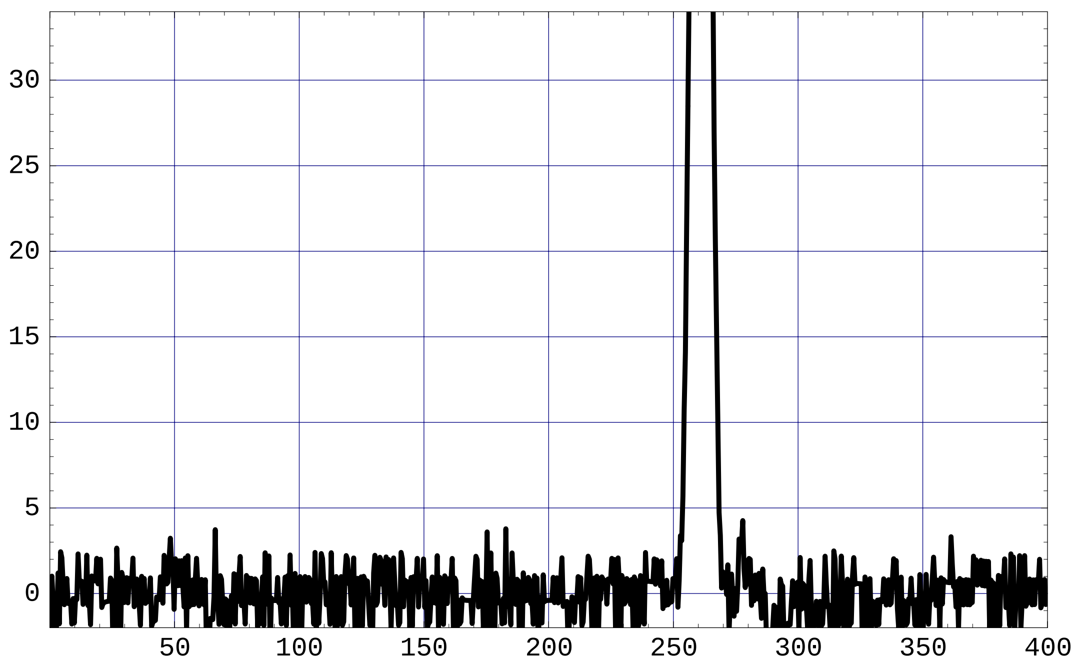
SBD RESPONSE AND A
TOY ALGORITHM FOR UNSCRAMBLING LONGITUDINAL PHASE SPACE

A.V.T 04-04-03

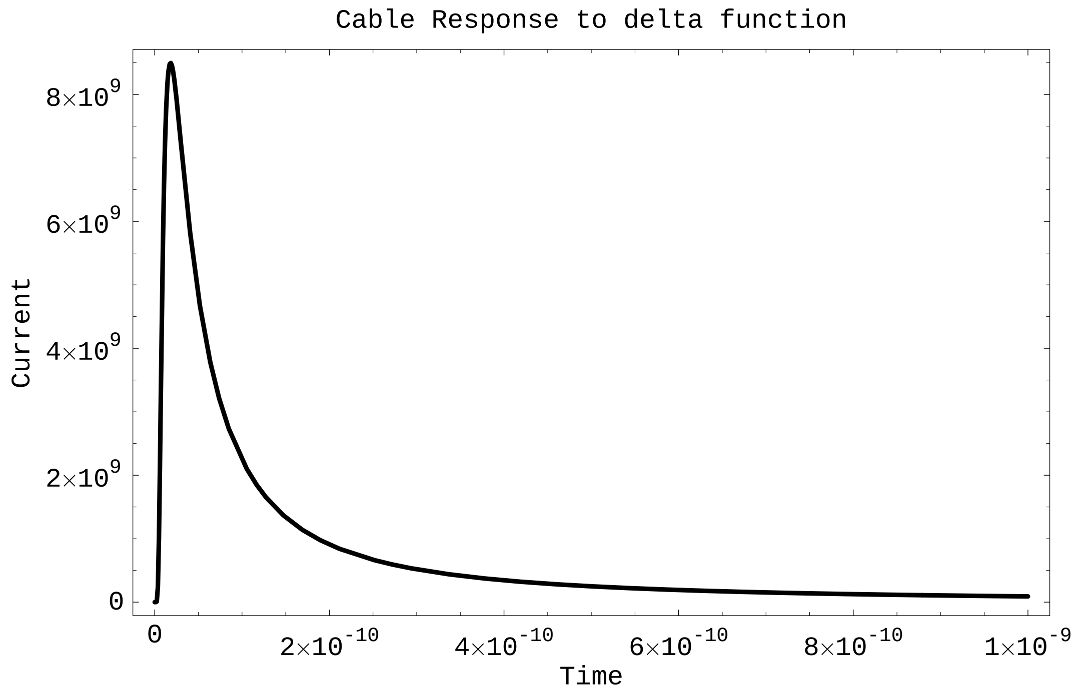
I. SBD response

1. S/N not good enough to study beam effects in a single bucket.



The rms noise on the baseline is 1.4 counts for both p and pbar channels.

2. Cable Dispersion adds a tail:



The delta function response is given by:

$$\frac{2.94306 \times 10^{-6}\, 2.71828^{-2.72112 \times 10^{-11}/t}}{t^{3/2}}$$

The total charge in the pulse at 200 ns is 98.7% and at 400 ns is 99.1%. The high freq response is set by the 0.2 ns peak in the above curve. The cable does not degrade the pulse…..the 1 ghz bandwidth of the scope does! Invest in a better scope if you want better information. The dispersive tail can be corrected. Bob Flora showed a filter to do this. It can be made exact by using the above response function. With the help of Flora, Crisp, Scarpini, Pordes, we measured the cable response and the tail description fits quite well.

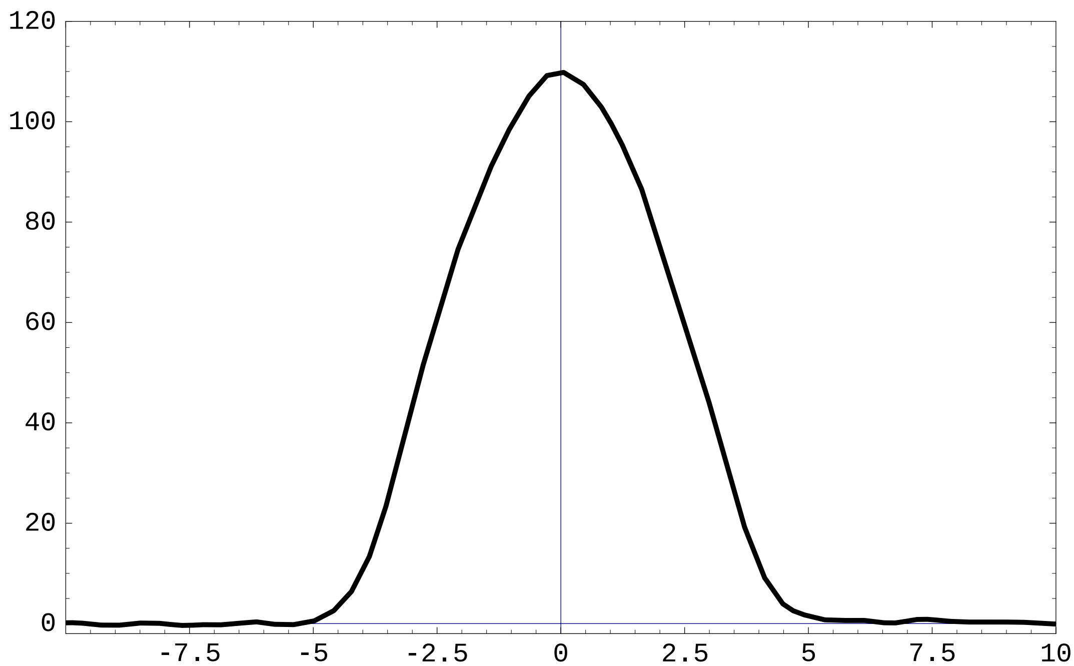
20	0.958398
40	0.970576
60	0.975974
80	0.979192
100	0.981388
120	0.98301
140	0.98427
160	0.985286
180	0.986127
200	0.986839
220	0.987451
240	0.987985
260	0.988457
280	0.988877
300	0.989254
320	0.989595
340	0.989906
360	0.99019
380	0.990452
400	0.990693

