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Quadratic Term in the Tevatron BPM Sum Signal

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Abstract

The data from the position grid study are used to examine how the BPM sum signal depends on the beam position. This effect is studied as a function of the beam position along the measured coordinate and also as a function of the beam position transverse to the measured coordinate. The behavior along the measured coordinate is approximately quadratic and the minimum of the parabola can be found to a precision of less than 100 μm. The position of this minimum depends on the beam position in the direction orthogonal to the measured coordinate. If the orthogonal coordinate is unknown, this contributes an error of order 200 μm to the position of the minimum. This last conclusion is a little uncertain because there are some outlier data points which have yet to be explained.

1 Introduction

At the April 26, 2004 meeting of the Tevatron BPM upgrade group, Jim Steimel presented some calculations which show how the BPM sum signal depends on the position of the beam. He also presented some measurements made by moving a wire inside a spare BPM. The measurements show the predicted effect. He also suggested that we could use this effect to determine the electrical center of the pickup.

In this note I will show the effect using Tevatron data acquired during a position grid study done on March 11, 2004. That data was previously described in Beams-doc-1076. Figure 1 shows the grid points as designed for the study.

For each of the 20 grid points I used the BPM data to extract the measured position and the measured sum signal, A+B. As described in Beams-doc-1076, the measured position was obtained by averaging 200 individual position measurements during a time when the position beam was observed to be stable. The error on the position was given by the RMS width of the 200 measurements. The mean sum signal, and the RMS width of the sum signal, were also determined from these same samples of 200 measurements. In the following, these RMS widths are used as the error on the sum and position data points.

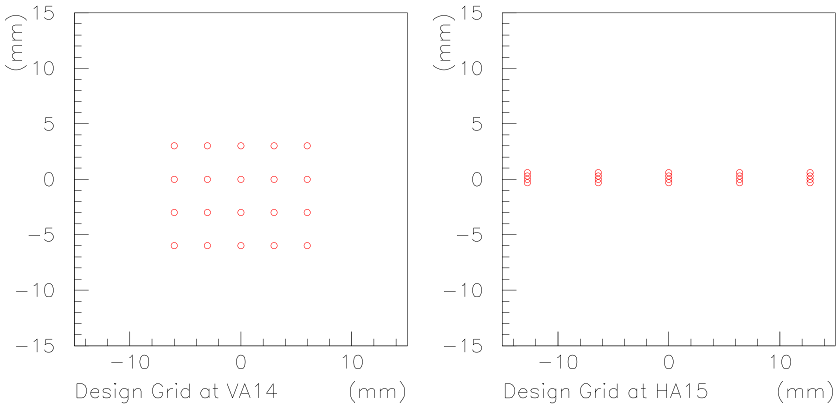

Figure 1: The circles show the designed position of the 20 grid points. The grid was designed to be square with a step size of 3 mm at VA14. The optics of the Tevatron produce a grid at HA15 with horizontal steps of 6.366 mm in the horizontal direction and 0.297 mm in the vertical direction. The origin, (0,0), refers to the beam position at the start of the study, not the true center of the pickups.

2 Results

The crosses in Figure 2a) show, for HA15, the measured sum signal as a function of the measured horizontal position. As indicated by the legend, the red crosses are the data points for which the nominal vertical position was -0.6 mm.[1] The other colors of points correspond to other nominal vertical positions. Both the horizontal and vertical error bars are much smaller than the crosses. The superimposed curves show the results of fitting each set of five data points using a parabola. The fit procedure and fit results will be discussed in Section 3.

Figure 2b) shows, for VA14, the measured vertical sum signal as a function of the measured vertical position. The five colors correspond to the five different nominal horizontal positions at VA14.

In all cases the minimum of the parabola is well defined. In the upcoming work it will be shown the the resolution on the position of the minimum varies between 50 and 100 μm. Therefore one can imagine using the knowledge of the minimum to calibrate the electrical center of the BPM. The issue which remains is to deal quantitatively with the orthogonal coordinate.

The remaining plots look at the same data in more detail.

Figure 3 shows each of the four data sets from Figure 2a) on its own plot; the data sets are distinguished by the nominal vertical position of the beam. The data points have both horizontal and vertical error bars, drawn to $\pm 3\,\sigma$.

