ENERGY LOSS OF A COASTING BEAM INSIDE THE RECYCLER RING

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References

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• 1. INTRODUCTION

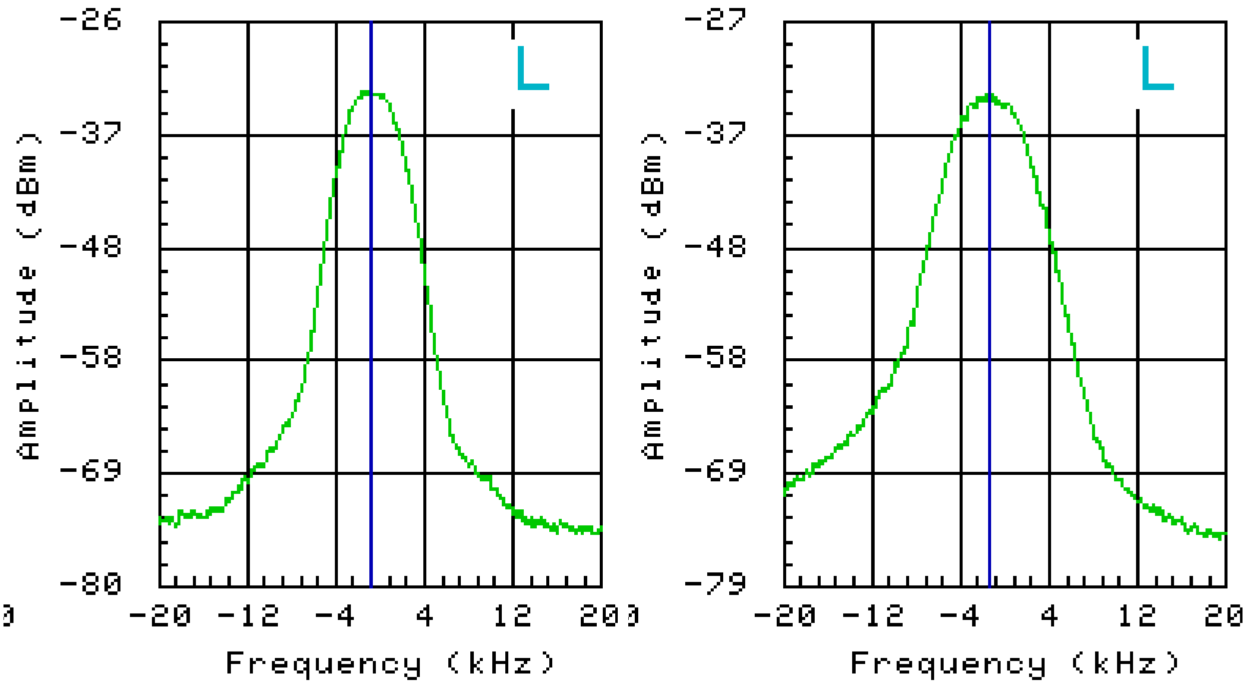


Figure 1: Digitized Schottky signals of the 19485th revolution harmonic (1.75 GHz). Comparison of the center of the initial signal at 12:29 (left) and that of the final signal at 13:30 (right) gives a shift of the revolution frequency of 0.032 Hz.

- August 2, 2003, Sergei Nagaitsev and Martin Hu:
  A 2×10^{11} proton beam circled in Recycler for 1 hr and 1.75 MHz Schottky signals measured.
  Emit. of 20 πmm-mr were first scraped to $\sim$10 πmm-mr
  Distribution shifted by 0.032 Hz in 61 min.
  With $\gamma_t = 19.968$,
  energy loss per particle: $\Delta E = 0.37$ MeV/hr
  or $U = 1.14$ meV per turn.
- Distribution shows a large increase in emittances.