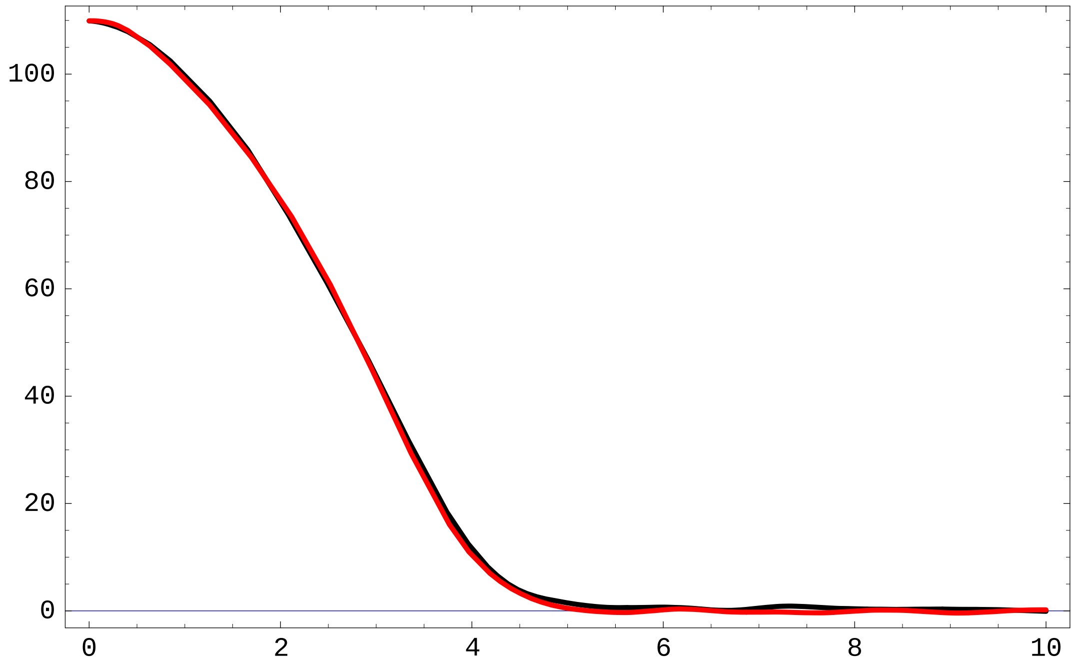
The following shows the effect of the correction on the raw data (the base line correction has been made, the time is in 0.5 ns increments):

To get better statistics, all 36 pulses are superimposed on top of each other. The technique is the following:

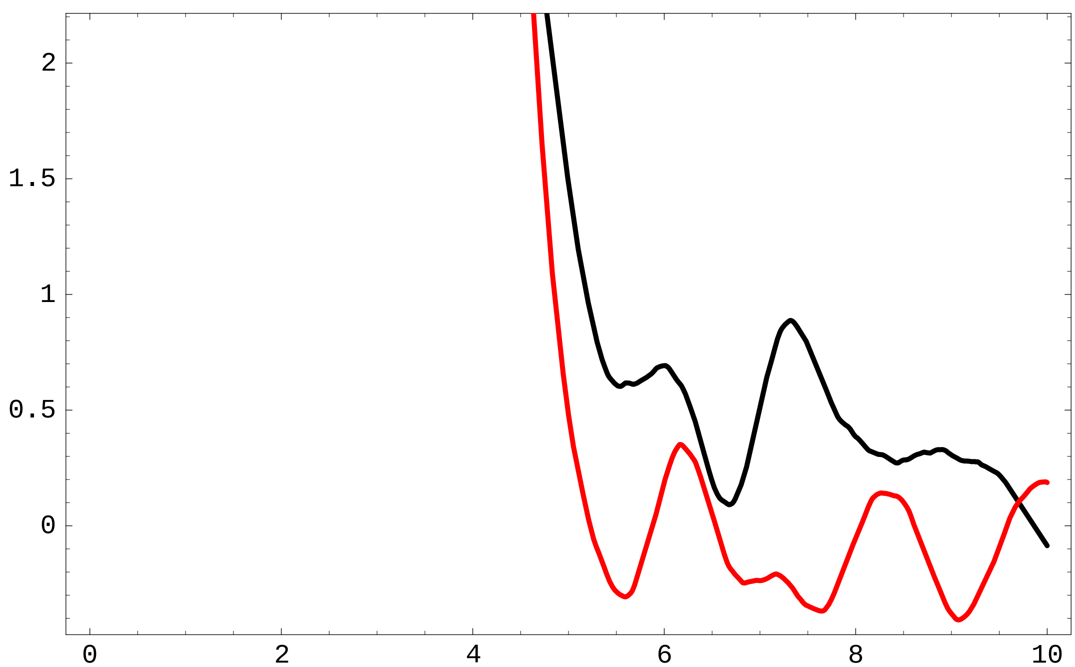
a. **Fit each bunch with a constant, slope, + a Gaussian to find the time. This gives a time to about 30 ps for the center of the pulse. Next make an interpolation function for the whole trace and use the times from the Gaussian fit for superimposing the pulses. The results for Store 2328 are shown below:**

Sum of 36 proton pulses at 980 GeV. The figure following shows the two halves plotted on top of each other.

