Effect of coupling on Tevatron 1.7 GHz Schottky tunes

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Abstract
It has been pointed out that the tunes measured by the 1.7 GHz Schottky pickups in the Tevatron are affected by coupling. Due to the width of the betatron bands at 1.7 GHz, it is not possible to resolve the two normal mode frequencies. Rather, the devices rely on geometry to resolve the horizontal and vertical motion, and the tunes are derived from the center of the frequency distribution in each plane. It is well known that in a coupled machine operating close to the coupling resonance, the normal modes may be inclined. This causes a mixing of the signal from the two normal modes into the horizontal and vertical signals of the Schottky. The purpose of this note is to study the effect this has on the measured tunes.

<u>Solution to equations of motion in coupled machine</u>
Assume a smooth machine model with constant focusing, and some small coupling. The equations of motion are given by

$$\ddot{x} + \Omega_x^2 x = -b\,\dot{y} - k_s y$$
$$\ddot{y} + \Omega_y^2 y = b\,\dot{x} - k_s x$$

where k_s is the skew quadrupole term, and b is the solenoid term. Using the standard ansatz

$$x = X(s)\, e^{j\Omega_x s}$$
$$y = Y(s)\, e^{j\Omega_y s}$$

where the amplitude functions are assumed to be varying slowly, one can expand the equations of motion and drop higher order terms, yielding

$$\dot{X} = j\frac{e^{-j\delta s}}{2\,\Omega}(k_s + j\,b\Omega)Y$$

$$\dot{Y} = j\frac{e^{j\delta s}}{2\,\Omega}(k_s - j\,b\Omega)X$$

where

$$\delta = \Omega_x - \Omega_y$$
$$\Omega = (\Omega_x + \Omega_y)/2$$

The solution to these equations can be written on the form

$$X = C_2 e^{\frac{j}{2}(\eta-\delta)s} - C_3\frac{\Omega(\eta-\delta)}{k_s - j\,b\Omega}e^{-\frac{j}{2}(\eta+\partial)s}$$

$$Y = C_3 e^{-\frac{j}{2}(\eta-\partial)s} + C_2\frac{\Omega(\eta-\delta)}{k_s + j\,b\Omega}e^{\frac{j}{2}(\eta+\delta)s}$$

$$\eta = \sqrt{\delta^2 + (k_s/\Omega)^2 + b^2}$$

which, inserted into the ansatz, can be rewritten as

$$x = C_2 e^{j(\Omega+\eta/2)s} - C_3 \frac{(\eta-\delta)}{\sqrt{(k_s/\Omega)^2 + b^2}} e^{j(\Omega-\eta/2+\phi)s}$$

$$y = C_3 e^{j(\Omega-\eta/2)s} + C_2 \frac{(\eta-\delta)}{\sqrt{(k_s/\Omega)^2 + b^2}} e^{j(\Omega+\eta/2-\phi)s}$$

$$\tan\phi = \Omega b / k_s$$

(Note that the term η-δ goes tends towards infinity for large negative values of δ.)

Tunes measured by 1.7 GHz Schottky

Now assume that the frequency (tune) is measured as the center of mass in a power spectrum of the motion in each plane, rather than in the normal modes, as is the case for the 1.7 GHz Schottky. For simplicity, consider the case of only a skew quadrupole term ($b=0$). The measured center frequencies will then be the weighted average of the two modes

$$\overline{\Omega}_x = \frac{(C_2)^2(\Omega+\eta/2) + \left(C_3 \frac{(\eta-\delta)}{\sqrt{(k_s/\Omega)^2}}\right)^2 (\Omega-\eta/2)}{(C_2)^2 + \left(C_3 \frac{(\eta-\delta)}{\sqrt{(k_s/\Omega)^2}}\right)^2} = \Omega + \eta/2 \frac{(C_2)^2 - (C_3)^2 \frac{(\eta-\delta)^2}{(k_s/\Omega)^2}}{(C_2)^2 + (C_3)^2 \frac{(\eta-\delta)^2}{(k_s/\Omega)^2}}$$

and

$$\overline{\Omega}_y = \frac{(C_3)^2(\Omega-\eta/2) + \left(C_2 \frac{(\eta-\delta)}{\sqrt{(k_s/\Omega)^2}}\right)^2 (\Omega+\eta/2)}{(C_3)^2 + \left(C_2 \frac{(\eta-\delta)}{\sqrt{(k_s/\Omega)^2}}\right)^2} = \Omega - \eta/2 \frac{(C_3)^2 - (C_2)^2 \frac{(\eta-\delta)^2}{(k_s/\Omega)^2}}{(C_3)^2 + (C_2)^2 \frac{(\eta-\delta)^2}{(k_s/\Omega)^2}}$$

Now make the further assumption that the power (emittance) in he two modes, is the same (ie $C_2=C_3$). Then the measured frequencies are

$$\overline{\Omega}_x = \Omega + \delta/2 = \Omega_x$$

$$\overline{\Omega}_y = \Omega - \delta/2 = \Omega_y$$

In other words, the observed tunes are equal to the uncoupled tunes. This perfect cancellation may not happen in a more complicate and realistic model. However, since the measured tune is essentially a weighted average, the effect is always to bring the observed tunes closer together, not further apart. Hence, the 1.7 GHz Schottky will tend to underestimate the coupling (tune split). It can never bring the tunes further apart or flip then.

