lattice properties to evaluate this result. The β to be used is the value at the center of the magnet (designated as β_{mid} above). Combining the results from many magnets is traditionally done by considering systematic effects (coherent shift due to average δb_2) and random effects due to the RMS width of the distribution. The former is obtained by multiplying the single magnet effect by the number of lattice locations involved, whereas, for the random effects, the RMS value of the expected change only multiplies the single magnet results by the square root on the number of locations.

We evaluate this result for the 4 types of gradient magnets in Table 3. If we add the values (with signs as shown) in column 5 (systematic) we find $\delta\nu = 0.0362$ for the sum of 4 location types or $\delta\nu = 0.0253$ for regular cells and $\delta\nu = 0.0109$ for dispersion suppressor cells. This is to be compared with result in Table 1 of MI-0160[7]. Note that rather than δb_2 (in units normalized to the dipole), MI-0160 specifies the change due for $\delta b_2/b_2$ so to compare with these results, one multiplies his result by $1/b_2$ which gives $\delta\nu = 0.03$ for regular cells (4.1 m magnets) and $\delta\nu = 0.0087$ for dispersion suppressor cells (2.7 m magnets in MI-0160 or 3 m magnets in the RRV11 lattice used here). The agreement between these is adequate considering the somewhat different lattices being considered.

Some care is required in expressing requirements which involved different magnet types. The systematic error may come from more than one source and they may affect the resulting tune with different signs. Consider the simple statement that one permits a systematic error of 1×10^{-4} (1 unit) of b_2 error. If this results from a magnet measurement probe calibration error then one would naturally give a defocusing magnet an error of -1 units when giving an error of 1 unit to the focusing gradient magnets. These errors will tend to cancel. If there is a source of systematic error which is a constant fraction of the dipole (some measurment feedthru), it will have opposite signs and the systematic errors will add. However, if the 'systematic' error is simply the mean (central value) of a random distribution resulting from choices made in correcting the series of magnets with, for example, end shims, any tendency toward cancellation is lost. Thus, one may prefer to specify the systematic limits individually for separate magnet series.

A.2 Tune Effect from End Shims

The lattice design assumed gradient magnet lengths of 177″ and 122″ for the regular cell and dispersion suppressor cells respectively. The fabrication plan is to create magnets with bare poles of this length and to add a 1″ end shim on each end to correct transverse field errors as measured with the rotating coil harmonics system. These end shims will spread the gradient beyond the design region. The constraints imposed on the fabrication is to correct the (naturally small) change in integrated field by retuning the quantity of ferrite and to achieve the desired integrated harmonics as measured by a coil which extends through the yoke length and beyond. Let us calculate some effects on focusing which will remain.

			1 magnet	Systm.	Random
Loc	β_{x-mid}	δb_2	$\delta\nu_x$	$\delta\nu_x$	$\delta\nu_x$
Type	m	units			
RGF	49.335	1	3.223E-4	0.0348	0.00335
RGD	13.522	-1	-0.883E-4	-0.0095	0.00092
RGS(f)	49.562	1	2.158E-4	0.0138	0.00173
RGS(d)	10.405	-1	-0.453E-4	-0.0029	0.00036

Table 3: Four types of lattice locations employ the three types of gradient magnets for the recycler (in RRV11). There are 108 locations for the RGF and RGD magnets and 64 focusing and 64 defocusing locations for RGS magnets. We tabulate here the parameters of these locations and the tune effect results for for gradient errors at each location. We use the β_{mid} value but acknowledge that the second order term modifies the average by a few percent. We choose the sign of δb_2 as one would obtain from a calibration error in a measurement coil. Different sources of systematic error might result in a different relative sign of error. Results for a single gradient magnet are in column 4, the systematic effect is obtained by multiplying by the number of locations and is shown in column 4 while the RMS (over an ensemble of machines) results from multiplying column 4 by the square root of the number of locations.

A.2.1 Tune Effect from Different Magnet Length

We note that by spreading the integrated field, the focusing will occur at different lattice positions. Assuming that the b_2 is constant (we will consider effects due to gradient end shims separately) there will remain a change in focusing due to the distribution of β along the magnet. We describe the transverse field as in Equation 1, but let $L' = L + 2L_{end}$ and choose B' such that $B'L' = B_0 L$. We will evalute Equation 9 again, extrapolating the description of β from fits to Equation 4. We observe that the contribution of the uniform component and the linear component will have no change for a symmetric length change. We focus on calculating the term proportional to β_{curv} but begin by calculating the tune change due to the gradient for a magnet of length L'.

$$\delta\nu = \int_{-L'/2}^{L'/2} \frac{\theta\beta(s)B'(s)}{4\pi(B_0 L)} ds \tag{23}$$

$$\delta\nu = \frac{\theta b_2}{4\pi a} \int_{-L'/2}^{L'/2} \frac{B'_0}{B_0 L} \beta_{mid}(1 + \beta_{sl}(\frac{2z}{L}) + \beta_{curv}(\frac{2z}{L})^2) \tag{24}$$

$$\delta\nu = \frac{\theta b_2}{4\pi a} \frac{B'_0}{B_0 L} \beta_{mid}[(L' + \beta_{curv}(\frac{4(L')^3}{12L^2}))] \tag{25}$$

where the term proportional to β_{sl} has dropped out by symmetry. The tuning to achieve the desired bend strength will result in having $B'_0 L' = B_0 L$

$$\delta\nu = \frac{\theta b_2}{4\pi a} \beta_{mid}[1 + \beta_{curv}(\frac{(L + 2L_{end})^2}{3L^2})] \tag{26}$$

$$\delta\nu = \frac{\theta b_2}{4\pi a} \beta_{mid}[1 + \frac{\beta_{curv}}{3}(1 + 4\frac{L_{end}}{L} + 4(\frac{L_{end}}{L})^2)]. \tag{27}$$

