Tevatron BPM Analog Filter Specifications

The minimum bandwidth of the analog filter on the front end of the BPM card is dictated by the single pass resolution requirement. The signal from the filter due to the passage of a single bunch must decay enough not to interfere with the signal from the following turn. This situation occurs only during injection conditions, when it is desired to get the first n-turns of data from the injection of the first bunch of a store. This implies only a single bunch in the machine, so the signal has a complete revolution to decay in time.

The requirement for BPM resolution in flash mode is 0.1mm. Assume for the worst case that beam comes in to the BPM on the first turn with an offset of 20mm. This means that the filter must have decayed in amplitude by 0.5% in 21μs. For simplicity, I will assume a second order bandpass filter is used.

The equation for the impulse response of a second order lowpass filter is given by:

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where ω_n is the cutoff frequency, and ξ is the damping factor.

The equation for the transient response of the second order filter is given by:

$$c(t) = \frac{\omega_n}{\beta} e^{-\xi\omega_n t} \sin(\omega_n \beta t)$$

$$\beta = \sqrt{1 - \xi^2} \ .$$

So the exponential decay of the signal must be fast enough to be at 0.5% of the initial value in one revolution of beam. If we assume a damping factor of 1.0 (Butterworth filter), then the cutoff frequency is determined by:

$$e^{-\omega_n * 21\mu s} = 0.005$$

$\omega_n = 2\pi * 40\text{kHz}$

The bandwidth necessary on a bandpass filter is approximately double that for the lowpass. This means that the minimum analog frontend bandwidth should be about 100kHz around 53MHz. We would actually have difficulty constructing a filter with such a narrow bandwidth. The minimum bandwidth specification is not restrictive.

The maximum bandwidth of the filters is determined by the sampling rate of the digitizer. We do not want signals aliased by the digitization process to interfere with the measurements around 53 MHz. The filter must provide about 70dB of rejection at any frequency that would interfere with the 53 MHz signal due to the digitization process. For example:

Digitization at 80 MHz:
70dB rejection at 27 MHz and 107 MHz

Digitization at 200 MHz:
70dB rejection at 147 MHz.