Summary on difference orbit measurements from week of May 17th, 2004

P. Piot[1], Y.-E. Sun[2], J. Wennerberg[1]
[1]*Fermilab, Batavia, IL 60510, USA*
[2]*University of Chicago*

June 3, 2004

Abstract

A series of difference orbit measurements were performed to try to characterize, with beam, the quadrupoles of the FNPL injector. The present note reports on the technique used and analysis of this first set of data.

1 Introduction

Let $\{\mathbf{X}, \mathbf{Y}\} = \{(\mathbf{x}, \mathbf{x}'); (\mathbf{y}, \mathbf{y}')\}$. be the coordinate of the beam center in the trace space. Given two locations along the direction of beam motion, f and 0 one as:

$$\begin{bmatrix} \mathbf{X}_f \\ \mathbf{Y}_f \end{bmatrix} = T^{0\to f} \begin{bmatrix} \mathbf{X}_0 \\ \mathbf{Y}_0 \end{bmatrix}, \tag{1}$$

wherein T designates the transfer matrix between the two considered location. A measure of the beam center in the configuration space (x, y) by the means of N beam position monitors (BPMs) can be related to the beam position coordinates at an upstream arbitrary position 0 via:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = R^{0\to i} \begin{bmatrix} \mathbf{X}_0 \\ \mathbf{Y}_0 \end{bmatrix}, \tag{2}$$

where $R^{0\to i}$ is the response matrix from position 0 to i. R can be written in term of T-matrix elements:

$$R^{0\to i} = \begin{bmatrix} T_{11}^{0\to i} & T_{12}^{0\to i} & T_{13}^{0\to i} & T_{14}^{0\to i} \\ T_{31}^{0\to i} & T_{32}^{0\to i} & T_{33}^{0\to i} & T_{34}^{0\to i} \end{bmatrix}. \tag{3}$$

Let's now consider the case when the initial condition is perturbed: $[\mathbf{X}_0, \mathbf{Y}_0]^t \to [\mathbf{X}_0 + \delta\mathbf{X}_0, \mathbf{Y}_0 + \delta\mathbf{Y}_0]^t$. The upstream orbit response, as measured on the BPMs is:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} \to \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \end{bmatrix} = R^{0\to i} \begin{bmatrix} \mathbf{X}_0 + \delta\mathbf{X}_0 \\ \mathbf{Y}_0 + \delta\mathbf{Y}_0 \end{bmatrix}. \tag{4}$$

It is more convenient to rewrite the previous equation by defining the orbit response $(\Delta x, \Delta y)$ as:

$$\begin{bmatrix} \Delta x_i \\ \Delta y_i \end{bmatrix} \doteq \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \end{bmatrix} - R^{0\to i} \begin{bmatrix} \mathbf{X}_0 \\ \mathbf{Y}_0 \end{bmatrix} = R^{0\to i} \begin{bmatrix} \delta\mathbf{X}_0 \\ \delta\mathbf{Y}_0 \end{bmatrix} \tag{5}$$

The latter equation is convenient since it relates a given impressed (and known) perturbation $[\delta\mathbf{X}_0, \delta\mathbf{Y}_0]^t$ to the corresponding orbit deviation from the nominal orbit (henceforth also refer to as the reference orbit).

2 orbit perturbation using the steerer HTA9-C

For all the measurement reported henceforth, the orbit was perturbed by impressing a kick using an horizontal steerer located downstream of the TESLA cavity (steerer HTA9-C). That means practically our orbit perturbation is $[0, \delta x', 0, 0]^t$. The first step is to calibrate steerer, that is to find the relation between excitation current and resulting $\delta x'$. The beam was transported along the injector and the steerers along the beamline were tuned to achieve beam positions within ± 2 mm on all the BPMs. The measured momentum was 15.5 MeV/c. Then the current of HTA9-C was scanned around the nominal value and the corresponding

