Fermilab

Beam Position Monitor
Design Note #1

EXPECTED CALIBRATION AND CALIBRATION TOLERANCES OF THE BEAM POSITION SYSTEM

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November 3, 1981

The purpose of this report is to consolidate various calculations and measurements on the calibration of the beam position system. Many pieces of equipment contribute to the calibration of the system, and the tolerances of each individual calibration affect the ultimate accuracy of the system. Specifically, this report derives the nominal calibration of both the position and intensity measuring channels, and examines the effect of various tolerances on the final accuracy.

This report is not meant to be an introduction to the beam position system. A thorough understanding of the various elements of hardware comprising the system is recommended before this report is read in detail. It is especially important that anyone who intends to make adjustments on any component in the analog signal processing portion understand the overall effect of the adjustment on the system.

To a certain extent this report presents algorithms and calibration constants which are design objectives rather than final values. The purpose of writing this report at this time is to understand the sensitivity of the system's accuracy to various calibration tolerances and to be able to specify what those tolerances must be in order that the design objectives be achieved. In addition, understanding the dependence of the system calibration on individual parameters will allow modification of these parameters if such modification leads to better accuracy, linearity, or matching of the system to the Energy Doubler requirements.

There are two basic calibration channels in this system. The primary one is the beam position system, which is expected to measure beam position with an accuracy of ±1 mm at ±25 mm displacements of the beam from the central orbit, and with correspondingly better accuracy at smaller displacements. This is supposed to work for beam intensities in the range 10^8 to 10^{11} protons per bucket. In addition, a second channel for measuring beam intensity to ±20% over the same intensity range is also discussed. Each channel is reviewed separately.

I. THE POSITION CHANNEL

The position channel includes the detector, the cable, the rf module, and the sample and hold/ADC in the analog box.

I-A. THE DETECTOR

The detector is discussed in IEEE NS Vol. NS-28 #3, pages 2290-3 (1981). Measurement of the position sensitivity presented in this paper show the response to be

$$.67x = \left(\frac{A}{B}\right)_{db} = 20 \log_{10} (A/B) \qquad (1)$$

where x is the displacement of the beam in mm from the detector centerline, and $(A/B)_{db}$ is the db ratio of the rf signal amplitudes out of the two detector output ports. In the case where the beam is also displaced in the y (orthogonal) direction, the relation becomes

(y also in mm)

$$.67x = \left(1 - \frac{y^2}{2830}\right)\left(\frac{A}{B}\right)_{db} \qquad (2)$$

I-B. THE CABLE

The cable being used to transport the rf signal back to the nearest Service Building where the electronics is located is a foam type RG8. The foam dielectric was chosen for two reasons: to minimize the attenuation (~1.3 db per 100' at 53 MHz) and to minimize the time skewing of the various signals arriving at each Service Building. Cable lengths typically range from 150' to 650' hence leading to expected attenuations of 2.0 and 8.5 db respectively. As cable pairs are expected to be the same length (within a few cm), the relative attenuation is expected to be zero. If one cable has attenuation $(A^1/A)_{db}$ and the other $(B^1/B)_{db}$, where the amplitudes A^1 and B^1 are outputs and A and B inputs, then it can be shown that

$$\left(\frac{A^1}{B^1}\right)_{db} = \left(\frac{A}{B}\right)_{db} + \left(\frac{A^1}{A}\right)_{db} - \left(\frac{B^1}{B}\right)_{db}$$

$$= \left(\frac{A}{B}\right)_{db} + \Delta(db) \qquad (3)$$

It is important that the signals A^1 and B^1 be in phase relative to each other within a few degrees in order that the phase difference not effect the amplitude measurement. In Appendix A, the amplitude to phase conversion technique is discussed and the sensitivity of the technique to phase differences is calculated. Specifically, if the phase difference can be held to $\pm 10^0$ max then the effect on the position measurement will be less than .2 mm for $x<\pm 25$ mm. This corresponds to about ±12 cm for 53 MHz in Foam-8 cable.

I-C. THE RF MODULE

The rf module (described in IEEE NS, Vol. NS-28, pages 2323-5 (1981) uses AM/PM (amplitude modulation to phase modulation) conversion followed by hard limiters, a double balanced mixer, a low pass filter and a low frequency amplifier to convert the AM signals to a voltage output. The equations are derived in Appendix A. Specifically

$$\frac{A^1}{B^1} = \tan\left(\frac{\theta_A - \theta_B}{2}\right) \quad (4)$$

where A^1 and B^1 are the amplitudes of the two rf input signals and θ_A and θ_B are the phases of the signals out of the AM/PM circuit.

A double balanced mixer and low pass filter is used to produce a voltage output such that

$$C_1 \left(V - V_0\right) + \frac{\pi}{4} = \frac{\theta_A - \theta_B}{2} \quad (5)$$

where C_1 and V_0 are calibration constants.

Hence the overall performance of the rf module is

$$\left(\frac{A^1}{B^1}\right)_{db} = \frac{20}{\ln 10} \ln \tan \{C_1 (V - V_0) + \frac{\pi}{4}\} \quad (6)$$

Nominal values for the calibration constants are

$C_1 = \pi/10$

$V_0 = 0$

I-D. THE ANALOG BOX

The analog channel circuit for the position signal is entirely contained on the daughter cards in the analog box. The circuit is described in detail in Appendix B. Specifically, the relation between the input voltage V and the 8-bit digitized number cut is of the form

$$V = \frac{C_2 - N}{C_3} \tag{7}$$

where nominal values for the constants are

$C_2 = 128.0$

$C_3 = 51.2$

I-E. OVERALL CALIBRATION

Combining the nominal value equations (1), (6), and (7) we get

$$x = \frac{20}{.67 \ln 10}\left(1 - \frac{y^2}{2830}\right) \ln \tan\left[\frac{\pi}{10}\left(\frac{128-N}{51.2}\right) + \frac{\pi}{4}\right]$$

$$= 12.96\left(1 - \frac{y^2}{2830}\right) \ln \tan\left[\frac{128-N}{163} + \frac{\pi}{4}\right] \tag{8}$$

The inverse relation is

$$128-163\left\{\tan^{-1}\exp\left\{\frac{x}{12.96}\left(1 + \frac{y^2}{2830}\right)\right\} - \frac{\pi}{4}\right\} \tag{9}$$

I-F. APPROXIMATIONS

As the ln tan and $\tan^{-1}$ exp expressions will be used often in converting between "user" (i.e., engineering) units and "raw" (i.e., microprocessor) units, more efficient approximations have been developed. Specifically the expressions

$$x = 12.96 \left[1.866 \left(\frac{128-N}{163}\right) + 2.548 \left(\frac{128-N}{163}\right)^3 \right] \quad (10)$$

and

$$N = 128 - 163 \left[0.4947 \left(\frac{x}{12.96}\right) - .0667 \left(\frac{x}{12.96}\right)^3 + .0063 \left(\frac{x}{12.96}\right)^5 \right] \quad (11)$$

are approximations for equations (8) and (9) which are accurate within ± 0.5 mm for $|x| < 24$ mm .

I-G. ALLOWABLE TOLERANCES

The design objective for the system accuracy is $\pm .5$ mm for $|x| < 25$ mm. This tolerance includes both electrical and mechanical tolerance contributions. If we assume that electrical and mechanical tolerances are equal, each individually should be $\sim \pm 0.35$ mm. The mechanical tolerance includes all dimensional tolerances related to the positioning of the detector relative to the magnetic centerline of the quadrupole as defined by the alignment lugs, the internal tolerances of the detector which may affect the relative signal output amplitudes, etc. The electrical tolerances include cable phase error and attenuation, and the calibration constants of both the rf module and the analog box.

It is important to note that all detector channels will be assumed to have the same electrical calibration. Only in this way is it possible to maintain the system calibration over long time periods when individual components may be exchanged or replaced from time to time.

We start with the following equations:

detector: $$x = \frac{1}{.67}\left(1 - \frac{y^2}{2830}\right)\left(\frac{A}{B}\right)_{db} \tag{2}$$

cable: $$\left(\frac{A}{B}\right)_{db} = \left(\frac{A^1}{B^1}\right)_{db} + \Delta(db) \tag{3}$$

rf module: $$\left(\frac{A^1}{B^1}\right)_{db} = \left(\frac{20}{\ln 10}\right) \ln\tan\{C_1(V - V_0) + \frac{\pi}{4}\} \tag{6}$$

analog box: $$V = \frac{C_2 - N}{C_3} \tag{7}$$

then

$$x = \frac{1}{.67}\left\{\frac{20}{\ln 10}\ln\tan\left[C_1\left(\frac{C_2 - N}{C_3} - V_0\right) + \frac{\pi}{4}\right] + \Delta(db)\right\}\left[1 - \frac{y^2}{2830}\right] \tag{12}$$

The rms error on x is then given by combining the contributions from all tolerances in quadrature:

$$\delta x = \left[\left(\frac{\partial x}{\partial C_1}\delta C_1\right)^2 + \left(\frac{\partial x}{\partial C_2}\delta C_2\right)^2 + \left(\frac{\partial x}{\partial C_3}\delta C_3\right)^2 + \left(\frac{\partial x}{\partial V_0}\delta V_0\right)^2 + \left(\frac{\partial x}{\partial N}\delta N\right)^2 + \left(\frac{\partial x}{\partial \Delta db}\delta(\Delta db)\right)^2\right]^{\frac{1}{2}} \tag{13}$$

We are ignoring both the cable phase error contribution and the effect of the y dependence which will be discussed separately.

