Analysis of the phase shift error between A and B signals in BPMs

BPM project

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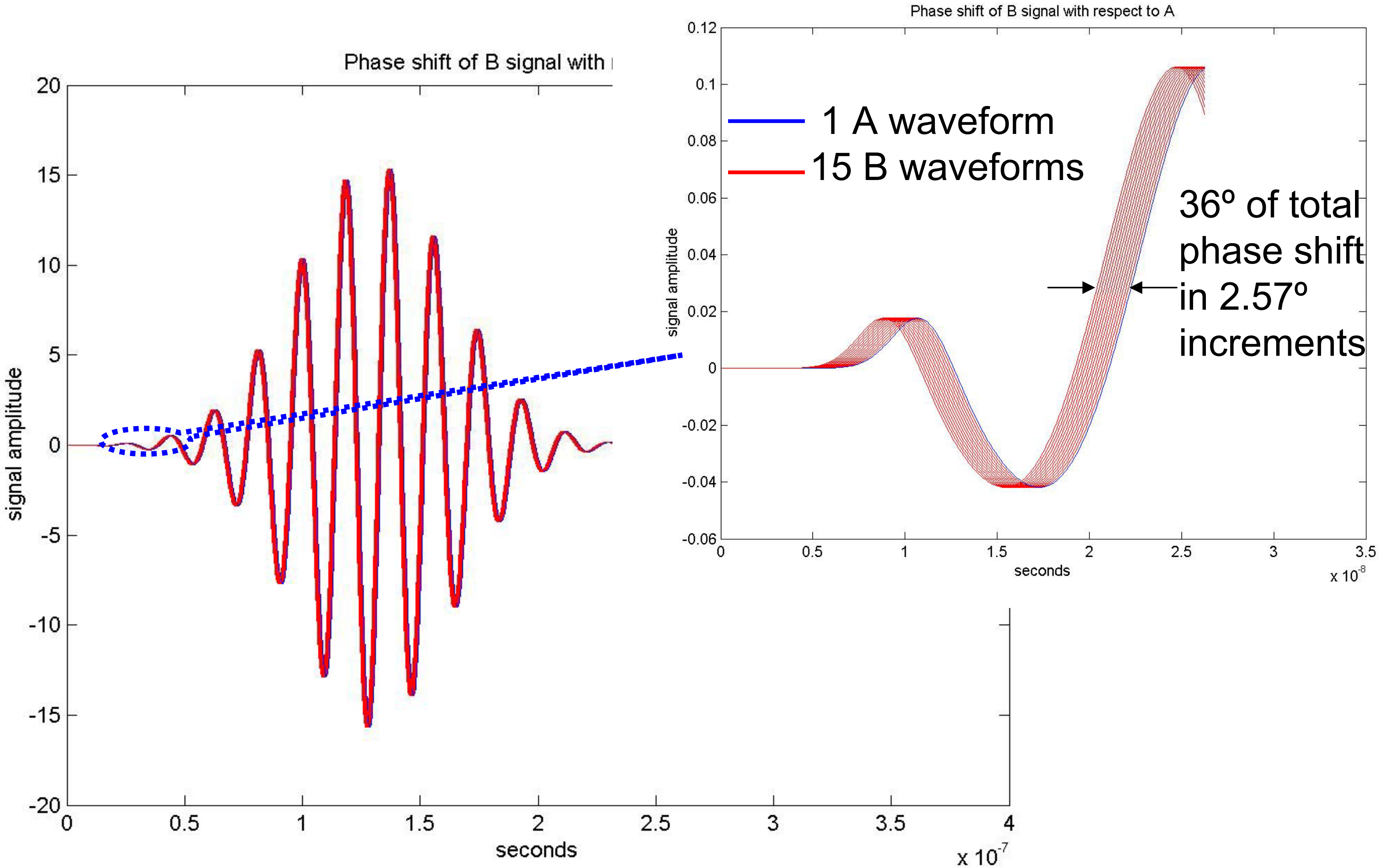
The phase shift problem

- A phase shift between A and B signals may arise do to different time propagation delays of A and B in the front-end electronics.
  - The propagation time in cables is ~1ns/ft.
  - A to B cable length mismatch of 1.5cm generates a phase shift of 1 degree.
- A and B signals are individually processed by analog filters. Different relative phases in the analog filters will generate an A to B phase shift.