- To reduce emittance growth due to IBS, experiment was repeated on August 25 with low intensity 0.088×10^{11} protons.
- In 46 min, frequency shifts by 0.024 Hz
  Per particle loss: $\Delta E = 0.036$ MeV
  or $U = 1.11$ meV per turn. (1.14 meV)
- The energy loss comes from three sources:
  1. synchrotron radiation,
  2. parasitic mode loss,
  3. interaction with residual gas.
- Want to estimate the various losses and compare the results with experimental measurement.

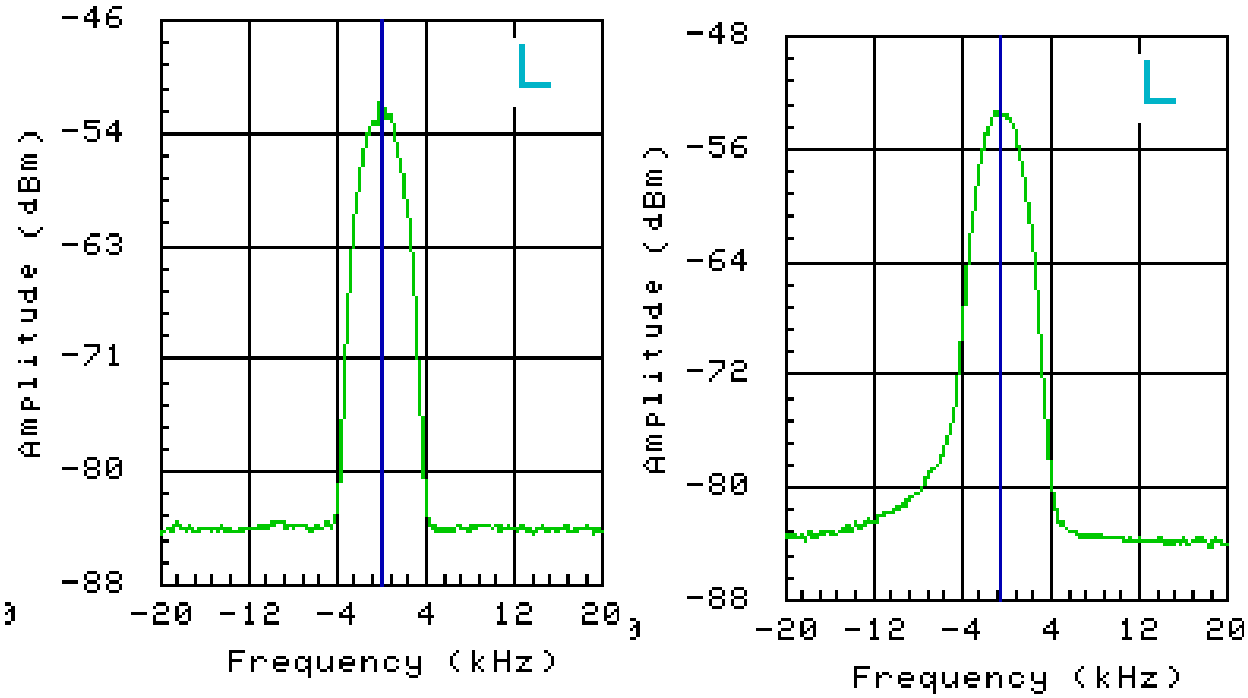


Figure 2: Digitized 1.75 GHz Schottky signals at the low intensity of 0.088×10^{11} protons. Comparison of the center of the initial signal at 11.41 (left) and the peak of the final signal at 12:27 (right) gives a shift of the revolution frequency of 0.024 Hz or 0.031 Hz per hour. Energy loss per particle is 0.36 MeV/hr or 1.11 meV/turn.