If we write the expression (12) in the following form

$$x = \left(12.96 \ln\tan\phi + \frac{\Delta(db)}{.67}\right)\left(1 - \frac{y^2}{2830}\right)$$

where $$\phi = C_1\left(\frac{C_2 - N}{C_3} - V_0\right) + \frac{\pi}{4} \tag{14}$$

The partial derivatives may be written

$$\frac{\partial x}{\partial C_1} = \frac{\partial x}{\partial \phi}\frac{\partial \phi}{\partial C_1} = 12.96\left(1 - \frac{y^2}{2830}\right)\left(\frac{2}{\sin 2\phi}\right)\left(\frac{C_2 - N}{C_3} - V_0\right) \quad (15)$$

$$\frac{\partial x}{\partial C_2} = \frac{\partial x}{\partial \phi}\frac{\partial \phi}{\partial C_2} = 12.96\left(1 - \frac{y^2}{2830}\right)\left(\frac{2}{\sin 2\phi}\right)\frac{C_1}{C_3} \quad (16)$$

$$\frac{\partial x}{\partial C_3} = \frac{\partial x}{\partial \phi}\frac{\partial \phi}{\partial C_3} = 12.96\left(1 - \frac{y^2}{2830}\right)\left(\frac{2}{\sin 2\phi}\right)\left(\frac{-C_1(C_2 - N)}{C_3^2}\right) \quad (17)$$

$$\frac{\partial x}{\partial V_0} = \frac{\partial x}{\partial \phi}\frac{\partial \phi}{\partial V_0} = 12.96\left(1 - \frac{y^2}{2830}\right)\left(\frac{2}{\sin 2\phi}\right)(-C_1) \quad (18)$$

$$\frac{\partial x}{\partial N} = \frac{\partial x}{\partial \phi}\frac{\partial \phi}{\partial N} = 12.96\left(1 - \frac{y^2}{2830}\right)\left(\frac{2}{\sin 2\phi}\right)\left(\frac{-C_1}{C_3}\right) \quad (19)$$

$$\frac{\partial x}{\partial \Delta db} = \frac{1}{.67} \quad (20)$$

In Table I, δx in mm is tabulated, as well as the individual contributions from each individual component, as a function of x.

The following average values and tolerances were used:

$C_1 = .314 \pm .003$

$C_2 = 128 \pm 0.5$

$C_3 = 52.8 \pm 0.5$ (measured value)

$V_0 = 0.00 \pm 0.01$

$\delta N = \pm 0.5$ (least count resolution)

$\Delta db = 0.0 \pm 0.1$

$y = 0$

In Fig. 1, δx is plotted against x. It is apparent that the size of δx is adequately small near x=0, the largest contribution being from the ±0.1 db tolerance on the relative cable attenuation. At large x, the two largest contributions arise from the tolerances on C_1 (the gain of the rf module) and C_3 (the conversion gain of the

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DB=  0.00 +/-   .10
C1=   .314  +/-  .003
V0= 0.000 +/-  .010
C2=128.00 +/-   .50
N=  +/-   .50
C3= 52.80 +/-   .50
Y=  0.00
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Input parameters

x(mm)	N	256-N	$\delta(\Delta db)$	δC_1	δV_0	δC_2	δC_3	δN	δx(mm)
			Contribution to δx(mm) from						
0.00	128.0	128.0	.15mm	0.00mm	.08mm	.08mm	0.00mm	.08mm	.20
1.00	121.5	134.5	.15	.01	.08	.08	.01	.08	.20
2.00	115.1	140.9	.15	.02	.08	.08	.02	.08	.21
3.00	108.7	147.3	.15	.03	.08	.08	.03	.08	.21
4.00	102.5	153.5	.15	.04	.09	.08	.04	.08	.21
5.00	96.3	159.7	.15	.05	.09	.08	.05	.08	.22
6.00	90.4	165.6	.15	.06	.09	.09	.06	.09	.23
7.00	84.6	171.4	.15	.07	.09	.09	.07	.09	.24
8.00	79.1	176.9	.15	.09	.10	.09	.09	.09	.25
9.00	73.8	182.2	.15	.10	.10	.10	.10	.10	.27
10.00	68.7	187.3	.15	.11	.11	.10	.11	.10	.28
11.00	63.9	192.1	.15	.13	.11	.11	.13	.11	.30
12.00	59.4	196.6	.15	.15	.12	.11	.15	.11	.32
13.00	55.0	201.0	.15	.17	.13	.12	.16	.12	.35
14.00	51.0	205.0	.15	.19	.13	.13	.18	.13	.38
15.00	47.1	208.9	.15	.21	.14	.13	.21	.13	.41
16.00	43.5	212.5	.15	.23	.15	.14	.23	.14	.44
17.00	40.2	215.8	.15	.26	.16	.15	.26	.15	.48
18.00	37.0	219.0	.15	.29	.17	.16	.28	.16	.52
19.00	34.1	221.9	.15	.32	.19	.18	.31	.18	.56
20.00	31.3	224.7	.15	.35	.20	.19	.35	.19	.61
21.00	28.8	227.2	.15	.38	.21	.20	.38	.20	.66
22.00	26.4	229.6	.15	.42	.23	.22	.42	.22	.72
23.00	24.2	231.8	.15	.46	.25	.23	.46	.23	.79
24.00	22.1	233.9	.15	.51	.27	.25	.50	.25	.86
25.00	20.2	235.8	.15	.56	.29	.27	.55	.27	.93

.036 CP SECONDS EXECUTION TIME.

Table I. Error analysis of position response based on input parameters as selected above.