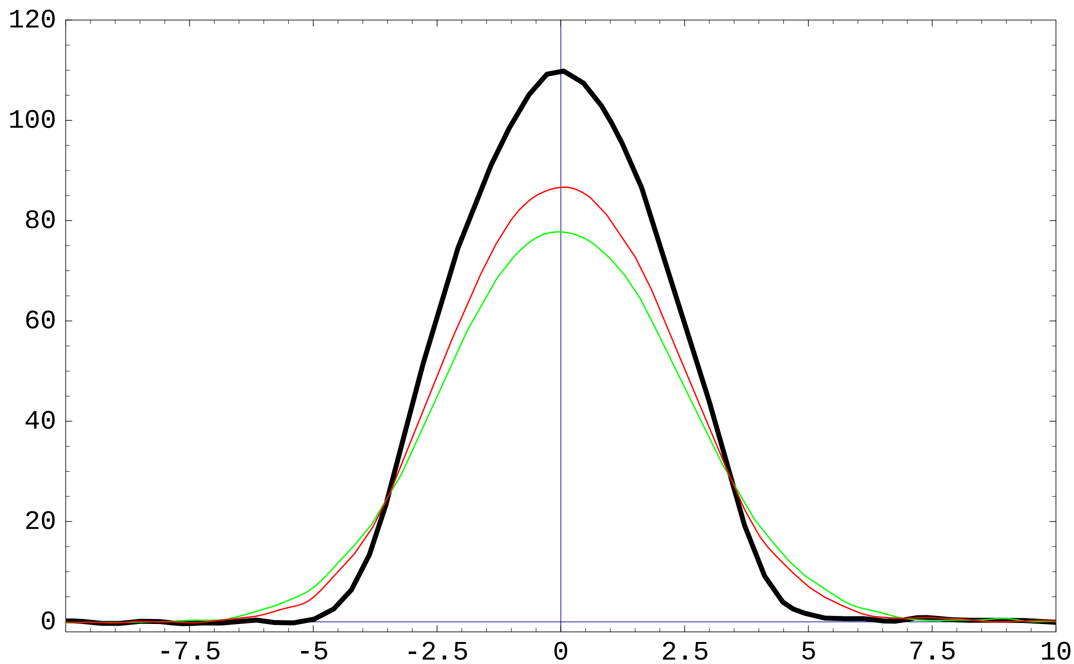




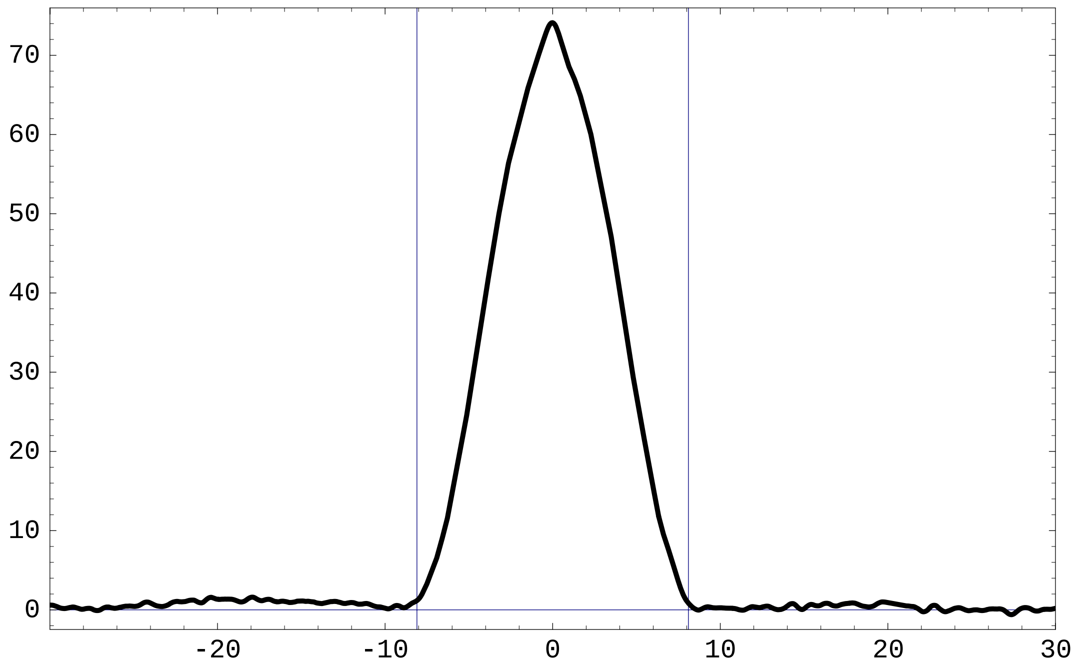
The following shows the above on an expanded scale showing the excellent correction for dispersion. Note the vertical scale.



The sum proton pulses during Store 2328 at 980 GeV:

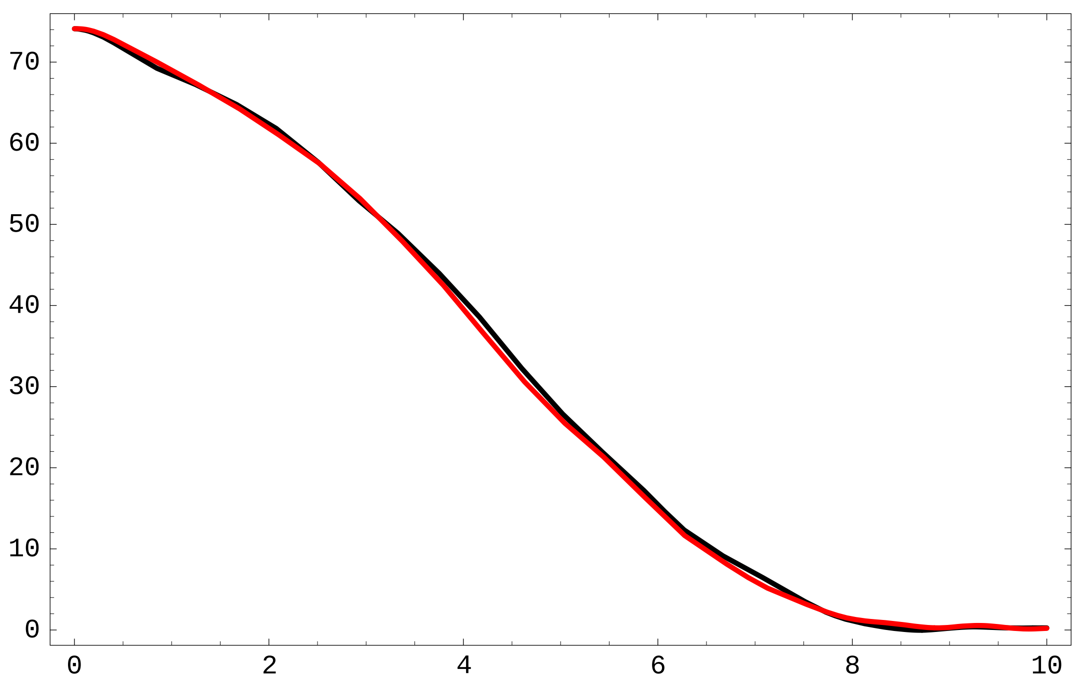


Same at 150 GeV just after pbar injection finished. The slight asymmetry and the very sharp peak is the result of non-equilibrium distribution. The lines are at the limit of what can be accelerated.



These two curves are at 150 GeV just after pbar loading.

