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Suppression of head-tail effects on turn-by-turn BPM measurements

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The study carried out for BLT proved that the head-tail motion excited by the machine chromaticity could seriously affect the performance of turn-by-turn BPM measurements. In this paper I consider how this problem can be alleviated for new Tevatron BPMs.

Let us consider the case when the bunch, which initially was on the central orbit, is excited by a kick. In the case of linear synchrotron motion and Gaussian longitudinal distribution one can write the following expressions presenting the dipole moment distribution along the bunch

$$D(t,f,m) \propto A\exp\left(im(t) + \frac{2in'f}{qa}\sin^2\left(\frac{\Omega_s t}{2}\right) - \frac{f^2}{2s_f^{\ 2}} - \frac{1}{2}\left(\frac{n's_f \sin(\Omega_s t)}{qa}\right)^2 \right) , \qquad (1)$$

and the bunch center of gravity

$$\overline{D}(t) \equiv \int_{-p}^{p} D(t,f)\,df = A\exp\left(im(t) - \left(\frac{\sqrt{2}n's_f}{qa}\right)^2 \sin^2\left(\frac{\Omega_s t}{2}\right)\right). \qquad (2)$$

Here f is the longitudinal coordinate changing in the range [-p, p] within one RF bucket, $n' = p\ dn/dp$ is the machine chromaticity, Ω_s is the synchrotron frequency, q is the harmonic number, s_f is the rms bunch length, a is the momentum compaction factor, and $m(t)$ is betatron phase. Figures 1 presents the development of the dipole moment along the bunch on time for m=0. One can see that the chromaticity causes averaging out of observable dipole moment with sequential recoherence after half synchrotron period. Nevertheless as one can see from Figure 2 the bunch center of gravity (average bunch dipole moment) is still small and will recohere only whole synchrotron period. As will be seen later these dipole moment oscillations originating from non-zero machine chromaticity cause the dependence of BPM measurement on type of BPM processing making difficult to process turn-by-turn BPM data.

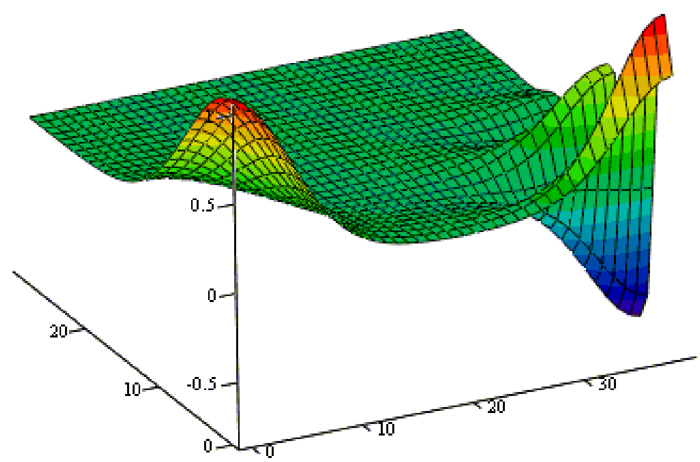

Figure 1. Development of dipole moment during one half of synchrotron period; n' = 12, s_f=0.7 rad, m = 0. The left axis presents longitudinal position scaled so that the range [0, 30] presents f from 0 to p. The right axis presents time so that the range [0, 40] presents half synchrotron period.

From the accelerator physics of point view the bunch center of gravity is the most important parameter characterizing turn-by-turn betatron oscillations. Therefore it is highly desirable that the turn-by-turn BPM measurements would be reporting the bunch center of gravity.

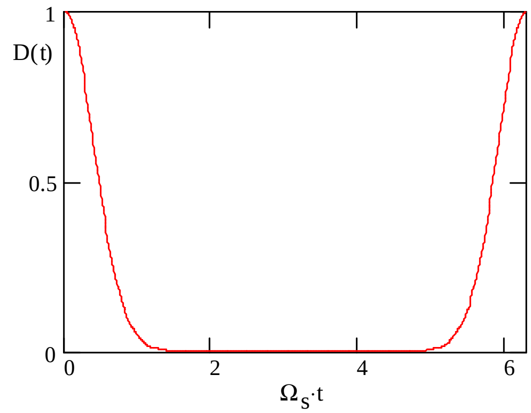

Figure 2. Dependence of betatron amplitude of the bunch center of gravity on time for one synchrotron period. All parameters are the same as in Figure 1.

