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Transverse Placement of Recycler Gradient Magnets

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Contents

Abstract

To bend the 8 GeV antiprotons on the Recycler design orbit, straight gradient magnets have been built to provide the design integral field on a straight path through the magnet as well as on the design orbit. This requires installation with a transverse displacement by $d/3$ at the magnet center where d is the sagitta. The transverse alignment of the gradient magnets for the Fermilab Recycler Ring used a slightly different set of numbers which will be described. Bend effects of design sextupole fields and the fields from end shims will be calculated.

1 Introduction

The Recycler Ring[1] uses gradient magnets for bending and most of the focusing in the ring. They are specified as straight (rectangular) magnets with a uniform dipole, quadrupole and sextupole components along the length. At the design stage, several notes were written exploring the requirements for placing these magnets in the ring. Norman Gelfand (MI-0200[2] and Steve Holmes (MI-0195[3], MI-0196[4]) explored issues concerned with the interaction between the specifications for straight magnets and the curved particle orbit which will be experienced in the Recycler. Issues of transverse placement were also discussed in MI-0207[5]. We will review these issues and document the alignment procedures used for Recycler Magnet installation through the end of 1999.

2 Mathematical Description

We obtain an explicit polynomial description of the lattice and magnets in the following way.

- Since the magnet design is for a straight hybrid permanent gradient magnet, the normalized harmonics of the design field are uniform along the length (neglecting end shim effects).

$$B_y(x) = B_0(1 + b_2\frac{x}{a} + b_3(\frac{x}{a})^2) \tag{1}$$

  where a is the reference (normalization) radius for the harmonic representation of the fields.

- The orbit through a single gradient dipole is adequately represented by a polynomial (parabola), since the deviation from a circular orbit is small (See Appendix A). The "natural" description for the design orbit thru a *curved* dipole would describe x as $x(s) = x_o$, where s is the parameter which identifies the coordinate along the (curved) orbit. For a uniform, straight dipole, if z is a rectilinear coordinate with zero at the magnet center, we describe the orbit by

$$x = x_0 - d(\frac{2z}{L})^2. \tag{2}$$

d is called the sagitta and is the distance from the chord to the arc of the circle. For a magnet of length L which deflects by an angle θ, the sagitta is given, in the small angle approximation, by

$$d = R(1 - \cos(\frac{\theta}{2})) = \frac{R\theta^2}{8} = \frac{L\theta}{8}. \tag{3}$$

We note that the path length difference between the straight line used for measurement and the circular orbit is $\delta s = R\theta - R(2\sin\frac{\theta}{2}) \approx L\theta^2/24 = 8.14 \times 10^{-5}m$ or 18.1 ppm for the regular cell gradient magnets in the Recycler.

2.1 Bend of Gradient Magnet

The Recycler gradient magnets were specified by the integrated fields which are measured by a straight harmonic probe which is inserted along and rotated about the transverse center of the gradient magnet. The field integral was selected to be that which would bend an 8 GeV (kinetic energy) proton or antiproton in a circle using 301 1/3 regular cell bends (assuming that the dispersion suppressor bends provide 2/3 of the deflection of a regular cell bend). Let us calculate the bend which will be achieved for a particle on a parabolic (circular) orbit. We integrate Equation 1 along the trajectory given by Equation 2. Let us at first neglect the small sextupole (b_3) component.

$$\int_{-L/2}^{L/2} Bd\ell = B_0L(1 - \frac{b_2}{a}(\frac{d}{3} - x_0)) \tag{4}$$

Thus, by placing the dipole at a transverse offset of $x_0 = d/3$, the integrated field on a circular orbit will be independent of the quadrupole term and have the same value as the integral on a straight line. Restating this description, we place the magnet center such that the circular orbit is at a displacement of $d/3$ to cancel the first order bend effects of the gradient. This places the design orbit at a displacement of $-2d/3$ at each end of the magnet.