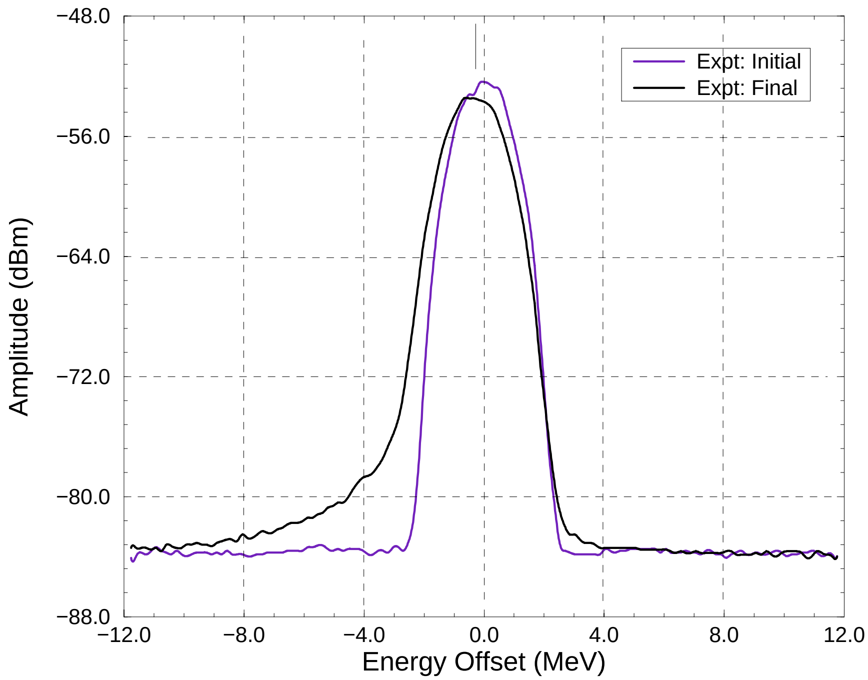


Figure 3: (color) Initial beam energy distribution (purple) centered at zero versus final beam energy distribution (black) peaked at −0.275 MeV as indicated by the black vertical bar above the distributions.

2. RADIATION LOSS

- $E = 8.91564$ GeV, $B\rho = 29.58$ T m.
  Dipole field: $B = 0.145$ T.
  Bending radius: $\rho = 203.97$ m.

$$U_{\rm rad} = \frac{e^2 Z_0 c \beta^3 \gamma^4}{3\rho} = 0.00024 \text{ meV/turn}$$

- Tiny compared with observed loss of 1.11 meV/turn.

3. PARASITIC MODE LOSS

- Loss is due to image current in beam pipe seeing impedance.

- dc component of beam is static because of
  no time dependency.
  E and B fields in the Maxwell equations are separated.
  There is no more Faraday's law.
  Static magnetic field of the beam's dc component does not induce any electric field on the surface or inside the wall of the beam pipe.
  There is no dc image current and no dc loss.

- In time domain, image pulse of each particle has rms width

$$\sigma_t = \frac{b}{\sqrt{2}\gamma\beta c}$$

- Total image current:

$$I(t) = \sum_{n=1}^{N} i_n(t) - i_{dc} \qquad\qquad i_n(t) = i_0(t - t_n)$$

- Spectrum:

$$\tilde{I}(\omega) = \begin{cases} 0 & \omega = 0\ , \\ \displaystyle\sum_{n=1}^{N} \tilde{i}_n(\omega) & \omega \neq 0\ , \end{cases}$$

- $\boxed{\text{Coasting beam} \Longrightarrow \langle \tilde{I}(\omega) \rangle = 0\ .}$

- $\boxed{\text{Energy loss} \propto |\tilde{I}(\omega)|^2}$

$$\langle |\tilde{I}(\omega)|^2 \rangle = \sum_{n=1}^{N} \langle |\tilde{i}_n(\omega)|^2 \rangle = N \langle |\tilde{i}_0(\omega)|^2 \rangle\ .$$

- Parasitic mode loss is a *single-particle effect* and is affected only by the particle's own wake.

  Does not depends on beam intensity.

  Has been confirmed at CERN ISR by monitoring energy loss with beam energy varying from 0.5 MeV to 32 GeV.

