Orbit shifts due to long-range beam-beam interactions

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1 Observations of bunch by bunch orbits

At 980 GeV the synchrotron light monitor at C11 shows the orbit positions of individual proton and anti-proton bunches. Figure 1 shows the positions of proton bunches while Figure 2 shows the positions of anti-proton bunches in Store 1787 (September 24, 2002). The maximum spread between proton

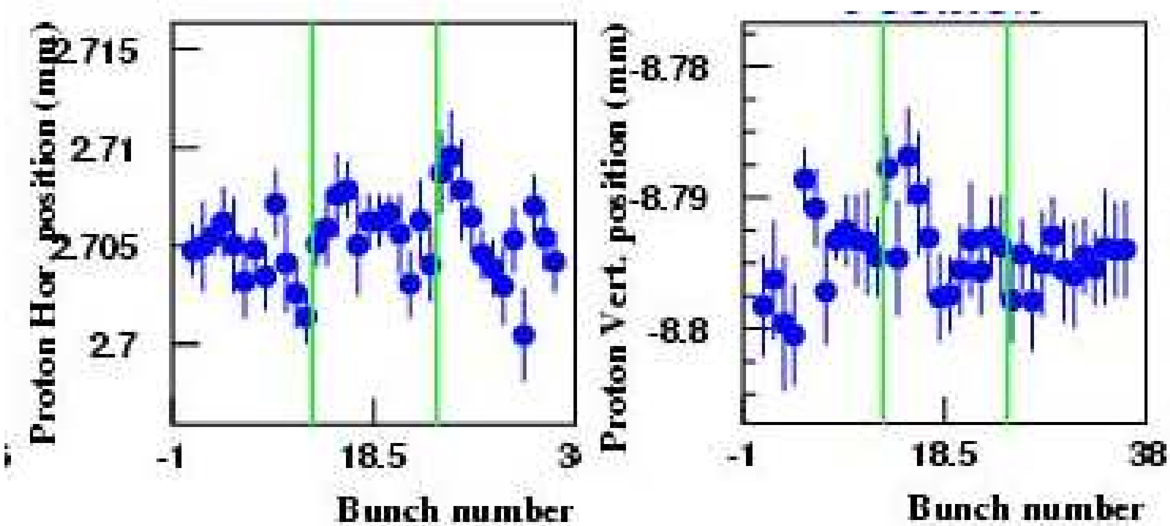

Figure 1: Positions of proton bunches observed at the synchrotron light monitor in Store 1787.

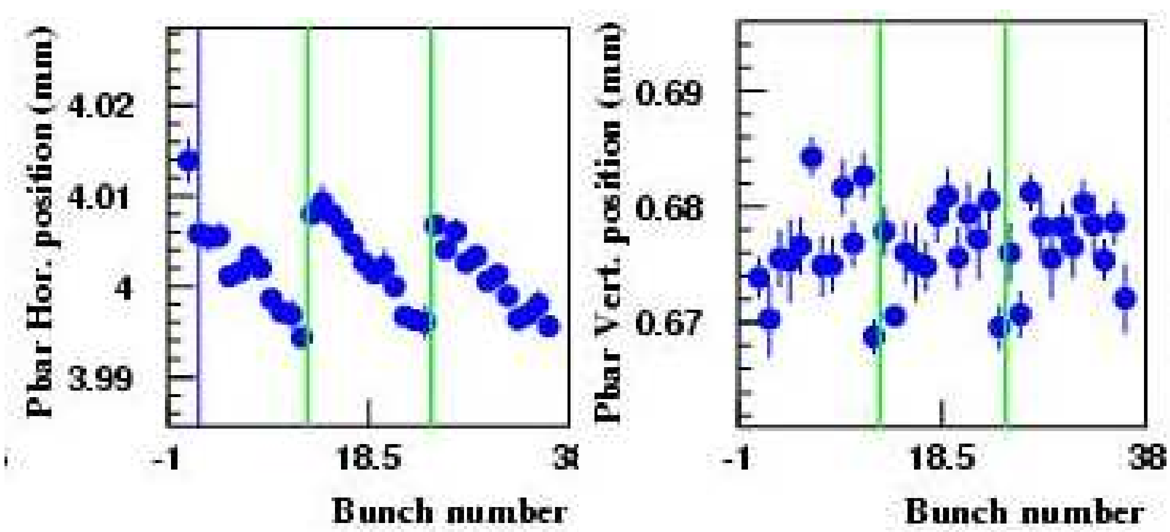

Figure 2: Positions of anti-proton bunches observed at the synchrotron light monitor in Store 1787.

bunches in a train is about 10 microns in both planes but the spread is very different between the trains. For example, the vertical spread is small (within the resolution of the synchrotron light monitor) in the third train and the horizontal spread is small in the first two trains. On the other hand, the horizontal position of the anti-proton bunches shows the same trend in all three trains - as expected from the three-fold symmetry. The maximum spread in horizontal position is about 30 microns (in the first train) and closer to 20 microns in the other trains. The vertical spread is smaller than 20 microns in all trains. An interesting feature seen in all trains is that the first and the last anti-proton bunch in each train are close together vertically.