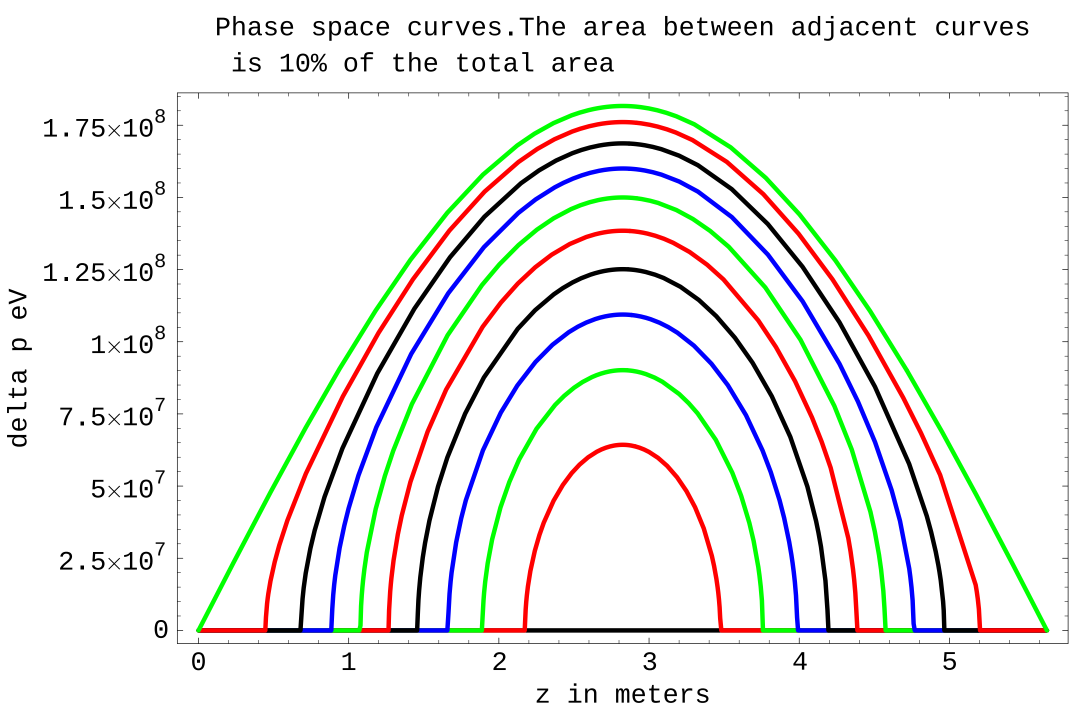
Plot the two halves on top of each other:



II. USE THE ABOVE DATE TO GET LONGITUDINAL PHASE SPACE DISTRIBUTION

Approach

Consider the longitudinal phase space plot shown in the figure below.



We know from Liouville's Theorem that the density along any of the trajectories shown will be constant. Thus if we wish the density at $\rho(z,p)$, it can be obtained from $\rho(z_0,p_0)$ where z_0 is at the synchronous phase, $\lambda/2$, and p_0 is on the same trajectory as (z,p). This is just a reflection of the fact that for an equilibrium distribution, there is only one variable on which the density depends. The natural variable to pick is the "action" or the area of the associated phase space "ellipse".

The connection between the two points is easy to calculate because the Hamaltonian is constant:

(0.1) $$p^2 + \frac{2v^2 E_s eV_{rf}}{\eta hc^2} Cos(2\pi z/\lambda) = p_0^2 + \frac{2v^2 E_s eV_{rf}}{\eta hc^2} Cos(2\pi z_0/\lambda)$$

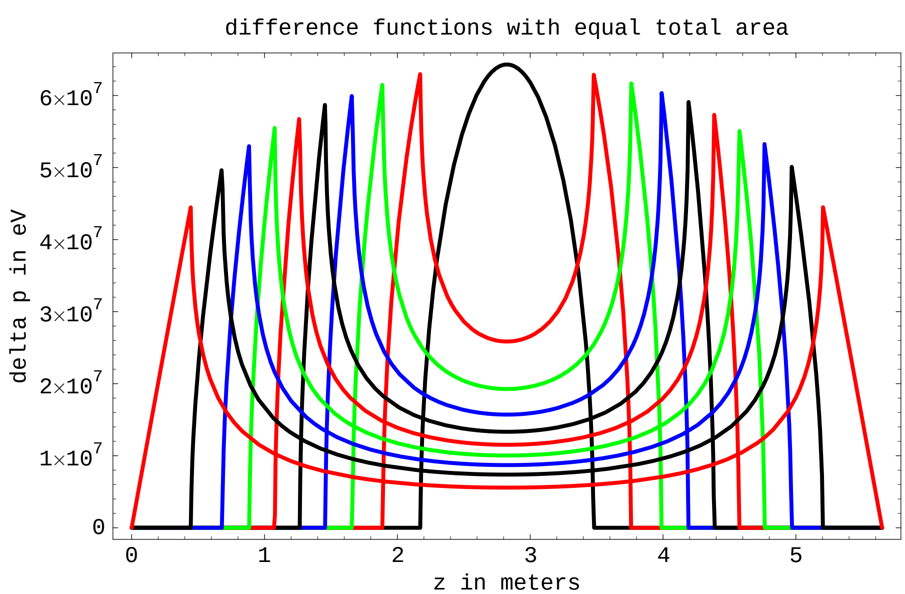
where p is the momentum deviation, λ is the wavelength, E_s is the energy, η is the slip factor, h is the harmonic number and V_{rf} is the cavity voltage. Applying this to the case where z_0 is on the center line:

(0.2)
$$p_0^2 = p^2 + \frac{2v^2 E_s e V_{rf}}{\eta h c^2}(Cos(2\pi z/\lambda)+1)$$

Thus the problem of finding the density function becomes one of finding the density along the p axis at the synchronous phase.

The expansion functions

The figure above was drawn with the area difference between curves 10% of the total area or in steps of 10% of the action. Approximately the density will be constant in these bands. This leads us to propose using these difference functions as a set of functions in which to expand the proton current. Since the density is constant, the total number of protons in a slice of z is just proportional to the height of the individual curves. The difference functions shown below correspond to the first figure.



The figure shows the case for ten curves, but in fact we will use of the order of fifty. The curves have equal areas and so we need a recipe to calculate the curves, ie we need to associate each curve with its action which is the average value of the action for the two bounding phase space curves. We need a formula that connects the action S with its corresponding phase curve. But the phase space curve is completely determined by specifying the value of p_0 on the central axis. So the relation we seek is given by using eq.1.2 above:

(0.3)
$$S = \int \sqrt{p_0^2 - \frac{2v^2 E_s eV_{rf}}{\eta hc^2}(Cos(2\pi z/\lambda)+1)dz}$$

This is an elliptic integral that is easily done and gives $S(p_0)$. In a practical sense it easiest to integrate over z from zero to λ and take the real part after the integral is done. However, the relation we have has to be inverted so that we have $p_0(S)$ so that we can generate the equally spaced curves inaction, S. This is done easily by making a numerical table of S vs p_0 , and from this table generate an interpolation function that gives $p_0(S)$. Once this function is known, the curves are easily generated.

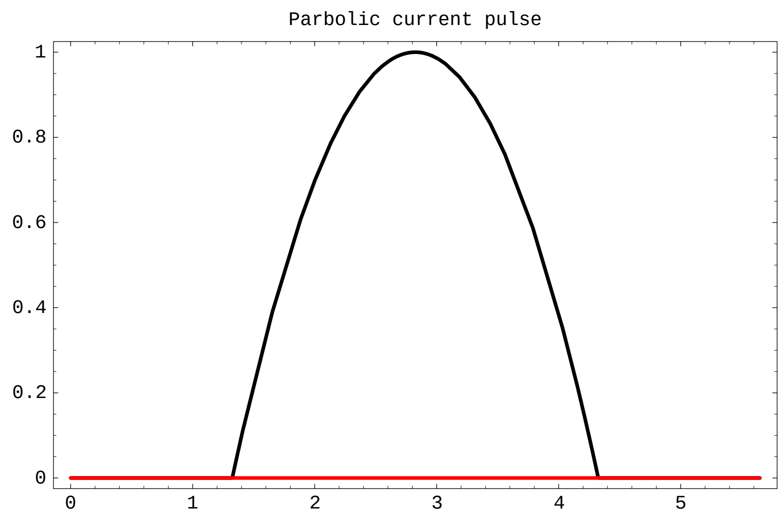
If we designate the difference functions by $F_i(z)$, then we have

