Beams-doc-1448-v1
November 12, 2004

How Mismatched Phases on the BPM Cables Affect Postion Measurements

Rob Kutschke, CD/EXP

Abstract

I have made a very crude model of phase mismatch between the A and B signals from a BPM and computed the error in the position measurement which results from this phase mismatch. In this crude model, a phase mismatch of 1 degree of 53.1 MHz, causes a shift in position of about 10 μm. A position bias due to phase mismatch has the right scale to explain the unresolved problem with the proton position which was described in Beams-doc-1197. However a phase mismatch produces too small an effect to explain the problem with the anti-proton measurement which was described in the same document.

1 The Model of the Waveform

The top plot in figure 1 shows the crude model of the signal on the BPM A cable, after the analog filter and before the digitizer. The model of the signal on the B cable is the same, up to a small phase shift. The signals have the same overall amplitude.

Mathematically each bunch is modeled as cosine wave, at the RF frequency, multiplied by a decaying exponential which almost runs out just as the next bunch arrives. The mathematics of the model is,

$$A(t) = \sum_{j=0}^{N-1} \cos(\omega_{RF}\ (t - t_j)) \exp\left(-(t - t_j)/\tau\right) \tag{1}$$

$$B(t) = \sum_{j=0}^{N-1} \cos(\omega_{RF}\ (t - t_j) + \delta) \exp\left(-(t - t_j)/\tau\right), \tag{2}$$

where t is time, N is the number of bunches, t_j is the start time of the j^{th} bunch and where the sum runs over only terms for which $t - t_j > 0$; that is, it only includes bunches which have already arrived. Also, f_{RF} is the RF frequency, $\omega_{RF} = 2\pi f_{RF}$, $T_{RF} = 1/f_{RF}$, δ is a phase shift and τ is the decay time of the analog filter. The start time of each bunch is separated by 21 RF periods, $t_j = 21\ jT_0$. In this model I chose $\tau = 5T_{RF}$ so that the wave form rings out after about 20 RF buckets. In this study δ will be varied from $0°$ to $10°$.

In the second plot in figure 1, smooth red curve shows a detail of the $A(t)$ waveform for small times. The blue histogram superimposed on this is the digitized version of this waveform, digitized at $7/5 f_{RF}$. In the third plot in figure 1, the smooth red curve is the model of $B(t)$, for a phase shift of $10°$, and the blue histogram is the digitized form of $B(t)$.

2 The Model of the Echotek

The model of the Echotek is equally crude. I down-convert the digitized data stream at $2/5\, f_{RF}$ but I do not model the detailed filtering of the Echotek. The filter in this model is just the sum of each cycle of the down-converter,

$$A_n = \sum_{k=0}^{n} A(t_k) \exp(-i\, \omega_D t_k) \qquad (3)$$

$$B_n = \sum_{k=0}^{n} B(t_k) \exp(-i\, \omega_D t_k) \qquad (4)$$

where A_n and B_n are complex numbers, k counts clicks of the digitizer clock, which is the same as ticks of the down-converter, t_k is the time at the k^{th} clock and $\omega_D = 2/5\, \omega_{RF}$ is the angular frequency of the down-converter. In this model, A_n and B_n for later values of n include all of the information from all of the previous data.

For each value of n, the position, in mm, was computed as,

$$P_n = 26 \frac{|A_n| - |B_n|}{|A_n| + |B_n|}. \qquad (5)$$

The plots in figure 2 show the time development of P_n for three different values of the phase shift, $\delta = \{0°, 1°, 10°\}$. The horizontal axis corresponds to the same time interval as the horizontal axis on the top plot of figure 1; all three plots have different vertical scales.

Note that the Echotek has a minimum decimation and cannot produce a time series of position with this granularity. The time interval shown in the figures is just a small fraction of the time for a single closed orbit measurement. On the other hand, for the existing measurements made in short-gate mode, the gate was open from about t=0 to t=40.

3 Discussion

The two major deficiencies of this model are that:

- The analog signal is overly simplified.
- The filter following the down-converter is too simple.

We can address the first item by using the 2 GHz scope data to build a better model of the waveform. I believe that Gustavo's simulation code can address the second issue.

My guess is that when these simplifications are improved, the gross picture presented in the figures will remain about the same, although the details are likely to change a lot. With this caveat let's discuss the gross features.