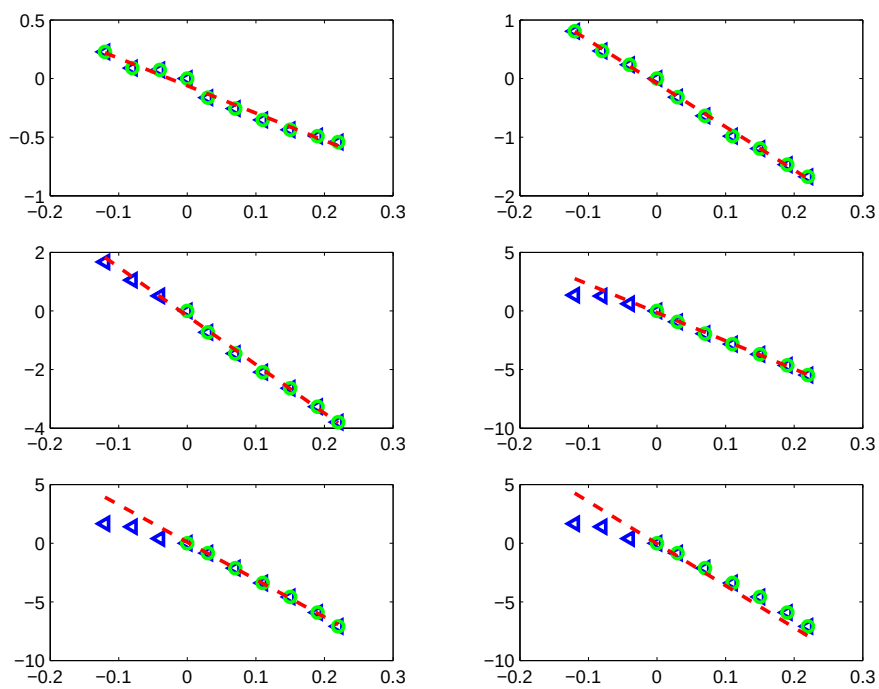

Figure 1: Absolute beam displacement in mm (as read by the BPM) versus HTA9-C excitation current in Ampère. The different plot corresponds from top left to bottom right to BPMs 2,3,4,5,6 and 7 respectively.

beam positions on all the BPMs were recorded. The measurements are reported in Fig. 1. To perform these measurements, the steerer was always scanned by decreasing the excitation current *a posteriori* to a degaussing of the steerer using an oscillatory degaussing procedure.

a linear fit of these data along with the knowledge of HTA9-C and the BPMs locations straightforwardly provide the angular kick per Ampère of excitation current. The value obtained is:

$$\mathcal{S} \doteq \left[\frac{dx'}{dI}\right]_{HTA9-C} = -4.06 \pm 0.52, \tag{6}$$

where the unit for $\mathcal{S}$ is mrad/A and the calibration holds for a beam momentum of 15.5 MeV/c. The computed divergence reported at each BPMs location is shown in Fig. 2.