(0.4)
$$I_p(z) = \sum_i C_i F_i(z)$$

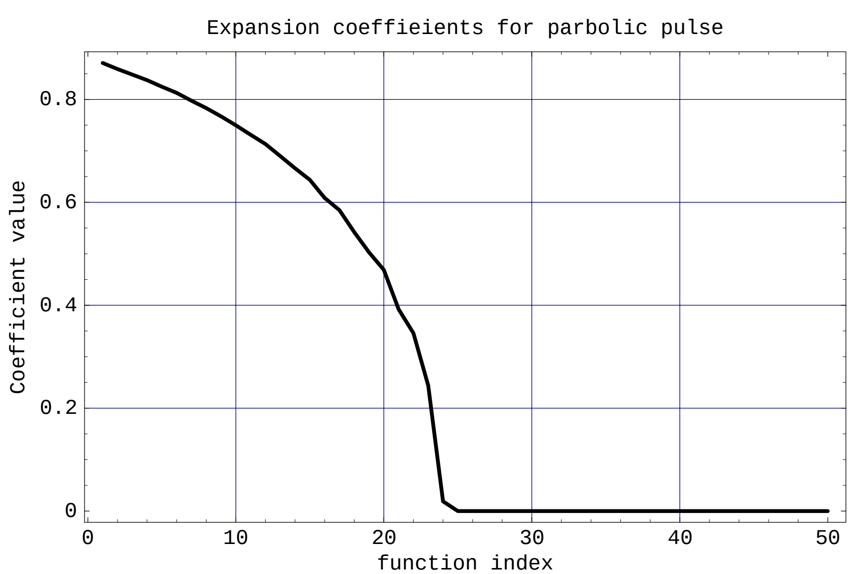
The C_i then give the density as a function of the index i which is directly related to the action and the density along the central axis. The constants are made by breaking up the proton current pulse into a number of discrete points and making a least squares fit using eq. 1.4.

Also note that we are considering equilibrium distributions (no dancing pulses….). Since the density depends on only one variable, we choose the action. We will apply the algorithm to the composite proton pulse at injection, which is obtained by superimposing all 36 proton bunches to make an ensemble as explained above.

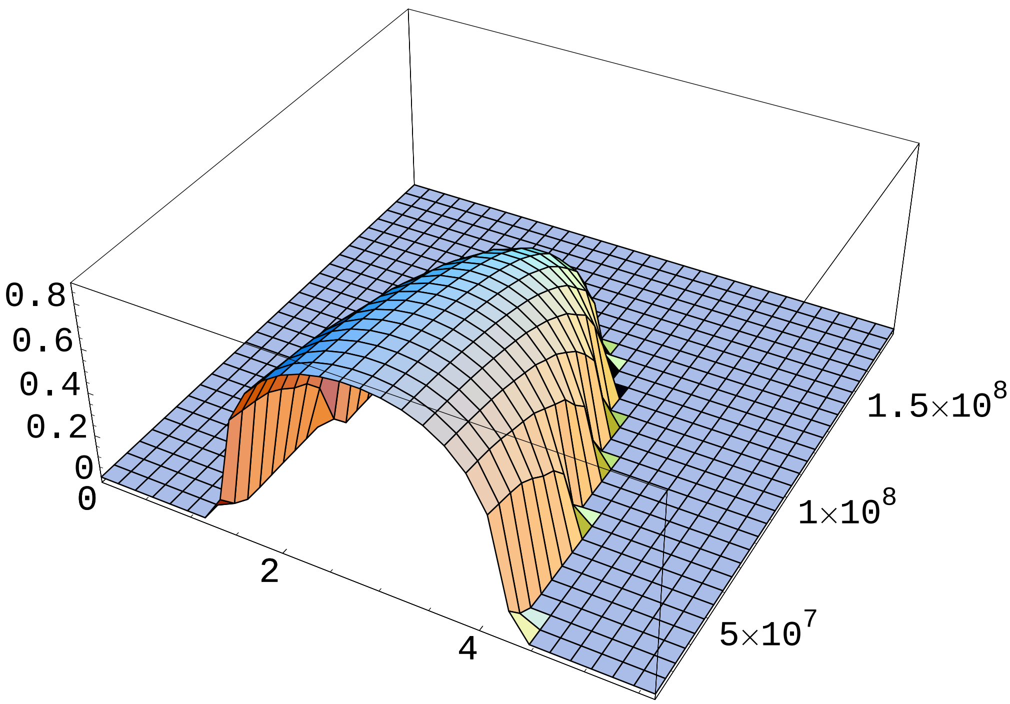
First consider an example of a parabolic proton pulse shown below:

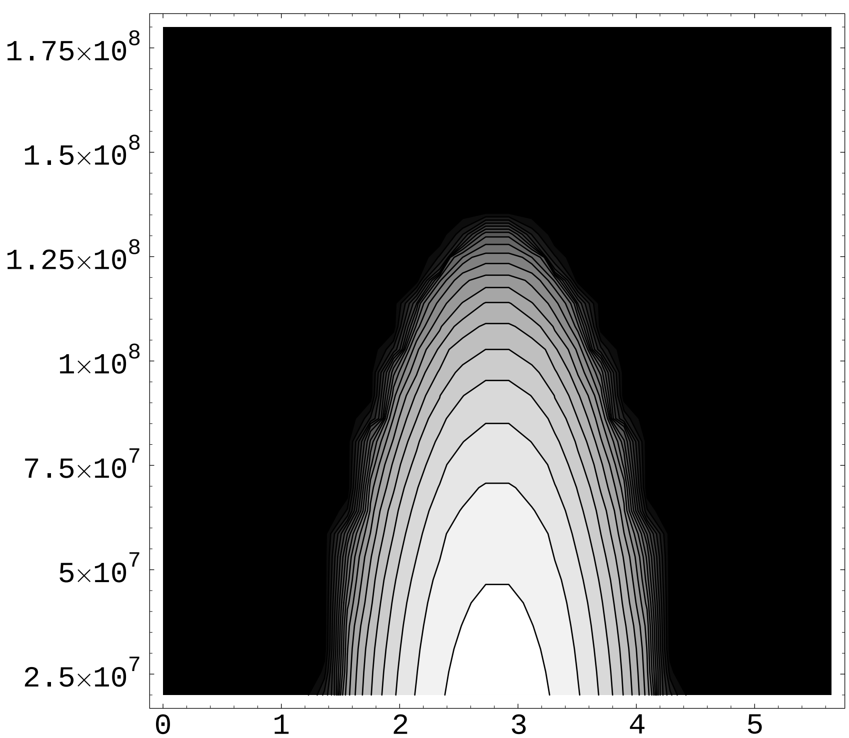


The z axis is in meters. The figure below shows the coefficients of the expansion. I have used 101 data points and 50 fitting functions and made an rms fit to the input data. The original proton current and the fit differ by less than 1%.