The spreads in positions are expected from the long-range beam-beam interactions. These spreads should be larger for the anti-protons since proton intensities are larger. These observed spreads can be tested against predictions from the model of the beam-beam interactions. Another important issue is the spread in positions at the collision points B0 and D0. If these are significant, there will be differences in bunch by bunch luminosities.

2 Theoretical orbit differences

The dipole part of the long-range beam-beam kicks changes the closed orbit. Since each bunch experiences differences strengths of these kicks, the closed orbit changes differently for each bunch.

The beam-beam kicks on the anti-protons may be calculated from the beam-beam potential due to the strong proton beam

$$U(x,y,s) = \frac{N_p r_p}{\gamma_p} \int_0^\infty \frac{dq}{[(2\sigma_x^2+q)(2\sigma_y^2+q)]^{1/2}} \left\{ 1 - \exp[-\frac{(x+D_x)^2}{2\sigma_x^2+q} - \frac{(y+D_y)^2}{2\sigma_y^2+q}] \right\} \tag{1}$$

Notation

N_p	Proton bunch intensity
r_p	classical proton radius
γ_p	Relativistic kinematic factor
σ_x, σ_y	Effective beam sizes of the proton beam
D_x, D_y	Distance of the proton beam centroid from the anti-proton

The change in closed orbit arises due to the amplitude independent part of the kick.

$$\Delta x'(0,0) \equiv -\frac{\partial U}{\partial x}(0,0) = \frac{2N_p r_p}{\gamma_p} D_x \int_0^\infty \frac{dq}{[(2\sigma_x^2+q)^3(2\sigma_y^2+q)]^{1/2}} \exp[-\frac{D_x^2}{2\sigma_x^2+q} - \frac{D_y^2}{2\sigma_y^2+q}] \tag{2}$$

$$\Delta y'(0,0) \equiv -\frac{\partial U}{\partial y}(0,0) = \frac{2N_p r_p}{\gamma_p} D_y \int_0^\infty \frac{dq}{[(2\sigma_x^2+q)(2\sigma_y^2+q)^3]^{1/2}} \exp[-\frac{D_x^2}{2\sigma_x^2+q} - \frac{D_y^2}{2\sigma_y^2+q}] \tag{3}$$

By a change of variables, these can be expressed in the computationally more convenient forms

$$\Delta x'(0,0) = \frac{N_p r_p}{\gamma_p}\frac{d_x}{\sigma_x} \int_0^1 \frac{dv}{[1+v(r^2-1)]^{1/2}} \exp[-\frac{1}{2}(d_x^2 + f d_y^2)v] \tag{4}$$

$$\Delta y'(0,0) = \frac{N_p r_p}{\gamma_p}\frac{d_y}{\sigma_y} \int_0^1 \frac{dv}{[1+v(r^2-1)]^{1/2}} f \exp[-\frac{1}{2}(d_x^2 + f d_y^2)v] \tag{5}$$

where

$$d_x = \frac{D_x}{\sigma_x}, \quad d_y = \frac{D_y}{\sigma_y}, \quad r = \frac{\sigma_y}{\sigma_x}, \quad f = \frac{r^2}{1+v(r^2-1)} \tag{6}$$

N_p	2.1×10^{11}
ϵ_x, ϵ_y (95%) [π mm-mrad]	20
$(\Delta p/p)_{rms}$	1.4×10^{-4}
(β_x, β_y) at synch. light. [m]	(51.1, 110.3)
(ψ_x, ψ_y) from B0 to synch. light	$2\pi \times (16.86, 17.07)$
ν_x, ν_y	0.583, 0.575

Table 1: Parameters used for the calculation of the closed orbit shifts at the synchrotron light monitor.