[1]Throughout this note I use the phrase "nominal position" to emphasize that the orthogonal transverse coordinate at each BPM is not measured, only calculated.

V (mm)	H Minimum (mm)	H (mm)	V Minimum (mm)
-0.6	1.899 ± 0.118	-6	0.605 ± 0.086
-0.3	1.102 ± 0.068	-3	0.541 ± 0.077
0.0	1.053 ± 0.054	0	0.418 ± 0.072
+0.3	1.016 ± 0.042	+3	0.242 ± 0.076
		+6	-0.097 ± 0.106
RMS:	0.36		0.19

Table 1: In the main body of the table, the left half reports the horizontal position of the minimum of the parabola for each of the four fits shown in Figure 3. The four fits are labeled by their nominal vertical positions. The right half of the main body of the table reports the corresponding quantities for the fits to the five parts of Figure 4. The bottom line reports the RMS spread of the positions. The error on each position was ignored when the RMS was computed.

Even with this magnification, the horizontal error bars are barely visible. The confidence level of each fit is reported as the number following "CL=" and all of the fits have an acceptable confidence level. The number following "min=" gives the horizontal position of the minimum of the parabola, as computed from the fit results. These minima are also summarized in Table 1.

Figure 4 shows a similar breakdown of the measurements of the vertical BPM, segregated by nominal horizontal beam position. All but one of these fits have very poor confidence levels. It is not yet understood why these fits are poor while the fits to the HA15 data have good confidence levels. At first I thought that, perhaps, the vertical BPM has better resolution so it can expose a deviation from quadratic behavior which is not resolvable with the horizontal BPM. However this is not the case. Although the position resolutions for the H and V BPMs are very different, the resolution on the sum signals is the same within about 25%. The reason for the difference on the position resolution is, presumably, real beam motion, which should not affect the resolution of the sum signal.

Another candidate explanation is that no correction was made for the drop in beam intensity with time. This effect should be larger for the VA14 data than for the HA15 15 data because of the order in which the grid points were measured. The five data points in each HA15 plot were acquired consecutively, within a few minutes. On the other hand, about 3 or 4 minutes elapsed between each of the four data points in one of the VA14 plots. This explanation has not yet been pursued quantitatively but the pattern of residuals in each plot is similar; in particular the earliest data point is always above the fitted curve.

Figure 5 summarizes how the minima reported above depend on the orthogonal transverse coordinate. That is, it shows how the horizontal position of the minimum, for an H measuring BPM, depends on the vertical position of the beam. And similarly for a V measuring BPM. These plots are a little difficult to interpret. The right hand plot shows a simple behavior over a wide range of the orthogonal coordinate. The left hand plot, however, shows a more complicated behavior over a much narrower range of orthogonal coordinate.

One might notice that the two most extreme points in the grid, (H,V) = $(\pm 6.366, -6)$, are used to compute the first data point in the left hand plot, the one point which seems to be an outlier. Therefore one might think that the outlier comes from higher order terms which are only important at large displacements. However the quality of that fit is very good, reducing the likelihood of this explanation.

In short, Figure 5 remains to be explained.

3 Fits

In this section the fit procedures will be described using the red data from Figure 2a) as an example. The same procedure was followed for the other fits. The red curve is the result of a fit in which the five red data points are modeled as a parabola,

$$S(x) = a_0 + a_1 x + a_2 x^2, \tag{1}$$

where x is the measured horizontal position, S is the predicted sum signal and where the coefficients a_i are to be determined from the fit. The fit was performed using the program `MINUIT` to minimize a χ^2 function defined as,

$$\chi^2 = \sum_i \frac{(S(x_i) - S_i)^2}{\sigma_i^2}, \tag{2}$$

where the sum runs over the five red data points, where x_i is the measured position, S_i is the measured sum signal and where σ_i is the error on S_i. This definition of χ^2 ignores the error on x_i; inspection of the horizontal and vertical error bars in Figures 3 and 4 shows this is a reasonable approximation for most of the data. I don't believe that including the errors on x_i would significantly improve the confidence levels of the fits to the VA14 data.