- Energy loss per particle per turn:

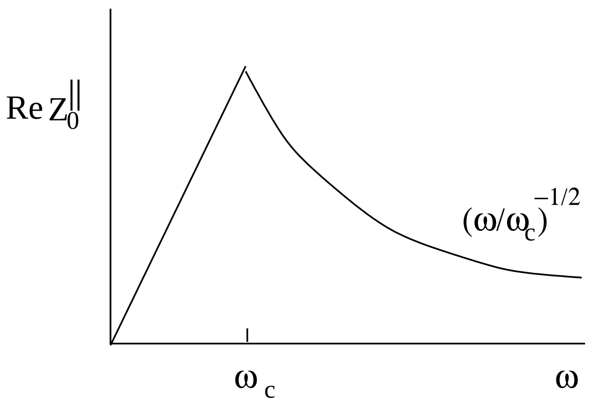
$$\boxed{U_{\text{para}} = -\frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\tilde{i}_0(\omega)\right|^2 \mathcal{R}e\, Z_0^{\parallel}(\omega)d\omega = -\frac{1}{\pi}\int_0^{\infty} \frac{\mathcal{R}e\, Z_0^{\parallel}(\omega)}{I_0(x)^2}d\omega}$$

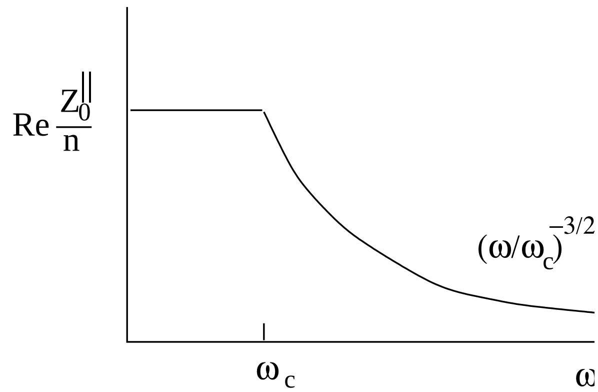
with $x = \sqrt{2}\sigma_t\omega$ and $I_0(x)$ modified Bessel function.

- Recycler has elliptic beam pipe: $3.806'' \times 1.75''$.
  Approximate as cylindrical with radius $b = 3.528$ cm.
  Rms length of the image pulse is $\boxed{\sigma_t = 8.78 \text{ ps}}$ (2.62 mm) or
  rms frequency spread is $1/(2\pi\sigma_t) = 18.11$ GHz.

- Knowledge of $\mathcal{R}e\, Z_0^{\parallel}$ up to several tens GHz will be required,
  but extremely difficult,
  because every mm variation of the vacuum chamber has to be taken into account.

- Assume diffraction model at high frequency and constant Z/n below cutoff.





- Diffraction model has been verified at SPS.
- For Recycler, $f_c = \omega_c/(2\pi) = 2.405c/(2\pi b) = 3.25$ GHz. But we need to estimate $\mathcal{R}e\, Z_0^{||}/n$ below cutoff.

3.1 Sources of Impedance

(1) *Resistive Wall*

For a stainless steel beam pipe with $\rho_{ss} = 7.4 \times 10^{-7}\,\Omega\,\text{m}$,

$$\mathcal{R}e\, Z_0^{\parallel}\Big|_{n=1} = \frac{1}{b}\sqrt{\frac{Z_0 \rho_{ss} R\beta}{2}} = 7.580\ \Omega\ .$$

This impedance increases as $\sqrt{\omega}$.

Per particle loss $U_{\text{wall}} = 0.00714$ meV.

Note: This is an overestimate, since resistive-wall impedance rolls off when $\omega b/(\gamma c) \sim 1$ or $f \sim 13$ GHz.

(2) *Bellows*

Recycler's bellows are shielded, and each treated as a small cavity of width $\ell \sim 1$ cm and depth $d \sim 1$ cm.

$$\mathcal{R}e\, Z_0^{\parallel} = \frac{Z_0}{2\pi^{3/2} b}\sqrt{\frac{c\ell}{\omega}}\ ,$$

according to the diffraction model.