The differential BPM signal is combined of two signals: the signal proportional to the bunch dipole moment, and its inversion delayed by $2L_{BPM}/c$. Figure 3 presents BPM signals immediately after the kick and after half synchrotron period. The betatron phase for the second case was chosen to be $p/2$ so that the dipole moment of the bunch would be equal to zero. We will use these BPM signals to analyze the response of BPM hardware to the signals. Below we will refer to these signals as Signal 1 and Signal 2.

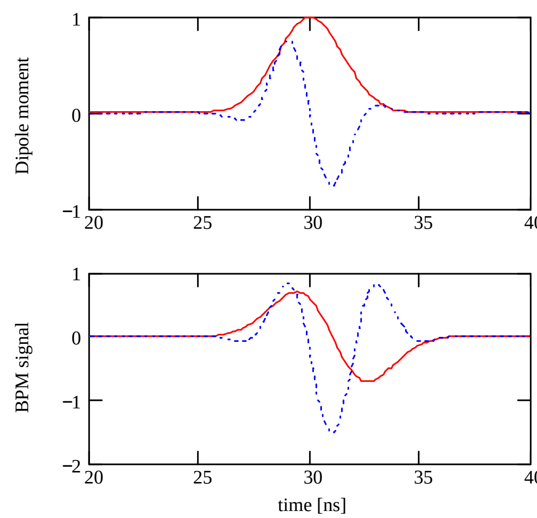

Figure 3. Dependence of bunch dipole moment (top) and BPM signal (bottom) on time for Signal 1 ($t = 0$, solid lines) and Signal 2 ($\Omega_s t = \pi$, dotted lines); $n' = 6$, $s_f = 0.5$ rad; $m = 0$ for Signal 1 and $m = \pi/2$ for Signal 2.

First we consider the response to 30 uncoalesced bunches for the electronics based on a narrow band filter around RF frequency. This is backbone of the existing BPM system. The results of simulations of the filter response to the BPM signal are presented in Figure 4. As one can see although the filter attenuates "head-tail" signal the attenuation is comparatively small. In particular for Signal 2, which has the average dipole moment equal to zero, the attenuation is only 4 times in comparison with Signal 1 presenting initial betatron amplitude. Note also that for given parameters the maximum betatron amplitude at half synchrotron period is ~0.18 of the initial amplitude. That means that at half synchrotron period the error in measuring "zero" bunch position is larger than the betatron amplitude. This relationship depends on the chromaticity and the time within synchrotron period but in most of practical cases the BPM error is between 5 and 25% of initial amplitude, which makes present turn-by-turn BPM measurements barely useful.

There are a few factors one needs to keep in mind to design a high accuracy turn-by-turn BPM system. First, the best high resolution ADCs (14 bits or above) have sampling rate not acceding ~100 MHz, which is not sufficient to digitize bunch shape directly and therefore analog preprocessing of the BPM signal is required. Second, the time interval between bunches at collisions is determined by time separation between protons and

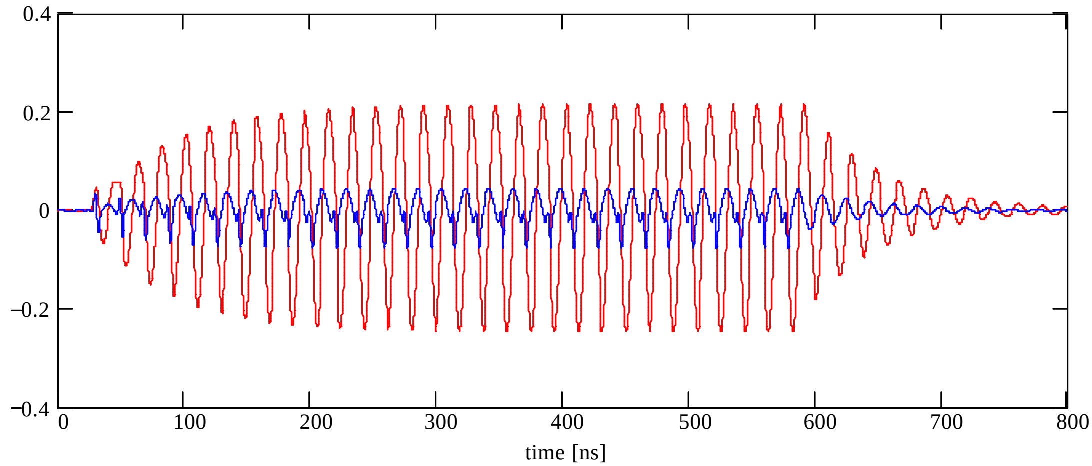

Figure 4. Response of the narrow band filter (Q = 10) to the BPM signals of 30 uncoalesced bunches; red line – Signal one, blue line - Signal 2;

pbars and is 198 ns. To prevent effects of one beam on another the analog preprocessing time has to be at least faster than that time. Third, the signal has to be preprocessed so that the longitudinal bunch displacements related to synchrotron motion would not affect BPM measurements.