The fit returned values for the parameters $\{a_0, a_1, a_2\}$, the 3×3 covariance matrix for these values, V, and the value of χ^2 at the minimum. This χ^2 was then expressed as a confidence level and reported on each part of Figures 3 and 4.

The value of x at which the parabola is at its minimum value, x_0, is given by,

$$x_0 = -\frac{a_1}{2a_2}. \tag{3}$$

And the error on x_0 is given by,

$$\sigma^2(x_0) = \left(\frac{\partial x_0}{\partial a_1}\right)^2 V_{2\,2} + \left(\frac{\partial x_0}{\partial a_2}\right)^2 V_{3\,3} + 2\left(\frac{\partial x_0}{\partial a_1}\right)\left(\frac{\partial x_0}{\partial a_2}\right) V_{2\,3}$$

$$= \frac{x_0^2}{a_1^2} V_{2\,2} + \frac{x_0^2}{a_2^2} V_{3\,3} - 2\frac{x_0^2}{a_1 a_2} V_{2\,3} \tag{4}$$

In the covariance matrix, a_1 is parameter 2 and a_2 is parameter 3.

Table 1 summarizes the measured minima for each of the nine fits.

4 Summary and Conclusions

This note has shown that the quadratic dependence of the sum signal on the measured beam position is clearly visible when the Tevatron BPMs are read out using modified Recycler BPM system. The position of the minimum can be determined with a precision which varies between 50 and 100 μm. The position of the minimum does depend on the position of the beam in the orthogonal transverse coordinate. The VA14 data suggest the behavior due to the orthogonal coordinate is simple to model and, if it is ignored entirely, will contribute an error of order 190 μm. to the position of the minimum. The HA15 data suggest that this error may be larger: the RMS of that sample is 360 μm but perhaps the true story of the resolution is told by the outlier datum?