For 516 such shielded bellows,

$$\mathcal{R}e\, Z_0^{\parallel} \approx 6000\sqrt{\frac{\omega_c}{\omega}}\ \Omega,$$

equivalent to having $\mathcal{R}e\, Z_0^{\parallel}/n \approx 0.167\ \Omega$ at cutoff.

(3) *RF Barriers*

4 rf barrier cavities have $Z_{\rm shunt} = 200\ \Omega$ for $n = 1$ to 500.

$\therefore \mathcal{R}e\, Z_0^{\parallel}/n$ decreases linearly from

$$200\ \Omega \text{ at } n = 1 \text{ to } 0.4\ \Omega \text{ at } n = 500 \text{ (50 MHz).}$$

$U_{\rm rf} = 2 \times 500 \times 200\, e^2 f_0 = 2.88 \times 10^{-6}$ meV.

(4) *Beam Position Monitors*

$M = 410$ split-can BPM's, each of length $\ell = 12''$.

$$\mathcal{R}e\left(\frac{Z_0^{\parallel}}{n}\right) = 2MZ_c\left(\frac{\phi}{2\pi}\right)^2 \frac{\sin^2 \omega\ell/c}{n} \lesssim 0.283\ \Omega \qquad \text{at cutoff}$$

(5) *Pump Ports*

Each pump port has opening $\ell = 5''$ and $h = 1.05''$.

EM waves are diffracted at upstream edge.

View as a cavity with opening angle $\phi \sim 2h/w = 0.55$ rad, diffraction model gives

$$\mathcal{R}e\, Z_0^{\parallel} = \frac{Z_0}{\pi^{3/2} w}\frac{\phi}{2\pi}\sqrt{\frac{c\ell}{\omega}} \longrightarrow 3980\sqrt{\frac{\omega_c}{\omega}}\ \Omega \quad \text{for 600 pump ports}$$

equivalent to having $\mathcal{R}e\, Z_0^{\parallel}/n = 0.110\ \Omega$ at cutoff,

	$\mathcal{R}e\, Z_0^{\parallel}/n$ at cutoff (Ω)
Resistive wall	negligible
Bellows	0.167
Barrier RF	negligible
BPM's	0.283
Pump ports	0.110

- It is reasonable to assume $(Z/n)_c \lesssim 1\ \Omega$ near cutoff.

  get $U_{\rm pm} \lesssim 0.056$ meV per turn,
  where $\sim 83.3\%$ from contribution above cutoff.

  Thus the parasitic mode loss amounts to only $\sim 5\%$ of the observed energy loss.

- Of course, it is still possible that $\mathcal{R}e\, Z_0^{\parallel}/n$ is much larger.

- Many small variations of the vacuum chamber may accumulate to give a large contribution at high frequencies that does not show up at low frequencies.
  In other words, $\mathcal{R}e\, Z_0^{\parallel}/n$ does not roll off as early as cutoff.

4. INTERACTION WITH RESIDUAL GAS

- Relativistic charged particles other than electron lose energy in residual gas primarily by ionization according to Bethe-Bloch equation

$$\boxed{-\frac{dE}{dx} = \frac{Kz^2}{\beta^2}\frac{Z^*}{A^*}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2\epsilon_{\max}}{I^2} - \beta^2 - \frac{\delta}{2}\right]}$$

	H_2	N_2	O_2	Mixture
Energy loss U_i (meV) at 0.5 nTorr	0.08	0.50	0.57	0.24
Vacuum pressure (nTorr) if $U_i = 1.144$ meV	7.17	1.14	1.01	2.42

↑

Assuming gas composition in CERN paper
64.1% H, 27.5% N, and 8.4% O by partial pressure
(or 11% H, 66% N, and 23% O by density)

5. LANDAU DISTRIBUTION

- Bethe-Bloch formula gives the mean rate of energy loss of a particle per matter *thickness* x through ionization.
- The spread in energy loss ϵ after traversing a thickness x follows the Landau distribution

$$f(x,\epsilon)d\epsilon = \frac{\varphi(\lambda)}{\xi}\,d\lambda\ , \qquad \varphi(\lambda) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{u\lambda+u\ln u}\,du$$