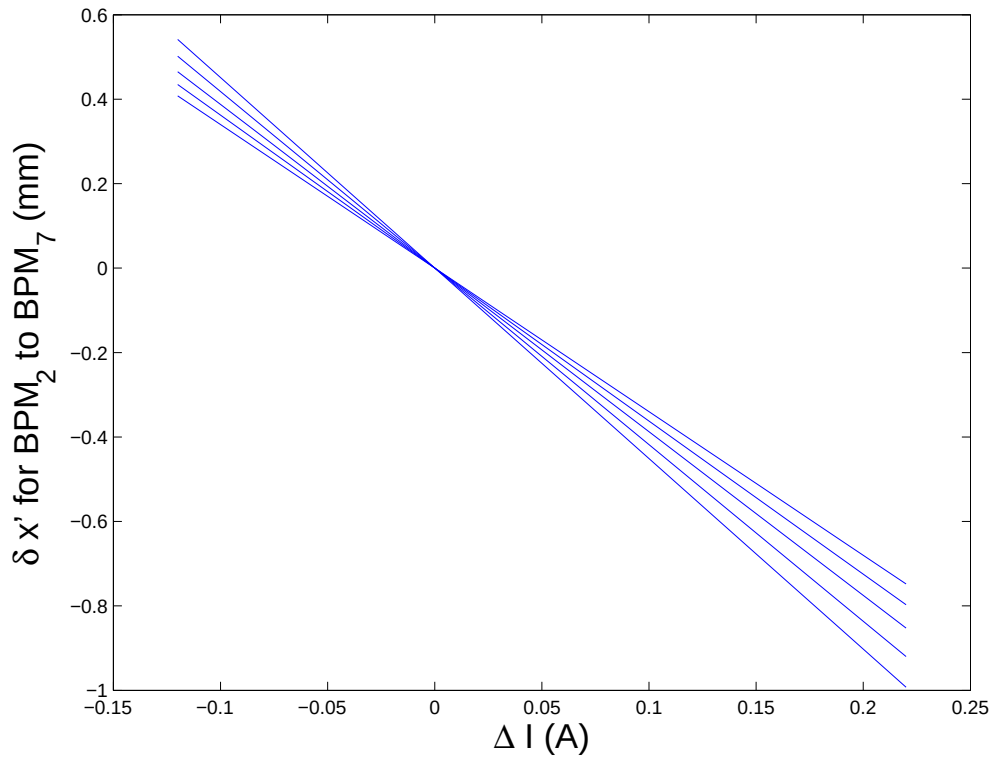

Figure 2: Computed divergence versus excitation current change for HTA9-C. The 6 lines corresponds to the 6 BPMs. The deviation of inferred divergence for the various BPM is coming from uncertainties in the measured beam position along with uncertainties in the exact location of the BPM.

3 Difference orbit measurement for various quadrupole excitations

Our main motivation is to check whether the evolution of the beam trajectory is consistent with our numerical prediction based on the magnetic measurement of one of the quadrupole for each of the quadrupole series in use at the FNPL injector. Given the gradient calibrations

reported in Table 1, we compute the normalized strength of the quadrupole as *:

$$k_1 1/\mathrm{m}^2 = 299.8 \frac{G[\mathrm{T/m}]}{p[\mathrm{MeV/c}]} = 8.92 I[\mathrm{A}], \tag{7}$$

wherein I is the quadrupole excitation current. The right hand side equality is given for a beam with p =15.5 MeV/c. So for the hereafter reported quadrupole excitations of ± 0.5 A, the corresponding strength used in the numerical calculation is ± 4.46 m^{-2}.

	field gradient	units
blue quadrupoles	0.461 ± 0.011	T/m/A
green quadrupoles	0.777 ± 0.029	T/m/A

Table 1: Quadrupoles calibration from the magnetic measurement performed by the technical division.

Next it is instructive to have an idea of what the difference orbit results are when one has one normal or skew quadrupole excited.

For the case of a normal quadrupole the 4×4 transfer matrix has the form:

$$T = \begin{bmatrix} \mathbf{F} & 0 \\ 0 & \mathbf{D} \end{bmatrix}, \tag{8}$$

and correspondingly,

$$R = \begin{bmatrix} F_{11} & F_{12} & 0 & 0 \\ 0 & 0 & D_{11} & D_{12} \end{bmatrix}. \tag{9}$$

For the case of a skew quadrupole the 4×4 transfer matrix has the form:

$$T = \frac{1}{2} \begin{bmatrix} \mathbf{F}+\mathbf{D} & \mathbf{F}-\mathbf{D} \\ \mathbf{F}-\mathbf{D} & \mathbf{F}+\mathbf{D} \end{bmatrix}, \tag{10}$$

and correspondingly,

$$R = \begin{bmatrix} F_{11}+D_{11} & F_{12}+D_{12} & F_{11}-D_{11} & F_{12}-D_{12} \\ F_{11}-D_{11} & F_{12}-D_{12} & F_{11}+D_{11} & F_{12}+D_{12} \end{bmatrix}. \tag{11}$$

From the latter response matrix, we see that the response difference orbit to a perturbation of the form $[0, \delta x', 0, 0]^t$ for the quadrupole excited to the value $+q$ is:

$$R = \begin{bmatrix} \delta x^+ \\ \delta y^+ \end{bmatrix} = \begin{bmatrix} (F_{12}+D_{12})\delta x' \\ (D_{12}-F_{12})\delta x' \end{bmatrix} \tag{12}$$