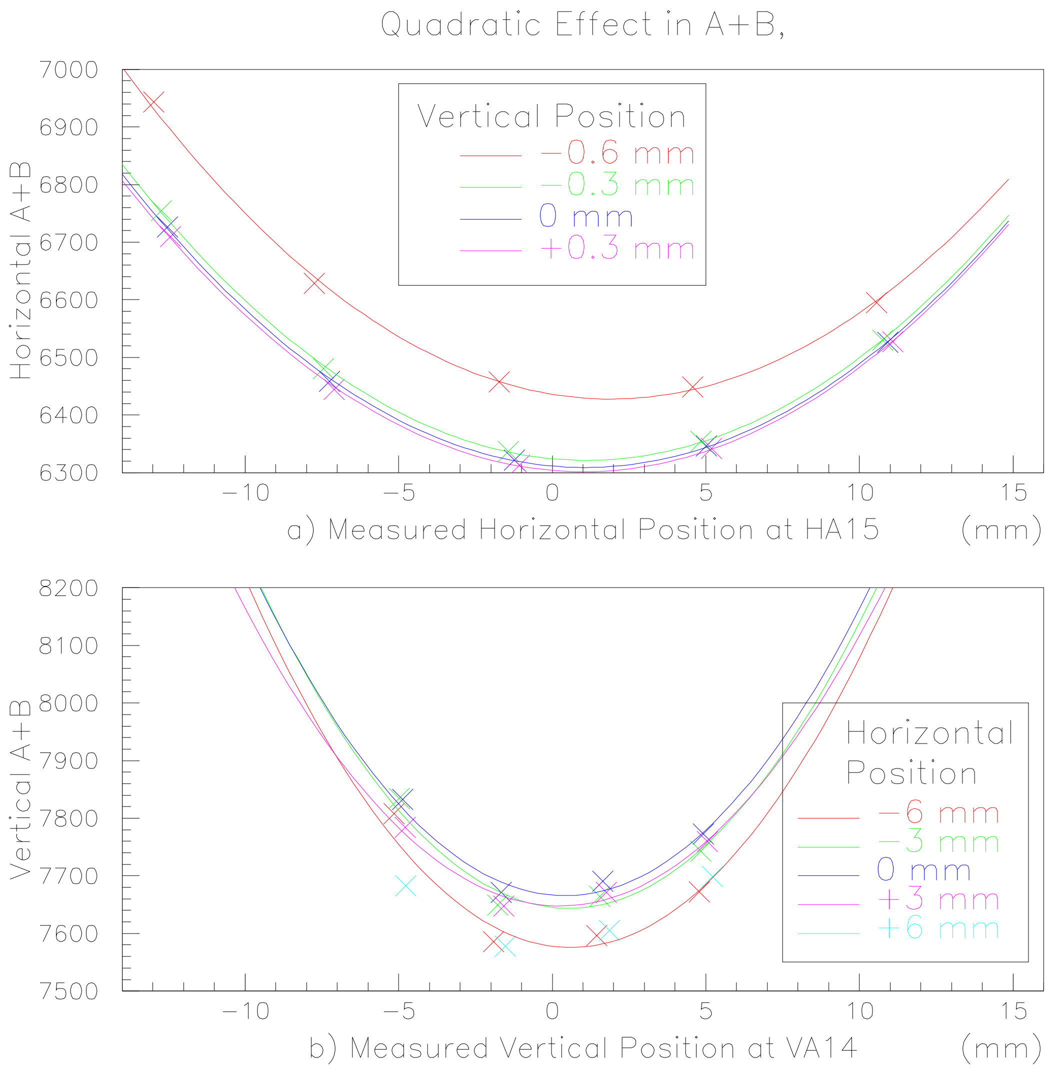

Figure 2: The top plot shows the measured sum signal at HA15 as a function of the measured position at HA15. The four different colors correspond to 4 different vertical positions at HA15. The bottom plot shows the measured sum signal at VA14 as a function of the measured vertical position at VA14. In both plots the superimposed curves are the result of a fit described in the text and the error bars on the points are smaller than the plot symbols.