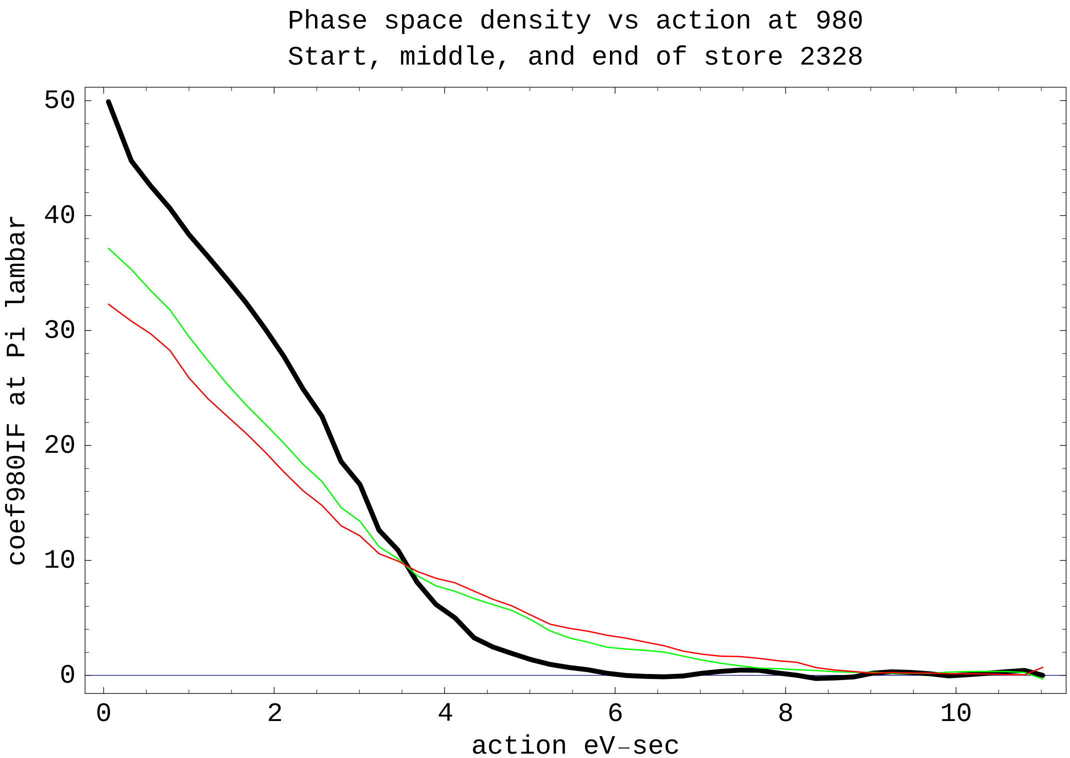
A reconstruction of the phase space density is shown in the next plot.


0.8
0.6
0.4
0.2
0
0
2
4
5×10^7
1×10^8
1.5×10^8


1.75×10^8
1.5×10^8
1.25×10^8
1×10^8
7.5×10^7
5×10^7
2.5×10^7
0
1
2
3
4
5

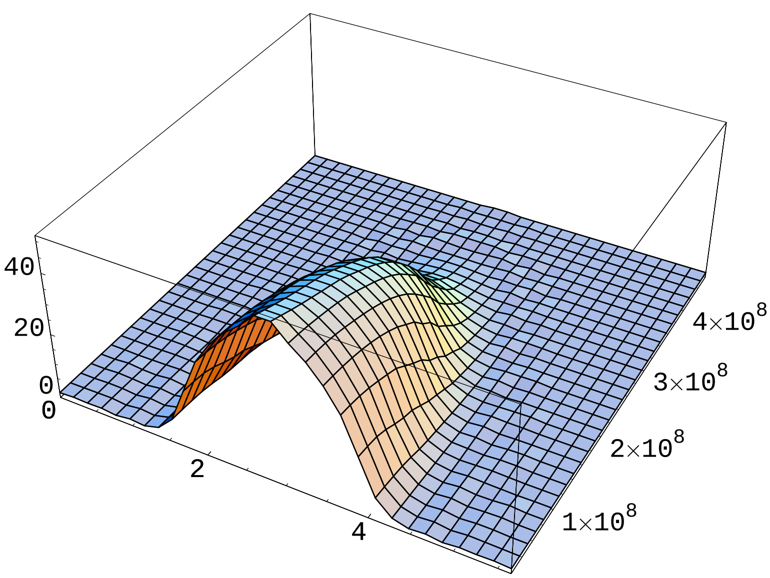
RUN 2328

A plot of the coefficients at 980 is:

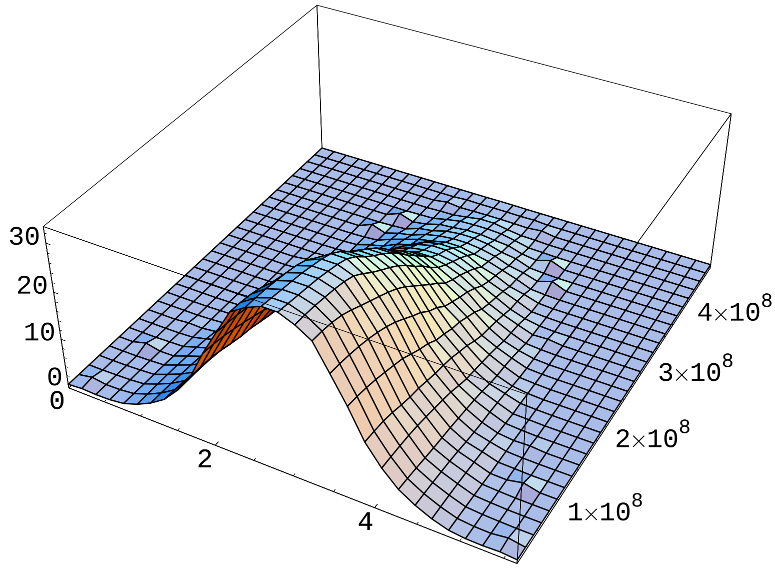


The horizontal scale is the index number and is proportional to the action.

The density plot is:

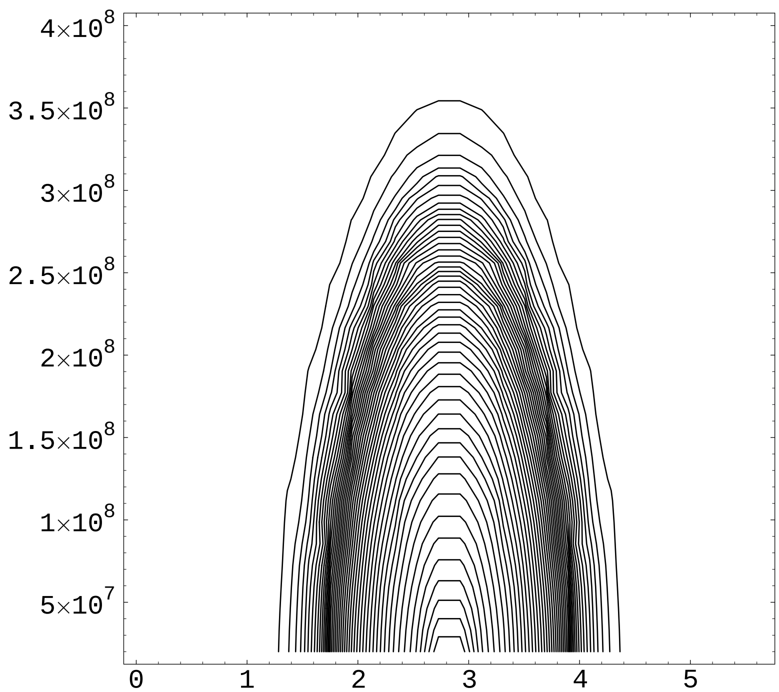


The z is in meters, p in eV, and vertical is relative density.

