Separator failure at A49
Analysis of Store 1253, April 26, 2002

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Contents

1 Luminosity drop

After about 13.5 hours into store 1253 (April 26, 2002), the voltage on the bottom plate of the horizontal separator at A49 dropped from a value of -90kV to -25kV. This immediately lowered the luminosity at CDF and D0. In this report we analyse the luminosity drop, compare the measured value with the expected drop and analyse the change in beam lifetimes and emittance growth rates.
What do we hope to learn?

- Why did the proton beam lifetime drop?

  Since the beam-beam forces on the protons are weak, this must be related to single beam dynamics.

- Why did the anti-proton lifetime drop?

  There may be some contribution of the beam-beam forces to this drop. Can we calculate how much is due to single beam effects and how much due to beam-beam?

- How well do we know the optics around the IR?

Initial emittances (p and pbar) ϵ_x, ϵ_y [πmm-mrad]	22, 21
Final emittances	unknown
Average emitt. growth rate [πmm-mrad/hr]	0.3-0.5
Length of store [hrs]	15
Estimated final emittances $(p, \bar{p})$ [πmm-mrad]	(26 - 30, 25 - 29)
Location of BPMS nearest to B0 [m]	7.5 upstream and downstream

B0	β_x = 0.35m, $\psi_x = 0$
BPM upstream of B0	β_x= 159.5 m, $\psi_x = 2\pi\times$ 20.337
BPM downstream of B0	β_x= 160.44 m, $\psi_x = 2\pi\times$ 0.238
A49H Separator	β_x= 867.67m, $\psi_x = 2\pi\times$ 20.329
D0	β_x = 0.35m, $\psi_x = 2\pi\times$
Horizontal tune	ν_x = 20.585
Observed Luminosity drop at B0	41.4%
Observed Luminosity drop at D0	42.3%
Total proton intensity before drop	5.78 $\times 10^{12}$

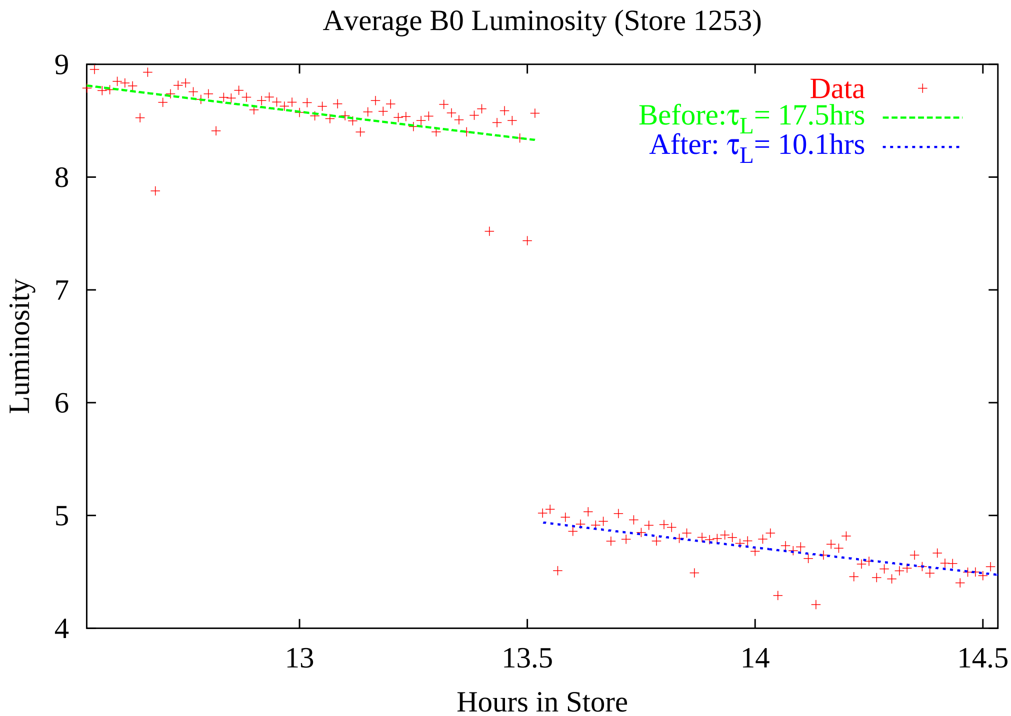

Figure 1: Average B0 Luminosity (raw data) and fits before and after the separator failure. The luminosity at B0 dropped by 41% right after the separator failure.