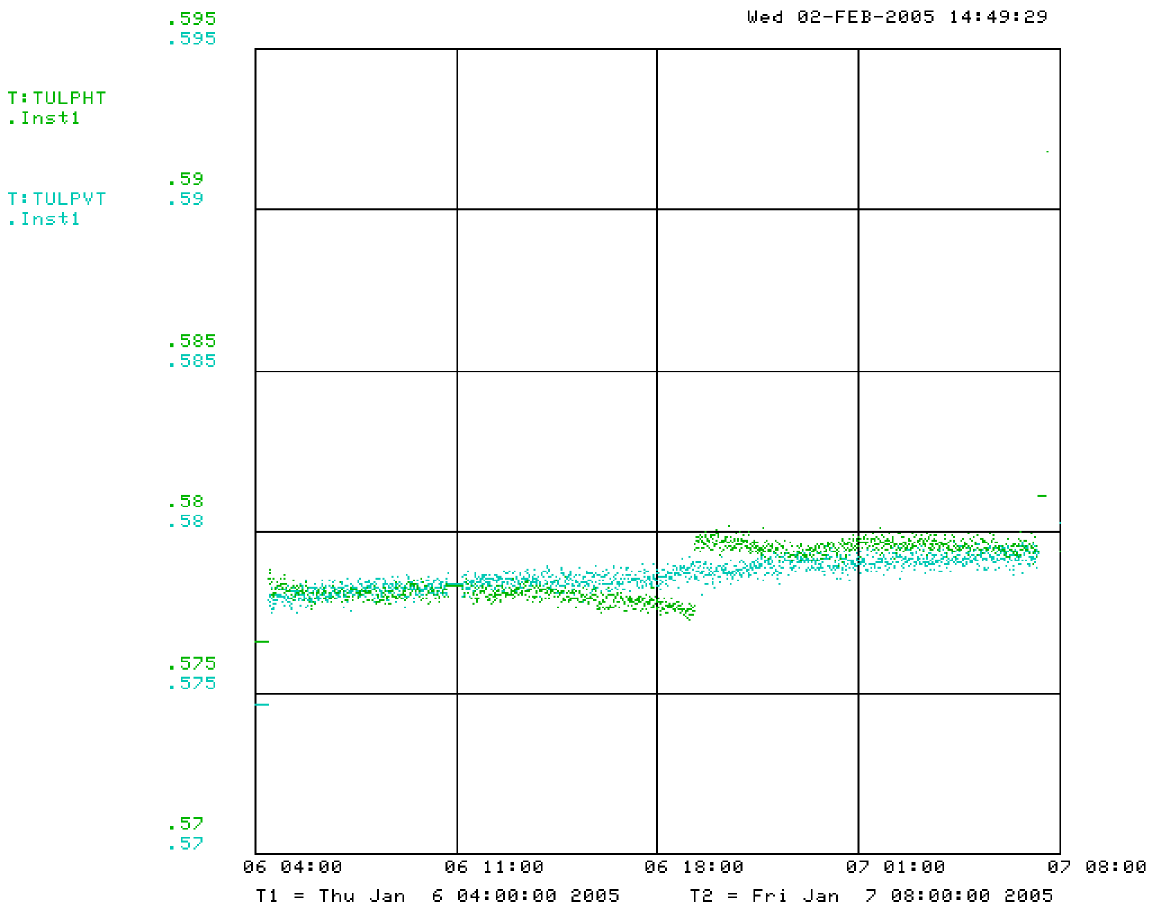

Figure 1 Observed proton tunes for store 3906. The observed tunes are right on top of each other. Halfway thru the store, the horizontal base tune was raised by 0.002. No effect was seen in the vertical tune. Unless the machine was completely uncoupled on the proton helix, which is possible but unlikely, this supports the conclusion that the instrument indeed measures the uncoupled tunes.