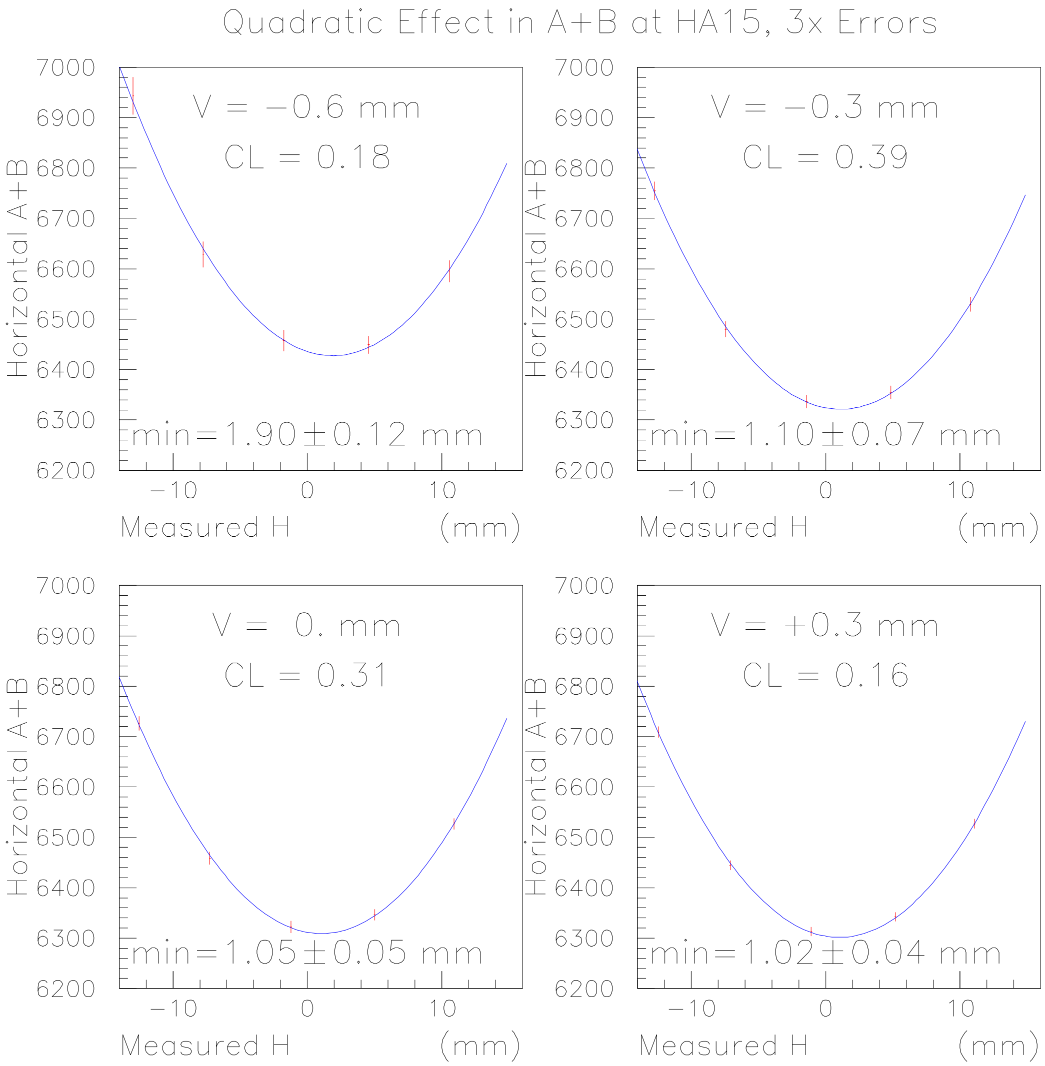

Figure 3: The data from the top plot of Figure 2 plotted separately for each vertical position. The vertical position is noted on each plot. The data points have both horizontal and vertical error bars that are drawn three times larger than their actual values. The superimposed curves, the confidence level (CL) and the position of the minimum are the results of a fit described in the text.