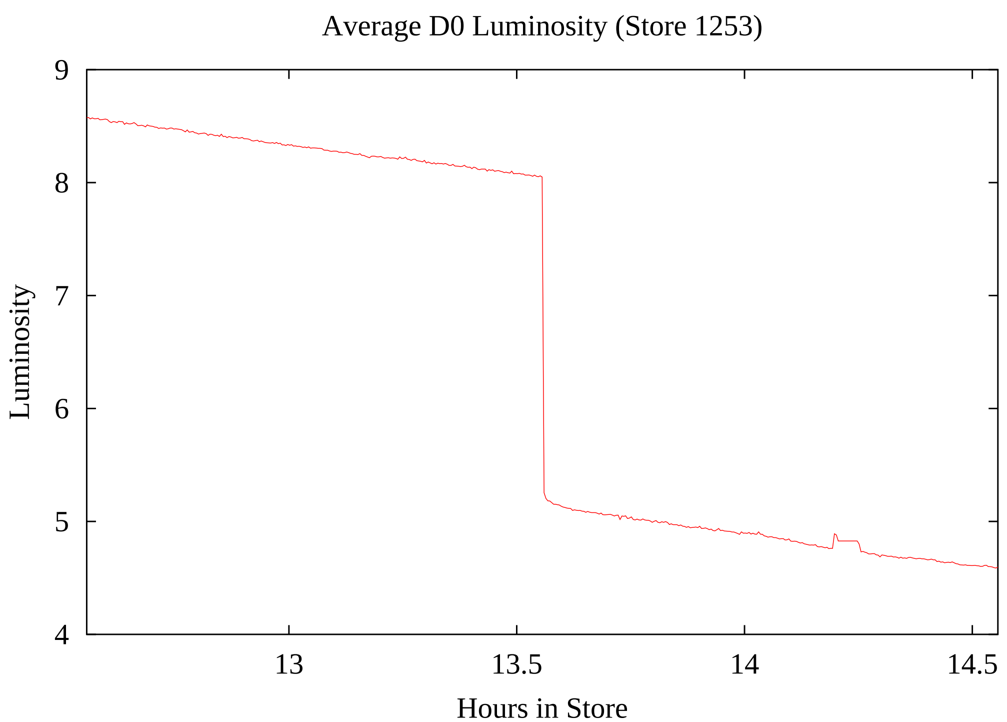

Figure 2: Average D0 Luminosity before and after the separator failure (without fits). The luminosity at D0 also dropped by 41% right after the separator failure.

1.1 Closed orbit shift at B0 and D0

The shift in the closed orbit due to a kick $\Delta\theta$ can be found from

$$\Delta x(s) = \frac{\Delta\theta\sqrt{\beta_{sep}\beta(s)}}{2\sin\pi\nu}\cos[\pi\nu - |\psi(s) - \psi_{sep}|] \tag{1}$$

The kick resulting from the electric field $\mathcal{E}$ across the separator plates of length L is given approximately by [1]

$$\Delta\theta = \frac{\mathcal{E}L}{E} \tag{2}$$

At 980 GeV, a change in voltage of 65 kV across the separator plates with a gap of 5cm results in a kick of

$$\Delta\theta = 3.41\mu\text{rad} \tag{3}$$

Using the above expressions we find that the proton's horizontal closed orbit would move by

$$\Delta x_{co}(B0)|_{sep} = 29.9\mu\text{m}, \qquad \Delta x_{co}(D0)|_{sep} = -30.3\mu\text{m} \tag{4}$$

The orbit separation d_{co} between the beams at the IPs would be twice the above value if the protons and pbars undergo the same but opposite changes in orbit. However the beam-beam kick with separated beams also induces an orbit shift and this will be larger for the pbars. Calculation of the beam-beam induced orbit kick requires that we know the separation between the beams, but that is precisely the quantity that we want to predict. We will approximate the beam-beam induced orbit kick by assuming that the beam separation is twice the shift in the proton orbit. In that case,

$$\Delta x'_{bmbm} = 8\pi\xi\epsilon\frac{x+d}{(x+d)^2+y^2}\left\{1 - \exp[-\frac{(x+d)^2+y^2}{2\sigma^2}]\right\} \tag{5}$$

Extracting the dipole part of the kick,

$$\Delta x'_{bmbm}(0,0) = 8\pi\xi\epsilon\frac{1}{d}\left\{1 - \exp[-\frac{d^2}{2\sigma^2}]\right\} \tag{6}$$

Since the sign of the pbar orbit offsets at B0 due to the separators have opposite signs, the kicks experienced by the anti-protons due to the dipole beam-beam kicks at B0 and D0 have opposite signs. Hence the contribution of the beam-beam kicks at B0 and D0 to the orbit shift at B0 is

$$\Delta x_{co}(B0)|_{BB} = \frac{\beta_0|\Delta x'_{bmbm}|}{2\sin\pi\nu_x}[\cos\pi\nu_x - \cos(\pi\nu_x - |\psi_x(D0) - \psi_x(B0)|)] \tag{7}$$

From the average proton bunch intensity of $N_p = 1.61\times 10^{11}$ and an expected proton emittance of 30π at this stage of the store (this number is found later by a self-consistent calculation), we find that the beam-beam parameter for anti-protons at this stage was

$$\xi = 3.92\times 10^{-3}$$

Hence the beam-beam kick using the value of the orbit offset found in Equation 4 is

$$\Delta x'_{bmbm}(B0) = 5.14\mu\text{rad} \tag{8}$$

while at D0, the kick has the opposite sign. Note that this kick is larger than the kick due to the change in the separator voltage, cf. Eq. (3). However because of the small beta function at the IPs, the change in orbit due to these beam-beam kicks is quite small,

$$\Delta x_{co}(B0)|_{BB} = -0.42\mu\mathrm{m} \tag{9}$$

using Equation (7). This is almost two orders of magnitude smaller than the orbit shift due to the separator and can be neglected.