- The distribution is characterized by only one parameter

$$\lambda = \frac{\epsilon}{\xi} - 1 + \beta^2 + C - \left[\ln\frac{2m_ec^2\beta^2\gamma^2\xi}{I^2} - \delta\right]\ , \qquad C = 0.577215\ldots$$

$$\xi = \frac{Kz^2Z^*}{2\beta^2A^*}\,x \qquad \textit{is a measure of the energy loss}$$

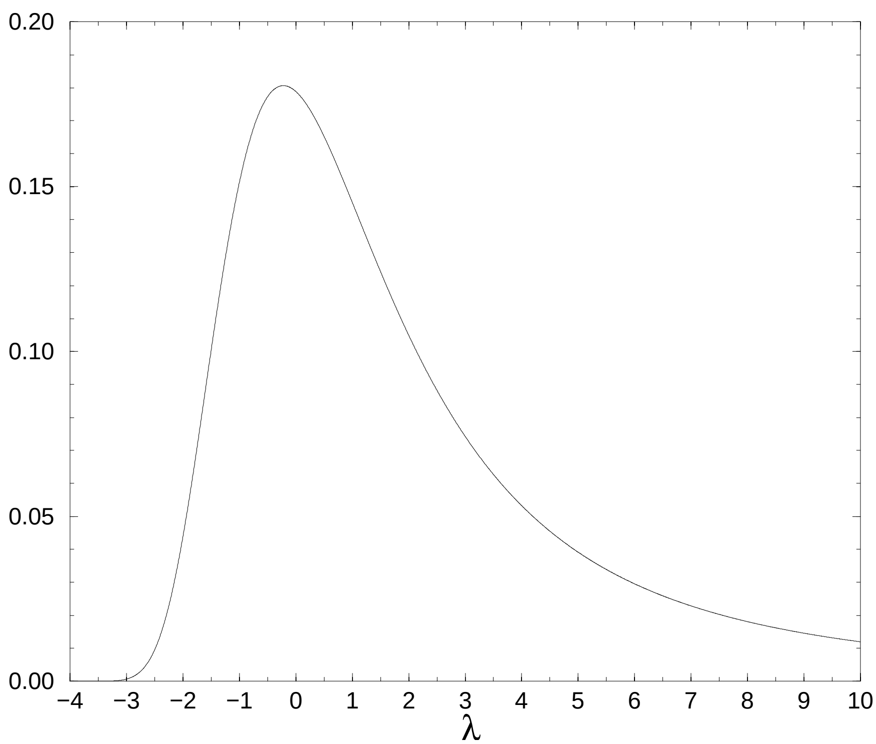
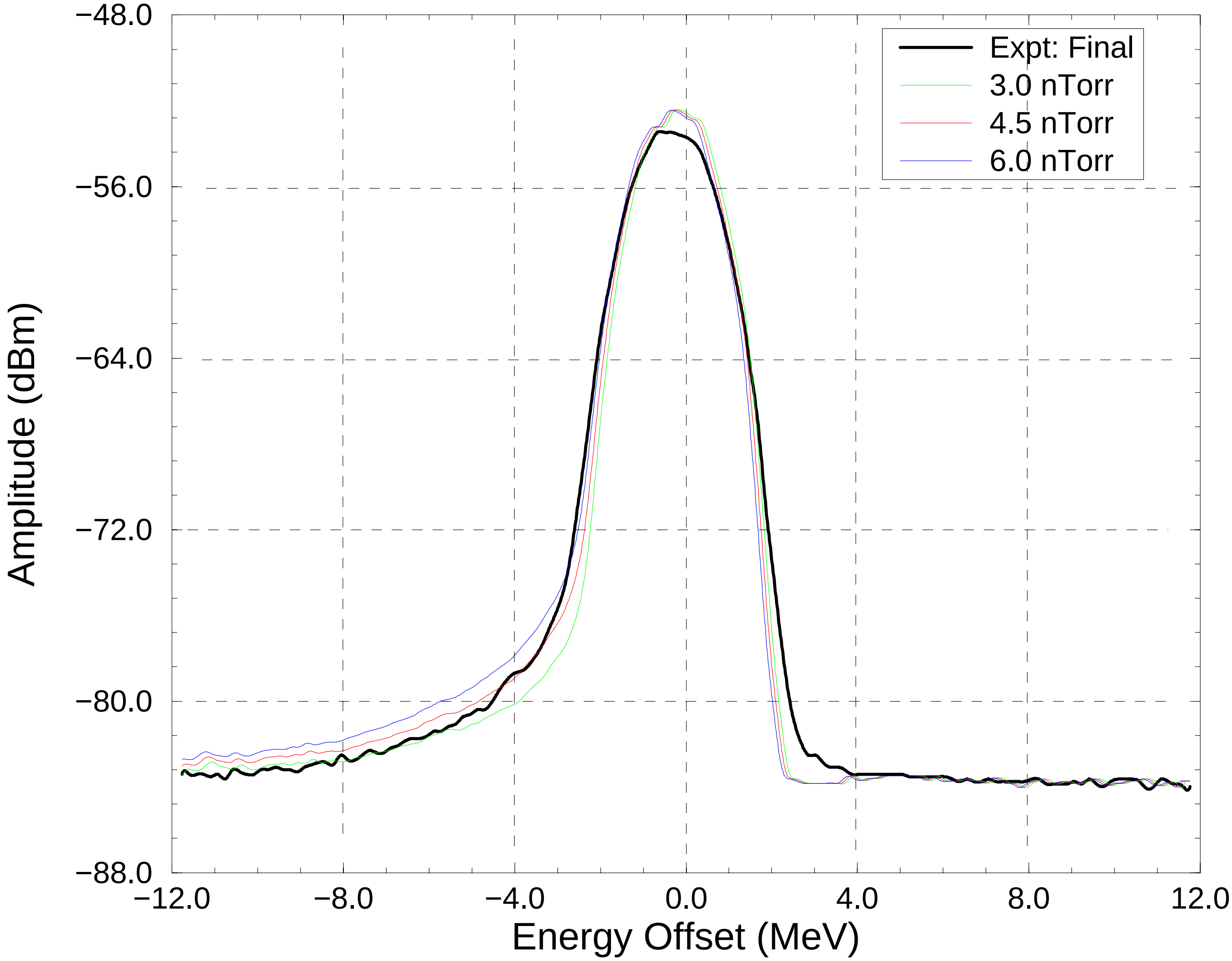
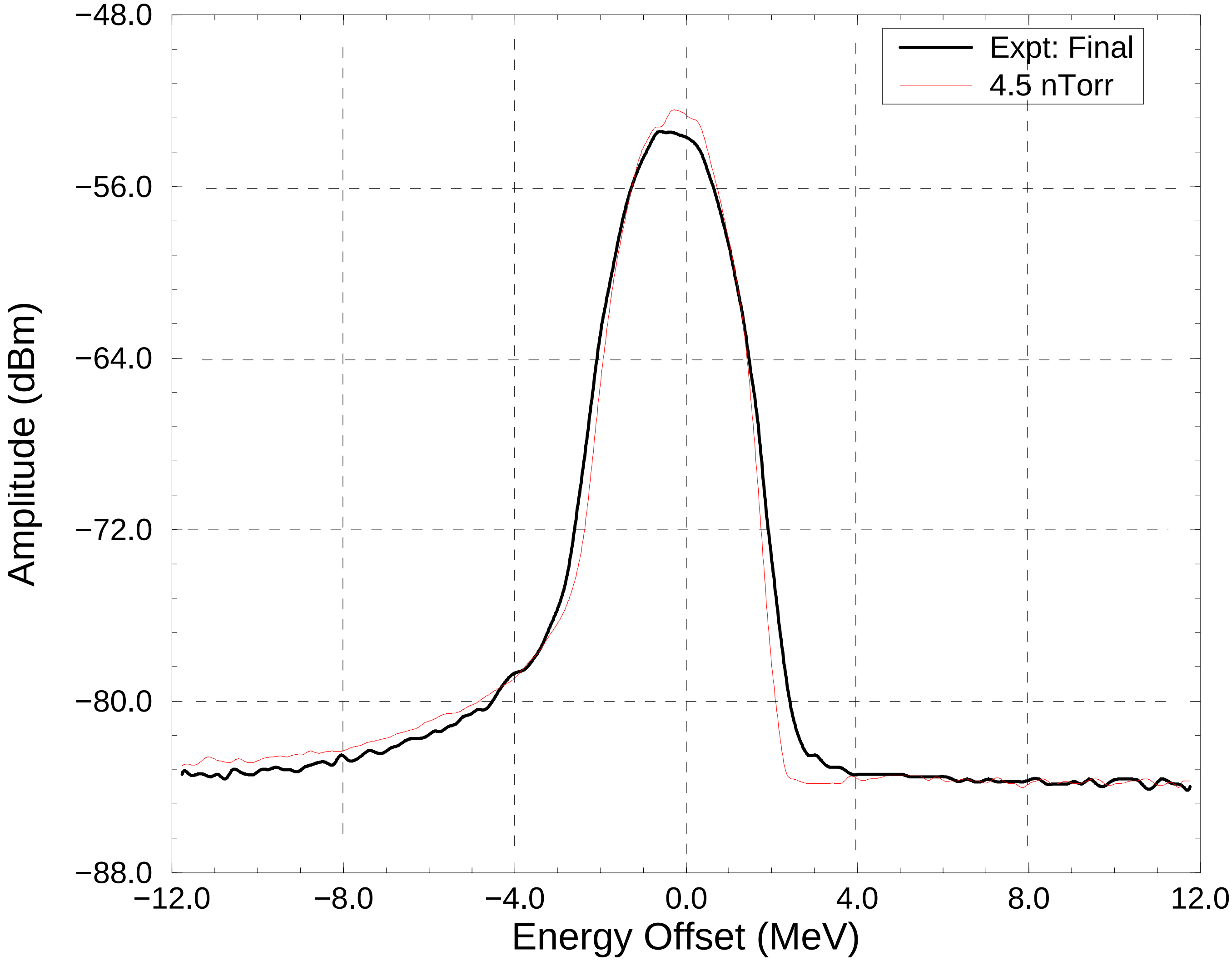


