Luminosity Ratio

$eV := 1.6 \cdot 10^{-19} joule$ $c := 2.9979 \cdot 10^{8} \cdot \frac{m}{sec}$

$mrad := 10^{-3}$

Parameters:

RF parameters: $\gamma := \frac{980}{0.938}$ $\delta := 0.43 \cdot 10^{-3}$ $f_{rf} := 53 \cdot 10^{6} \cdot Hz$ $E_s := 980 \cdot 10^{9} \cdot eV$

(bucket height)

Initial emittances: $S_0 := 3.5 \cdot eV \cdot sec$ $\varepsilon_0 := 20 \cdot \pi \cdot 10^{-6} \cdot m$

Emittance growth rates: $Sdot := 0.15 \cdot \frac{eV \cdot sec}{hr}$ $\varepsilon dot := 0.8 \cdot \pi \cdot \frac{mm \cdot mrad}{hr}$

Time interval:

$\Delta t := 2 \cdot min$ $i := 0 .. 750$ $t_i := i \cdot \Delta t$ $S_i := S_0 + Sdot \cdot t_i$ $\varepsilon_i := \varepsilon_0 + \varepsilon dot \cdot t_i$

End of Store Parameters: $S_{length(S)-1} = 7.25\, eV \cdot sec$ $\varepsilon_{length(\varepsilon)-1} = 40\pi \cdot 10^{-6} \cdot m$

$t_{length(t)-1} = 25\, hr$

Bunch Evolution: $\sigma_p := \sqrt{\frac{\delta \cdot f_{rf} \cdot S}{6 \cdot E_s}}$ $\sigma_\phi := \sqrt{\frac{2 \cdot f_{rf} \cdot S}{3 \cdot \delta \cdot E_s}}$ $\sigma_z := \frac{c}{2 \cdot \pi \cdot f_{rf}} \cdot \sigma_\phi$

Hourglass Effect:
(minima located @ δz)

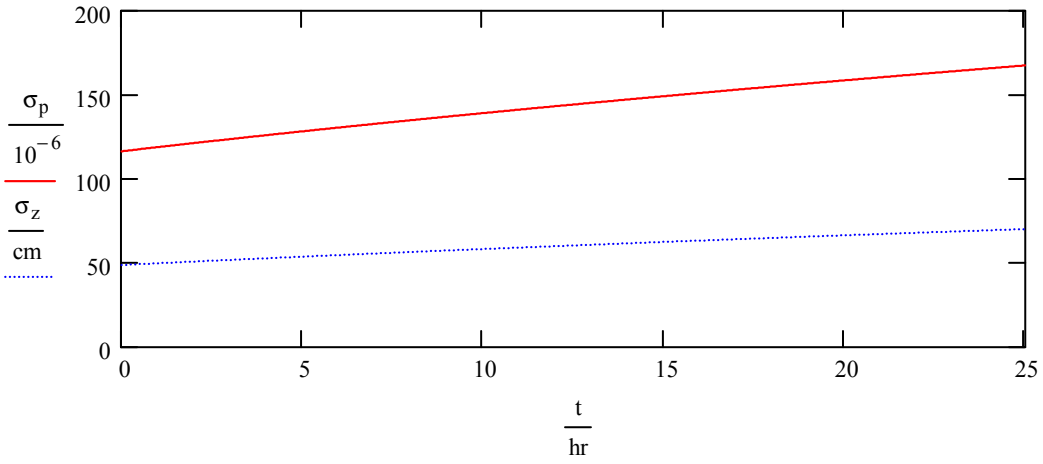
$$H(\delta z_x, \delta z_y, \beta_x, \beta_y, \sigma_z) := \frac{\sqrt{2}}{\sqrt{2 \cdot \pi} \cdot \sigma_z} \cdot \int_{-8 \cdot m}^{8 \cdot m} \frac{1}{\sqrt{1 + \left(\frac{z - \delta z_x}{\beta_x}\right)^2}} \cdot \frac{1}{\sqrt{1 + \left(\frac{z - \delta z_y}{\beta_y}\right)^2}} \cdot e^{\frac{-z^2}{\sigma_z^2}} dz$$