The predicted luminosity in terms of the luminosity $\mathcal{L}_0$ before the separator failure is found from

$$\mathcal{L} = \mathcal{L}_0 \exp[-\frac{d_{co}^2}{2(\sigma_p^2 + \sigma_{\bar{p}}^2)}] \tag{10}$$

where $d_{co} \approx 2\Delta x_{co}|_{sep}$. This calculation depends on the emittances at the time of the failure. With the initial emittances and average emittance growth rates shown in Table 1, the proton emittances after 13.5 hrs were likely to be in the range (26, 30)πmm-mrad and anti-proton emittances (25, 29)πmm-mrad. We find using the orbit shifts in Eq. 4 (virtually the same at B0 and D0) and the low and high end of the emittance range,

$$\begin{aligned} \frac{\Delta\mathcal{L}}{\mathcal{L}} &= 0.47, \quad (\epsilon_p = 26, \epsilon_{pbar} = 25) \\ \frac{\Delta\mathcal{L}}{\mathcal{L}} &= 0.42 \quad (\epsilon_p = 30, \epsilon_{pbar} = 29) \end{aligned} \tag{11}$$

These values are to be compared with the observed relative drops in luminosity of 0.414 at B0 and 0.423 at D0. This suggests that the emittances were more likely at the higher end of the quoted range.

Another test of the optics is to propagate the measured orbit changes at the BPMs closest to B0 back to B0 using

$$\Delta x(s_2) = \sqrt{\frac{\beta(s_2)}{\beta(s_1)}} \frac{\cos[\pi\nu - |\psi(s_2) - \psi_{sep}|]}{\cos[\pi\nu - |\psi(s_1) - \psi_{sep}|]} \Delta x(s_1) \tag{12}$$

This expression does not depend on the kick angle at the separator nor upon the beta function at the separator.

From Figure 3 we observe that just before the failure, the proton horizontal position was relatively steady at 1.349 mm, then it falls for about 15 minutes after which it stabilizes at 1.217 mm. Similary at the downstream BPM, the horizontal positions at these same times are -6.375 mm and -6.214 mm. This slow decay in the position is related to the long integration time (about 15 mins) of these CPMs.

Hence observed shifts at the CPMs are

$$\text{Upstream}: \quad \Delta x_U^{obs} = -0.132 \text{ mm} \qquad\qquad \text{Downstream}: \quad \Delta x_D^{obs} = 0.161 \text{ mm} \tag{13}$$

Propagating these orbit shifts to the IP, we find using the upstream BPM that the expected orbit shift at B0 and relative luminosity drop is

$$\Delta x_{co}(B0) = 0.0285 \text{ mm}, \qquad \Rightarrow \frac{\Delta\mathcal{L}}{\mathcal{L}}(B0) = 0.384 \tag{14}$$