When there is no phase shift, the system measures the correct position at all times. When there is a phase shift, however, the system almost always produces a biased answer.

1. There are oscillations in the measured position as each new bunch arrives.
2. These oscillations die down in about 5 to 10 RF periods and the measured position because quasi-stable until the next bunch arrives.
3. The initial oscillations are smaller on each successive bunch.
4. As we add more and more bunches, it appears that the measurement will stabilize but will it stabilize at the correct answer or at some other value?

If we are smart about avoiding the oscillations which occur near the start of a bunch, it seems likely that, for phase shifts of a few degrees, the bias will be, at most of order 10 μm. This is true for both short gate and long gate mode.

Beams-doc-1197 presented a measurement of coasting beam in both closed orbit mode and short gate mode. It found a difference in measured proton position of about 50 μm and a difference in measured anti-proton position of about 600 μm. I don't believe that any particular care was taken with phase matching for this exercise. So it seems likely that a phase mismatch can explain the difference in measured proton position. However the difference in anti-proton position seems larger than can be explained by this effect alone.

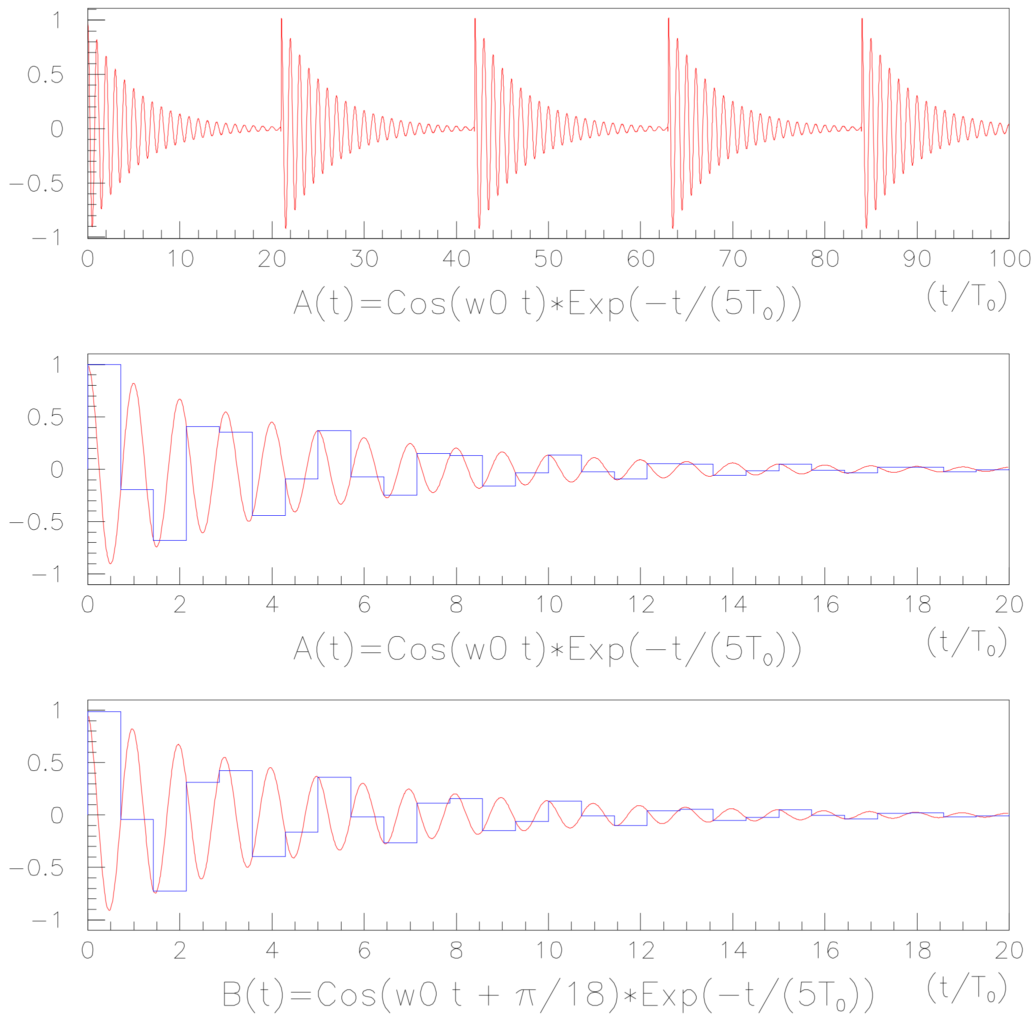

Figure 1: The top plot shows the crude model of the signal on the A cable, after the analog signal but before the digitizer. The horizontal axis is time, in units of the RF period; the vertical axis is in arbitrary units. The red curve in second plot shows a detail of the first plot. The red curve in the third plot shows the model of the B waveform, which is shifted by $10°$ of RF phase from the A waveform. In the last two plots the superimposed blue histograms show the digitized version of the waveforms, digitized at $7/5\, f_{RF}$.