To address these issues I propose the following preprocessing scheme. First, we perform integration of the BPM signal. Taking into account that the BPM plates are shorter than the bunch length ($2L_{BPM}$ = 0.6 m versus $2\sigma_s \geq 1$ m) we can consider that BPM signal is a derivative of the dipole moment density and, consequently, the integration yields the signal proportional to the dipole moment density. To get the signal proportional to the bunch center of gravity we need to integrate the signal again. The requirement to have the final signal independent on the longitudinal bunch motion determines that after the integration the integrator discharge should no be smaller than about 1 bit resolution during the time corresponding to the uncertainty of sampling time. Taking the uncertainty time of 1 ns we obtain that the

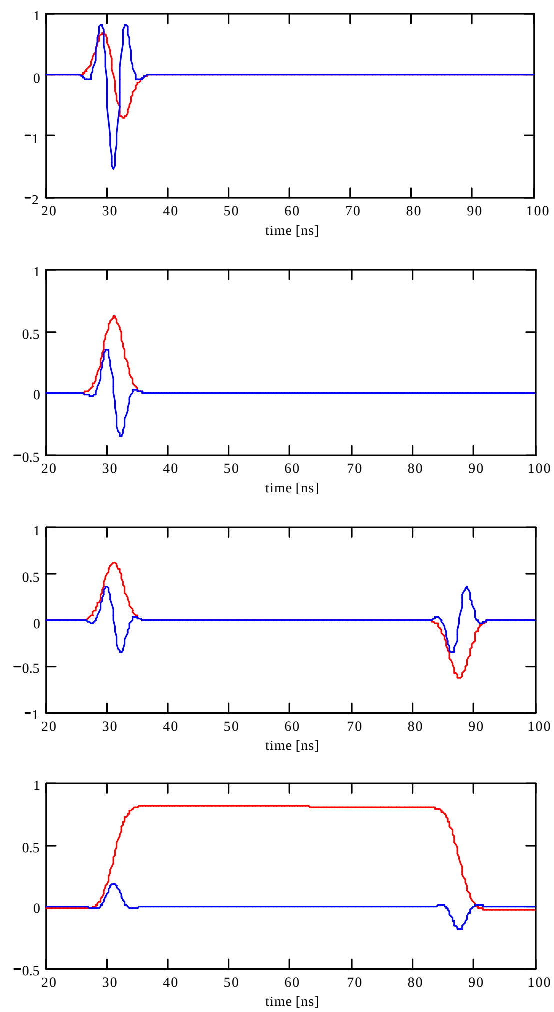

Figure 5. Signals at different stages of analog preprocessing; top to bottom: (1) initial BPM signals, (2) after first integration, (3) after delay and addition, (4) final signal; red line – Signal 1, blue line - Signal 2.

integration time needs to be ~2^{13}·1 nc ~ 10 μs. To prevent coupling between bunches and to form a nice rectangular pulse to be digitized I propose that after first integration we add up the signal and its delayed inversion. Then we perform the second integration. The delay time should be sufficiently large so that the top of the pulse would be well determined. Choosing delay time to be integer number of bucket-to-bucket spacing also allows one to form a nice pulse in the case of large number of uncoalesced bunches. The frequency response of analog preprocessing can be presented by the following expression,

$$\mathrm{K}(w) = \frac{1}{1+iwt_2}\left(1-e^{-iwT_{delay}}\right)\frac{1}{1+iwt_1} \quad . \qquad (3)$$

Figure 5 presents signals of a single coalesced bunch at different stages of the analog preprocessing. Delay time is chosen to be 3 buckets and integration time of both integrators to be 10 μs. Figure 6 presents output signal of 30 uncoalesced bunches. As one can see the proposed solution completely eliminates parasitic effects of head tail motion on BPM measurements.