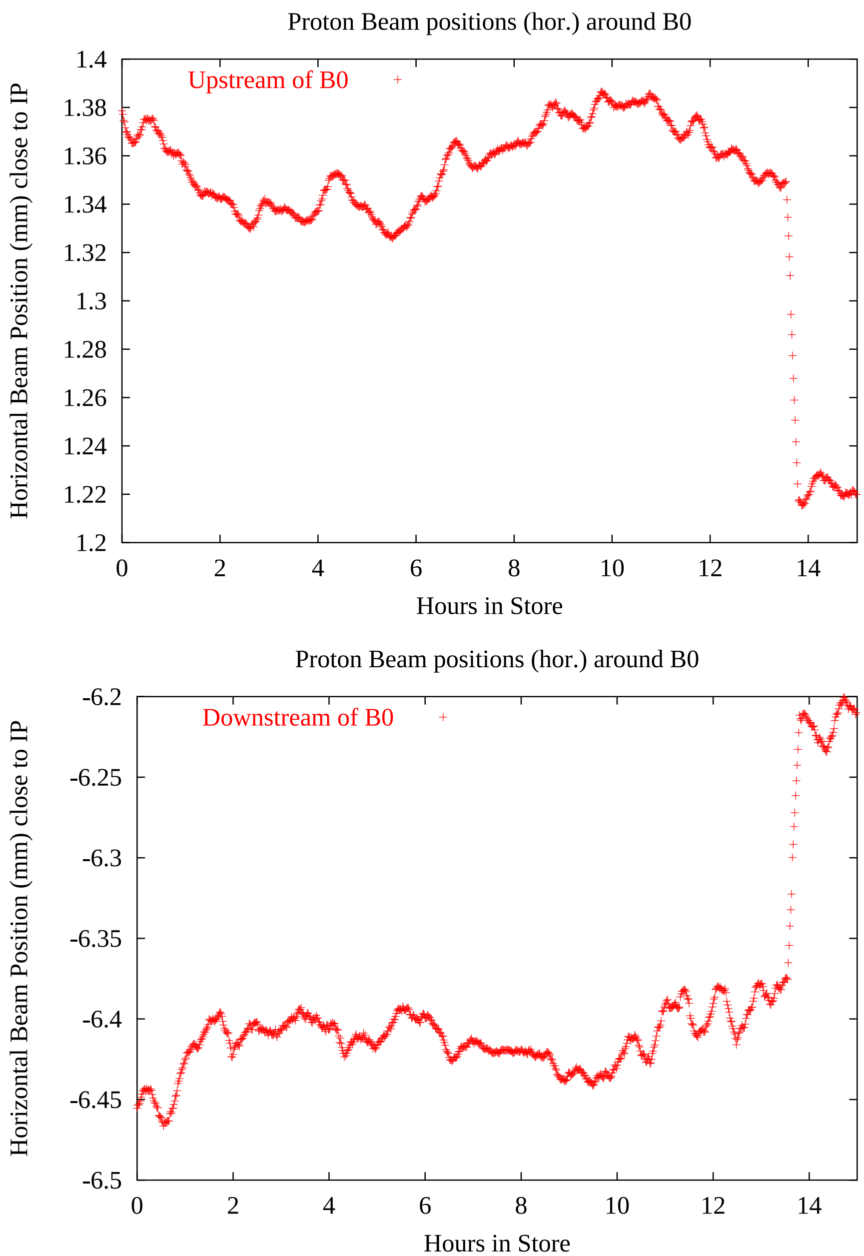

Figure 3: Proton horizontal beam position at BPMs upstream and downstream of B0. The jump in position coincides with the separator failure

while using the downstream BPM, we find

$$\Delta x_{co}(B0) = 0.0241 \text{ mm}, \qquad \Rightarrow \frac{\Delta \mathcal{L}}{\mathcal{L}}(B0) = 0.293 \tag{15}$$

This calculation shows that the upstream BPM was more consistent with the observed luminosity drop. This may indicate either more errors downstream from the IP to the CPM (this is unlikely since there is only the detector between the IP to the CPM) or that this downstream CPM reading was less reliable.

Unfortunately at the time of this store, CPM readings of the pbar orbits were not available. Had they been available, differences between the deflections of the proton and anti-proton beams could have been used to determine the strength of the beam-beam effects on the anti-proton orbits.

2 Lifetimes and emittance growth times

The luminosity in terms of beam parameters is

$$\mathcal{L} = \frac{3\gamma f_{rev} M_b N_p N_{\bar{p}}}{\pi \beta^* (\epsilon_{N,p} + \epsilon_{N,\bar{p}})} \mathcal{H}(\frac{\beta^*}{\sigma_s}) \tag{16}$$

where $\epsilon_{N,p}, \epsilon_{N,\bar{p}}$ are the 95% emittances of the beams and $\mathcal{H}$ is the hourglass form factor

$$\mathcal{H}(z) = \sqrt{\pi} z e^{z^2}(1 - \Phi(z)), \qquad \Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^2 e^{-t^2} dt \tag{17}$$

The average bunch length in Store 1253 was 2.6 nano-seconds or $\sigma_s = 78$cm. With $\beta^* = 35$cm, the hourglass reduction factor for $z \equiv \beta^*/\sigma_s = 0.45$ is $\mathcal{H}(z) = 0.51$. As a point to note, the hour-glass reduction factor would be $\mathcal{H}(z) = 0.74$ or the luminosity about 30% greater if the bunch length were 37cm as specified for Run II design parameters.

The luminosity lifetime can be calculated from the beam parameters by taking the logarithmic derivatives,

$$\frac{1}{\mathcal{L}}\frac{d\mathcal{L}}{dt} = \frac{1}{\mathcal{H}}\frac{d\mathcal{H}}{dz}\frac{dz}{dt} + \frac{1}{N_p}\frac{dN_p}{dt} + \frac{1}{N_{\bar{p}}}\frac{dN_{\bar{p}}}{dt} - \frac{1}{\epsilon_{N,p} + \epsilon_{N,\bar{p}}}\frac{d}{dt}(\epsilon_{N,p} + \epsilon_{N,\bar{p}}) \tag{18}$$

Defining the luminosity and intensity lifetimes as

$$\frac{1}{\tau_{\mathcal{L}}} = -\frac{1}{\mathcal{L}}\frac{d\mathcal{L}}{dt}, \qquad \frac{1}{\tau_p} = -\frac{1}{N_p}\frac{dN_p}{dt}, \qquad \frac{1}{\tau_{\bar{p}}} = -\frac{1}{N_{\bar{p}}}\frac{dN_{\bar{p}}}{dt} \tag{19}$$

while the longitudinal bunch length and transverse emittance growth times are

$$\frac{1}{\tau_s} = \frac{1}{\sigma_s}\frac{d\sigma_s}{dt}, \qquad \frac{1}{\tau_{\epsilon_\perp}} = \frac{1}{\epsilon_{N,p} + \epsilon_{N,\bar{p}}}\frac{d}{dt}(\epsilon_{N,p} + \epsilon_{N,\bar{p}}) \tag{20}$$

Then the luminosity lifetime is

$$\frac{1}{\tau_{\mathcal{L}}} = \frac{z}{\mathcal{H}}\frac{d\mathcal{H}}{dz}\frac{1}{\tau_s} + \frac{1}{\tau_p} + \frac{1}{\tau_{\bar{p}}} + \frac{1}{\tau_{\epsilon_\perp}} \tag{21}$$

Using the expression for the hour-glass form factor $\mathcal{H}$, this can be written as

$$\frac{1}{\tau_{\epsilon_\perp}} = \frac{1}{\tau_{\mathcal{L}}} - (1 - \frac{2z}{\mathcal{H}} + 2z^2)\frac{1}{\tau_s} - \frac{1}{\tau_p} - \frac{1}{\tau_{\bar{p}}} \tag{22}$$

This expression can be used to calculate the emittance growth time $\tau_{\epsilon_\perp}$ from the measured values of the other time scales.