Let us now consider the term which is linear in L_{end}

$$\delta\nu = \frac{\theta b_2}{3\pi a} \beta_{mid}\beta_{curv}(\frac{L_{end}}{L}). \tag{28}$$

Evaluating this for RGF series locations we find that $1''$ end shims on both ends of an RGF will produce a tune change of $\delta\nu_x = -7.60 \times 10^{-5}$ so 108 of them will create $\delta\nu_x = -.0082$,

A.2.2 Tune Effect from Lumped End Correctors

Localizing a harmonics correction in the end shim will necessarily place the sextupole field at a β and x which are not characteristic of the design. The result in Equation 17 for constant b_3 will apply to the body sextupole of the Recycler gradients. But the extruded poles for the regular cell gradient magnets are sufficiently different from the design as to require end shims with significant sextupole correction. Let us calculate the tune effect due to placing these shims on the magnet centerline and thereby at a significant x with respect to the design orbit. Similarly, there will be effects from the lumped quadrupole correction due to the variation of β along the magnet. We note that the quadrupole field at the end shims is given by

$$B'(x, z = L/2) = B_0\left(\frac{b_{2end}}{a} + \frac{b_{3end}}{a}\left(\frac{x}{a}\right)\right) \tag{29}$$

where b_{2end} and b_{3end} are the localized harmonic components and x is

$$x = x_{offset} + x_{slope}z + d\left(\frac{1}{3} - \left(\frac{2z}{L}\right)^2\right). \tag{30}$$

We frequently discuss the end shim harmonic values by describing the effect (δb_j) they will have on the integrated harmonics as follows

$$B_0\ \delta b_j L = B_0 b_{j\ end} L_{end} \tag{31}$$

so

$$b_{j\ end} = \delta b_j \frac{L}{L_{end}}. \tag{32}$$

Note that for many magnets the correction will be applied to two ends, doubling the effect. Let us now evaluate the tune change equation for the end correctors only.

$$\delta\nu = \int_{-L'/2}^{L'/2} \frac{\theta\beta(s)\delta B'(s)}{4\pi(B_0 L)} ds \tag{33}$$

$$\delta\nu = \frac{\theta}{4\pi(B_0 L)}[\beta(s)B_0(\frac{\delta b_2}{a}\frac{L}{L_{end}} + \frac{\delta b_3}{a}\frac{L}{L_{end}}\frac{x}{a})]L_{end} \tag{34}$$

where the evaluation of the integration limits is to be carried out for evaluation of β and x within the [].

$$\delta\nu = \frac{\theta}{4\pi}[\beta(s)(\frac{\delta b_2}{a} + \frac{\delta b_3}{a}\frac{x}{a})] \tag{35}$$

We need to evaluate this expression at $z = L/2$ (or $L'/2$ but we will neglect that difference). The effect of a pair of quadrupole end shims which together create δb_2 will be

$$\delta\nu = \frac{\theta}{4\pi}[\beta_{mid}(1 + \beta_{curv})(\frac{\delta b_2}{a})] \tag{36}$$

Comparing to Equation 16 for the design uniform gradient, we note that for a uniform gradient magnet, the term in β_{curv} has a relative weight of 1/3. Since the δb_2 which is being created by the end shims is to correct the deficit in the body field, we take 2/3 of the term in β_{curv}

$$\delta\nu = \frac{\theta}{4\pi}[\frac{2}{3}\beta_{mid}\beta_{curv}(\frac{\delta b_2}{a})]. \tag{37}$$

When evaluated for one RGF we find $\delta\nu_x = -1.066 \times 10^{-5}$ for a quadrupole correction of 1×10^{-4} (1 unit). Thus a systematic correction of 1 unit on all RGF's will shift the horizonal tune by $\delta\nu_x = -.0011$

Returning to Equation 35 and considering the sextupole correction end shim we find

$$\delta\nu = \frac{\theta}{4\pi}[\beta(s)\frac{\delta b_3}{a}\frac{x}{a}] \tag{38}$$

Again we note that the terms with β_{sl} will cancel (for symmetric) correctors while we get

$$\delta\nu = \frac{\theta}{4\pi}[\beta_{mid}(1 + \beta_{curv})\frac{\delta b_3}{a}\frac{x_{offset} + d(\frac{1}{3} - 1)}{a}] \tag{39}$$

Comparing with Equation 17 and saving only the difference we have

$$\delta\nu = \frac{\theta}{4\pi}[\beta_{mid}\beta_{curv}\frac{\delta b_3}{a}\frac{\frac{1}{3}x_{offset} - \frac{22}{45}d}{a}] \tag{40}$$

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