*in the herein reported experiment only the blue quadrupole were used

changing the sign of the quad excitation is equivalent to the permutation $\mathbf{D} \leftrightarrow \mathbf{F}$. Hence we have the corresponding response orbit:

$$R = \begin{bmatrix} \delta x^- \\ \delta y^- \end{bmatrix} = \begin{bmatrix} (F_{12} + D_{12})\delta x' \\ -(F_{12} - D_{12})\delta x' \end{bmatrix} = \begin{bmatrix} \delta x^+ \\ -\delta y^+ \end{bmatrix}. \tag{13}$$

Comparing the orbit perturbations resulting from a $\delta x'$ for two opposite excitations of a skew quadrupole will result in the same orbit response in the x-plane and the opposite orbit response in the y-plane.

Our results are reported in the Figures 3, 4, 5, 6 and 7. First we note that generally, the difference orbit values does not match with the simulated one though the sign of the difference orbit does. We also note (see circles on Fig. 3) that the orbit downstream the beamline has a kick. Using a linear fit on the two parts of the difference orbit we estimate this parasitic kick to be located around 4-5 m from the HTA9-C steerer. There error bar on our estimate of the kick location is large, but we expect the kick to be before or at the magnetic bunch compressor. It is finally worth mentioning that measurement of transverse beam spot of an angular-momentum-dominated beam start to appear skewed at X7 and X8, which also suggest the presence of asymmetric focusing field in that area.

Further investigation using magnetic measurement is on-going.

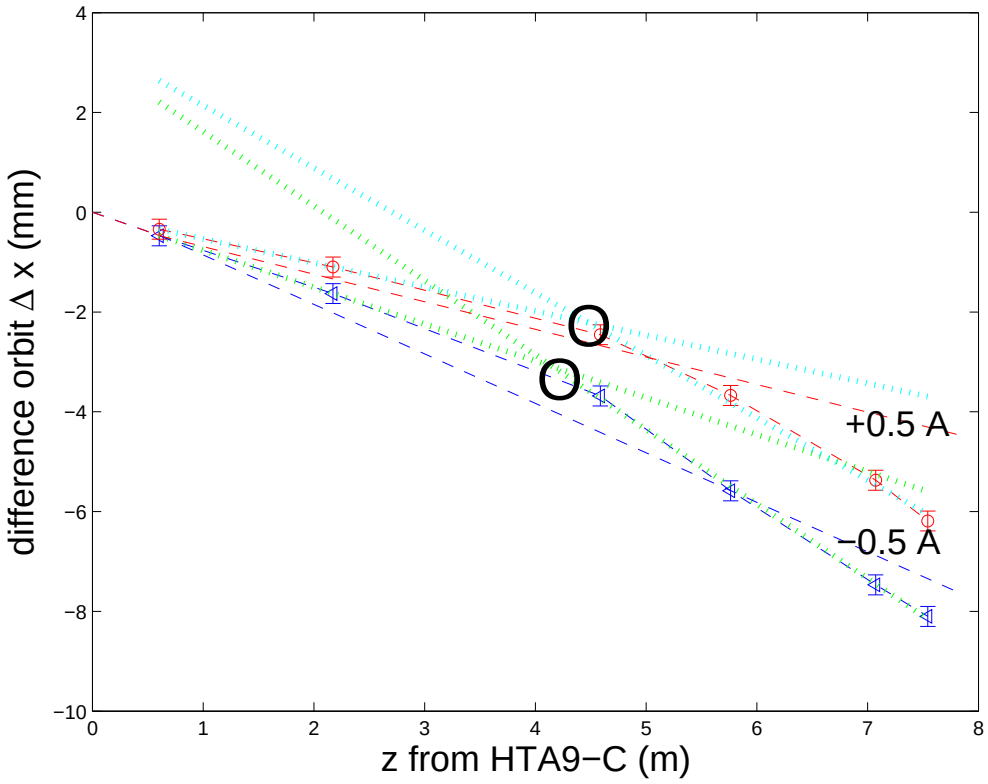

Figure 3: Difference orbit measurement for two excitations of Q1AX03. Diamonds and circles are beam positions on the BPM for the quadrupole excited to -0.5 and +0.5 Ampère respectively. Dashed lines correspond to expected difference orbits for 0.5 (red) and -0.5 A (blue). The green and cyan dashed line are the results of a linear fit of the first part of the orbit defined by the two first BPMs and the second part of the orbit defined by the four last BPMs.