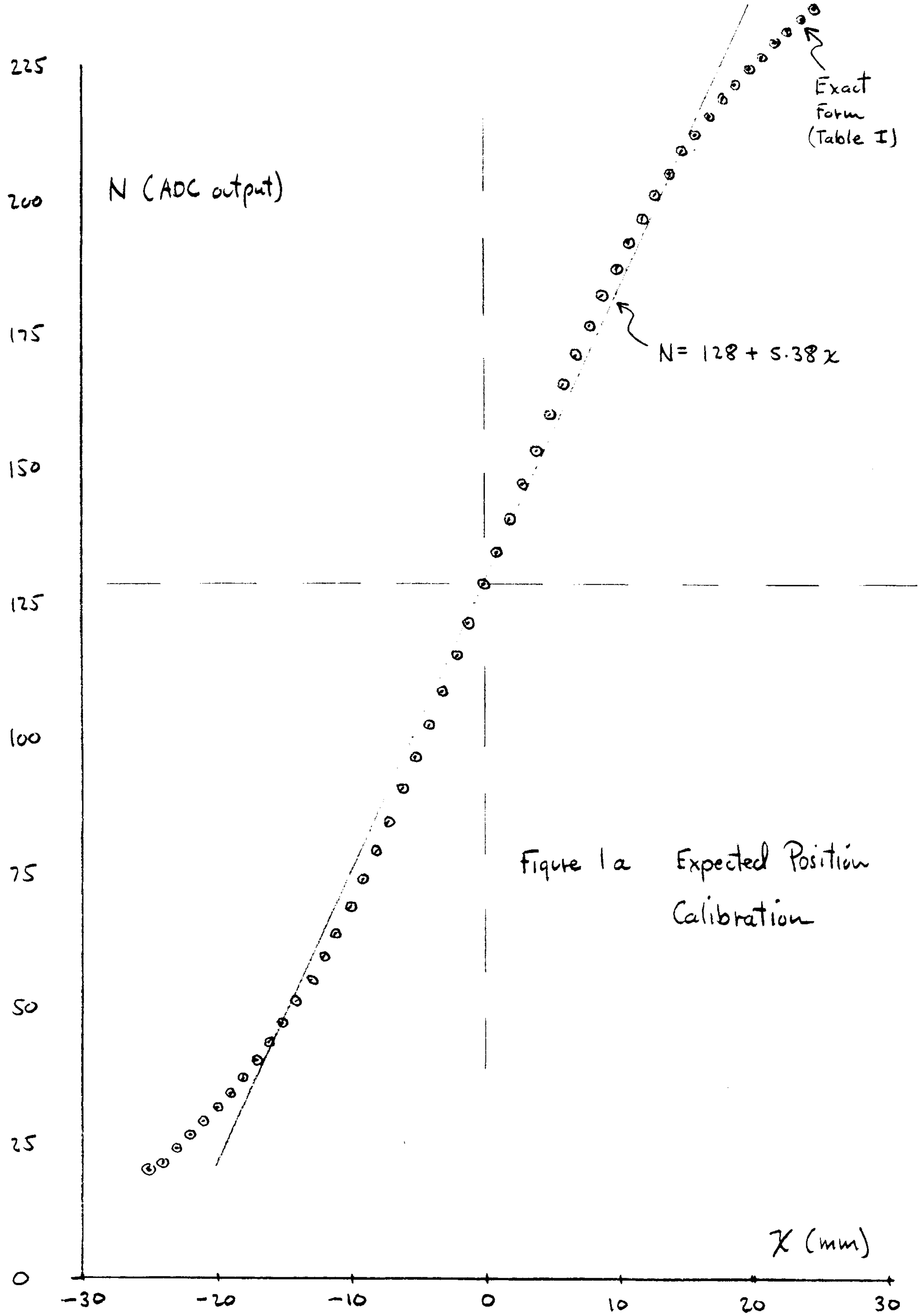

Figure 1a Expected Position Calibration

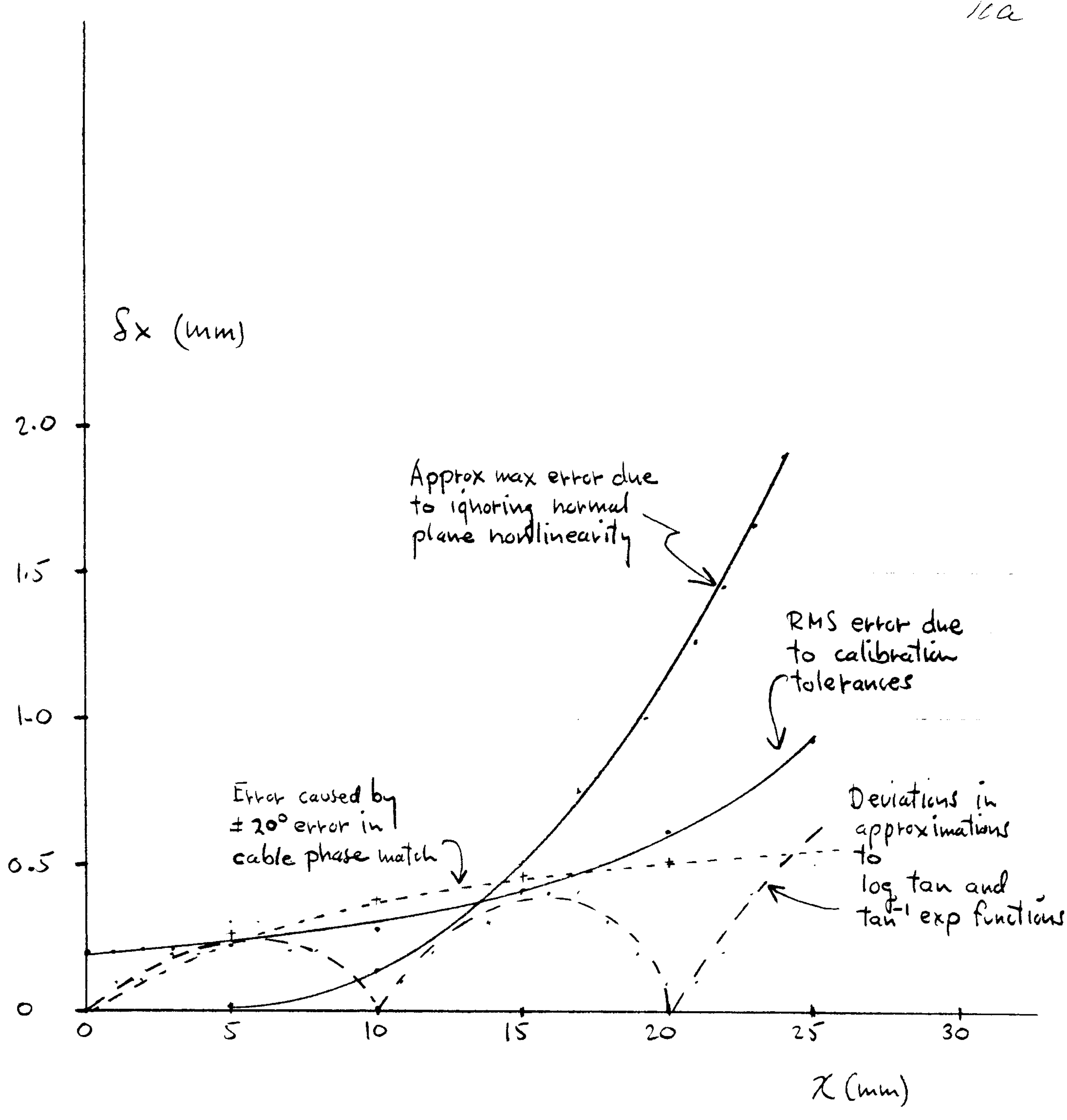

Figure 1b: Contributing Errors in Beam Position Measurement. Mechanical alignment tolerances are not included.

ADC). If each of these are held to ±1%, then the minimum error on x is about ±3%. Other contributions raise it to about ±4%. At present it appears that there may be a large temperature coefficient in the double balanced mixer in the rf module (-0.4% per ^{0}C) which would make the contribution from C_3 four times larger for $\delta T = \pm 10^0$C.

I-H. THE NORMAL PLANE DEPENDENCE OF POSITION SIGNAL

The position signal for an x plane detector is dependent on the orthogonal coordinate y as is shown in Eq. (2). To correct for this it is necessary to extrapolate the measurements of the orthogonal coordinate from adjacent detectors. As roughly 25% of the detectors in any service building have no adjacent detector (in that building) the problem of making this correction has been left to the Host.

An interim solution is to ignore this coupling. The following analysis shows how serious the effect would be.

If we assume the beam orbit to be contained within a certain excursion r from the center of the detector, where $r^2 = x^2 + y^2$, then it is possible to calculate the maximum effect of the uncertainty in y or x.

Since $$\frac{\delta x}{x} = \frac{y^2}{2830} = \frac{r^2 - x^2}{2830} \tag{21}$$

$$\frac{d\,\delta x}{dx} = \frac{d}{dx}\left[\frac{x(r^2 - x^2)}{2830}\right] = \frac{r^2 - 3x^2}{2830} = 0 \tag{22}$$

$$\therefore x = r/\sqrt{3} \tag{23}$$

and $$\delta x = \frac{2r^3}{2830\sqrt{27}} = \left(\frac{r}{19.4}\right)^3 \tag{24}$$

If a maximum deviation δx of 1 mm were allowed, this would lead to an uncertainty of ±.5 mm. This occurs at r = 19 mm. This uncertainty is plotted in Fig. 1.

I-J. THE ACCURACY OF APPROXIMATIONS

The accuracy of the approximations x(N) and N(x) [Eqs. (10) and (11)] are presented in Table II.

Specifically

$$N_1(x) = C_2 - \frac{C_3}{C_1}\left\{\tan^{-1}\left[\exp\left\{\frac{x}{12.96}\left(1+\frac{y^2}{2830}\right)\right\}\right] + C_1V_0 - \frac{\pi}{4}\right\} \qquad (25)\ \text{(Exact)}$$

$$N_2(x) = C_2 - \frac{C_3}{C_1}\left\{C_1V_0 + .4947\left[\frac{x}{12.96}\left(1+\frac{y^2}{2830}\right)\right] - .0667\left[\frac{x}{12.96}\left(1+\frac{y^2}{2830}\right)\right]^3 + .0063\left[\frac{x}{12.96}\left(1+\frac{y^2}{2830}\right)\right]^5\right\} \qquad (26)\ \text{(Approx)}$$

$$x_1(N) = 12.96\left(1-\frac{y^2}{2830}\right)\ln\tan\left\{C_1\left[\frac{(C_2-N)}{C_3} - V_0\right] + \frac{\pi}{4}\right\} \qquad (27)\ \text{(Exact)}$$

$$x_2(N) = 12.96\left(1-\frac{y^2}{2830}\right)\left\{1.866\left[\frac{C_1(C_2-N)}{C_3} - C_1V_0\right] + 2.548\left[\frac{C_1(C_2-N)}{C_3} - C_1V_0\right]^3\right\} \qquad (28)\ \text{(Approx)}$$

The table presents

$x_{12}(x) = x_1[N_2(x)]$ and $x_{12}(x) - x$

$x_{21}(x) = x_2[N_1(x)]$ and $x_{21}(x) - x$

$x_{22}(x) = x_2[N_2(x)]$ and $x_{22}(x) - x$

The reason for needing these approximations $N_2(x)$ and $x_2(N)$ is that there will be many conversions between user units and raw units in the BPM operation, and calculation of the ln tan function or the $\tan^{-1}$ exp function may be time inefficient.

II. INTENSITY CALIBRATION

The intensity calibration depends on the detector sensitivity, the cable attenuation, the rf module calibration and the conversion gain of the log amplifier/ADC in the analog box.

Table II - Accuracy of Approximations (Series Expansions) in mm.

x (mm)	x_{12}	$(x_{12}-x)$	x_{21}	$(x_{21}-x)$	x_{22}	$(x_{22}-x)$
0.0	0.0	(0.0)	0.0	(0.0)	0.0	(0.0)
1.0	1.0	(-.0)	.9	(-.1)	.9	(-.1)
2.0	2.0	(-.0)	1.9	(-.1)	1.9	(-.1)
3.0	3.0	(-.0)	2.8	(-.2)	2.8	(-.2)
4.0	4.0	(-.0)	3.8	(-.2)	3.8	(-.2)
5.0	5.0	(-.0)	4.8	(-.2)	4.7	(-.3)
6.0	6.0	(-.0)	5.8	(-.2)	5.7	(-.3)
7.0	7.0	(-.0)	6.8	(-.2)	6.8	(-.2)
8.0	8.0	(-.0)	7.8	(-.2)	7.8	(-.2)
9.0	9.0	(-.0)	8.9	(-.1)	8.9	(-.1)
10.0	10.0	(.0)	10.0	(-.0)	10.0	(-.0)
11.0	11.0	(.0)	11.0	(.0)	11.1	(.1)
12.0	12.0	(.0)	12.1	(.1)	12.2	(.2)
13.0	13.1	(.1)	13.2	(.2)	13.3	(.3)
14.0	14.1	(.1)	14.3	(.3)	14.3	(.3)
15.0	15.1	(.1)	15.3	(.3)	15.4	(.4)
16.0	16.1	(.1)	16.3	(.3)	16.4	(.4)
17.0	17.0	(.0)	17.3	(.3)	17.4	(.4)
18.0	18.0	(-.0)	18.3	(.3)	18.3	(.3)
19.0	19.0	(-.0)	19.3	(.3)	19.2	(.2)
20.0	19.9	(-.1)	20.2	(.2)	20.1	(.1)
21.0	20.9	(-.1)	21.1	(.1)	20.9	(-.1)
22.0	21.9	(-.1)	21.9	(-.1)	21.8	(-.2)
23.0	22.9	(-.1)	22.7	(-.3)	22.6	(-.4)
24.0	24.0	(.0)	23.5	(-.5)	23.5	(-.5)
25.0	25.2	(.2)	24.2	(-.8)	24.4	(-.6)

.034 CP SECONDS EXECUTION TIME.

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$N_1(x)$ exact $\qquad$ $x_1(N)$ exact

$N_2(x)$ approx $\qquad$ $x_2(N)$ approx

$$x_{12} = x_1[N_2(x)]$$

$$x_{21} = x_2[N_1(x)]$$

$$x_{22} = x_2[N_2(x)]$$

II-A. DETECTOR

The detector signal output is calculated in IEEE NS, Vol. NS-28, #3, page 2290 (1981). The calculated response is (see Fig. 2 of reference).

$$I = C_1 V_p \text{ ppb} . \quad \text{where } C_1 \sim 2.17 \times 10^{10} \text{ ppb/peak volt} \qquad (29)$$

where I is the number of protons per bucket and V_p is the peak voltage output at 53 MHz. This is the signal amplitude output of each electrode when the beam is centered. As is shown in the above reference, the output voltage is somewhat dependent on the bunch width.

II-B. THE CABLE

The amplitude attenuation in the Foam-8 cable at 53 MHz is approximately 1.3 db/100 ft. If V_p and $V_p{}^1$ are the signal amplitudes at the detector and at the far end of the cable respectively, then

$$V_p = V_p{}^1 \exp\ (L/C_2) \quad \text{where} \quad C_2 \sim \frac{2000}{1.3 \ln 10} = 668 \qquad (30)$$

L ranges from about 150 ft to 650 ft.

II-C. THE RF MODULE

The rf module uses 1/2 the total input signal power (the summation of the signals from the two electrodes) to provide a voltage output proportional to the input signal amplitude. The circuit utilizes a double balanced mixer in a synchronous detector circuit to minimize the low level nonlinearities. The gain of the circuit is adjusted at two amplitudes:

V_p(in)	V_{out}
.178V (-5 dbm)	V_{RFH} (= -.500 volts)
.00178V (-45 dbm)	V_{RFL} (= - .005 volts)

This leads to the following response function

$$V_p(\text{in}) = C_3 V_{out} + C_4 \quad (31)$$

where $$C_3 = \frac{.178 - .00178}{V_{RFH} - V_{RFL}} \quad (= - .356 \text{ for } V_{RFH} = 100\, V_{RFL} = .500V) \quad (32)$$

and $$C_4 = .178 \left[1 - \frac{.99\, V_{RFH}}{V_{RFH} - V_{RFL}}\right] \quad (= 0 \text{ for above conditions}) \quad (33)$$

II-D. THE ANALOG BOX

The analog box includes a buffer amplifier, a logarithmic amplifier, and an ADC. There is no temperature compensation in the log amp. The ADC is shared with the position measurement system.