With this in mind, for the rest of this presentation, we will describe the orbit with this term built into the description.

$$x = x_{offset} + x_{slope}z + d(\frac{1}{3} - (\frac{2z}{L})^2). \qquad (5)$$

3 Magnet and Alignment Details

Gradient magnets for the recycler[6] consist of a pair of shaped poles, flux sources consisting of ferrite and compensator, and an outer shell for flux return. The slot length of a recycler gradient is set by the length of the shell but the length over which the bending occurs is determined by the pole length. The pole consists of steel pieces which are uniform in transverse dimensions with a fixed length set by the design. To this pole is added at each end a field shaping shim whose length varies across the radial direction to create a correction to the integral field. We designate the fixed length as L in the above formulas, recognizing that the bend effects are modified by the longitudinal distribution of the bricks as well as the effects of the variable length end shims[1] but these variations are small and will be unimportant for this set of calculations.

Let us at this point document the design features of the Recycler. The magnet design was specified on 2/16/98 in http://wwwfermi3.fnal.gov/recycler/magnets/gradient_list.html and this information was then entered into the MTF database where it was accessed for magnet measurement. The values stored there are reported in the database

[1] The central value (at $x = 0$) of the end shim length is also fixed.

	Length	dipole	quadrupole		sextupole	
Magnet	L	B_0	B_2	b_2	B_3	b_3
Name	m	$Tesla$	T/m	@1″	T/m^2	@1″
RGF	4.4958	0.13752	.3355	.06197	0.1853	8.696e-04
RGD	4.4958	0.13752	-.3238	-.05981	-0.3209	-15.05e-04
SGF	3.0988	0.13301	.66816	.1276	0.0000	0.0000
SGD	3.0988	0.13301	-.68236	-.1303	0.0000	0.0000

Table 1: A Current Set of Design Properties of Gradient Magnets for the Recycler Ring. These are restated from the Recycler Magnet Web page. Normalized harmonics are quoted at a reference radius of 1″.

report at
http://www-ap.fnal.gov/MagnetData/PAGE/series_report_main.html. At this point we will document the features which these specifications imply. For a proton (antiproton) kinetic energy of 8 GeV, the corresponding momentum is 8.88889 GeV/c. The magnetic rigidity is $B\rho = 29.6501$ T-m. The integrated bend field is $\int Bd\ell = 2\pi B\rho = 186.297$ T-m. The regular cell bends (RGF and RGD) each provide 1/(301 1/3) of this or 0.618243 T-m. The dispersion suppressor bends (SGF and SGD) provide 2/3 of that or 0.412162 T-m. Precisely these numbers appear in the Recycler specification and the MTF Database. The pole length is specified as $L = 177''$ for the regular cell magnets and $L = 122''$ for the dispersion suppressor cells. The approximation used in this note will assume that the specified bend is achieved on a circular arc of chord length L. Known corrections due to longitudinal offsets and Mean Squared Length differences are second order for this calculation and have a small effect.

3.1 Offset Desired

Series	RGF / RGD	SGF / SGD	Units	Comments
Offset	0.024475012	0.010793394	feet	For Installation 1999
Offset	7.46	3.29	mm	For Installation 1999
$2d/3$	7.81	3.59	mm	displacement for parabola
x_{offset}	-0.35	-0.30	mm	For Equation 5
Corr Offset	7.87 7.76	3.61 3.56	mm	Inc. Quartic Terms
x_{offset}	-0.41 -0.30	-0.32 -0.27	mm	w/ Quartic for Eq 5

Table 2: The radial displacement of the design ends of the poletip in the accelerator reference frame (LTCS). The first row gives the prescription from MI-0196 translated to survey feet which was specified to the alignment group. The second row gives this in mm. A displacement from a parabolic orbit of $2d/3$ eliminates the first order bend change due to the gradient. x_{offset} is defined above to specify the radial displacement from the desired design orbit. The displacement shown on the line labeled Corr Offset is the correct shift including the first order orbit shape changes (orbit in gradient field). See Appendix A. The specification used for alignment was based on MI-0196 which corresponded to a shorter magnet design.