A to B signal shift simulation

- The phase of B is advanced with respect to the phase of A in steps of 2.57º (degrees) in the interval [0º,36º].
- The simulation computes the error in position calculation as a function of the phase shift.

A and shifted B plots at high sampling frequency



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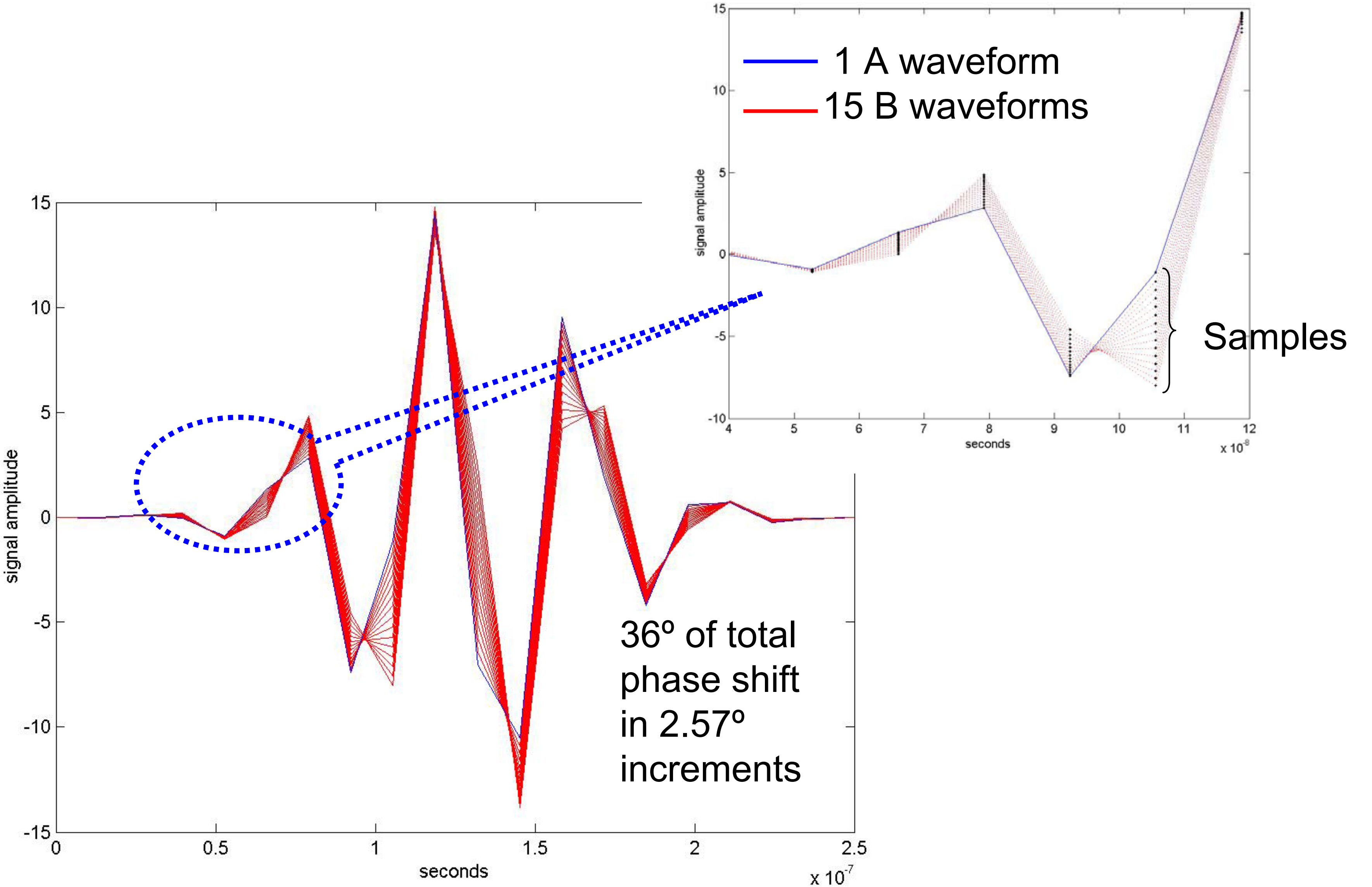
Plots of A and shifted B, sampled at 74MHz