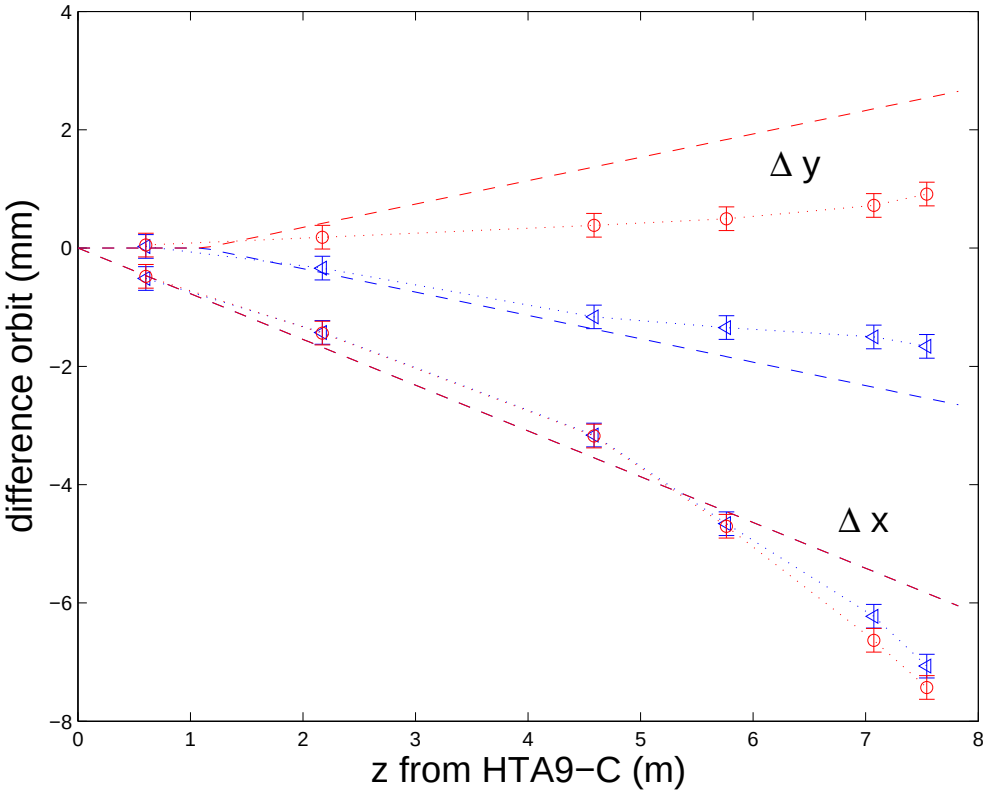

Figure 4: Difference orbit measurement for two excitations of S2AX03. Diamonds and circles are beam positions on the BPM for the quadrupole excited to -0.5 and +0.5 Ampère respectively. Dashed lines correspond to expected difference orbits for 0.5 (red) and -0.5 A (blue). The four top curves correspond to y-plane difference orbit, the four bottom curves correspond to x-plane difference orbits. (Note that two expected difference orbit curves are superimposed)