Figure 4: The Landau distribution $\varphi(\lambda)$. The distribution is skew and has a long tail for larger energy loss (larger λ). The distribution peaks at $\lambda = -0.223$ and has a full-width-at-half-maximum of $\lambda_{\mathrm{FWHM}} = 2.6$.

- Landau distribution is valid provided that
  (1) energy loss is much less than the maximum energy loss
  (2) and the matter traversed is *thin*.
- Parasitic mode loss is shift of distribution
  Loss to residual gas makes distribution asymmetric.
- Hope to fit distribution to determine how much loss is due to parasitic mode loss and how much is due to residual gas.



- We assume a mixture of 90% H_2 and 10% CO by partial pressure.
- It appears that vacuum pressure of 4.5 nTorr fits very well. Deviation may come from IBS.



Expt: Final
4.5 nTorr
−48.0
−56.0
−64.0
−72.0
−80.0
−88.0
Amplitude (dBm)
−12.0
−8.0
−4.0
0.0
4.0
8.0
12.0
Energy Offset (MeV)

CONCLUSION

1. Impedance estimation says parasitic mode loss $\lesssim 5\%$.

2. Computation of ionization loss using Bethe-Bloch formula reveals vacuum pressure much higher than the expected 0.5 nTorr.

3. Fit to Landau distribution implies
   (a) Parasitic mode loss is very small.
   (b) Loss is dominated by residual gas interaction.
   (c) The mean vacuum pressure is about 4.5 nTorr.