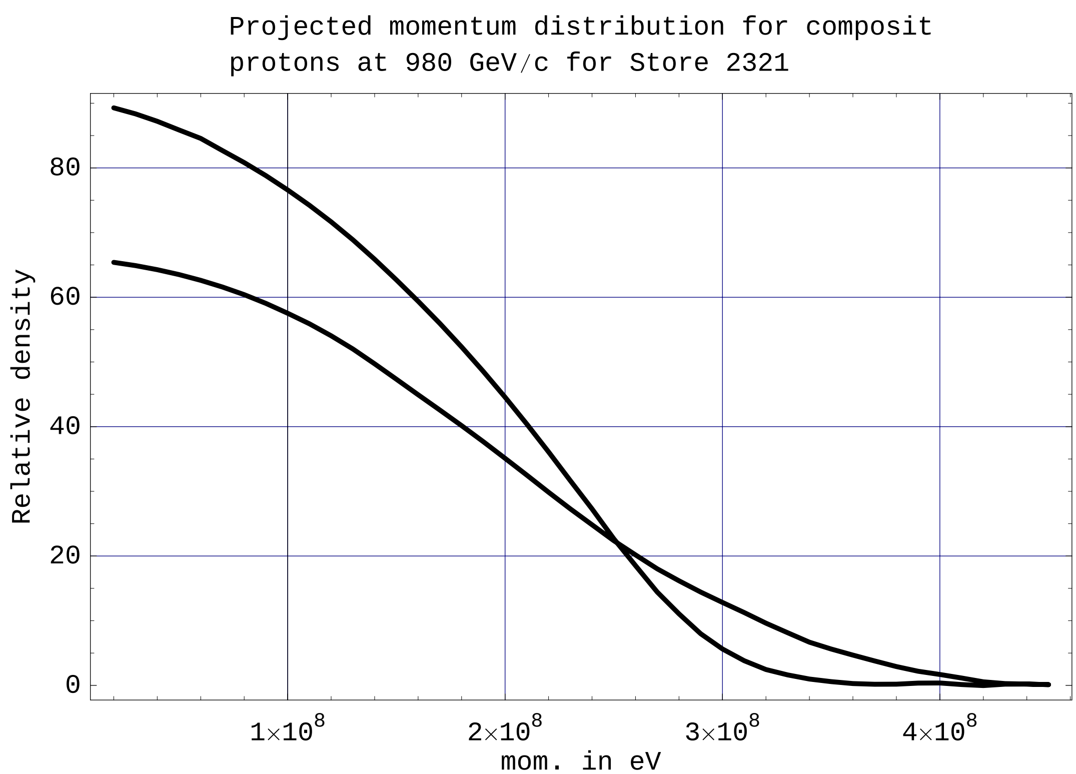


At the end of the store

A contour plot just after acceleration:

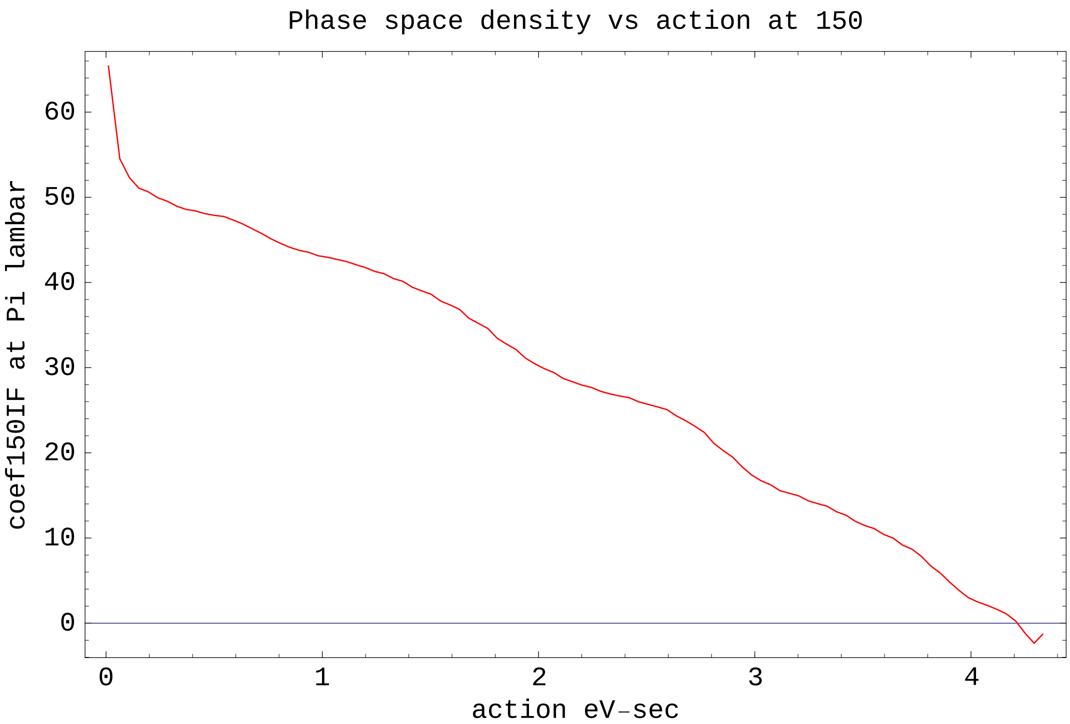


The above can be projected out onto the momentum axis:

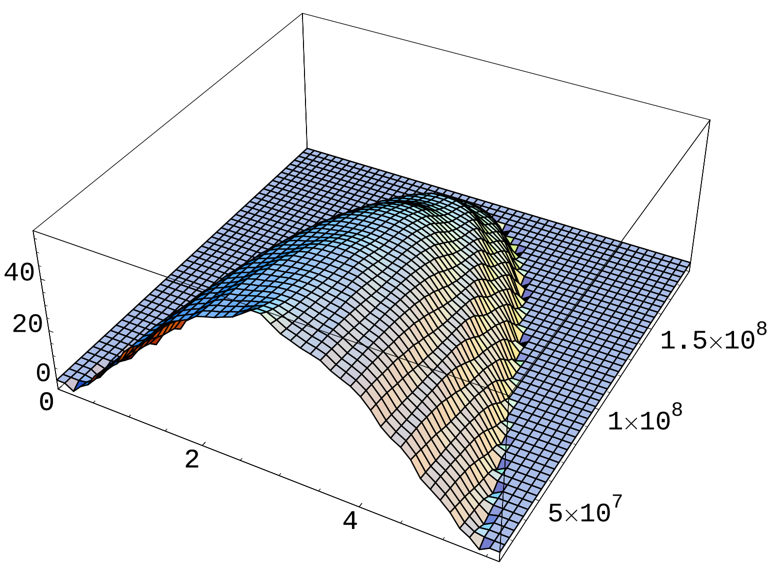


The momentum scale depends on eta and Vrf. We need accurate values for these. Does the sync frequency provide a constraint?