The buffer amplifier has the response (see Fig. 2)

$$V_{in} = V_1 - V_{0S1} \quad (34)$$

where V_{0S1} is the voltage offset in IC1, and V_1 the output voltage.

The logarithmic amplifier consists of a 2N2060 matched pair of NPN transistors in a transdiode configuration and three RCA CA3140 operational amplifiers. The log dependence is based on the fact that the current-voltage relationship in the transdiodes is given approximately by

$$I = I_0 \exp \frac{qV}{kT} \quad (35)$$

where I_0 is the saturation current

q = electron charge = 1.602×10^{-19} Coulombs

k = Boltzmann constant = 1.3805×10^{-23} Joules/^{0}K

T = junction temperature in ^{0}K.

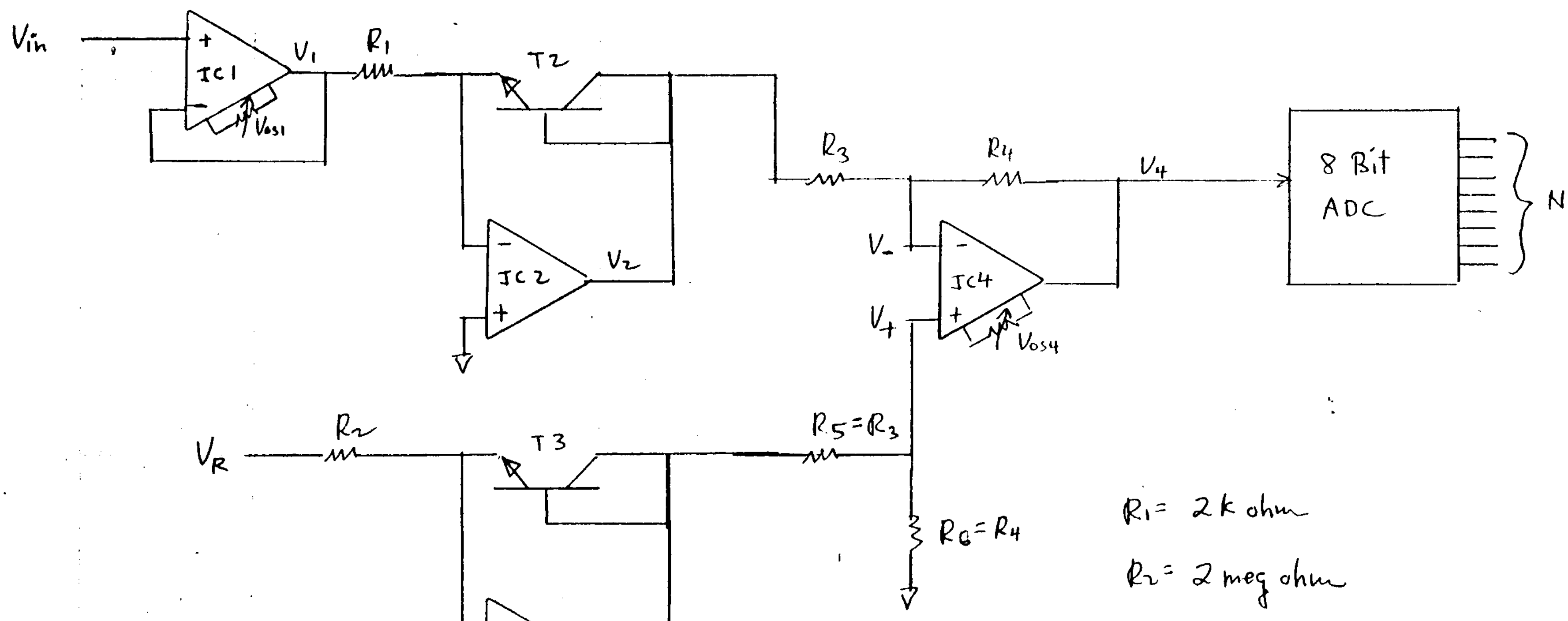


Figure 2 Analog Box Intensity Channel

R_1 = 2 k ohm

R_2 = 2 meg ohm

R_3 = 2 k ohm

R_4 = 4.99 k ohm

R_5 = R_3 = 2 k ohm

R_6 = R_4 = 4.99 k ohm

IC1 - IC4 CA3140E

T2,3 2N2060

This leads to the following equation for the log amplifier response (see Appendix B-4)

$$V_{in} = \frac{1}{C_5} \exp \frac{N}{C_6} + C_7 \tag{36}$$

where nominal values of the coefficients are

$C_5 = 200$

$C_6 = 32.1$ (at $T = 20^\circ C$); 34.3 (at $T = 40^\circ C$)

$C_7 = 0$

II-E. OVERALL CALIBRATION

We may now combine the above equations to yield

$$I = C_1 \left\{ C_3 \left[\frac{1}{C_5} \exp\left(\frac{N}{C_6}\right) + C_7 \right] + C_4 \right\} \exp\left(\frac{L}{C_2}\right)$$

$$= \left\{ \frac{C_1 C_3}{C_5} \exp\left(\frac{N}{C_6}\right) + C_1 C_3 C_7 + C_1 C_4 \right\} \exp\left(\frac{L}{C_2}\right) \tag{37}$$

Using nominal values of the coefficients, this becomes at $40^\circ C$

$$I = 3.86 \times 10^7 \exp\left(\frac{N}{34.3}\right) \exp\left(\frac{L}{668}\right) \text{ protons per bucket} \tag{38}$$

where N is the ADC output count and L the Foam-8 cable length in feet.

II-F ALLOWABLE TOLERANCES

The overall system variance, not including mechanical, is the quadrature sum of all the contributions from the tolerances of individual parameters:

$$\frac{dI}{I} = \frac{1}{I}\left\{\sum_{i=1}^{7}\left(\frac{\partial I}{\partial C_i}\,dC_i\right)^2 + \left(\frac{\partial I}{\partial N}\,dN\right)^2 + \left(\frac{\partial I}{\partial L}\,dL\right)^2\right\}^{\frac{1}{2}} \tag{39}$$

the partial derivatives are

$$\frac{\partial I}{\partial C_1} = \frac{I}{C_1} \tag{40}$$

$$\frac{\partial I}{\partial C_2} = \frac{-I}{C_2}\,\frac{L}{C_2} \tag{41}$$

$$\frac{\partial I}{\partial C_3} = \frac{1}{C_3}\left[I - C_1C_4\exp\left(\frac{L}{C_2}\right)\right] \tag{42}$$

$$\frac{\partial I}{\partial C_4} = C_1\exp\left(\frac{L}{C_2}\right) \tag{43}$$

$$\frac{\partial I}{\partial C_5} = \frac{1}{C_5}\left[I - (C_1C_3C_7 + C_1C_4)\exp\left(\frac{L}{C_2}\right)\right] \tag{44}$$

$$\frac{\partial I}{\partial C_6} = \frac{-N}{C_6{}^2}\left[I - (C_1C_3C_7 + C_1C_4)\exp\left(\frac{L}{C_2}\right)\right] \tag{45}$$

$$\frac{\partial I}{\partial C_7} = C_1C_3\exp\left(\frac{L}{C_2}\right) \tag{46}$$

$$\frac{\partial I}{\partial N} = \frac{1}{C_6}\left[I - (C_1C_3C_7 + C_1C_4)\right]\exp\left(\frac{L}{C_2}\right) \tag{47}$$

$$\frac{\partial I}{\partial L} = \frac{I}{C_2} \tag{48}$$

We analyze the overall system tolerance using the following values and tolerances for the individual parameters. In particular, we assume that the temperature is stable to $\pm 10^{0}$C and the cable length is estimated to ±20 ft.

Variable	Value	Tolerance	
C_1	2.17×10^{10}	±10%	detector sensitivity
C_2	668	±10%	cable attenuation
C_3	.356	± 5%	rf module calibration
C_4	0	±.0004	
C_5	200	± 5%	analog box calibration at 40°C
C_6	34.3	± 3%	
C_7	0	±.0005	
N	variable	±1	least count
L	variable	±20	20 ft cable length

In Table III the analysis is presented. For low intensity the predominant source of error is C_4, which causes a ±23% error in $\delta I/I$ at 1×10^8 ppb, assuming its tolerance can be held to ±400μV. At the high end, C_6 contributes a ±23% error due to a temperature spread of ±10°C. Note that the intensity output is off scale for 1×10^8 ppb and L = 650 ft, and also for 1×10^{11} ppb, L = 150 ft. In order to assure proper operation at 1×10^8 ppb, it is desirable to raise C_5 to about 300 from its present value of 200. This will add about 14 counts to all N values. The optimum way to do this is to raise R_2 from its present value of 2 megohms to about 3 megohms An alternate choice is to lower V_R to about 4 volts from 5 (see Appendix C, Eq. 15 for details). In any case, the expected errors in measuring intensity will for the most part be in the ±20% to±30% range.

Table III - Tolerances and RMS errors on Intensity Measurements

a) Maximum cable length (650')

b) T= 40°C ± 10°C

```
C1=  .217E+11+/-  .217E+10
C2= 668.0+/-  66.8
 3= .356+/- .018
C4=0.0000+/- .0004
C5= 200.0+/-  10.0
C6= 34.30+/- 1.029
C7=0.0000+/- .0005
N=+/-1.0
L=  650.+/-   20.
```

		Error contribution to δI/I due to tolerance in									
I(ppb)	N	C_1	C_2	C_3	C_4	C_5	C_6	C_7	N	L	δI/I
.100E+09	-1.	.10	.10	.05	.23	.05	-.00	.10	.03	.03	.30
.200E+09	23.	.10	.10	.05	.11	.05	.02	.05	.03	.03	.21
.500E+09	54.	.10	.10	.05	.05	.05	.05	.02	.03	.03	.18
.100E+10	78.	.10	.10	.05	.02	.05	.07	.01	.03	.03	.18
.200E+10	102.	.10	.10	.05	.01	.05	.09	.01	.03	.03	.19
.500E+10	133.	.10	.10	.05	.00	.05	.12	.00	.03	.03	.20
.100E+11	157.	.10	.10	.05	.00	.05	.14	.00	.03	.03	.21
.200E+11	181.	.10	.10	.05	.00	.05	.16	.00	.03	.03	.23
.500E+11	212.	.10	.10	.05	.00	.05	.19	.00	.03	.03	.25
.100E+12	236.	.10	.10	.05	.00	.05	.21	.00	.03	.03	.26

.020 CP SECONDS EXECUTION TIME.