If we used the prescription above to set the magnets, the ends ($z = \pm L/2$) would be displaced so that in the magnet's reference frame (as we are using for these calculations), the design orbit enters at $-2d/3$ (inside). In the tunnel reference frame, the orbit is fixed and we displace RGF or RGD magnets (outside) by 7.81 mm and SGF or SGD magnets by 3.59 mm. As shown above, this will eliminate bending effects due to the gradient. In Table 2 we show these values and the similar numbers assuming the orbit calculated for a gradient (see Appendix A). Since the installation used values from MI-0196, we calculate the orbit offset we expect in the magnet frame. Corrections for the effects of sextupole body field and other effects will be shown below.

3.2 Prescription for Magnet Placement

The procedure for installation of the gradient magnets in the tunnel[7] involved several steps. The lattice program MAD was used with version RR19 of the Recycler lattice description to produce a survey output file in terms of the Main Injector LTCS (Local Tunnel Coordinate System) metric site coordinate system. These coordinates specified the design central orbit trajectory and were given at the magnetic pole tips of the gradient magnets and quads, and at the center of the bpm assembly. The design orbit had a circumference of 3319.418828 meters. These site coordinates were converted into survey feet (39.37/12) and transmitted to the Survey and Alignment group to be transferred to the ceiling of the Main Injector enclosure and used for initial placement of the gradient magnets. These pole-tip marks on the ceiling specified the design longitudinal bend center of all the gradient magnets.

The magnet fabrication determined the relation between the iron poles and the fiducial cups (survey plugs) which were built into 4 locations at each end of each magnet. For radial positioning, adequate precision was obtained by fabrication. For the vertical position, the pole shape was sufficiently accurate but the tolerance buildup including the vertical depth of the fiducial cup required measurement. A procedure was implemented and documented in the magnet fabrication traveler to measure (redundantly) the vertical offset between the back face of the pole and the reference face of the fiducial cup. James Volk supervised this work and prepared a summary for all the magnets which was then passed to the survey and alignment group. The results were histogrammed, revealing that the corrections were typically less than 0.25 mm (0.01$''$).

The longitudinal offset (z_cent_off) of the bend center with respect to the physical pole tip length was measured for each gradient magnet[8][9]. During the initial placement of the gradient magnets in the tunnel, this longitudinal offset was applied to displace the magnet along the tangent to the design orbit at the bend center[10]. The initial transverse placement of the gradient magnets placed the magnet centerline on the design orbit trajectory without taking into account any sagitta correction.

The MAD survey output file was further processed to provide final alignment specifications. The transverse offsets (sagitta correction) applied to the Recycler magnets used the prescription described in note MI-0196[4]. Table 2 describes the transverse displacements (offset) used for the RGF and RGD magnets (regular arc cells) and the SGF and SGD magnets (dispersion suppressor cells). Individual gradient magnet longitudinal offsets (z_cent_off) were taken from the z-scan measurements. The program read the MAD survey output file. Transverse offsets were applied to shift the magnet centerline at the bend center perpendicular to the tangent of the reference orbit. Since the stands provided no longitudinal adjustment, up to 8 mm of error in longitudinal placement was permitted. Since the offsets were specified to the laser tracker with respect to the local tangent, the horizontal and vertical setting tolerance of 0.25 mm were not compromised. The longitudinal offset was applied parallel to the tangent. Coordinates for the pole tip corners were calculated. A revised file was produced of site coordinates (X,Y,Z) at each pole-tip which represented the shifted location of the center of the magnet steel. These new coordinates were then transferred to the survey and alignment group where Babatunde Oshinowo used these revised site coordinates, the design locations of the four fiducial cups at each end, and the measured vertical offsets to calculate the site coordinates of the fiducial cups. These were installed into the laser tracker[2] software for final survey.