Figure 4 shows the proton and anti-proton bunch intensities an hour before and after the separator failure. There is a clear change in the intensity lifetimes before and after the failure.

Figure 5 shows the bunch length as a function of time over the store. There is evident growth in the bunch length over the 15 hours of the store but from the data now available (bunch lengths every 15 minutes) it is not possible to discern a change in the growth of the bunch length after the separator failure. We will assume that the growth rates of the bunch length were the same an hour before and an hour after the failure. Table 1 shows the measured lifetimes and growth times and the calculated transverse emittance growth time before and after the failure.

	Before separator failure	After separator failure
Luminosity lifetime [hrs]	17	10
Proton lifetime [hrs]	198	100
Anti-proton lifetime [hrs]	64	36
Average Bunch length [nsec]	2.6	2.6
Bunch length growth time [hrs]	81.5	81.5
Transverse Emittance growth time [hrs]	24.5	15.3

Table 1: Lifetimes and growth times from data 1 hour before and an hour after the separator failure. The transverse emittance growth rates are calculated from Equation 22.

2.1 Beam lifetime

The separator failure changed the beam orbits around the ring. Figure 6 and 7 show the changes in the proton orbit in units of the rms beam size as calculated by MAD. The maximum horizontal orbit change is 1.8σ radially outwards while the rms orbit change is 0.59σ. In the vertical plane, the maximum orbit shift is about 1.8σ downwards but the rms orbit change is smaller, 0.24σ, as expected.

Figure 8 shows the proton horizontal orbit change at all the long shadow collimators in the ring. `More here.`

We have seen that the emittance growth rate increased and the orbits changed significantly after the separator failure. Could these two phenomena explain the sharp drop in beam lifetime?

We proceed by assuming that the beam density distribution function evolves according to the diffusion equation. For simplicity we will consider the transverse distribution function can be decoupled as the product of horizontal ρ_x and vertical ρ_y distribution functions and consider only the evolution of ρ_x,

$$\frac{\partial}{\partial t}\rho_x = \frac{\partial}{\partial J_x}\left[D(J_x)\frac{\partial \rho_x}{\partial J_x}\right] \tag{23}$$