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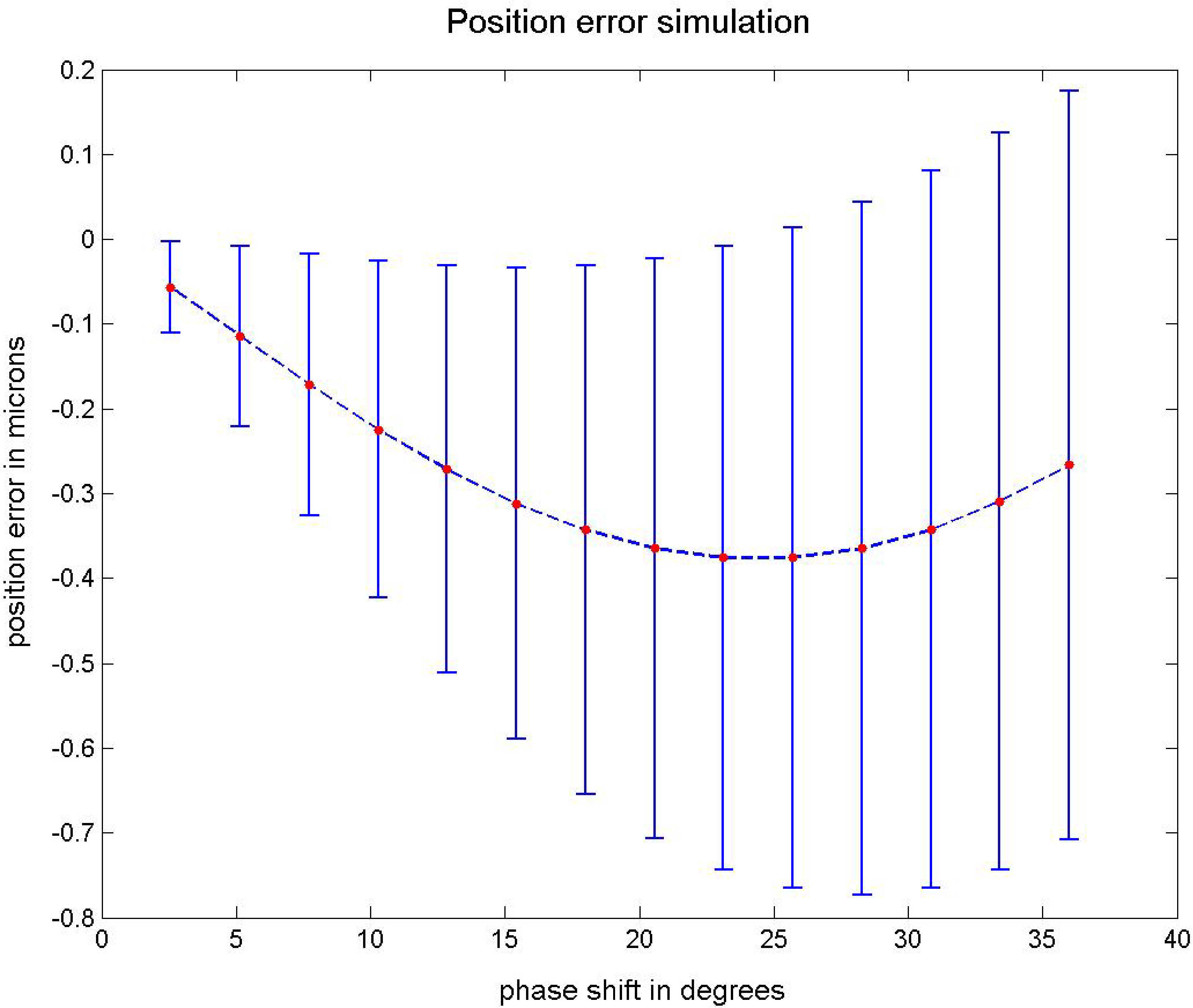