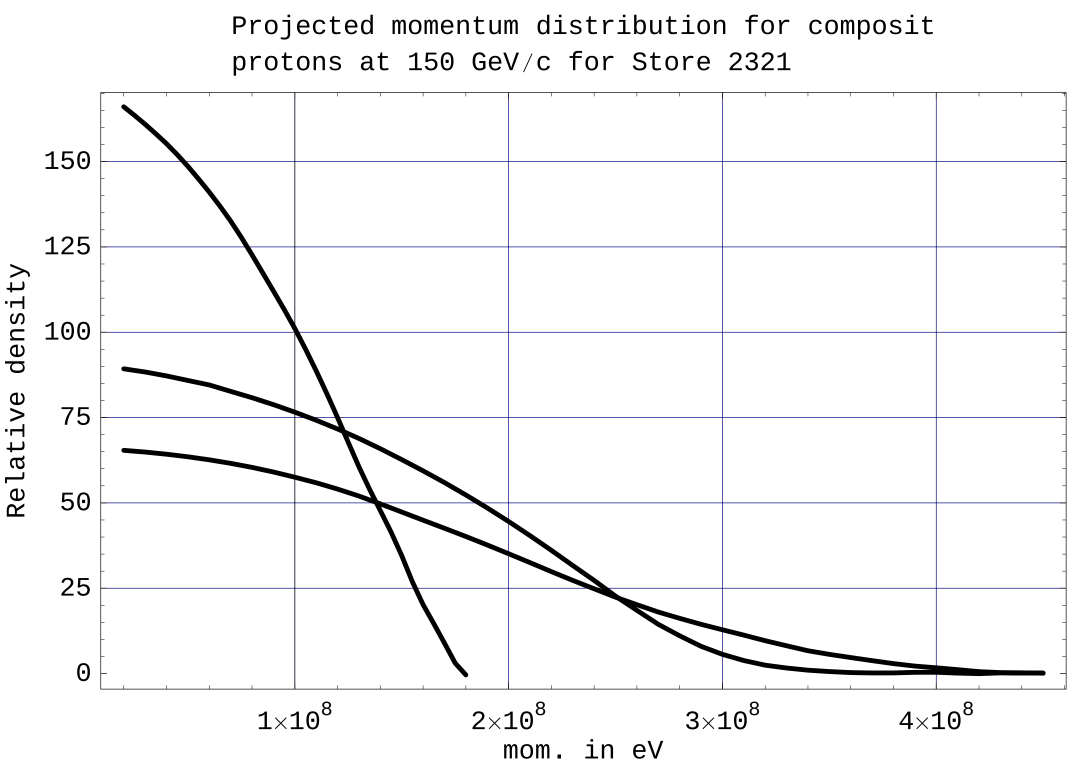
The above plots have been at 980. The following is at 150 just before acceleration.



The peak for the first coefficient is caused by a non-equilibrium distribution seen in the proton current. The distribution is much broader than at 150:

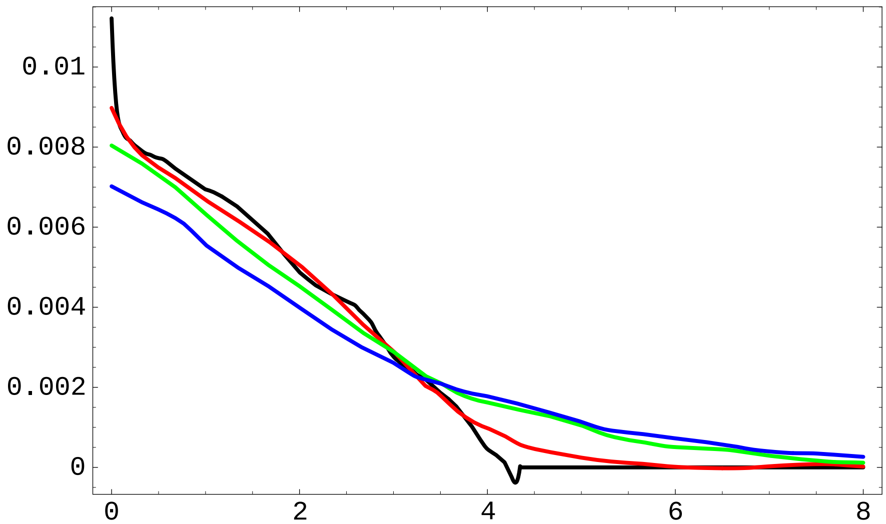


The momentum distribution before and after acceleration and at the end of the store:



Since the phase space density is an invariant, we can compare at different times. We must normalize for the particle loss with time:

Comparison of charge/ev-sec vs action
The charge loss has been normalized out
Black 150
Red 980 end of acceleration
Green, middle of store
Blue end of store



The black curve is at 150, the red just after 980. The normalized areas are in the ratio of .0197 / .0200.

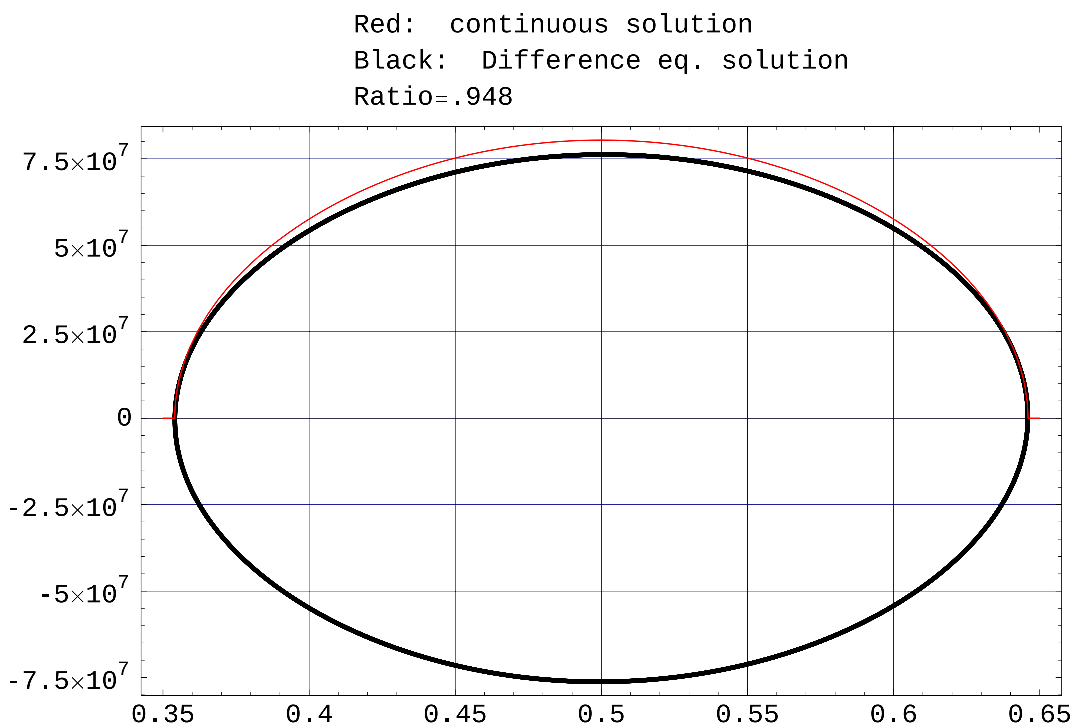
RMS CALCULATION

From the momentum distribution, we can get the RMS and compare it with the linear prediction.

Ratio of Predicted / observed = 1.102 150 GeV just before accel.

1.63 980 GeV just after acceleration

There is also an effect from the discreet nature of the acceleration:



The black curve comes from solving the difference equation for the motion at 150 GeV. The difference in max energy excursion seems to be independent of the initial conditions. The same 5% effect is seen for different starting positions and is also the same at 980 GeV.

These two effects are in the same direction and can make the actual sigma smaller by 15% than predicted and will have an important effect in calculating emittances.