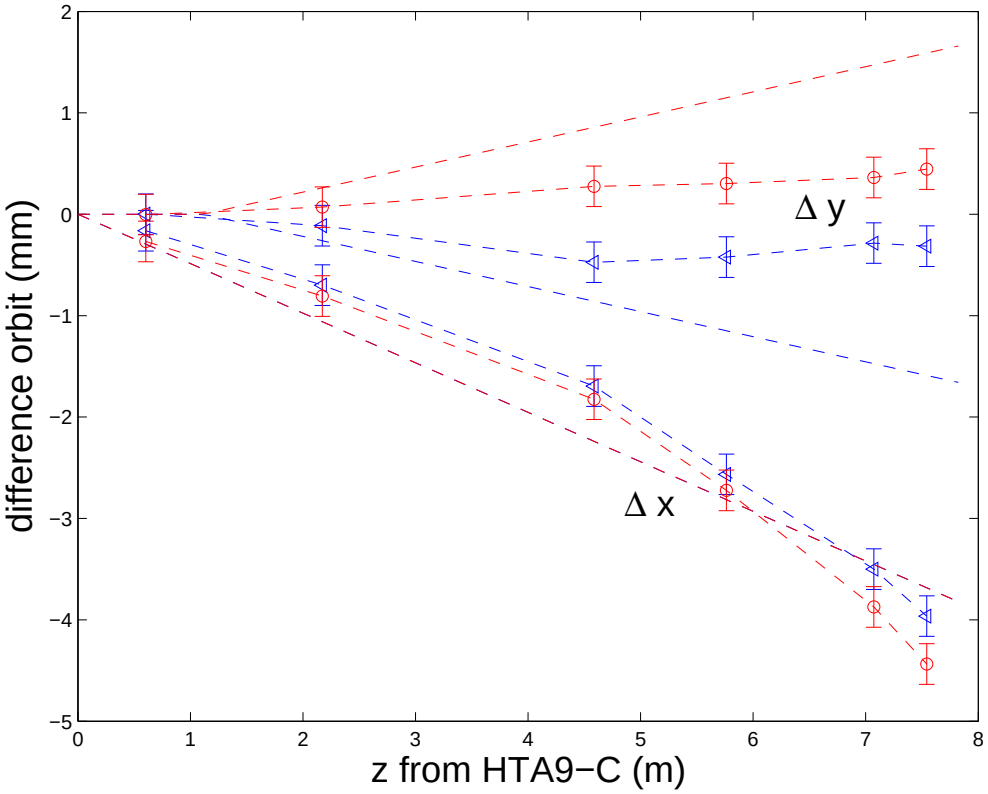

Figure 5: Difference orbit measurement for two excitations of S1AX04. Diamonds and circles are beam positions on the BPM for the quadrupole excited to -0.5 and +0.5 Ampère respectively. Dashed lines correspond to expected difference orbits for 0.5 (red) and -0.5 A (blue). The four top curves correspond to y-plane difference orbit, the four bottom curves correspond to x-plane difference orbits. (Note that two expected difference orbit curves are superimposed)