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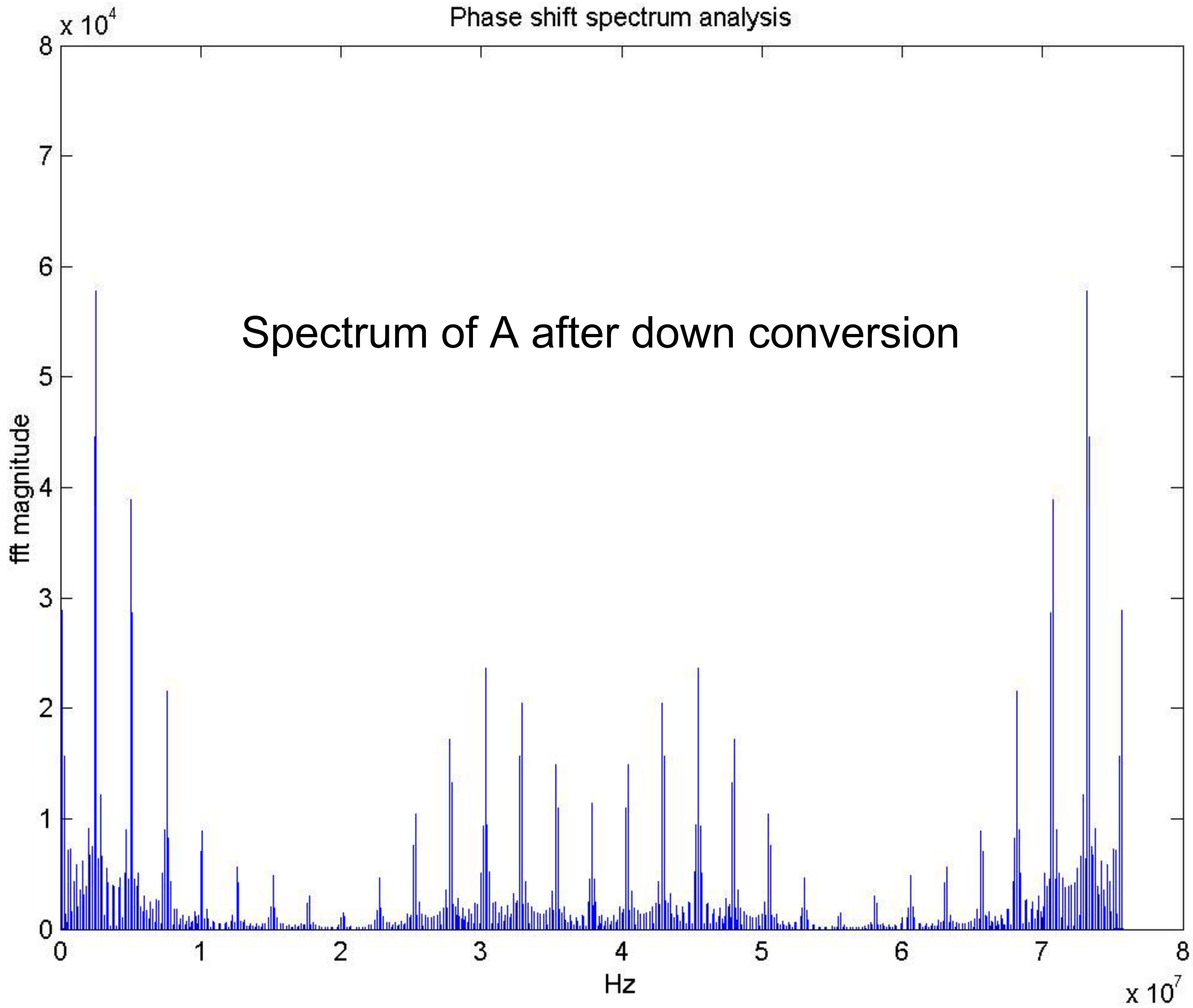
Spectrum analysis