4 Bend Effects for Magnets as Installed

We have described the Recycler gradient magnet fabrication and installation along with a description of the desired offset for an ideal gradient

[2]The recycler alignment used an interferometric laser tracker system. Model SMX Tracker4000 was employed with associated Insight software. Its properties as well as the specifications and procedures for recycler alignment are documented by O'Sheg Oshinowo[7].

magnet. Let us now calculate some effects due to higher order fields, end shim corrections and non-ideal magnet placement.

4.1 Bend of Gradient Magnet with Sextupole

If we include the design sextupole term and integrate the field (Equation 1) along the circular orbit (Equation 5), we find

$$\int_{-L/2}^{L/2} Bd\ell = B_0L(1 + b_2\frac{x_{offset}}{a} + b_3[\frac{4d^2}{45a^2} + \frac{x_{offset}^2}{a^2} + \frac{L^2x_{slope}^2}{12a^2}]). \quad (6)$$

If we substitute for the sagitta with Equation 3, we have

$$\int_{-L/2}^{L/2} Bd\ell = B_0L(1 + b_2\frac{x_{offset}}{a} + b_3[\frac{L^2\theta^2}{720a^2} + \frac{x_{offset}^2}{a^2} + \frac{L^2x_{slope}^2}{12a^2}]). \quad (7)$$

We can solve Equation 6 for the offset for which the integral is B_0L. We find

$$x_{offset} = \frac{-ab_2 \pm \sqrt{(ab_2)^2 - (16/45)b_3^2d^2}}{2b_3} \approx -\frac{4}{45}\frac{b_3d^2}{ab_2} \quad (8)$$

for the case where $x_{slope} = 0$. Evaluating this for the RGF gradient magnet we require only a displacement of $x_{offset} = -6.74 \times 10^{-6}$ m to have the same integral on the circular orbit as on the central straight path. Alternatively, $(\int Bd\ell/B_0L) - 1 = 1.645 \times 10^{-5}$ on the an orbit with $x_{offset} = 0$.

4.2 Bend Correction with End Shims

The error fields in the Recycler are corrected using shims which are attached at the ends of the poles. Note that the dipole field is adjusted (changing the permanent magnet material which drives the flux) after modifying the end shims so that no dipole field need be ascribed to the end field. (Therefore there is not a term $(1 + \cdots)$ in Equation 9.) The installation displacement by z_cent_off allows one to achieve the desired bend center. The error correction applied by the end shims was selected to create the design field on the straight measurement path. Let us calculate its effect on the bend field seen by particles on the expected circular orbit.

We describe the end contributions as if they occured at the points $\pm L/2 - z_{off}$ (where z_{off} = z_cent_off) with normalization using an effective length,

L_e. L_e is sufficiently small compared to the betatron wavelength that we can apply the correction as if it were at a point. For $z = +L/2 - z_{off}$,

$$B = B_0(b_{2l}\frac{L}{L_e}\frac{x}{a} + b_{3l}\frac{L}{L_e}(\frac{x}{a})^2) \tag{9}$$

For $z = -L/2 - z_{off}$,

$$B = B_0(b_{2o}\frac{L}{L_e}\frac{x}{a} + b_{3o}\frac{L}{L_e}(\frac{x}{a})^2) \tag{10}$$

were we have normalized the harmonics to the field integrated over the magnet length L. The subscripts of 'l' (LEAD or LABEL end) and 'o' (OTHER end) designate the positions of the end shims whose field is being described. L_e is the effective length of the end but is used here only for a normalizing factor. We multiply the end field by L_e to get the integrated bend and evaluated the field at the orbit position corresponding to $z = -L/2$. For the 'OTHER' end we obtain

$$\int_{OTHER} Bd\ell = B_0L[\tag{11}$$
$$b_{2o}(L\theta(\frac{1}{12a} + \frac{z_{off}}{2aL} + \frac{z_{off}^2}{2aL^2}) + (\frac{-x_{offset}}{a} + \frac{x_{slope}(L+2z_{off})}{2a}))$$
$$+\frac{b_{3o}}{144a^2L^2}(L^2\theta + 6L^2x_{slope} + 6\theta z_{off}^2 - 12Lx_{offset} + (\theta + 2x_{slope})6Lz_{off})^2]$$