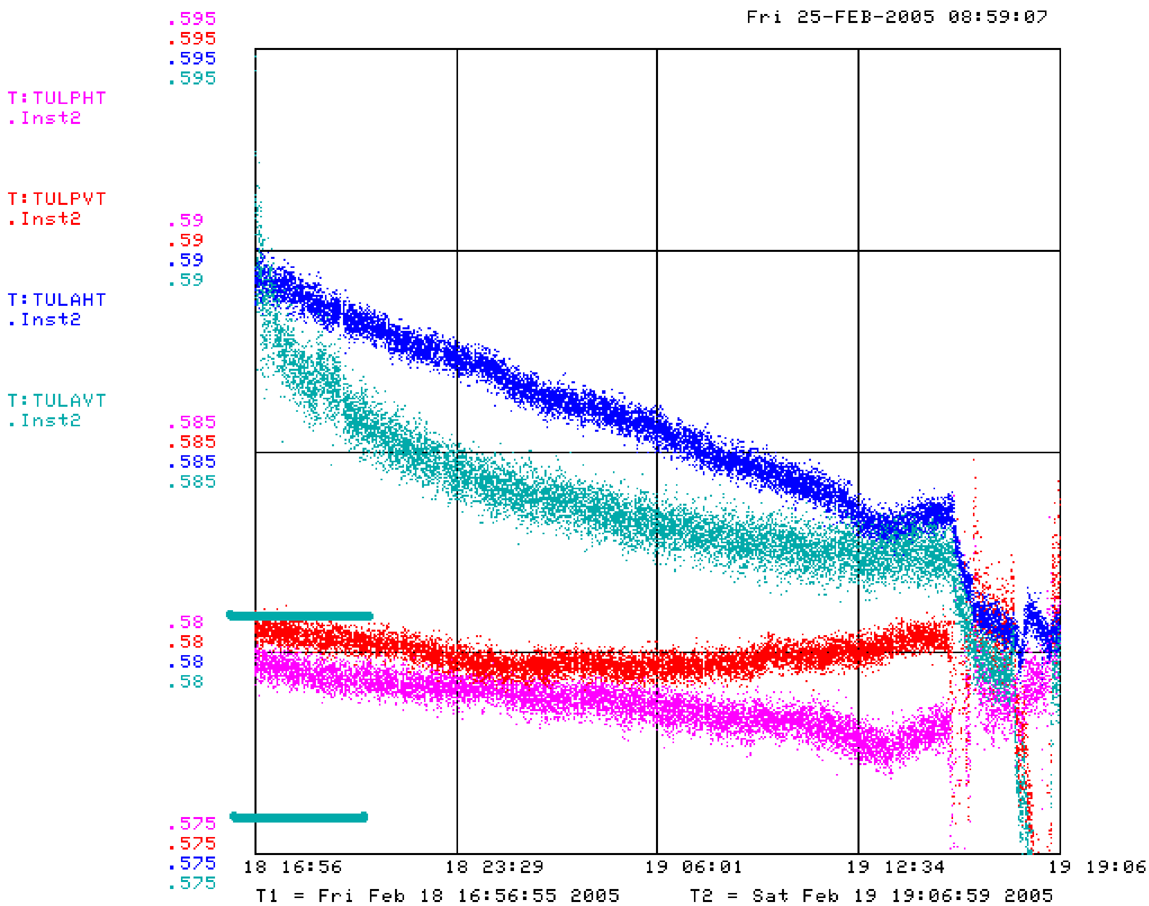

Figure 2. Observed tunes for store 3989. The horizontal lines indicate the proton tunes as observed in the 21.4MHz Schottky spectra. The normal mode tune split was clearly much higher than indicated by the 1.7 GHz Schottky.

To see what happens in a real machine, Valerie Lebedev's Tevatron model (which is based on differential orbit measurements) was used to calculate the normal mode tunes and coupled beta functions at the Schottky location (actually at the nearby flying wire, since the Schottky pick-ups are not in the model). Using these values in the above formula yield estimated 1.7 Schottky tunes very close to the uncoupled ones.

Conclusions

It is found that in a very simplified model of coupling, the 1.7 GHz Schottky method of measuring tunes (which relies on finding the center the frequency distribution in each plane rather than the locating the normal frequencies) exactly cancels the effect of the machine coupling, yielding the uncoupled machine tunes as a result. In a more realistic model, or a real machine, the cancellation may not be perfect, although calculations based on the coupled Tevatron indicate that it is a very good approximation. In general, the 1.7 GHz Schottky tends to underestimate the tune split due to coupling. This should be kept in mind when analyzing the datalogged results from the Schottky OAC.