/

a) Minimum cable length (150')

b) T= 40°C ± 10°C

```
C1=  .217E+11+/-  .217E+10
C2= 668.0+/-  66.8
C3= .356+/- .018
C4=0.0000+/- .0004
C5= 200.0+/-  10.0
C6= 34.30+/- 1.029
C7=0.0000+/- .0005
N=+/-1.0
L=  150.+/-   20.
```

		Error contribution to δI/I due to tolerance in									
I(ppb)	N	C_1	C_2	C_3	C_4	C_5	C_6	C_7	N	L	δI/I
.100E+09	25.	.10	.02	.05	.11	.05	.02	.05	.03	.03	.18
.200E+09	49.	.10	.02	.05	.05	.05	.04	.02	.03	.03	.15
.500E+09	80.	.10	.02	.05	.02	.05	.07	.01	.03	.03	.15
.100E+10	104.	.10	.02	.05	.01	.05	.09	.00	.03	.03	.16
.200E+10	128.	.10	.02	.05	.01	.05	.11	.00	.03	.03	.17
.500E+10	159.	.10	.02	.05	.00	.05	.14	.00	.03	.03	.19
.100E+11	183.	.10	.02	.05	.00	.05	.16	.00	.03	.03	.21
.200E+11	207.	.10	.02	.05	.00	.05	.18	.00	.03	.03	.22
.500E+11	238.	.10	.02	.05	.00	.05	.21	.00	.03	.03	.25
.100E+12	262.	.10	.02	.05	.00	.05	.23	.00	.03	.03	.26

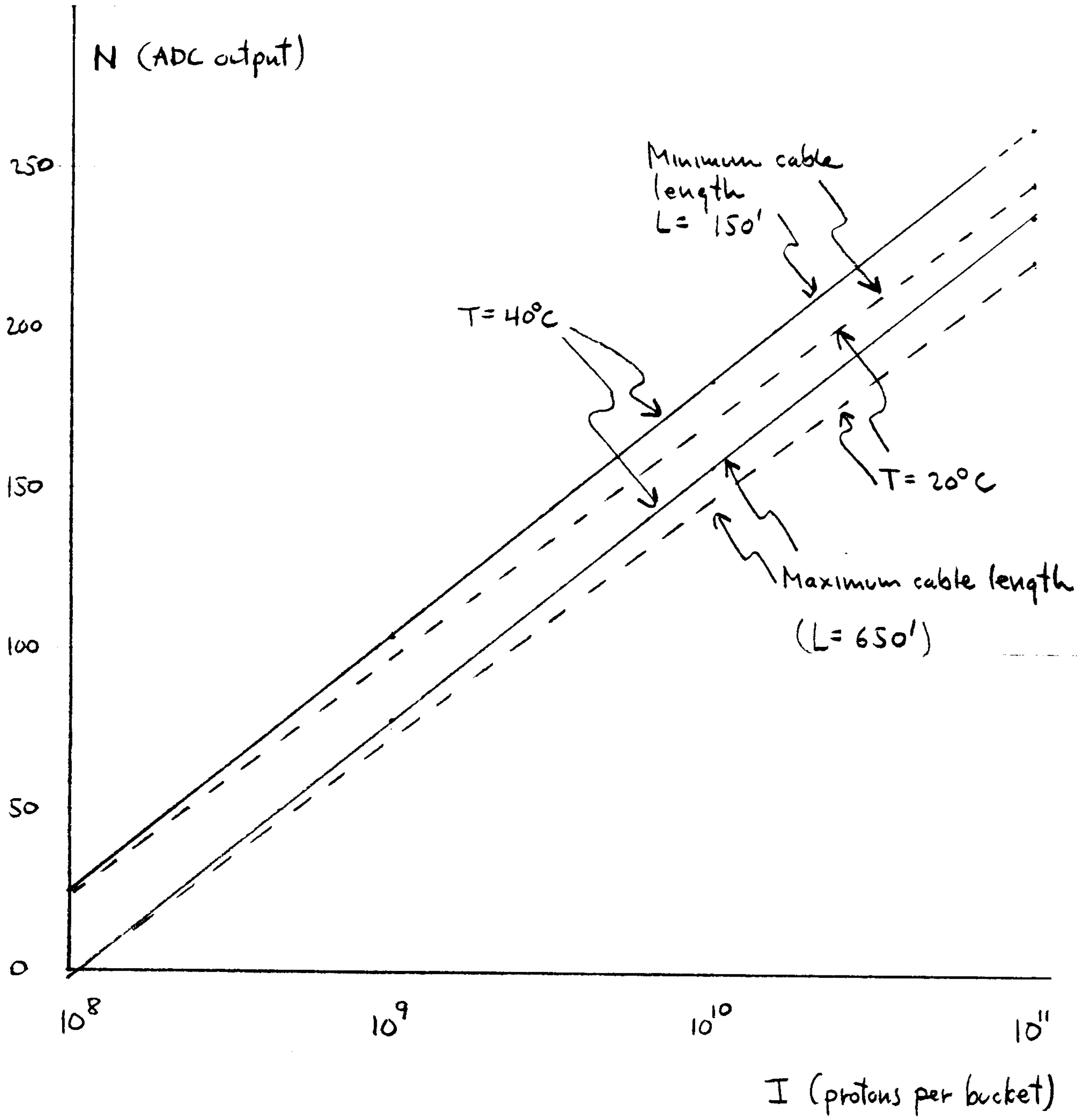


Figure 3a Calibration of Intensity Channel.

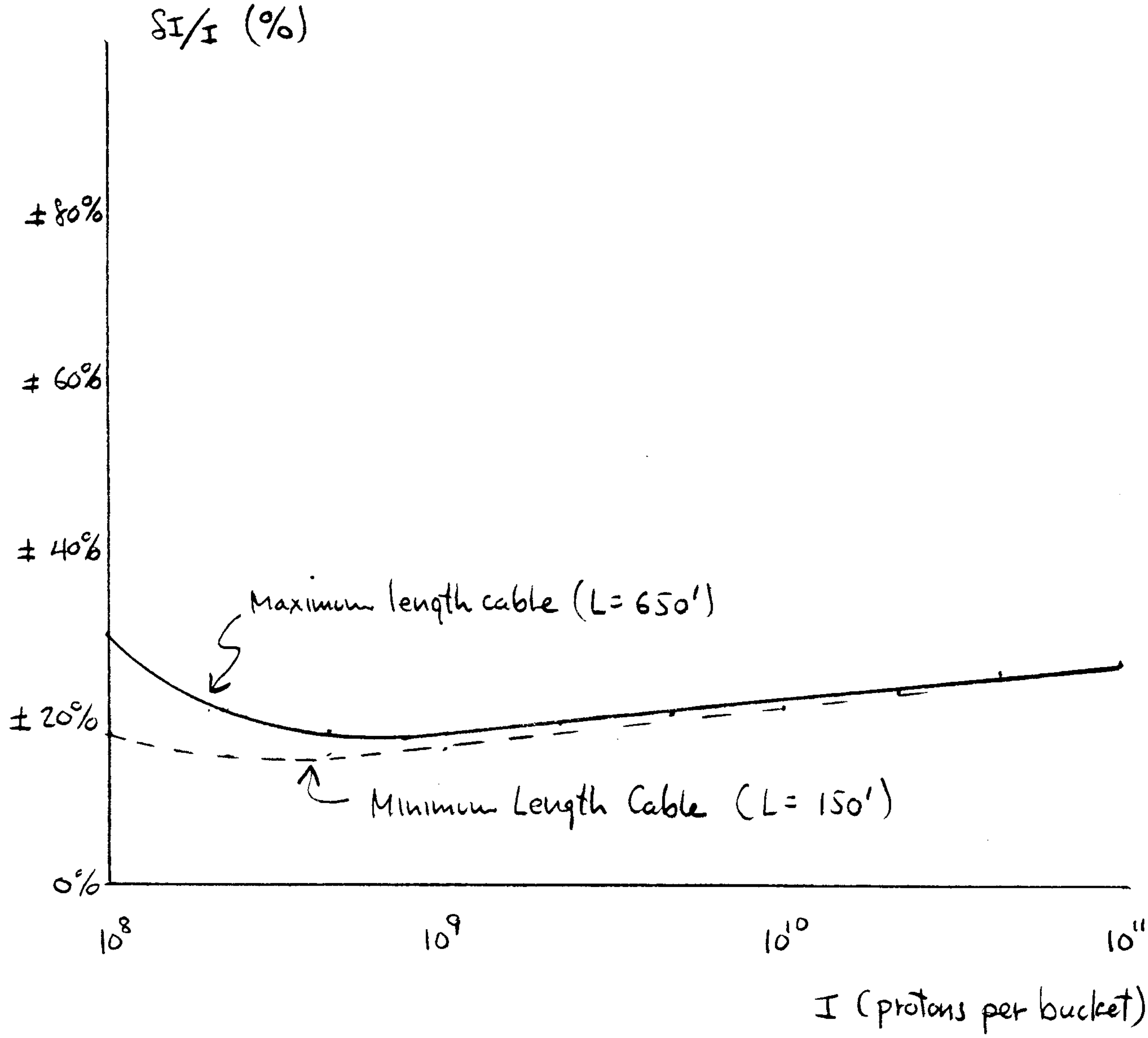


Figure 3b Expected RMS % error in Intensity Measurement

APPENDIX A: AMPLITUDE TO PHASE CONVERSION

Consider two rf signals of amplitudes A and B, and phase difference ϕ, applied to the inputs of a passive amplitude-to-phase conversion circuit as shown in Fig. A-1. The amplitude-to-phase conversion circuit is made up of four power splitter/combiners and three 90^0 delay lines. Specifically

$$V_1 = A\sin(wt + \phi/2) \tag{A1}$$

$$V_2 = B\sin(wt - \phi/2) \tag{A2}$$

The outputs of the amplitude-to-phase conversion circuit are, neglecting the approximately 6 db losses in the circuit:

$$V_3 = \tfrac{1}{2}[A\sin(wt + \tfrac{\phi}{2}) + B\sin(wt - \tfrac{\phi}{2} - \tfrac{\pi}{2})] \tag{A3}$$

$$V_4 = \tfrac{1}{2}[A\sin(wt + \tfrac{\phi}{2} - \pi) + B\sin(wt - \tfrac{\phi}{2} - \tfrac{\pi}{2})]$$

$$= \tfrac{1}{2}[-A\sin(wt + \tfrac{\phi}{2}) + B\sin(wt - \tfrac{\phi}{2} - \tfrac{\pi}{2})] \tag{A4}$$