A similar result would can be obtained for the 'LEAD' end. Typically, $z_{off} < 0.05$ m. This makes all terms involving z_{off} small compared to the other terms in the Equation 12. Let us evaluate it after setting $z_{off} = 0$. We will employ the L and θ appropriate for the regular cell magnets.

$$\int_{OTHER} Bd\ell = B_0L[\tag{12}$$
$$b_{2o}(-0.307556 + 39.3701x_{offset} - 88.5x_{slope}$$
$$+b_{3o}(0.09459 - 24.217x_{offset} + 1550.003x_{offset}^2$$
$$+54.437x_{slope} - 6968.5x_{offset}x_{slope} + 7832.25x_{slope}^2)$$

4.3 Magnitudes for Installation and Closed Orbit Effects

For most strong focusing rings, the longitudinal distribution of fields is simple enough that there is no need to separately consider the effects of installation offsets from the offsets created by orbit distortions. We are in a position to determine if this is the case for the Recycler Gradient magnets. The end shims which have been created typically provide $< 5 \times 10^{-4}$ quadrupole and sextupole correction when normalized to the integrated magnet strength as shown here. We can examine Equation 12 to see what to expect for various errors. Placing these corrections at the ends of the pole (longitudinal effect alone) gives a coefficient of b_{2o} of -0.307556 and a coefficient of b_{3o} of 0.09459. Assuming the specified alignment tolerance of 0.25 mm (probably the installation achieved about 0.15 mm) we conclude that $x_{slope} < 55$ μRadian. Including the design specification difference of 0.35 mm and adding linearly an installation error of 0.15 mm we expect $x_{offset} < 0.5$ mm. On the other hand the aperture of the Recycler Ring will accommodate orbit errors >10 mm and corresponding angles of $\sim$400μRadian. With these larger errors, we find a coefficient of b_{2o} of -0.737 and a a coefficient of b_{3o} of 0.543 for $x_{offset} = -10$ mm while the other sign for x_{offset} gives small coefficients. Although systematic changes in bending of $\sim 2 \times 10^{-4}$ or so are not negligible, they do not justify further effort at this point. Note that although the installation effects can add systematically, the orbit dependent closed orbit effects will mostly cancel when averaged over the ring.

5 Conclusions

We have shown that the bend field of a straight gradient magnet will be the same on the (nearly) circular design orbit as on a straight path through the transverse center of the magnet, provided it is installed with the design orbit displaced by $+d/3$ at the center. This placement will make the bend independent of the gradient strength. The difference between this plan and the specification used for magnet offset during installation is documented to be about -0.35 mm for RGx and -0.30 for SGx magnets resulting in a bend field correction of 8×10^{-4} for RGx and 15×10^{-4} for SGx magnets. Since these effects are of opposite sign for F and D magnets, the momentum orbit effects should cancel but some closed orbit effects are expected. Table 2 shows the change in displacement required because the orbit is not circular. Bend effects due to the lumped end corrections for quadrupole and sextupole are a bit smaller. Bend corrections for design sextupole field, for longitudinal placement to correct bend center errors and for displacement and angle dependence of the end shim corrections have been shown to be small for the parameters of the recycler magnets.

6 Acknowledgments

We would like to thank Norman Gelfand, Leo Michelotti, Shekhar Mishra, Thornton Murphy, Babatunde Oshinowo, and James Volk for helpful discussions, suggestions for the text and assistance in locating references.

A Orbit Through a Gradient Dipole

It is well known that the orbit of a charged particle moving in a uniform dipole field is described by a circle. For the short arc section contained within a single bending magnet of a strong focusing accelerator, one can use the parabolic approximation to the circle without significant error. Gradient fields must change the orbit. We will use recursive approximations to discover the magnitude of these effects.