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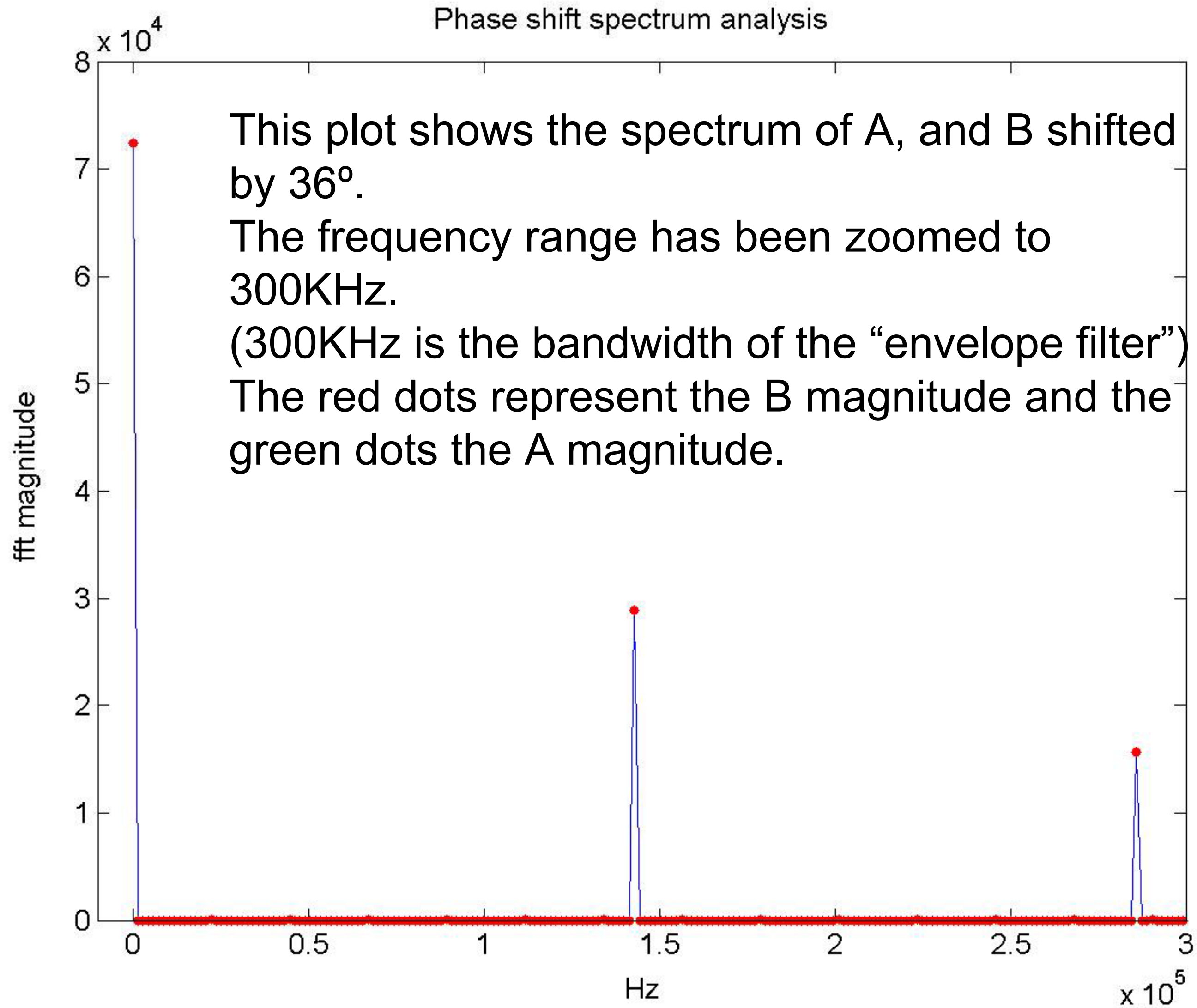
Spectrum analysis