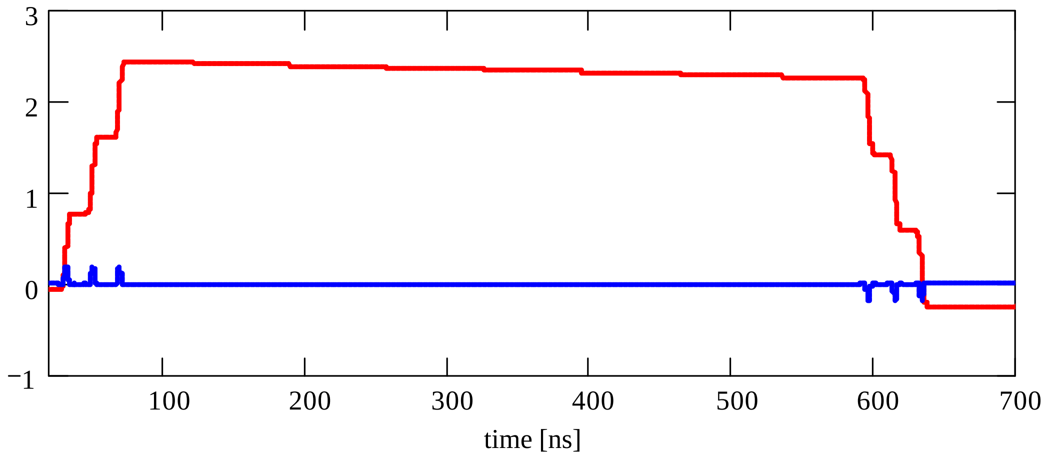

Figure 6. Response of analog preprocessing on the signal of 30 coalesced bunches; red line – Signal 1, blue line - Signal 2.

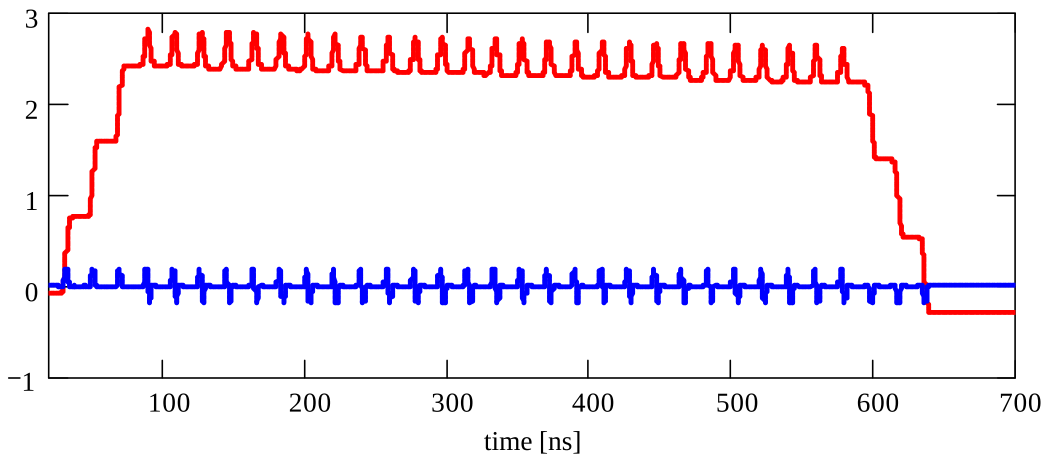

Figure 7. Response of analog preprocessing on the signal of 30 coalesced bunches for the case when delay time has an error of 2 nc; red line – Signal 1, blue line - Signal 2.

Digitizing signals after analog preprocessing and applying appropriate digital filter should allow getting high quality turn-by-turn BPM measurements. It is important to keep delay time close to the bunch spacing in the case of uncoalesced bunches. Figure 7 presents output signal of 30 uncoalesced bunches when the delay time has an error of 2 ns of the total delay time of 56 ns. In the case if the bunch dancing is present the longitudinal bunch oscillations are not directly correlated and have amplitude of about 3 ns. That can significantly perturb the signal in the manner similar presented in Figure 7. Nevertheless if the digitization of both BPM plates is well synchronized it can be easily filtered out digitally.

Another advantage of proposed scheme is that the integration reduces dependence of output signals on the bunch length, and, in the case of three bucket delay, minimizes difference between coalesced and uncoalesced bunches.