We assume that V_3 and V_4 may be written in the form

$$V_3 = C_3\sin(wt + \theta_3) \tag{A5}$$

$$V_4 = C_4\sin(wt + \theta_4) \tag{A6}$$

Hence

$$C_3\sin wt\cos\theta_3 + C_3\cos wt\sin\theta_3$$

$$= \tfrac{1}{2}[A\sin wt\cos\tfrac{\phi}{2} + A\cos wt\sin\tfrac{\phi}{2} - B\cos wt\cos\tfrac{\phi}{2} - B\sin wt\sin\tfrac{\phi}{2}]$$

$$= \tfrac{1}{2}[(A\cos\tfrac{\phi}{2} - B\sin\tfrac{\phi}{2})\sin wt + (A\sin\tfrac{\phi}{2} - B\cos\tfrac{\phi}{2})\cos wt] \tag{A7}$$

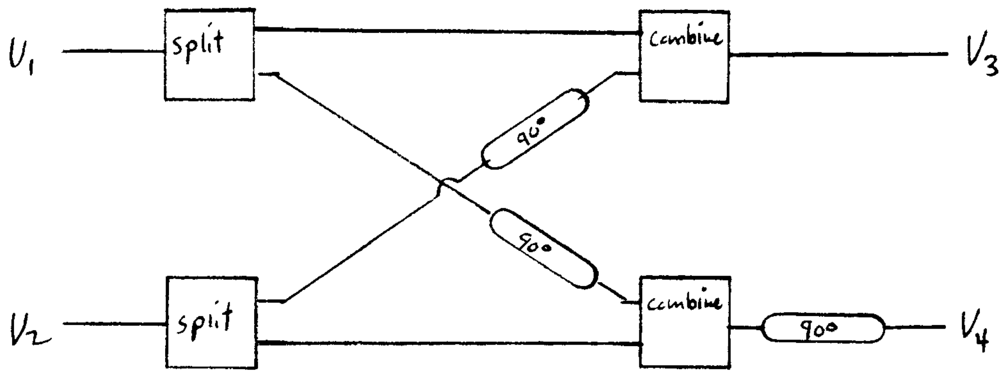


Figure A—1a. Basic Amplitude to Phase Conversion Block Diagram

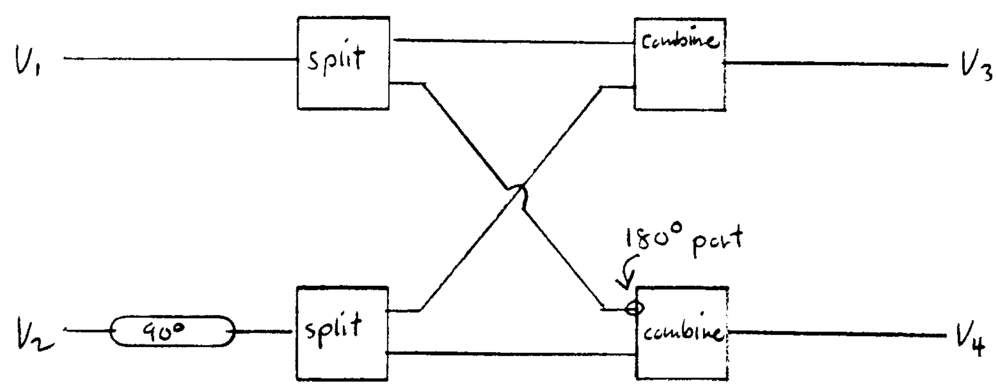


Figure A—1b. Modified Amplitude to Phase Conversion Circuit using only one delay line and a 180° port.

Hence $$C_3 \cos\theta_3 = \tfrac{1}{2}(A \cos \tfrac{\phi}{2} - B \sin \tfrac{\phi}{2}) \tag{A8}$$

and $$C_3 \sin\theta_3 = \tfrac{1}{2}(A \sin \tfrac{\phi}{2} - B \cos \tfrac{\phi}{2}) \tag{A9}$$

But $$C_3 = \left\{\tfrac{1}{4}(A \cos \tfrac{\phi}{2} - B \sin \tfrac{\phi}{2})^2 + \tfrac{1}{4}(A \sin \tfrac{\phi}{2} - B \cos \tfrac{\phi}{2})^2\right\}^{\frac{1}{2}}$$

$$= \tfrac{1}{2}\left\{A^2 \cos^2 \tfrac{\phi}{2} + B^2 \sin^2 \tfrac{\phi}{2} - 2AB \cos \tfrac{\phi}{2} \sin \tfrac{\phi}{2} + A^2 \sin^2 \tfrac{\phi}{2}\right.$$

$$\left. + B^2 \cos^2 \tfrac{\phi}{2} - 2AB \sin \tfrac{\phi}{2} \cos \tfrac{\phi}{2}\right\}^{\frac{1}{2}}$$

$$= \tfrac{1}{2}\left\{A^2 + B^2 - 2AB \sin\phi\right\}^{\frac{1}{2}} \tag{A10}$$

Hence

$$\theta_3 = \cos^{-1} \frac{A \cos \phi/2 - B \sin \phi/2}{[A^2 + B^2 - 2AB \sin \phi]^{\frac{1}{2}}}$$

$$= \tan^{-1} \left\{\frac{A \sin \phi/2 - B \cos \phi/2}{A \cos \phi/2 - B \sin \phi/2}\right\} \tag{A11}$$

Similarly

$$C_4 = \tfrac{1}{2}\{ A^2 + B^2 + 2AB \sin \phi \}^{\frac{1}{2}} \tag{A12}$$

and

$$\theta_4 = \cos^{-1} \frac{A \cos \phi/2 + B \sin \phi/2}{[A^2 + B^2 + 2AB \sin \phi]^{\frac{1}{2}}}$$

$$= \tan^{-1} \left\{\frac{A \sin \phi/2 + B \cos \phi/2}{A \cos \phi/2 + B \sin \phi/2}\right\} \tag{A13}$$

when $\phi = 0$, we note that $C_3 = C_4$, and in addition the relative phase shift between V_3 and V_4 is

$$\theta_4 - \theta_3 = 2 \tan^{-1} \left[\frac{B^1}{A^1}\right] \tag{A14}$$

inverting,

$$\frac{B^1}{A^1} = \tan \left(\frac{\theta_4 - \theta_3}{2}\right) \tag{A14}$$

In an actual measurement, a cable pair with a relative phase shift ϕ will yield a relative phase shift $\theta_4 - \theta_3$:

$$\theta_4 - \theta_3 = \tan^{-1}\left\{\frac{A\sin\phi/2 + B\cos\phi/2}{A\cos\phi/2 + B\sin\phi/2}\right\} - \tan^{-1}\left\{\frac{A\sin\phi/2 - B\cos\phi/2}{A\cos\phi/2 - B\sin\phi/2}\right\}$$

$$= \tan^{-1}\left\{\frac{\sin\phi/2 + (B/A)\cos\phi/2}{\cos\phi/2 + (B/A)\sin\phi/2}\right\}$$

$$- \tan^{-1}\left\{\frac{\sin\phi/2 - (B/A)\cos\phi/2}{\cos\phi/2 - (B/A)\sin\phi/2}\right\} \qquad \text{(A16)}$$

To estimate beam position, (B^1/A^1) will be computed using Eq. (A15) which assumes $\phi = 0$, and using the value of $\theta_4 - \theta_3$ given in Eq. (A16).

Specifically

$$\frac{B^1}{A^1} = \mathrm{Tan}\left\{\tfrac{1}{2}\tan^{-1}\left[\frac{\sin\phi/2 + B/A\cos\phi/2}{\cos\phi/2 + B/A\sin\phi/2}\right]\right.$$

$$\left. - \tfrac{1}{2}\tan^{-1}\left[\frac{\sin\phi/2 - (B/A)\cos\phi/2}{\cos\phi/2 - (B/A)\sin\phi/2}\right]\right\} \qquad \text{(A17)}$$

As beam position detectors seem to be linear in

$$20\log\left(\frac{B}{A}\right) = \left(\frac{B}{A}\right)_{db}$$

an estimate of the error in the position measurement due to the phase error ϕ can be estimated by calculating $(B^1/A^1)_{db}$ as a function of $(B/A)_{db}$ and ϕ. This is shown in Table A-I. As the detector sensitivity is about 0.67 db/mm, the error in estimating position can be held under 0.2 mm for $x \le \pm 25$ mm if ϕ is less than $\pm 10^0$. The corresponding effect on the signal amplitude for $\phi = \pm 10^0$ is about ±17% as can be seen in Eqs. (A10) and (A12). This phase error represents about ±12 cm of foam dielectric coaxial cable.

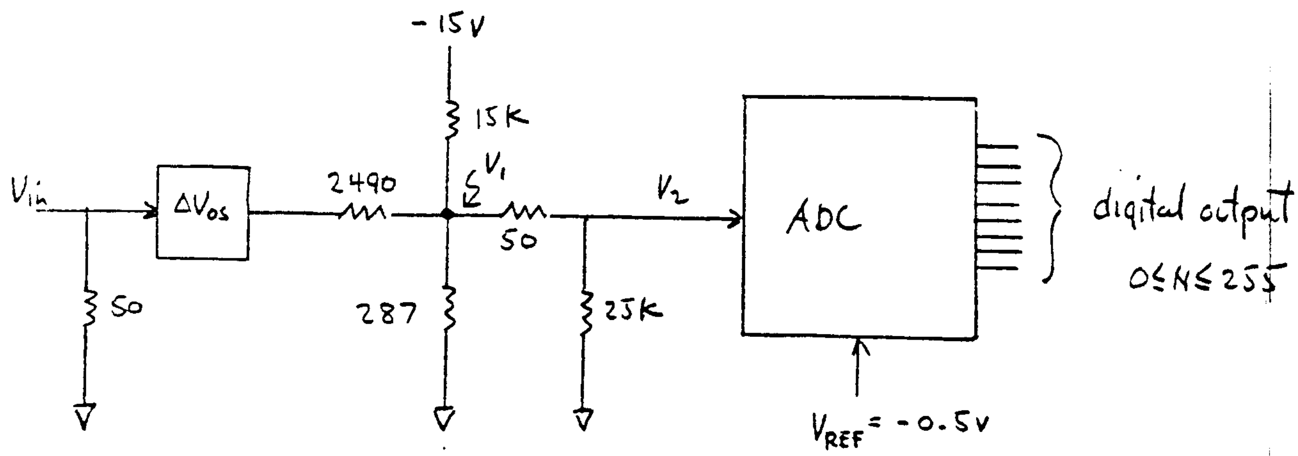
Table A—I. Tabulated values of $(B'/A')_{db}$ as a function of $(B/A)_{db}$ and phase error φ.