To describe the orbit in the magnet frame, we begin with an expression for a circle of radius R. A tangent to the circle defines the z axis with the origin at the tangent point. The x axis is radial with the center located along the negative x direction (negative curvature). A rectangular gradient magnet is parallel to the z axis with its centerline at $x = 0$. A circle displaced by x_0 at the center of the magnet (coordinate origin) is described by

$$x = x_0 + \sqrt{R^2 - z^2} - R. \tag{13}$$

We expand in terms of $(z/R)^2$ which gives us an equation for x *vs.* z similar to Equation 2.

$$x = x_0 - d(\frac{2z}{L})^2 - \frac{d\theta^2}{16}(\frac{2z}{L})^4 + \cdots. \tag{14}$$

At the magnet end ($z = L/2$) the x^4 term (correction to the parabola) is smaller than the parabolic term by the ratio $\theta^2/16$ so for $\theta \sim 0.02$ this correction is 25 ppm of the sagitta or about 0.25μm for an RGF.

As an alternative to a complete solution to the orbit in a gradient dipole field, we will observe that the deviations will be small and use recursion to obtain a next order correction to the dipole orbit. Let us begin by establishing an equation for the bending in a gradient field.

$$d\theta = -\frac{Bdz}{B\rho}(1 + b_2\frac{x}{a}) \tag{15}$$

$$\frac{d^2x}{dz^2} = -\frac{1}{\rho}(1 + b_2\frac{x}{a}) \tag{16}$$

We obtain our recursive solution by substituting Equation 2 into the quadrupole (b_2) term in Equation 15 and integrating twice.

$$\frac{dx}{dz} = -\frac{L}{2\rho}((1 + \frac{b_2x_0}{a})\frac{2z}{L} - \frac{b_2d}{3a}(\frac{2z}{L})^3). \tag{17}$$

We fix the integration constant by assuming that the angle at the magnet center is zero. Remember that for the circle, $L/\rho = \theta$. Examining this result at $Z = L/2$, we see that if $x_0 = d/3$, the linear and cubic terms cancel such that the bend is precisely $\theta/2$ for half the magnet.

$$x = x_0 - \frac{L^2}{8\rho}((1 + \frac{b_2x_0}{a})(\frac{2z}{L})^2 + \frac{L^2db_2}{48a}(\frac{2z}{L})^4) \tag{18}$$

$$x = x_0 - d(1 + \frac{b_2x_0}{a})(\frac{2z}{L})^2 + \frac{b_2d^2}{6a}(\frac{2z}{L})^4, \tag{19}$$

where we have used the relations between sagitta, chord length and radius: $d = L\theta/8 = L^2/8\rho$. The correction to the parabolic term is b_2x_0/a. Thus, the parabolic curvature is determined by the dipole field at $z = 0$. For the RGx magnets this correction to the parabolic curvature is $0.154b_2$ which we evaluate as -0.00919725 (0.9%) for an RGD.

At the magnet end, $Z = L/2$, we wish to know what displacement the beam will have experienced, since we wish to displace the dipole appropriately. We find with no gradient, $-2d/3 = -0.00781193$ m is the exit point when $x_0 = d/3$. Evaluating Equation 19 with the same x_0 and the parameters for and RGF (RGD) magnet we have the exit point at x =-0.00786777 m (-0.00775805 m). This shift of $+56$ μm (-54 μ) is the result of the difference between the change in the quadratic term $(b2d^2/(3a))$ for RGD magnets of 0.000107772 m and the quartic term $(b2d^2/(6a))$ of amplitude 0.0000538862 m. For the SGF (SGD) magnets the exit position is at $-2d/3$ =-0.00358966 m ignoring the gradient whereas it is -0.00361394 m (-0.00356487 m) when we account for the gradient.

Using these results we have the following values of x_{offset}: RGF: -.0408 mm RGD:-0.298 mm SGF: -0.324 mm SGD -0.275 mm. This difference is about twice the difference found in MI-0200.