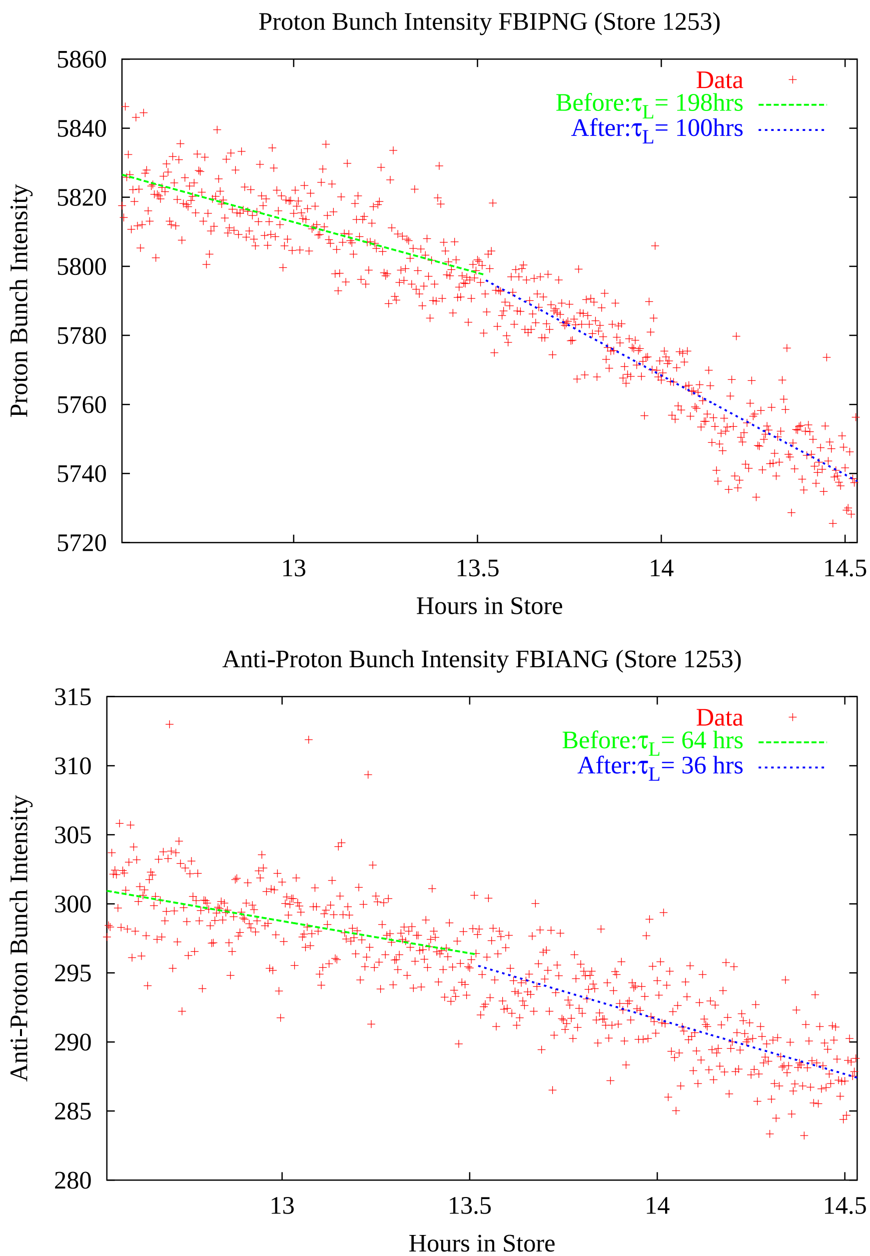

Figure 4: Proton bunch intensity (top) and anti-proton bunch intensity (bottom) before and after the separator failure.

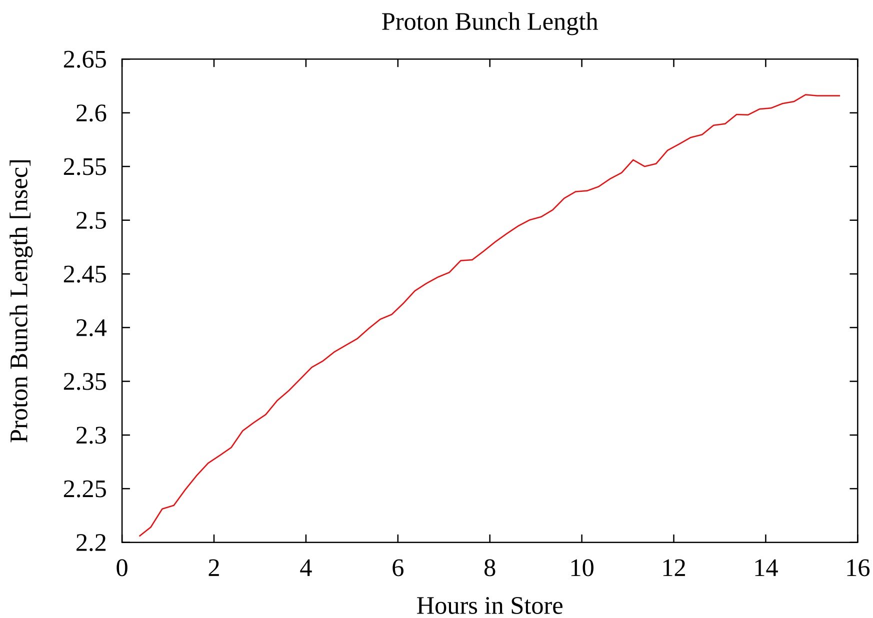

Figure 5: Proton bunch length during the store. The separator failure occurred about at 13.5 hours into the store.

Here $D(J_x)$ is the diffusion coefficient in the action $J_x = [x^2 + (\beta_x x' + \alpha_x)^2]/\beta_x$. The limiting physical aperture is assumed to be in the horizontal plane at an amplitude A_x and corresponding action at the aperture is $J_A = A_x^2/\beta_x$. Under the assumption of independence of the treansverse planes, the total number of particles in the beam at time t can be written as $N(t) = N_x(t)N_y(t)$ where N_x is defined by

$$N_x(t) = \int_0^{J_A} \rho_x(J_x, t)\, dJ_x \tag{24}$$

and a similar expression for N_y. We assume that particles that diffuse out to the aperture J_A are lost.

The beam emittance ϵ_x is related to the average action which is defined as

$$\langle J_x \rangle = \int_0^{J_A} J_x \rho_C(J_x, t)\, dJ_x \tag{25}$$

where ρ_C is the conditional density which accounts for the particle number changing in time and hence is defined as $\rho_C = \rho_x/N_x$. From the definition it is clear that ρ_C is normalized to unity; $\int_0^{J_A} \rho_C\, dJ_x = 1$. It follows from the diffusion equation that the average action evolves as

$$\frac{d}{dt}\langle J_x \rangle = \int_0^{J_A} D'(J_x)\rho_C\, dJ_x + [J_A - \langle J_x \rangle] D(J_A) \frac{\partial \rho_C}{\partial J_x}(J_A) \tag{26}$$