This plot shows the spectrum of A, and B shifted by 36º.
The frequency range has been zoomed to 300KHz.
(300KHz is the bandwidth of the “envelope filter”)
The red dots represent the B magnitude and the green dots the A magnitude.

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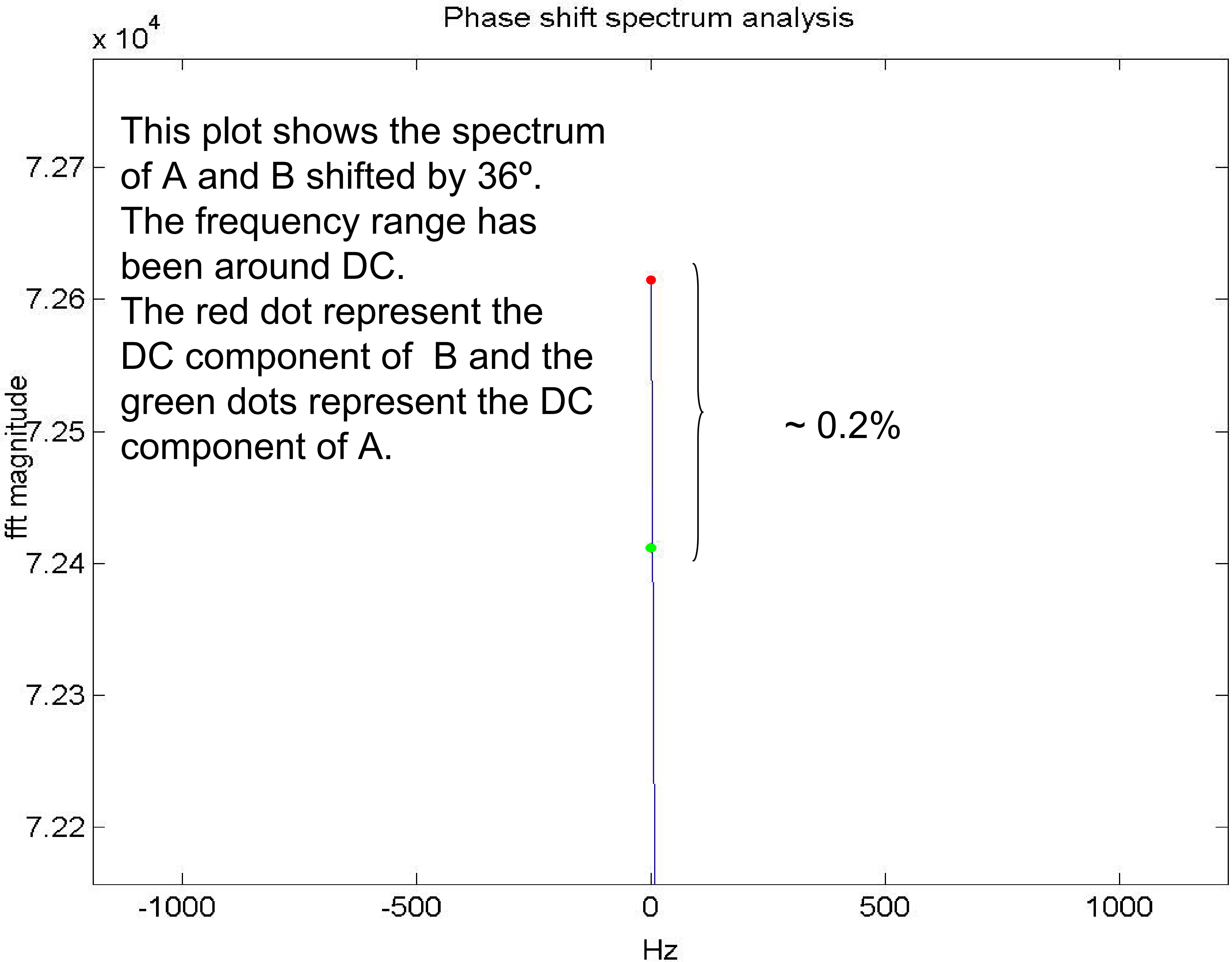