Using Mathematica for the calculation, the process is equally simple when including a sextupole term. The resulting equation for the orbit contains terms up to Z^6 and the lower order terms have a sextupole correction.

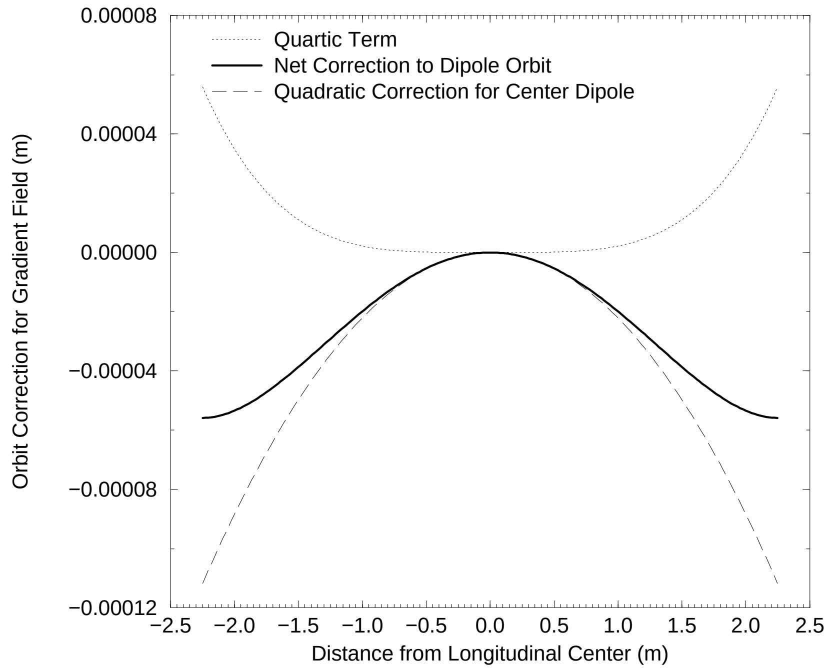

Figure 1: Orbit changes due to the gradient field in an RGF magnet. The parabolic orbit shape due to the dipole field has been subtracted. The remaining parabolic shape is proportional to the x displacement at $z = 0$. (In fact, it depends on the dipole field on the orbit at $z = 0$.) The quartic term partly cancels this shift.

The numerical result is -0.14 μm for the RGF and +0.14 μm for the RGD. As expected, these corrections are too small to consider.

B Other Offset Results

In addition to the numerical integration results by Norman Gelfand (MI-0200), Leo Michelotti[3] has used a C++ accelerator model to obtain offsets

[3] Private communication. March 2000

using integration through a magnet after dividing it into 16 parts. This was done on different occasions with slightly different input.

	RGF	RGD	SGF	SGD	Offset in mm.
MI-0259 (dipole)	7.81	7.81	3.59	3.59	parabola
MI-0259 (gradient)	7.87	7.76	3.61	3.56	inc. quartic
Michelotti (RRV18)	7.83263	7.77949	3.59519	3.56908	16 July 1998
Michelotti (RRV ??)	7.833	7.777	3.477	3.453	29 July 1998
Holmes MI-0196	7.46	7.46	3.29	3.29	Used for Survey thru March 2000
Gelfand (MI-0200)	7.44	7.39	3.30	3.28	

The orbit calculations I am making analytically for MI-0259 do not agree in detail with the results by Norman Gelfand (MI-0200) or Leo Michelotti (private communication). At present, we have not found the reason for the differences. Leo Michelotti has suggested that the number of magnet steps in his calculation may not be sufficient for 50 micron accuracy. The MI-0200 results, like the MI-0196 results are for an earlier design of the Recycler Gradient Magnets.

Since the magnet installation has a maximum error tolerance of 0.25 mm, differences of 0.05 mm are not significant for machine operation. We make these comparisons as a check on our methods. After examining our results we have not found any problems with these analytic results.

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