Crossing Angle Effect:
(α is total x-angle, assumed Horizontal)

$$C(\alpha, \sigma_z, \sigma_x) := \frac{1}{\sqrt{1 + \alpha^2 \cdot \frac{\sigma_z^2}{4\sigma_x^2}}}$$

Offset Effect:
(Δ is total offset, assumed Horizontal)

$$O(\Delta, \sigma_x) := e^{\frac{-\left(\frac{\Delta}{2}\right)^2}{\sigma_x^2}}$$



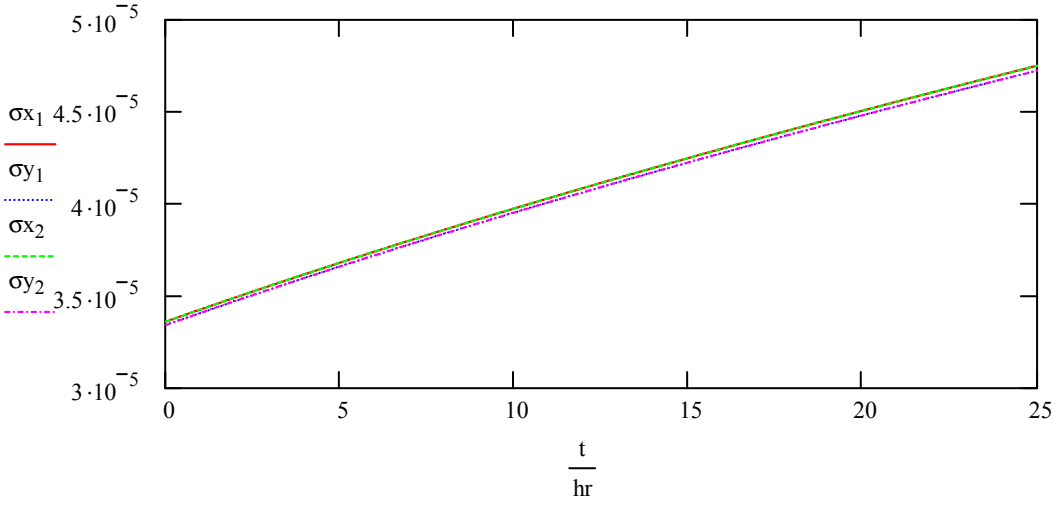
Static IP Parameters: $\beta x_1 := 35 \cdot cm$ $\beta y_1 := 35 \cdot cm$ $\beta x_2 := 35 \cdot cm$ $\beta y_2 := 35 \cdot cm$

$D_1 := 3 \cdot cm$ $D_2 := 3 \cdot cm$

Transverse Beam Size Evolution:

$$\sigma x_1 := \sqrt{\frac{\beta x_1 \cdot \varepsilon}{6 \cdot \pi \cdot \gamma} + (D_1 \cdot \sigma_p)^2} \qquad \sigma x_2 := \sqrt{\frac{\beta x_2 \cdot \varepsilon}{6 \cdot \pi \cdot \gamma} + (D_2 \cdot \sigma_p)^2}$$

$$\sigma y_1 := \sqrt{\frac{\beta y_1 \cdot \varepsilon}{6 \cdot \pi \cdot \gamma}} \qquad \sigma y_2 := \sqrt{\frac{\beta y_2 \cdot \varepsilon}{6 \cdot \pi \cdot \gamma}}$$



Hourglass....