If the density falls sufficiently rapidly and the aperture is far enough away from the beam, then $\partial \rho_C(J_A)/\partial J_x \to 0$ and the second term in the above equation can be dropped. With this simplification,

$$\frac{d}{dt}\langle J_x \rangle = \int_0^{J_A} D'(J_x)\rho_C\, dJ_x \tag{27}$$

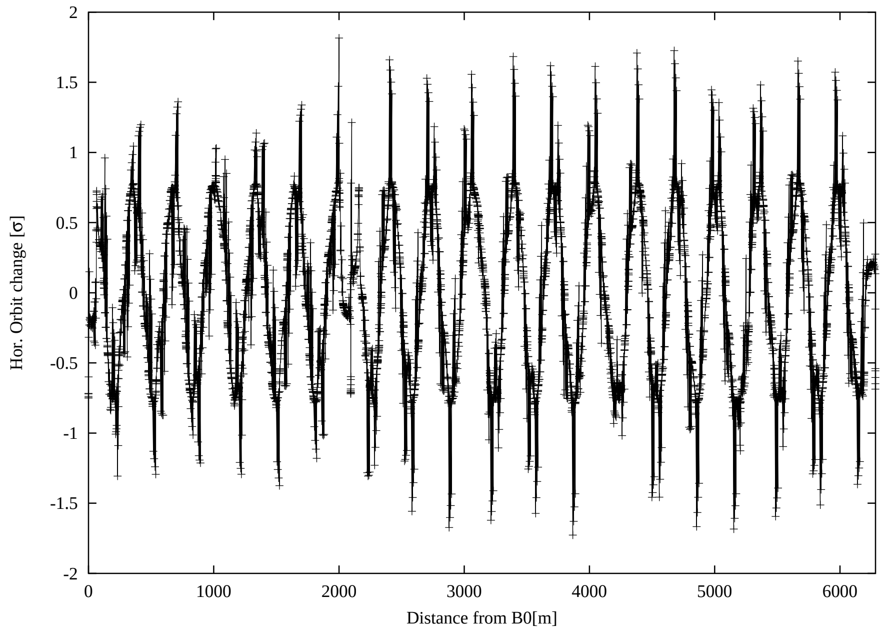

Figure 6: Change in the horizontal proton orbit (in units of the rms beam size) around the ring due to the separator failure. The rms orbit change is 0.59σ.

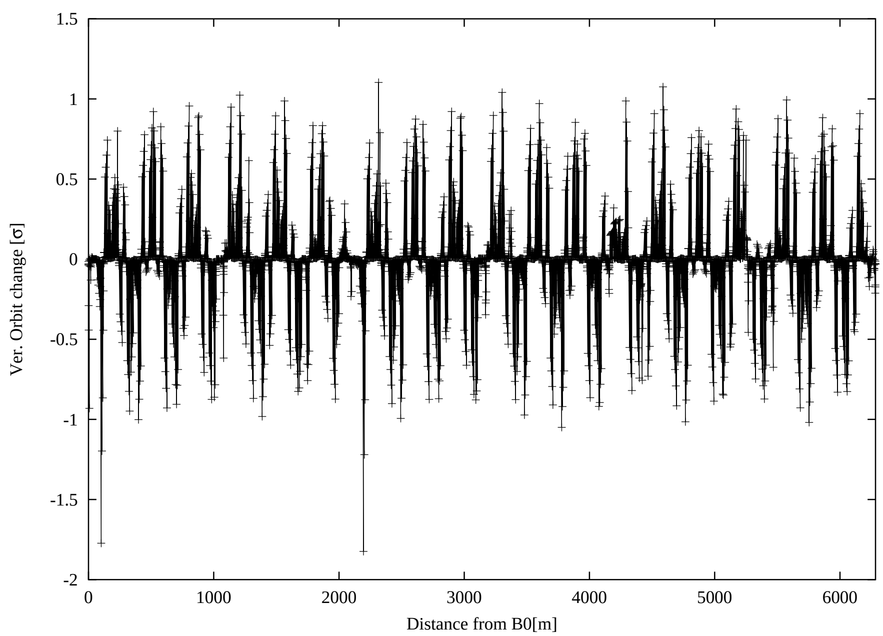

Figure 7: Change in the vertical proton orbit (in units of the rms beam size) around the ring due to the separator failure. The rms orbit change is 0.24σ.

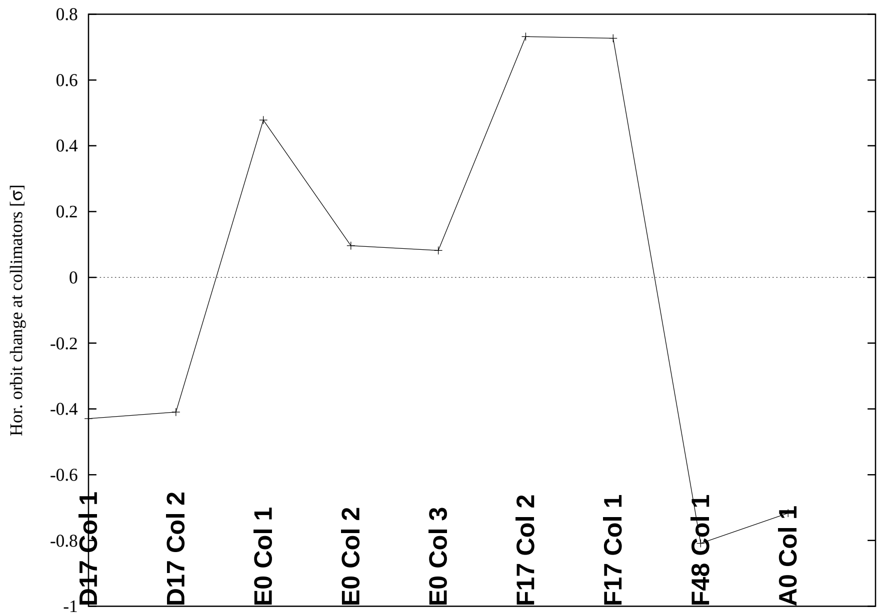

Figure 8: Change in the horizontal proton orbit (in units of the rms beam size) at the collimators due to the separator failure. The largest change was at the F17 collimators where the protons moved by about 0.73 σ to the radial outside.