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Model of the shift problem

Signal model:

$$a(t) = A.\left[e^{-(t/\sigma_s)^2} - e^{-(t-t_0)^2/\sigma_s^2}\right] \qquad b(t) = B.\left[e^{-(t+\Delta t/\sigma_s)^2} - e^{-(t+\Delta t-t_0)^2/\sigma_s^2}\right]$$

Fourier transforms:

$$A(\omega) = \sqrt{\pi} A \sigma_s.\left[1 - e^{-j\omega t_0}\right].e^{-(\pi\sigma_s\omega)^2} \qquad B(\omega) = \sqrt{\pi} B \sigma_s.\left[e^{-j\omega\Delta t} - e^{-j\omega(t_0+\Delta t)}\right].e^{-(\pi\sigma_s\omega)^2}$$

Ringing filter model for time and frequency domains:

$$h(t) = h_0 e^{-(t-t_0)^2/\sigma^2}.\cos(\omega_c t + \phi) \qquad H(\omega) = h_0 \sqrt{\pi}\sigma\, e^{-\frac{\sigma^2\omega_c^2}{4}} e^{-\pi^2\sigma^2\left(\omega - \frac{\omega_c}{4\pi^2}\right)^2}.\cos(\omega_c t_0 + \phi)$$

Signal at the output of the ringing filter (frequency domain)

$$U_A(\omega) = U_{0A}.\left[1 - e^{-j\omega t_0}\right].e^{-\pi^2\sigma^2\left(\omega - \frac{\omega_c}{4\pi^2}\right)^2}$$ where $$U_{0A} = H_0\sqrt{\pi} A \sigma_s e^{-\pi^2\sigma_s^2\left(\frac{\omega_c}{4\pi^2}\right)^2}$$

$$U_B(\omega) = U_{0B}.\left[e^{-j\omega\Delta t} - e^{-j\omega(t_0+\Delta t)}\right].e^{-\pi^2\sigma^2\left(\omega - \frac{\omega_c}{4\pi^2}\right)^2}$$

where $$U_{0B} = H_0\sqrt{\pi} B \sigma_s e^{-\pi^2\sigma_s^2\left(\frac{\omega_c}{4\pi^2}\right)^2}$$

The shift shows as phase information

Model of the shift problem

Let's assume that the signal is filtered using an ideal filter with a cutoff freq. = ω_0 (e.g. ω_0=2*π*300KHz) and unit gain.

$$V_A(\omega) = U_A(\omega).W(\omega) = U_{0A}.\left[1 - e^{-j\omega t_0}\right].e^{-\pi^2\sigma^2\left(\omega - \frac{\omega_c}{4\pi^2}\right)^2}$$

$$V_B(\omega) = U_B(\omega).W(\omega) = U_{0B}.\left[e^{-j\omega\Delta t} - e^{-j\omega(t_0 + \Delta t)}\right].e^{-\pi^2\sigma^2\left(\omega - \frac{\omega_c}{4\pi^2}\right)^2}$$