$\delta z_{x1} := 0 \cdot cm \qquad \delta z_{y1} := 0 \cdot cm$

$\delta z_{x2} := 0 \cdot cm \qquad \delta z_{y2} := 0 \cdot cm$

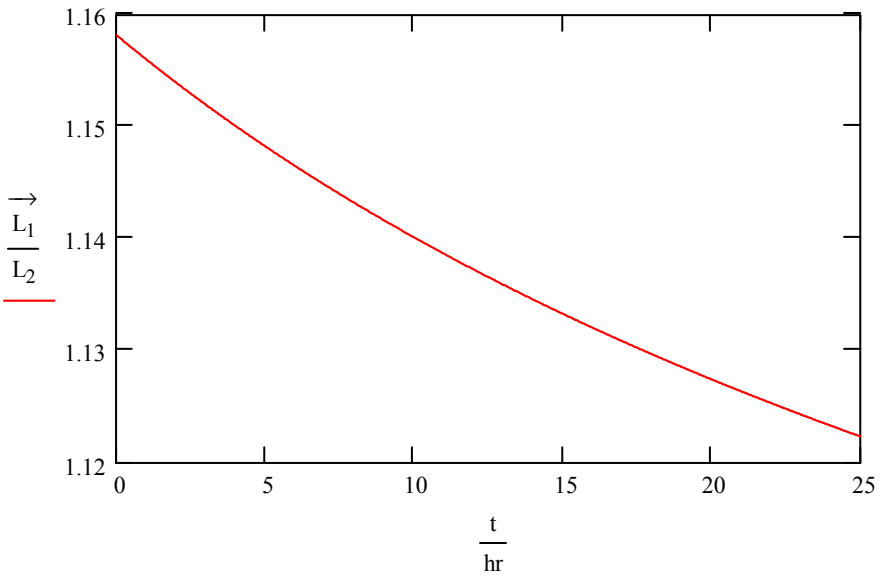
(total separation between x,y minima is $\delta z_y - \delta z_x$)

$$L_1 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_1}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_1}{6 \cdot \pi \cdot \gamma} + D_1^{\,2} \cdot \sigma_p^{\,2}}} \cdot H\left(\delta z_{x1}, \delta z_{y1}, \beta x_1, \beta y_1, \sigma_z\right) \right)^{\rightarrow}$$

$$\varepsilon dot = 0.8 \frac{\pi \cdot mm \cdot mrad}{hr}$$

$$L_2 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_2}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_2}{6 \cdot \pi \cdot \gamma} + D_2^{\,2} \cdot \sigma_p^{\,2}}} \cdot H\left(\delta z_{x2}, \delta z_{y2}, \beta x_2, \beta y_2, \sigma_z\right) \right)^{\rightarrow}$$

$$Sdot = 0.15 \frac{eV \cdot sec}{hr}$$



$\beta x_1 = 0.32\,m$

$\beta y_1 = 0.32\,m$

$D_1 = 0.03\,m$

$\beta x_2 = 0.42\,m$

$\beta y_2 = 0.42\,m$

$D_2 = 0.03\,m$

$S_0 = 3.5\,eV \cdot sec$

$S_{last(S)} = 7.25\,eV \cdot sec$

$\varepsilon_0 = 20\,\pi \cdot mm \cdot mrad$

$\varepsilon_{last(\varepsilon)} = 40\,\pi \cdot mm \cdot mrad$

$\frac{L1_0}{L2_0} = 1.158$

$\frac{L1_{last(L1)}}{L2_{last(L2)}} = 1.122$

Check: $H(0 \cdot cm, 0 \cdot cm, 35 \cdot cm, 35 \cdot cm, 60 \cdot cm) = 0.595$

Hourglass....