$(B/A)_{db}$	$\varphi=2°$	4°	6°	8°	10°	12°	14°	16°	18°	20°
1.0	1.00	1.00	1.01	1.01	1.02	1.02	1.03	1.04	1.05	1.06
2.0	2.00	2.00	2.01	2.02	2.03	2.04	2.06	2.08	2.10	2.13
3.0	3.00	3.01	3.02	3.03	3.04	3.06	3.09	3.12	3.15	3.18
4.0	4.00	4.01	4.02	4.04	4.06	4.08	4.11	4.15	4.19	4.24
5.0	5.00	5.01	5.02	5.04	5.07	5.10	5.14	5.18	5.23	5.29
6.0	6.00	6.01	6.03	6.05	6.08	6.12	6.16	6.21	6.27	6.33
7.0	7.00	7.01	7.03	7.06	7.09	7.13	7.18	7.23	7.29	7.37
8.0	8.00	8.02	8.03	8.06	8.10	8.14	8.19	8.25	8.32	8.40
9.0	9.00	9.02	9.04	9.07	9.10	9.15	9.20	9.27	9.34	9.42
10.0	10.00	10.02	10.04	10.07	10.11	10.16	10.22	10.28	10.36	10.45
11.0	11.00	11.02	11.04	11.07	11.11	11.16	11.22	11.29	11.37	11.46
12.0	12.00	12.02	12.04	12.07	12.12	12.17	12.23	12.30	12.39	12.48
13.0	13.00	13.02	13.04	13.08	13.12	13.17	13.24	13.31	13.40	13.49
14.0	14.00	14.02	14.04	14.08	14.12	14.18	14.24	14.32	14.40	14.50
15.0	15.00	15.02	15.04	15.08	15.12	15.18	15.25	15.32	15.41	15.51
16.0	16.01	16.02	16.05	16.08	16.13	16.18	16.25	16.33	16.42	16.52
17.0	17.01	17.02	17.05	17.08	17.13	17.18	17.25	17.33	17.42	17.52
18.0	18.01	18.02	18.05	18.08	18.13	18.19	18.25	18.33	18.42	18.52
19.0	19.01	19.02	19.05	19.08	19.13	19.19	19.26	19.33	19.43	19.53
20.0	20.01	20.02	20.05	20.08	20.13	20.19	20.26	20.34	20.43	20.53

.038 CP SECONDS EXECUTION TIME.

APPENDIX B - ANALOG BOX DAUGHTER CARD POSITION CHANNEL

The position channel on the analog box daughter card includes a 50 ohm termination, a FET switch, a storage capacitor, a voltage follower, a resistor attenuation and offset network, a FET multiplexer switch and an 8-bit ADC. For the purposes of calibration, the circuit may be represented as follows:



The 2490, 287, and 15K resistors are 1% metal film, the series 50 ohm resistor represents the FET "on" resistance, and the 25K the ADC input impedance.

The voltages V_{in} and V_1 are related as follows:

$$\frac{V_{in} - V_{os} - V_1}{2490} + \frac{-15 - V_1}{15{,}000} + \frac{-V_1}{287} + \frac{-V_1}{25{,}050} = 0 \qquad \text{(B1)}$$

or

$$V_{in} = 2490\ V_1 \left[\frac{1}{2490} + \frac{1}{15,000} + \frac{1}{25,050}\right] + \frac{15 \cdot 2490}{15,000} + \Delta V_{os}$$

$$= 9.9414\ V_1 + 2.490 + \Delta V_{os} \qquad \text{(B2)}$$

Also, $$\frac{V_2}{V_1} = \frac{25,000}{22,050} \qquad \text{(B3)}$$

Hence: $$V_{in} = 9.9613\ V_2 + 2.49 + \Delta V_{os} \qquad \text{(B4)}$$

The ADC calibration is expected to be

$$\frac{V_2}{-.500} = \frac{N}{255} \qquad \text{(B5)}$$

$$\therefore V_{in} = \frac{9.9613\ (-.5)N}{255} + 2.49 + \Delta V_{os}$$

$$= \frac{127.5 + 512\ \Delta V_{os} - N}{51.2} \qquad \text{(B6)}$$

The offset adjustment will be used to set this to

$$V_{in} = \frac{128.0 - N}{51.2} \qquad \text{(B7)}$$

For reasons not completely understood at present, the measured value of the constant in the denominator is about 52.8.

APPENDIX C - ANALOG BOX DAUGHTER CARD INTENSITY CHANNEL

The intensity channel on the analog box daughter card includes a 50 ohm termination, a FET switch, a storage capacitor, a buffer amplifier, a log amplifier, a FET multiplexer switch and an 8-bit ADC (shared with the position channel described in Appendix B. The basic circuit is shown in Fig. C-1.

The input circuit is a voltage follower circuit with the response

$$V_{in} = V_1 - V_{os1} \tag{C1}$$

where V_{os1} is an adjustable offset.

The current-voltage relation for the two transdiode connected NPN transistors is

$$I = I_0 \left[\exp\left(\frac{qV}{kT}\right) + 1\right] \tag{C2}$$

where q is the electron charge, k is Boltzmann's constant, T is the junction temperature in ^{0}K, and I_0 the reverse bias saturation current. I_0 is normally in the picoamp range, allowing the following approximation when $I >> I_0$ (i.e., a few microamps):

$$V = \frac{kT}{q} \ln (I/I_0) \tag{C3}$$

Hence in the circuit in the figure we can write the equations for V_2 and V_3:

$$V_2 = \frac{kT}{q} \ln \frac{V_1}{R_1 I_{02}} \tag{C4}$$

and

$$V_3 = \frac{kT}{q} \ln \frac{V_R}{R_2 I_{03}} \tag{C5}$$

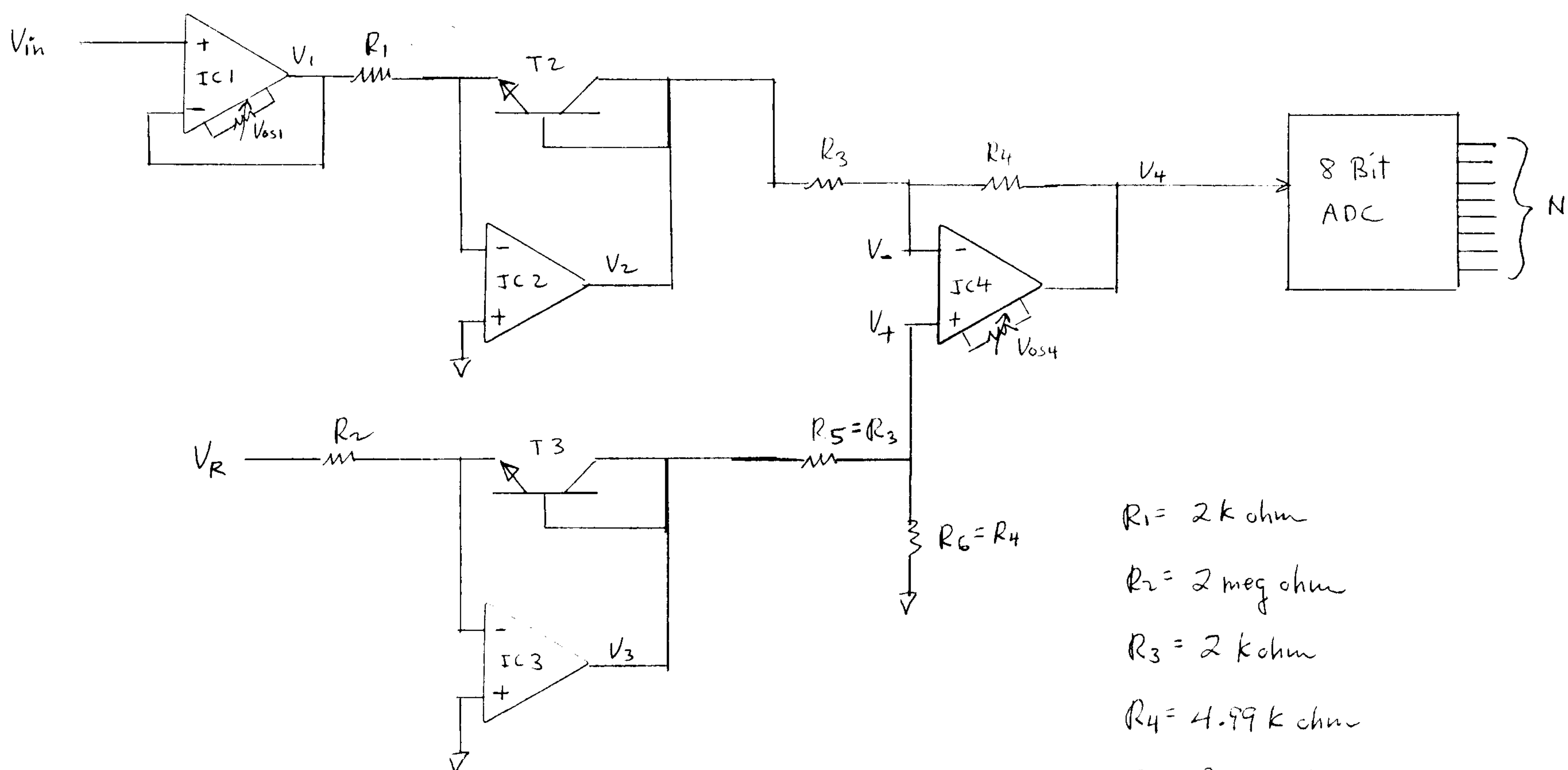


Figure C-1: Analog Box Intensity Channel

$R_1 = 2$ k ohm

$R_2 = 2$ meg ohm

$R_3 = 2$ k ohm

$R_4 = 4.99$ k ohm

$R_5 = R_3 = 2$ k ohm

$R_6 = R_4 = 4.99$ k ohm

IC1 - IC4 CA3140E

T2,3 2N2060

IC4 is a common mode rejection circuit. Specifically, since

$$\frac{V_2 - V_-}{R_3} = \frac{V_- - V_4}{R_4} \tag{C6}$$

and

$$\frac{V_+}{V_3} = \frac{R_6}{R_5 + R_6} \tag{C7}$$

and

$$V_+ = V_- \tag{C8}$$

We have

$$V_4 = \frac{R_4}{R_3}\left[V_3 - V_2 - V_{os4}\right] \tag{C9}$$

Hence,

$$V_2 = V_3 - V_{os4} - \frac{R_3}{R_4} V_4 \tag{C10}$$

The ADC calibration is (N is the 8-bit conversion output)

$$\frac{V_4}{-.500} = \frac{N}{255} \tag{C11}$$

Combining Eqs. (C1), (C4), (C5), (C10) and (C11):

$$N = 510\,V_4 = (-510)\left(\frac{R_4}{R_3}\right)\left[V_3 - V_2 - V_{os4}\right]$$

$$= (-510)\,\frac{R_4}{R_3}\left\{\frac{kT}{q}\ln\left(\frac{V_R}{R_2 I_{03}}\right) - \frac{kT}{q}\ln\left(\frac{V_{in} + V_{os}}{R_1 I_{02}}\right) - V_{os4}\right\}$$

$$= -510\,\frac{R_4}{R_3}\,\frac{kT}{q}\left\{\ln\left[\frac{V_R\,R_1}{(V_{in} + V_{os})\,R_2}\right]\right.$$

$$\left. + \ln\left[\frac{I_{02}}{I_{03}}\right] - \frac{q}{kT}V_{os4}\right\} \tag{C12}$$