If the diffusion coefficient increases linearly with the action,

$$D(J_x) = D_0 J_x \tag{28}$$

then it follows

$$\frac{d}{dt}\langle J_x \rangle = D_0 \tag{29}$$

Since we observe that the proton emittance increases nearly linearly with time during a store, we may justifiably assume that Equation (28) is valid.

We can now calculate time scales associated with the diffusive motion. The *mean escape time* for particles to travel to the absorbing boundary at J_A is defined as

$$t_{esc} = \int_0^{J_A} \frac{J_x}{D(J_x)}\, dJ_x \tag{30}$$

Assuming Equation (28), we obtain

$$t_{esc} = \frac{J_A}{D_0} \tag{31}$$

This escape time is related to the beam lifetime, the one observable time scale.

The lifetime can be calculated by a more complete analysis as in Edwards and Syphers [2]. The diffusion equation can be solved analytically when the diffusion coefficient is linear or quadratic in the action. With a linear dependence as assumed above, the density at time t is

$$\rho_x(J_x, t) = \sum_n c_n J_0(\lambda_n \sqrt{\frac{J_x}{J_A}}) \exp[-\frac{\lambda_n^2}{4}\frac{D_0 t}{J_A}] \tag{32}$$

where

$$c_n = \frac{1}{J_1^2(\lambda_n) J_A} \int_0^{J_A} \rho_0(\frac{J_x}{J_A}) J_0(\lambda_n \sqrt{\frac{J_x}{J_A}})\, dJ_x \tag{33}$$

J_0, J_1 are the zeroth and first order Bessel functions, the λ_n's are the n'th roots of J_0 and ρ_0 is the initial density. For an initially Gaussian distribution in phase space, the distribution in action is an exponential,

$$\rho_0(J_x) = \alpha \exp[-\frac{\alpha J_x}{J_A}]\,, \qquad \alpha = \frac{A^2}{2\sigma_x^2} \tag{34}$$

Assuming that the beam is sufficiently far from the aperture, the coefficients c_n simplify in this case to

$$c_n = \frac{1}{J_1^2(\lambda_n)} \exp[-\frac{\lambda_n^2}{4\alpha}] \tag{35}$$

Keeping only the first and dominant term in the solution for the density Equation 32, the partial number of particles N_x in the beam simplifies to

$$N_x(t) \simeq \frac{2}{\lambda_1 J_1(\lambda_1)} \exp[-\frac{\lambda_1^2}{2}(\frac{\sigma_x}{A})^2] \exp[-\frac{\lambda_1^2}{4}\frac{D_0 t}{J_A}] \tag{36}$$

From this it follows that the lifetime defined as

$$t_L = -\frac{N_x}{dN_x/dt} = \frac{4}{\lambda_1^2}\frac{J_A}{D_0} \simeq 0.7\frac{J_A}{D_0} \tag{37}$$

This expression is very close to the mean escape time t_{esc} calculated in Equation 31.

We will now use Equation 37 to relate the beam lifetimes before and after the separator failure. Equating

$$D_0 \equiv \frac{d\epsilon_x}{dt} = \frac{\epsilon_0}{\tau_\epsilon} \tag{38}$$

where ϵ_0 is the initial emittance and τ_ϵ is the emittance growth time. Then

$$t_L = 0.7\frac{J_A}{\epsilon_0}\tau_\epsilon \tag{39}$$

Before the separator failure, the beam aperture was approximately 6σ at one or more of the collimators. Hence before the failure, $J_A = (6\sigma)^2/\beta_x$. After the failure the beam moved closer to the physical aperture by 0.7σ. Hence $J_A = (5.3\sigma)^2/\beta_x$ after the failure.

$$\Rightarrow \frac{t_L(\text{after})}{t_L(\text{before})} = \frac{J_A(\text{after})}{J_A(\text{before})}\frac{\tau_\epsilon(\text{after})}{\tau_\epsilon(\text{before})} \tag{40}$$

From Table 1 we find that $\tau_\epsilon(\text{before}) = 24.5$hrs and $\tau_\epsilon(\text{after}) = 15.3$hrs. Hence

$$\frac{t_L(\text{after})}{t_L(\text{before})} = (\frac{5.3}{6})^2\frac{15.3}{24.5} = 0.49 \tag{41}$$

From Table 1 we find that the ratio of the lifetimes is $= 100/198 = 0.51$. Hence the predictions of the one dimensional theory are in very good agreement with the observed drop in lifetime.

The question remains why the emittance growth rate increased when the beam moved horizontally after the failure. It could be due to increased nonlinearities or possibly some other source.

No matter what the source of emittance growth, we have shown that the drop in lifetime is consistent with diffusive emittance growth and the beam center moving closer to a physical aperture.

References

[1] Handbook of Accelerator Physics

[2] D. Edwards and M.J. Syphers,*An Introduction to the Physics of High Energy Accelerators*, John Wiley