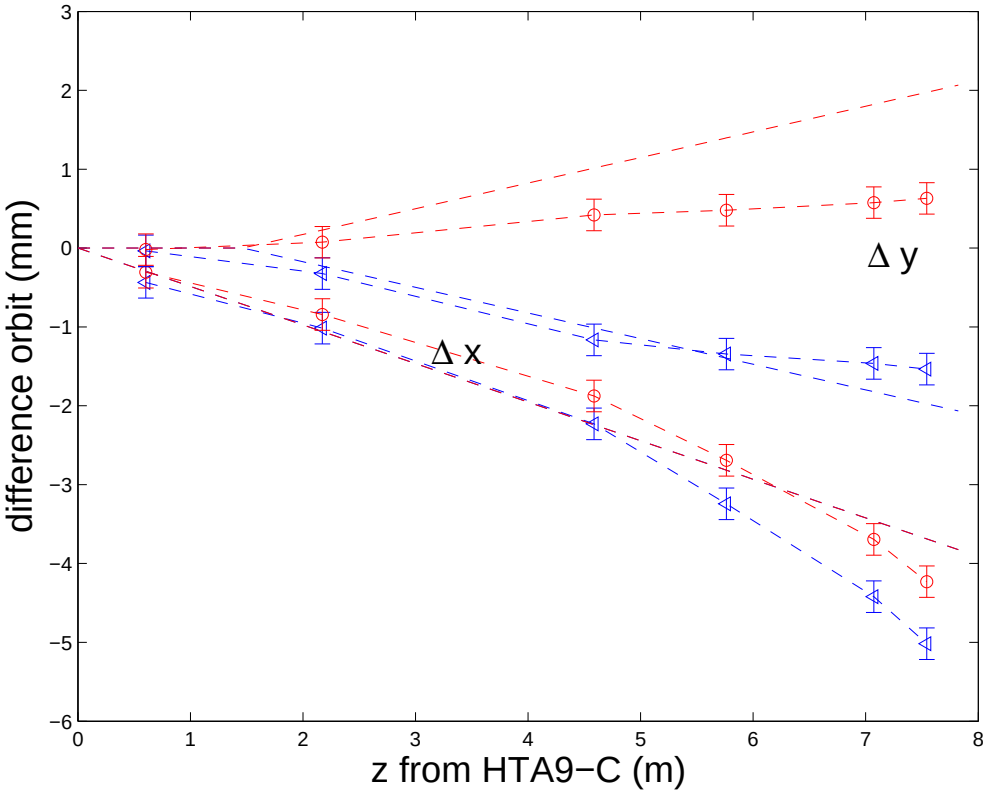

Figure 6: Difference orbit measurement for two excitations of S1AX05. Diamonds and circles are beam positions on the BPM for the quadrupole excited to -0.5 and +0.5 Ampère respectively. Dashed lines correspond to expected difference orbits for 0.5 (red) and -0.5 A (blue). The four top curves correspond to y-plane difference orbit, the four bottom curves correspond to x-plane difference orbits. (Note that two expected difference orbit curves are superimposed)

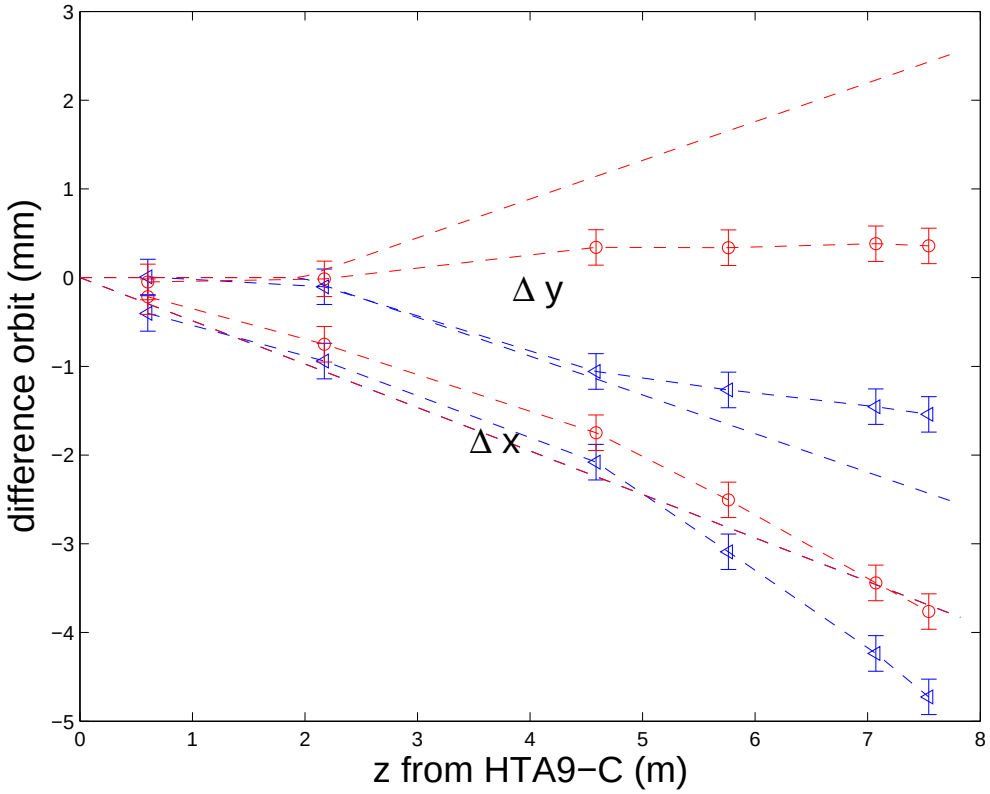

Figure 7: Difference orbit measurement for two excitations of S1AX06. Diamonds and circles are beam positions on the BPM for the quadrupole excited to -0.5 and +0.5 Ampère respectively. Dashed lines correspond to expected difference orbits for 0.5 (red) and -0.5 A (blue). The four top curves correspond to y-plane difference orbit, the four bottom curves correspond to x-plane difference orbits. (Note that two expected difference orbit curves are superimposed)