This may be written with V_{in} as the explicit variable:

$$V_{in} = -\left(\frac{R_1 V_R}{R_2}\right) \exp\left[\frac{q}{kT} V_{os4} + \ln \frac{I_{02}}{I_{03}}\right] \exp\left[\frac{R_3}{R_4} \frac{q}{kT} \frac{N}{510}\right] - V_{os1} \tag{C13}$$

The nominal value of this equation (with the V_{os4} set to cause the first exponent to equal zero) is

$$V_{in} = \frac{-1}{C_5} \exp\left(\frac{N}{C_6}\right) + C_7 \tag{C14}$$

Using $R_1 = 2K$ ohm

$R_2 = 2$ megohm

$V_R = -5$ volts

$R_3 = 2K$ ohm

$R_4 = 4.99K$ ohm

$q = 1.602 \times 10^{-19}$ Coulombs

$k = 1.38 \times 10^{-23}$ Joules/^{0}K

$T = 313^0$K (40^0C)

$\frac{kT}{q} = 27.0$ mV at 40^0C

The constants are:

$$C_5 = \frac{R_2}{R_1 V_R} \exp\left[\frac{-q}{kT} V_{os4} - \ln\left(\frac{I_{02}}{I_{03}}\right)\right] = 200 \quad \text{(See Note 1)} \tag{C15}$$

$$C_6 = \frac{510\ R_4}{R_3} \frac{kT}{q} = 32.1 \text{ at } T = 20^0\text{C}$$

$$= 34.3 \text{ at } T = 40^0\text{C}$$

$$C_7 = V_{os1} = 0$$

Note 1. The base-emitter voltage of the NPN pair in the 2N2060 is matched to better than ±5mV for a given emitter current. Using Eq. (C3) it is easy to show that $\ln \frac{I_{02}}{I_{03}}$ is less than ±5/27 = ±.19. Hence a ±5mV range on adjusting V_{os4} is adequate to set the exponent to zero.

Appendix D Computer Programs

```
p0:*
      PROGRAM BPMERR(INPUT,OUTPUT)
      IMPLICIT REAL(N)
      PI=2.*ASIN(1.0)
      DB=0
      EDB=.1
      C1=.314
      EC1=.003
      V0=0
      EV0=.010
      C2=128.
      EC2=.5
      EN=.5
      C3=52.8
      EC3=.5
      Y=0
      EY=10.
      X=-1.
      PRINT 50,DB,EDB,C1,EC1,V0,EV0,C2,EC2,EN,C3,EC3,Y
50    FORMAT(" DB=",F6.2," +/-",F6.2,/," C1=",F7.3,"  +/-",F6.3,/
     $  " V0=",F6.3," +/-",F6.3,/," C2=",F6.2," +/-",F6.2,/
     $  " N=  +/-",F6.2,/," C3=",F6.2," +/-",F6.2,/," Y=",F6.2,///)
      DO 10 I=1,26
      X=X+1.
      TMP=(1.+Y**2/2830)*X/12.96
      PHI=ATAN(EXP(TMP))
      N=C2-C3*(PHI+C1*V0-PI/4.)/C1
      N2=256.-N
      T1=1.-Y**2/2830.
      T2=12.96*2./SIN(2.*PHI)
      E1=1.5*T1*EDB
      E2=T1*T2*((C2-N)/C3-V0)*EC1
      E3=T1*T2*C1*EV0
      E4=T1*T2*(C1/C3)*EC2
      E5=T1*T2*(C1*(C2-N)/C3**2)*EC3
      E6=T1*T2*(C1/C3)*EN
      ERR2=E1**2+E2**2+E3**2+E4**2+E5**2+E6**2
      ERR=SQRT(ERR2)
      PRINT 100,X,N,N2,E1,E2,E3,E4,E5,E6,ERR
100   FORMAT(1X,F6.2,2X,2F6.1,2X,6F7.2,2X,F8.2)
10    CONTINUE
      STOP
      END
??
```

A) Program for calculating table I

```
 r,*
   8 FILE(S) PROCESSED.
/ice
 DIT: APPROX
.? p0:*
      PROGRAM APPROX(INPUT,OUTPUT)
      IMPLICIT REAL(N)
      PI=2.*ASIN(1.0)
      DB=0
      EDB=.1
      C1=.314
      EC1=.003
      V0=0
      EV0=.010
      C2=128.
      EC2=.5
      EN=.5
      C3=52.8
      EC3=.5
      Y=0
      EY=10.
      X=-1.
      DO 10 I=1,26
      X=X+1.
      TMP=(1.+Y**2/2830)*X/12.96
      PHI=ATAN(EXP(TMP))
      N1=C2-C3*(PHI+C1*V0-PI/4.)/C1
      Z=X/12.96
      N2=C2-(C3/C1)*(.4947*Z-.0667*Z**3+.0063*Z**5)
      ARG=C1*((C2-N2)/C3)+PI/4.
      X12=12.96*(1.-Y**2/2830)*ALOG(TAN(ARG))
      Z2=(C2-N1)*C1/C3
      X21=12.96*(1.866*Z2+2.548*Z2**3)
      Z3=(C2-N2)*C1/C3
      X22=12.96*(1.866*Z3+2.548*Z3**3)
      DX12=X12-X
      DX21=X21-X
      DX22=X22-X
      PRINT 100,X,X12,DX12,X21,DX21,X22,DX22
100   FORMAT(1X,F8.1,3(2X,F6.1," (",F4.1,")",))
10    CONTINUE
      STOP
      END
?? eu
FILE: APPROX
/replace,approx
/
```

B) Program for calculating Table II

```
      PROGRAM BPIERR(INPUT,OUTPUT)
      IMPLICIT REAL (L-Z)
      DIMENSION RI(10)
      C1=2.17E10
      DC1=2.17E9
      C2=668.
      DC2=66.8
      C3=.356
      DC3=.0178
      C4=0
      DC4=.0004
      C5=200.
      DC5=10.
      C6=34.3
C THIS VALUE (34.3) CORRESPONDS TO 40 DEGREES C.
      DC6=1.029
      C7=0
      DC7=.0005
      DN=1.
      L=150.
      DL=20.
      PRINT 50,C1,DC1,C2,DC2,C3,DC3,C4,DC4,C5,DC5,C6,DC6,C7,DC7,
     $DN,L,DL
50    FORMAT(" C1=",E10.3,"+/-",E10.3,/," C2=",F6.1,"+/-",F6.1,/,
     $" C3=",F5.3,"+/-",F5.3,/," C4=",F6.4,"+/-",F6.4,/,
     $" C5=",F6.1,"+/-",F6.1,/," C6=",F6.2,"+/-",F6.3,/,
     $" C7=",F6.4,"+/-",F6.4,/," N=+/-",F3.1,/,
     $" L=",F6.0,"+/-",F6.0,///)
      DATA RI/1.0E8,2.0E8,5.0E8,1.0E9,2.0E9,5.0E9,
     $  1.0E10,2.0E10,5.0E10,1.0E11/
      DO 10 I=1,10
      TMP=RI(I)*EXP(-L/C2)-C1*C3*C7-C1*C4
      TMP=C5*TMP/(C1*C3)
      N=C6*ALOG(TMP)
      EC1=DC1/C1
      EC2=L*DC2/C2**2
      ELC2=EXP(L/C2)
      EC3=(RI(I)-C1*C4*ELC2)*DC3/(C3*RI(I))
      EC4=C1*ELC2*DC4/RI(I)
      EC5=(RI(I)-(C1*C3*C7+C1*C4)*ELC2)*DC5/(RI(I)*C5)
      EC6=N*DC6*(RI(I)-(C1*C3*C7+C1*C4)*ELC2)/(RI(I)*C6**2)
      EC7=C1*C3*ELC2*DC7/RI(I)
      EN=(RI(I)-(C1*C3*C7+C1*C4)*ELC2)*DN/(C6*RI(I))
      EL=DL/C2
      EI2=EC1**2+EC2**2+EC3**2+EC4**2+EC5**2+EC6**2+EC7**2+EN**2+EL**2
      EI=SQRT(EI2)
      PRINT 100,RI(I),N,EC1,EC2,EC3,EC4,EC5,EC6,EC7,EN,EL,EI
100   FORMAT(1X,E10.3,F6.0,2X,10F5.2)
10    CONTINUE
      STOP
      END
??
```

c) Program for calculating Table III

```
ice
EDIT: PHIERR
?? p0:*
      PROGRAM PHIERR(INPUT,OUTPUT)
      DIMENSION DELDB(10),ADEL(10),X(10)
      PI=2.*ASIN(1.0)
      A=1.0
      DB=0
      DO 10 I=1,20
      DB=DB+1.
      B=10.**(DB/20.)
      PHI=0
      DO 20 J=1,10
      PHI=PHI+2.0
      PH2=PHI/2.
      COS=COSD(PH2)
      SIN=SIND(PH2)
      TAN4=(SIN+B*COS)/(COS+B*SIN)
      TAN3=(SIN-B*COS)/(COS-B*SIN)
      TH4=ATAN(TAN4)
      TH3=ATAN(TAN3)
      THAV=0.5*(TH4-TH3)
      IF(THAV.LE.0)THAV=THAV+PI/2.
      DEL=TAN(THAV)
      DELDB(J)=20.*ALOG10(DEL)
20    CONTINUE
      PRINT 100,DB,(DELDB(J),J=1,10)
100   FORMAT(1X,F5.1,2X,10F7.2)
10    CONTINUE
      STOP
      END
?? eu
FILE: PHIERR
/
```

D) Program for calculating Table A – I.