$\delta z_{x1} := 0 \cdot cm \qquad \delta z_{y1} := 0 \cdot cm$

$\delta z_{x2} := -20 \cdot cm \qquad \delta z_{y2} := 20 \cdot cm$

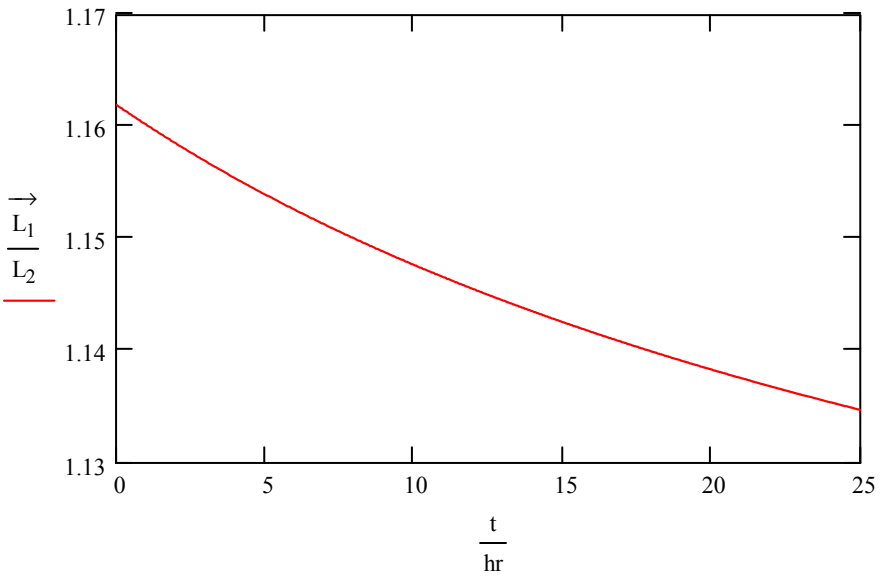
(total separation between x,y minima is $\delta z_y - \delta z_x$)

$$L_1 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_1}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_1}{6 \cdot \pi \cdot \gamma} + D_1^{\,2} \cdot \sigma_p^{\,2}}} \cdot H\left(\delta z_{x1}, \delta z_{y1}, \beta x_1, \beta y_1, \sigma_z\right) \right)^{\rightarrow}$$

$$\varepsilon dot = 0.8 \frac{\pi \cdot mm \cdot mrad}{hr}$$

$$L_2 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_2}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_2}{6 \cdot \pi \cdot \gamma} + D_2^{\,2} \cdot \sigma_p^{\,2}}} \cdot H\left(\delta z_{x2}, \delta z_{y2}, \beta x_2, \beta y_2, \sigma_z\right) \right)^{\rightarrow}$$

$$Sdot = 0.15 \frac{eV \cdot sec}{hr}$$



$\beta x_1 = 0.35\,m$

$\beta y_1 = 0.35\,m$

$D_1 = 0.03\,m$

$\beta x_2 = 0.35\,m$

$\beta y_2 = 0.35\,m$

$D_2 = 0.03\,m$

$S_0 = 3.5\,eV \cdot sec$

$S_{last(S)} = 7.25\,eV \cdot sec$

$\varepsilon_0 = 20\,\pi \cdot mm \cdot mrad$

$\varepsilon_{last(\varepsilon)} = 40\,\pi \cdot mm \cdot mrad$

$\frac{L1_0}{L2_0} = 1.162$

$\frac{L1_{last(L1)}}{L2_{last(L2)}} = 1.135$

Check: $H(0 \cdot cm, 0 \cdot cm, 35 \cdot cm, 35 \cdot cm, 60 \cdot cm) = 0.595$

Hourglass....