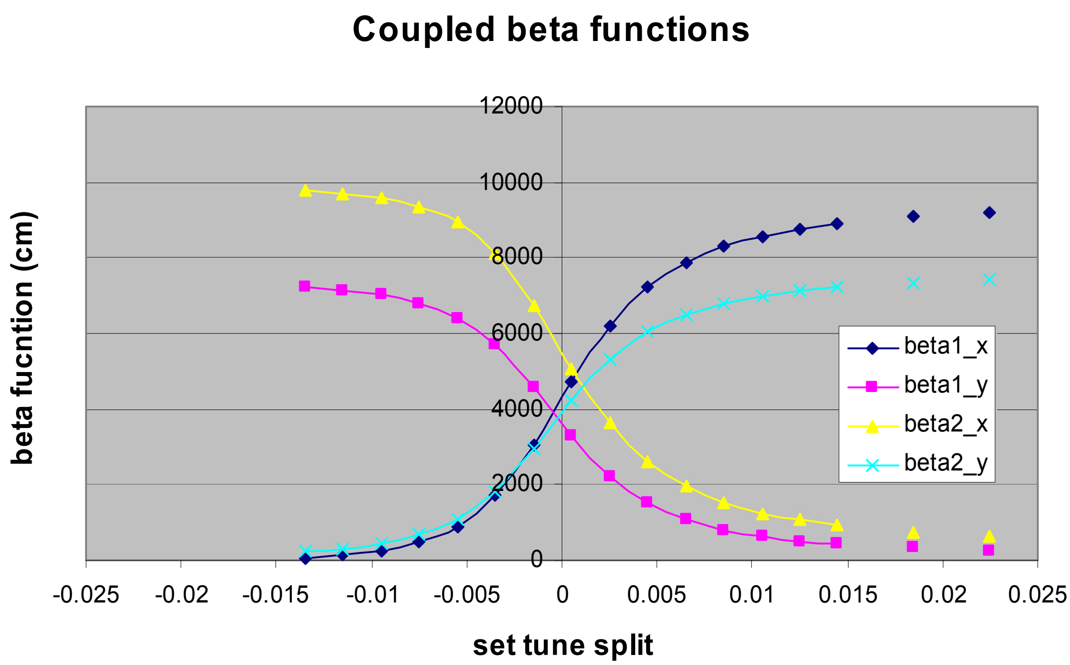

Figure 3. Projected normal mode beta functions at E17 as a function of tune split, calculated with OPTIM using Valerie Lebedev's Tevatron model. The tunes crossed at about 0.580.

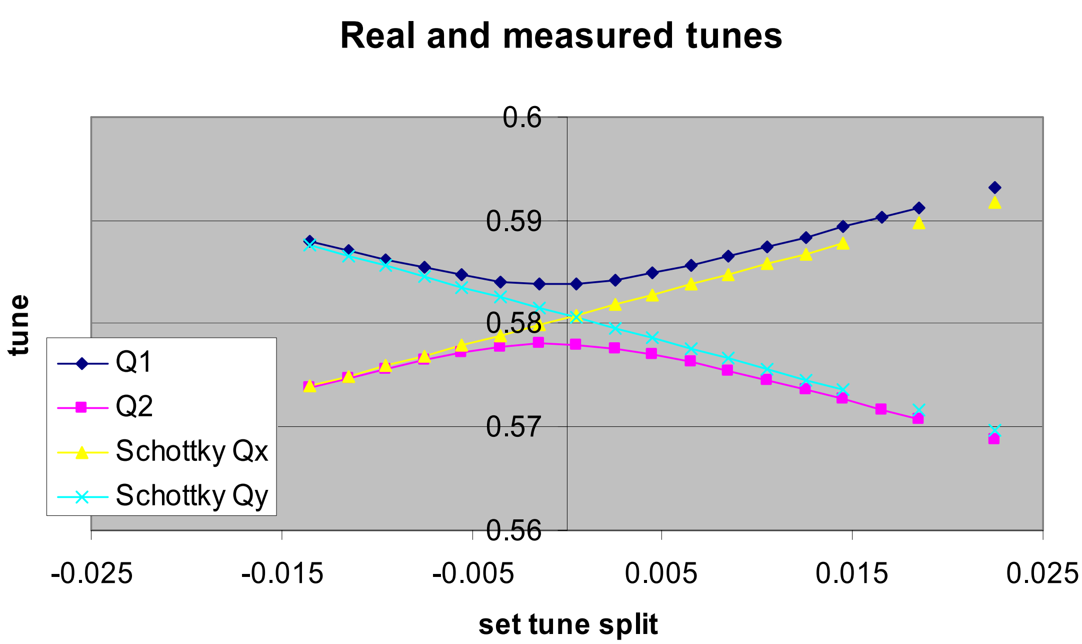

Figure 4. Normal mode tunes calculated with OPTIM using Valerie Lebedev's Tevatron model. The tunes expected to be seen by the 1.7 GHz Schottky (calculated using the formula in this note, assuming identical emittances) is also indicated. It is very close to the uncoupled tune. Varying the emittance ratio within reasonable limits (± 50%) did not give any significant effect.