The change in closed orbit at a specified location s can then be found from

$$\Delta x(s) = \frac{\sqrt{\beta_x(s)}}{2\sin\pi\nu_x}\sum_{i=1}^{72}\Delta x_i'\sqrt{\beta_{x;i}}\cos[|\psi_x(s)-\psi_{x;i}|-\pi\nu_x] \quad (7)$$

$$\Delta y(s) = \frac{\sqrt{\beta_y(s)}}{2\sin\pi\nu_y}\sum_{i=1}^{72}\Delta y_i'\sqrt{\beta_{y;i}}\cos[|\psi_y(s)-\psi_{y;i}|-\pi\nu_y] \quad (8)$$

where the sums are over all the beam-beam kicks experienced by the anti-proton. Each bunch experiences 72 beam-beam kicks including the two head-on kicks at B0 and D0 which do not change the closed orbit if the collisions are exactly centered. Overall there are 138 different locations for the beam-beam kicks. Figure 3 shows the beam separations and Figure 4 shows the beta functions at all of these locations at collision optics.

Table 1 shows the beam and optics parameters relevant for Store 1787 shown in the previous section. The rms momentum spread of protons is used in calculating the effective beam size. For simplicity we have used the same intensity and emittance for all proton bunches. The intensity data at the time the positions were recorded on the synchrotron light monitor shows that the bunch intensity varied between $(1.8-2.2)\times 10^{11}$ with the majority of bunches around 2.1×10^{11}. Emittance variations between proton bunches were around 10%. A more accurate calculation would use the individual emittances and intensities of the proton bunches.

Figures 5 and 6 shows the calculated positions of the twelve anti-proton bunches in a train relative to bunch 1. Comments

- The spread in horizontal positions is about 27 microns and increases monotonically with the bunch number, except for bunch 2 which is displaced to the other side of other bunches. Bunches 6, 7 and 8 are clustered together horizontally.
- The vertical spread is smaller, about 19 microns. Bunch 12 is close to bunch 1 vertically.

These features of the calculated spreads are in good agreement with the observed spreads seen in Figure 2. One feature which is not observed is the near monotonic increase in vertical spread with bunch number, except for bunch 12. Coupling effects which were not included could have some influence on the observed orbits.

3 Impact on Luminosity

Collisions of all bunches will not be centered at the IPs if each bunch has a different closed orbit. This will lead to differences in bunch by bunch luminosities.

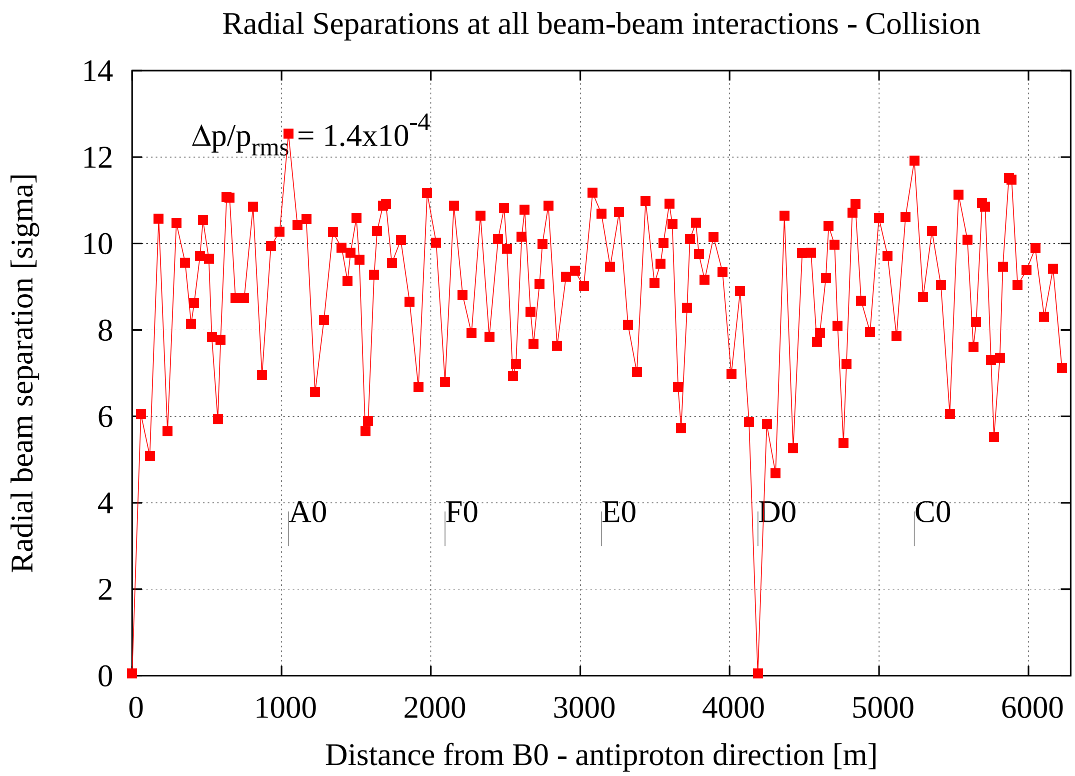

Figure 3: Beam separations at all locations of the beam-beam kicks.