$\delta z_{x1} := 0 \cdot cm \qquad \delta z_{y1} := 0 \cdot cm$

$\delta z_{x2} := 0 \cdot cm \qquad \delta z_{y2} := 0 \cdot cm$

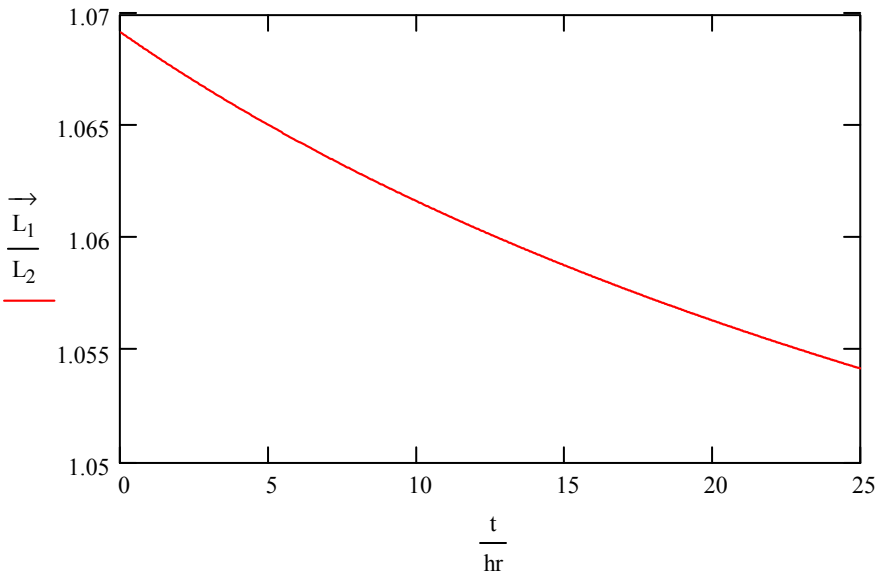
(total separation between x,y minima is $\delta z_y - \delta z_x$)

$$L_1 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_1}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_1}{6 \cdot \pi \cdot \gamma} + D_1^{\,2} \cdot \sigma_p^{\,2}}} \cdot H\left(\delta z_{x1}, \delta z_{y1}, \beta x_1, \beta y_1, \sigma_z\right) \right)^{\rightarrow}$$

$$\varepsilon dot = 0.8 \frac{\pi \cdot mm \cdot mrad}{hr}$$

$$L_2 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_2}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_2}{6 \cdot \pi \cdot \gamma} + D_2^{\,2} \cdot \sigma_p^{\,2}}} \cdot H\left(\delta z_{x2}, \delta z_{y2}, \beta x_2, \beta y_2, \sigma_z\right) \right)^{\rightarrow}$$

$$Sdot = 0.15 \frac{eV \cdot sec}{hr}$$



$\beta x_1 = 0.32\,m$

$\beta y_1 = 0.35\,m$

$D_1 = 0.03\,m$

$\beta x_2 = 0.35\,m$

$\beta y_2 = 0.41\,m$

$D_2 = 0.03\,m$

$S_0 = 3.5\,eV \cdot sec$

$S_{last(S)} = 7.25\,eV \cdot sec$

$\varepsilon_0 = 20\,\pi \cdot mm \cdot mrad$

$\varepsilon_{last(\varepsilon)} = 40\,\pi \cdot mm \cdot mrad$

$\frac{L1_0}{L2_0} = 1.069$

$\frac{L1_{last(L1)}}{L2_{last(L2)}} = 1.054$

Check: $H(0 \cdot cm, 0 \cdot cm, 35 \cdot cm, 35 \cdot cm, 60 \cdot cm) = 0.595$

Crossing Angle....