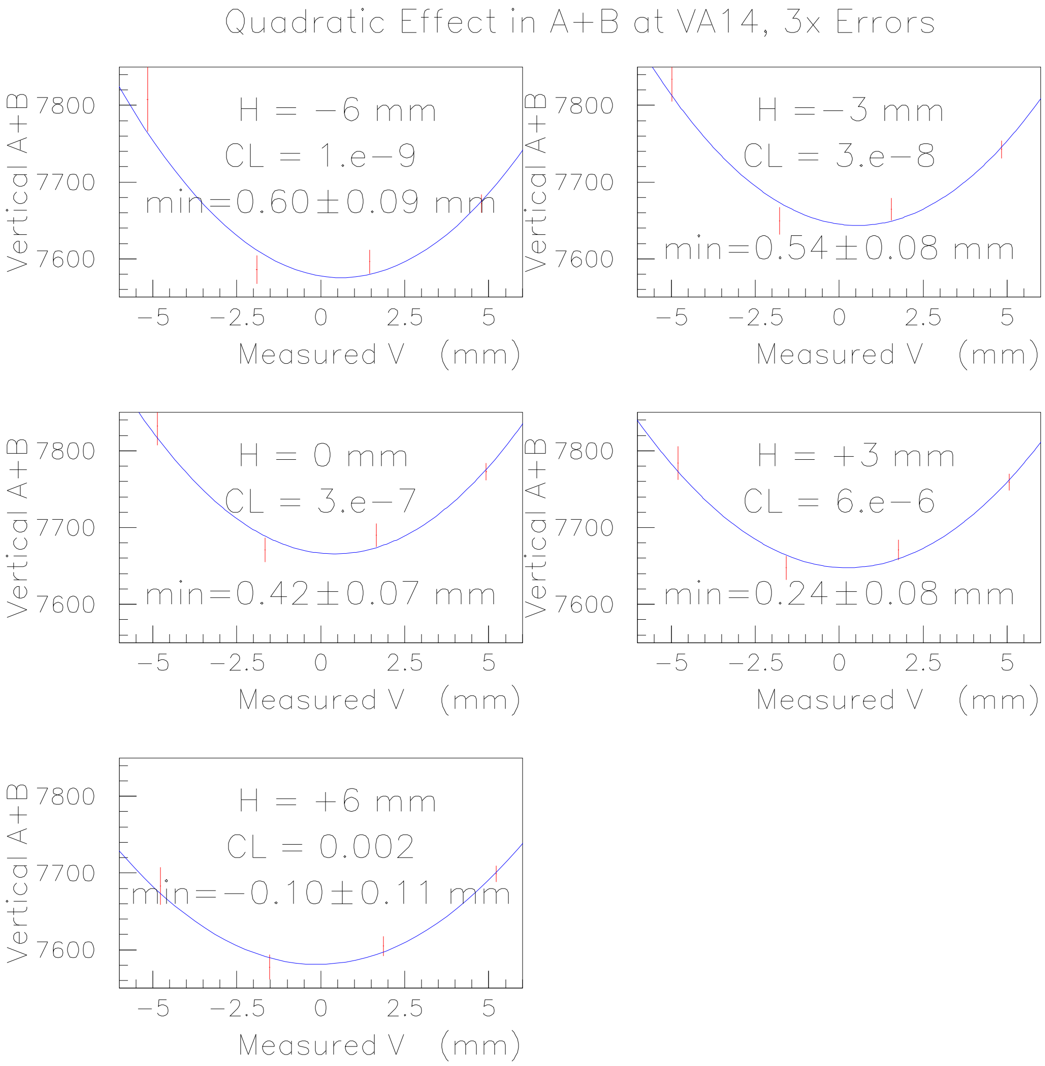

Figure 4: The data from the bottom plot of Figure 2 plotted separately for each horizontal position. The horizontal position is noted on each plot. The data points have both horizontal and vertical error bars that are drawn three times larger than their actual values. The superimposed curves, the confidence level (CL) and the position of the minimum are the results of a fit described in the text.

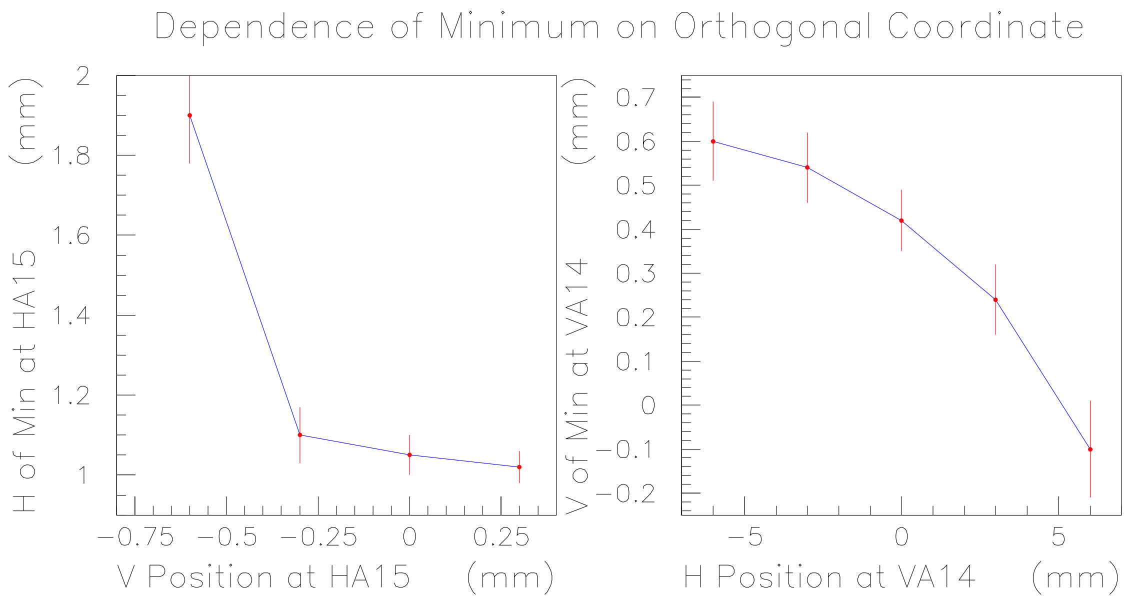

Figure 5: Summary of how the minima depend on the orthogonal transverse coordinate. The left hand figure shows the horizontal position of each minima from Figure 3 plotted against the nominal vertical position of the beam. The right hand figure shows the vertical position of each minima from Figure 4 plotted against the nominal horizontal position of the beam.