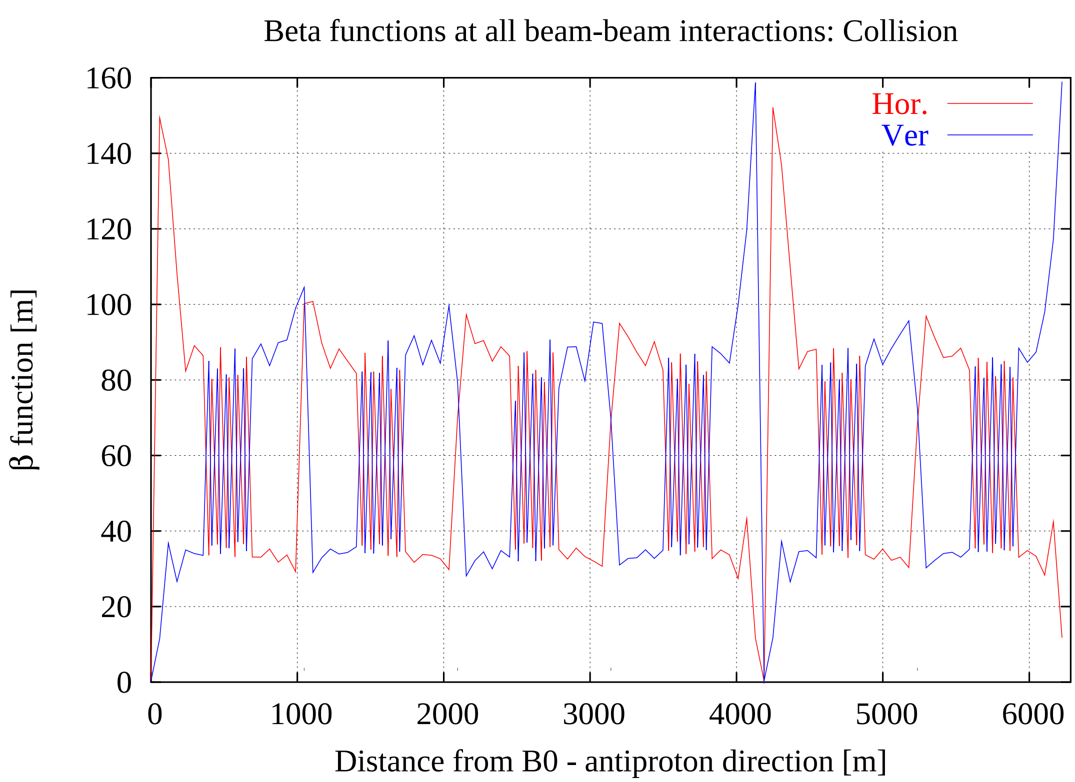

Figure 4: Beta functions at all locations of the beam-beam kicks.