$\alpha_1 := 0 \cdot 10^{-6}$

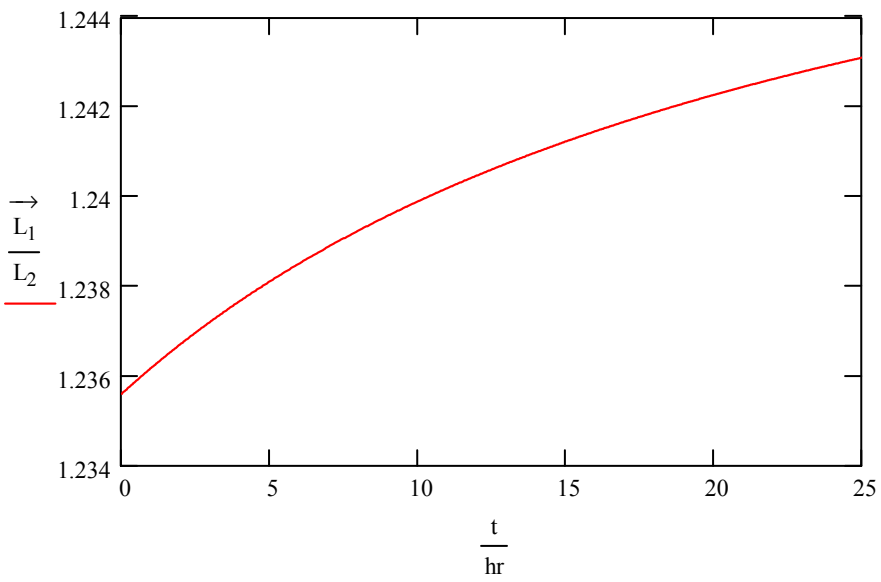
$\alpha_2 := 100 \cdot 10^{-6}$

$$\overrightarrow{L_1 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_1}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_1}{6 \cdot \pi \cdot \gamma} + D_1^{\,2} \cdot \sigma_p^{\,2}}} \cdot C(\alpha_1, \sigma_z, \sigma x_1) \right)}$$

$\varepsilon dot = 0.8 \frac{\pi \cdot mm \cdot mrad}{hr}$

$$\overrightarrow{L_2 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_2}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_2}{6 \cdot \pi \cdot \gamma} + D_2^{\,2} \cdot \sigma_p^{\,2}}} \cdot C(\alpha_2, \sigma_z, \sigma x_2) \right)}$$

$Sdot = 0.15 \frac{eV \cdot sec}{hr}$



$\beta x_1 = 0.35\,m$

$\beta y_1 = 0.35\,m$

$D_1 = 0.03\,m$

$\beta x_2 = 0.35\,m$

$\beta y_2 = 0.35\,m$

$D_2 = 0.03\,m$

$S_0 = 3.5\,eV \cdot sec$

$\varepsilon_0 = 20\,\pi \cdot mm \cdot mrad$

$\frac{L1_0}{L2_0} = 1.236$

$S_{last(S)} = 7.25\,eV \cdot sec$

$\varepsilon_{last(\varepsilon)} = 40\,\pi \cdot mm \cdot mrad$

$\frac{L1_{last(L1)}}{L2_{last(L2)}} = 1.243$

Offset ....

$\Delta_1 := 0 \cdot 10^{-6} \cdot m$

$\Delta_2 := 30 \cdot 10^{-6} \cdot m$

$$L_1 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_1}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_1}{6 \cdot \pi \cdot \gamma} + D_1^{\,2} \cdot \sigma_p^{\,2}}} \cdot O(\Delta_1, \sigma x_1) \right)$$