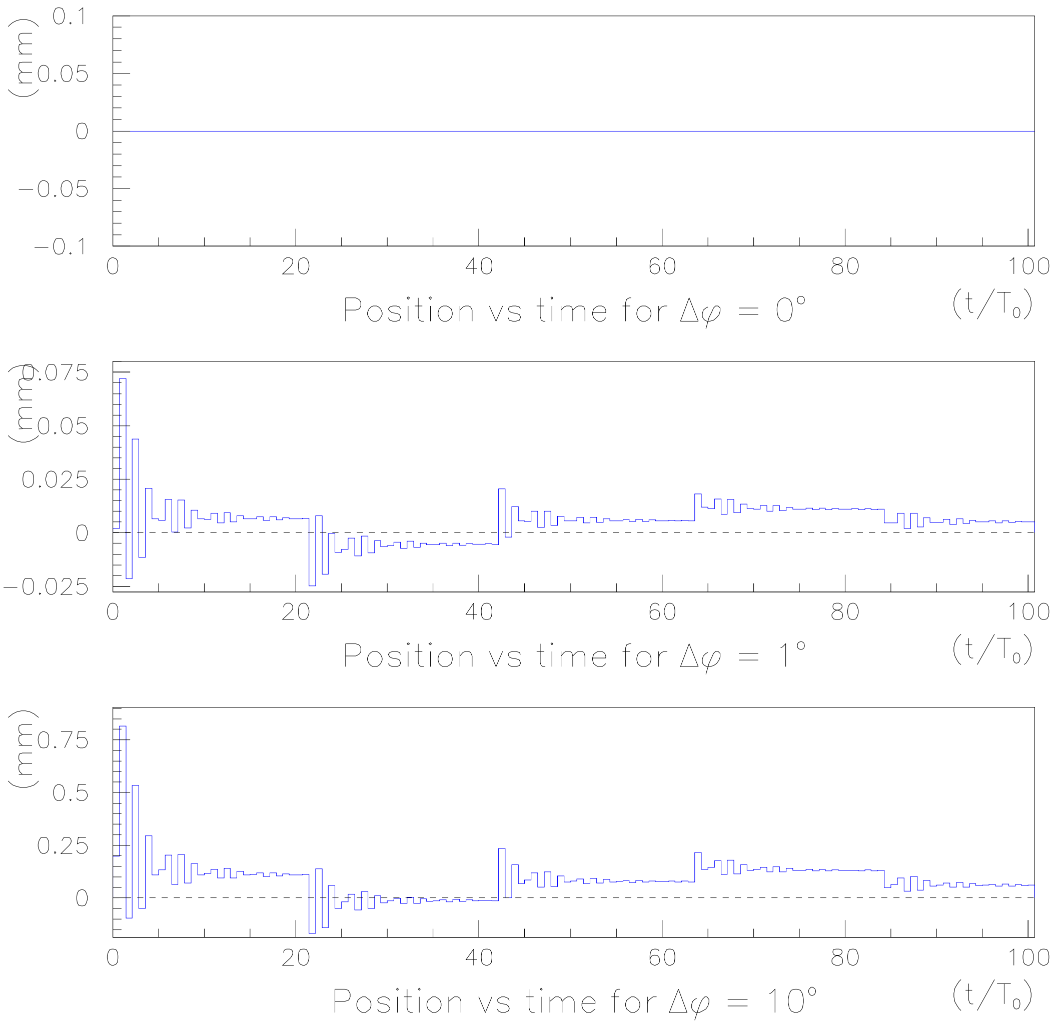

Figure 2: The beam position, as a function of time, computed as described in the text. The horizontal axis is time, in units of the RF period, and the vertical axis is the beam position in mm. The three plots are distinguished by different phase shifts of the B signal relative to the A signal. In all three plots the correct answer is zero, as is indicated by the dashed horizontal line in the last two plots.