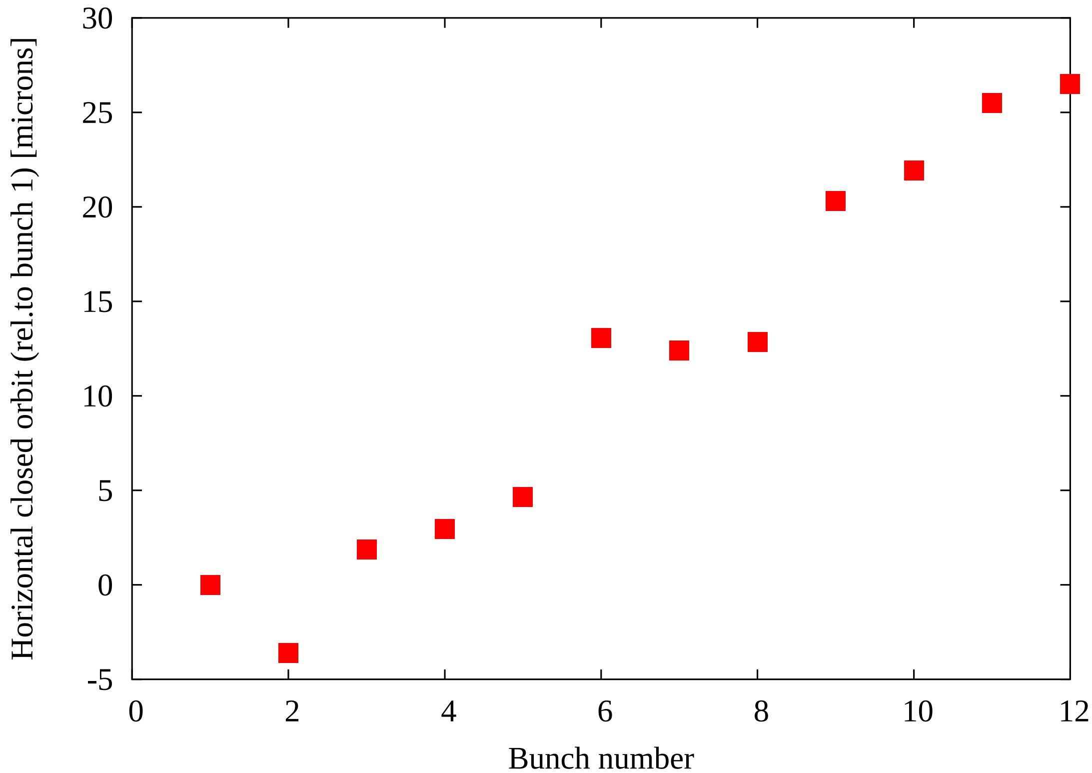

Figure 5: Theoretical horizontal separation between anti-proton bunches relative to bunch 1 at the synchrotron light monitor (C11).

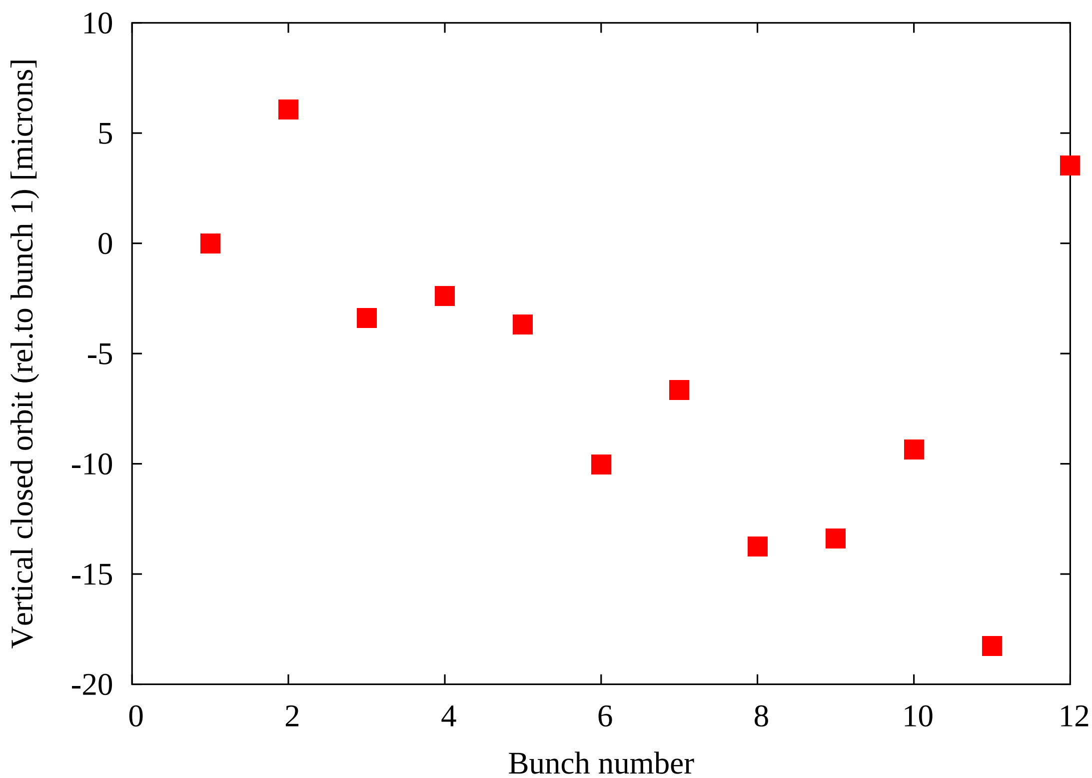

Figure 6: Theoretical vertical separation between anti-proton bunches relative to bunch 1 at the synchrotron light monitor (C11).