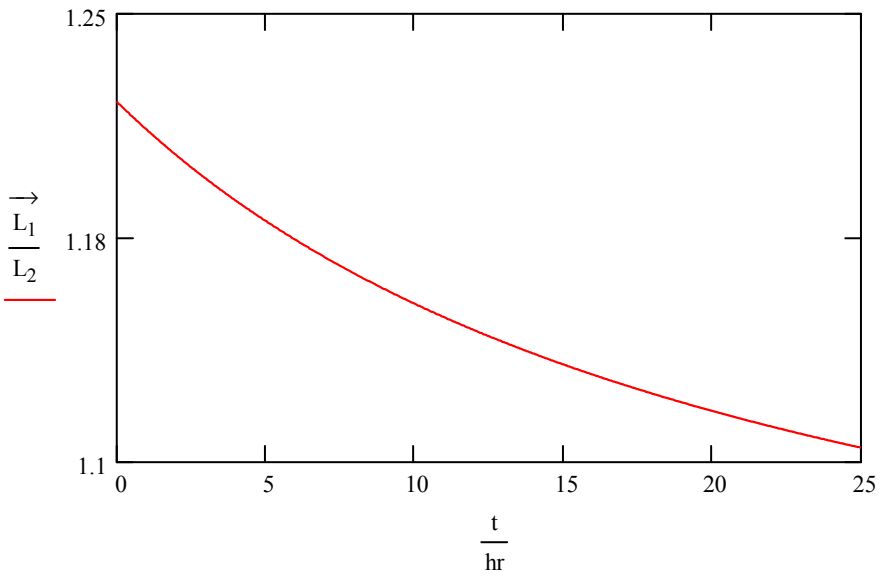
$\sigma x1_0 = 33.599\,10^{-6} \cdot m$

$\sigma x2_0 = 33.599\,10^{-6} \cdot m$

$$L_2 := \left(\frac{1}{\sqrt{\frac{\varepsilon \cdot \beta y_2}{6 \cdot \pi \cdot \gamma}} \cdot \sqrt{\frac{\varepsilon \cdot \beta x_2}{6 \cdot \pi \cdot \gamma} + D_2^{\,2} \cdot \sigma_p^{\,2}}} \cdot O(\Delta_2, \sigma x_2) \right)$$

$\varepsilon dot = 0.8 \frac{\pi \cdot mm \cdot mrad}{hr}$

$Sdot = 0.15 \frac{eV \cdot sec}{hr}$



$\beta x_1 = 0.35\,m$

$\beta y_1 = 0.35\,m$

$D_1 = 0.03\,m$

$\beta x_2 = 0.35\,m$

$\beta y_2 = 0.35\,m$

$D_2 = 0.03\,m$

$S_0 = 3.5\,eV \cdot sec$

$\varepsilon_0 = 20\,\pi \cdot mm \cdot mrad$

$\frac{L1_0}{L2_0} = 1.221$

$S_{last(S)} = 7.25\,eV \cdot sec$

$\varepsilon_{last(\varepsilon)} = 40\,\pi \cdot mm \cdot mrad$

$\frac{L1_{last(L1)}}{L2_{last(L2)}} = 1.105$

Luminous Region

$\beta_{x1} := 30 \cdot cm \quad \beta_{y1} := 30 \cdot cm \quad \delta z_{x1} := 0 \cdot cm \quad \delta z_{y1} := 0 \cdot cm$

$\beta_{x2} := 45 \cdot cm \quad \beta_{y2} := 45 \cdot cm \quad \delta z_{x2} := 0 \cdot cm \quad \delta z_{y2} := 0 \cdot cm$

$\sigma_z := 60 \cdot cm$

$$dLdz_1(z) := 2 \cdot \frac{1}{\sqrt{\beta_{x1} \cdot \beta_{y1}}} \cdot \frac{\sqrt{2}}{\sqrt{2 \cdot \pi} \cdot \sigma_z} \cdot \frac{1}{\sqrt{1 + \left(\frac{z - \delta z_{x1}}{\beta_{x1}}\right)^2}} \cdot \frac{1}{\sqrt{1 + \left(\frac{z - \delta z_{y1}}{\beta_{y1}}\right)^2}} \cdot e^{\frac{-z^2}{\sigma_z^2}}$$

$$L_1 := \int_{-8 \cdot m}^{8 \cdot m} dLdz_1(z)\, dz$$

$$dLdz_2(z) := 2 \cdot \frac{1}{\sqrt{\beta_{x2} \cdot \beta_{y2}}} \cdot \frac{\sqrt{2}}{\sqrt{2 \cdot \pi} \cdot \sigma_z} \cdot \frac{1}{\sqrt{1 + \left(\frac{z - \delta z_{x2}}{\beta_{x2}}\right)^2}} \cdot \frac{1}{\sqrt{1 + \left(\frac{z - \delta z_{y2}}{\beta_{y2}}\right)^2}} \cdot e^{\frac{-z^2}{\sigma_z^2}}$$

$$L_2 := \int_{-8 \cdot m}^{8 \cdot m} dLdz_2(z)\, dz$$

$$\Sigma_z := \frac{\sigma_z}{\sqrt{2}} \cdot H(0 \cdot cm, 0 \cdot cm, 35 \cdot cm, 35 \cdot cm, \sigma_z)$$

$z := -500 \cdot cm, -499.9 \cdot cm .. 500 \cdot cm$

$$\frac{L_1}{L_2} = 1.215$$