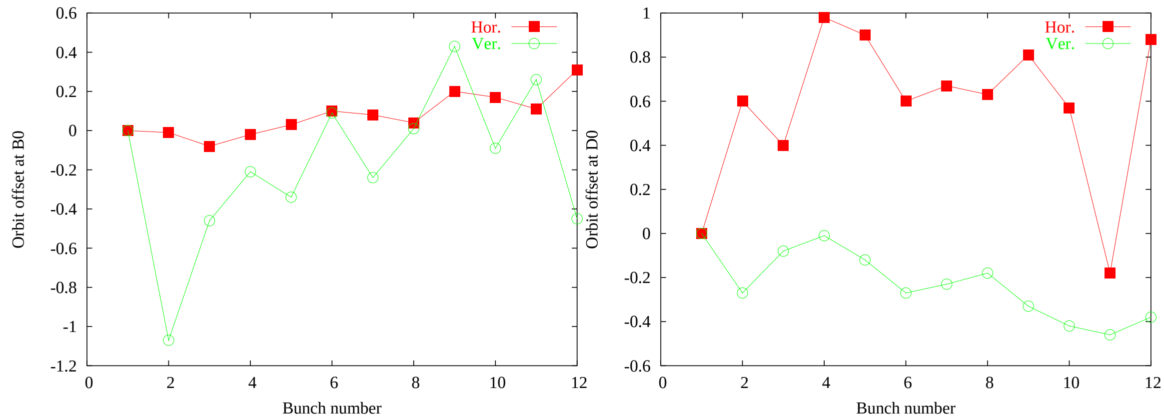

Figure 7: Orbit differences (relative to bunch 1) at B0 (left figure) and at D0 (right figure).

Taking the hour-glass effect into account ($\beta^* = 35$cm, $\sigma_z = 60$ cm), the single bunch luminosity of beams that are offset by $(\Delta x, \Delta y)$ at the IP is given by

$$\mathcal{L} = \frac{N_p N_{pbar} f_{rev}}{\pi} \frac{1}{\sqrt{(\epsilon_{x;p} + \epsilon_{x;pbar})(\epsilon_{y;p} + \epsilon_{y;pbar})}} \frac{1}{\sqrt{\pi}\sigma_{z;eff}} \int_0^\infty \frac{dt}{1+t^2} \exp[-t^2 z^2 - \frac{\Delta x^2}{4\sigma_{x;h}^2} - \frac{\Delta y^2}{4\sigma_{y;h}^2}] \tag{9}$$

where

$$\begin{aligned} z &= \frac{\beta^*}{\sigma_{z;eff}}, \qquad \sigma_{z;eff}^2 = \frac{1}{2}(\sigma_{z;p}^2 + \sigma_{z;pbar}^2) \\ \sigma_{x;h}^2 &= \frac{1}{2}\beta^*(\epsilon_{x;p} + \epsilon_{x;pbar})[1+t^2], \qquad \sigma_{y;h}^2 = \frac{1}{2}\beta^*(\epsilon_{y;p} + \epsilon_{y;pbar})[1+t^2] \end{aligned} \tag{10}$$

Here we have assumed the beams are round but not necessarily matched at the IPs.

Figure 7 shows the differences in orbits (relative to bunch 1) at B0 and D0. The orbit differences are at most around 1 micron and are small compared to the average beam size (33 microns).

Using these calculated orbit differences and the expression (9) for the luminosity, we find that there is virtually no difference in the bunch by bunch luminosities at both B0 and D0.

$$\mathcal{L}(\text{bunch } i) = \mathcal{L}(\text{bunch } 1), \quad \text{to better than } 0.01\% \qquad i = 2, ...12 \tag{11}$$

Mainly this is due to the small offsets at the IPs. The hourglass effect which makes the beam size larger away from the IP also reduces the overlap between the beams and hence reduces the impact of the offsets as well.

4 Conclusions

The calculated beam offsets due to the long-range beam-beam interactions at the synchrotron light monitor are found to be in good agreement with the observed spreads in positions. It may be possible to explain the differences between the three trains by a more accurate calculation which uses intensities and emittances of individual proton bunches.

The offsets of the beams at the IPs B0 and D0 are found to be of the order of 1 micron at most. These small offsets have almost no impact on the bunch by bunch luminosities.