BPM Note #3
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11/30/81

Interpretation of E.D. Beam Position Detector Measurements in Industrial #1.

The purpose of this BPM Note is to compare the results of the beam position detector sensitivity tests made in Industrial Bldg. #1 using Q. Kerns' RF cavity system inducing T.E.M. signals on a .004" dia wire with the calculated response for an ideal stripline detector. These tests are reviewed in BPM Design Note #2.

In UPC Report #101 (R. Shafer, 5/18/79) it was shown that for grounded pickup plates of radius a, and a subtended angle φ for a centered beam, the induced image charge per unit length on the inside surface per unit length is given by

$$Q_A = -\frac{Q\varphi}{2\pi}\left\{1 + \frac{4}{\varphi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r_0}{a}\right)^n \cos n\theta_0 \sin\frac{n\varphi}{2}\right\} \tag{1}$$

$$Q_B = -\frac{Q\varphi}{2\pi}\left\{1 + \frac{4}{\varphi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r_0}{a}\right)^n \cos n\theta_0 \sin\left[n\left(\pi+\frac{\varphi}{2}\right)\right]\right\} \tag{2}$$

where Q is the charge per unit length in the beam, and its position is given as r_0, θ_0. This geometry is shown in Figure 1.

The important assumption here is that the entire circle at radius $r=a$ is at ground potential. This is not the case in the actual beam position detector, however, for two reasons. In the

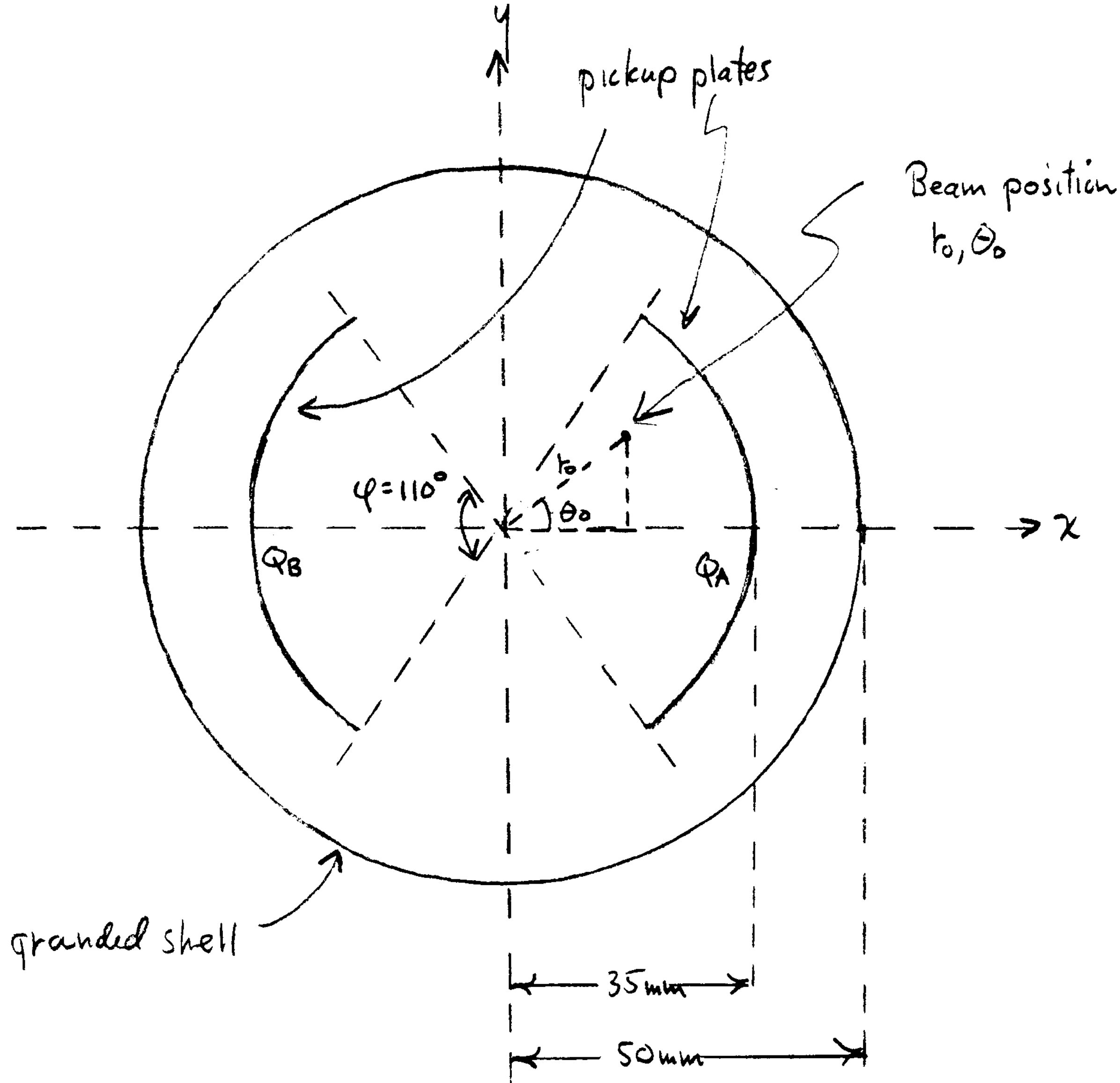


Figure 1. Actual detector geometry. In the original calculation the pickup plates were assumed to be granded, and the space between the electrodes was assumed to have granded arcs. In the actual detector the electrodes each form a 50 ohm transmission line with the grand plane (when the electrodes are driven with equal amplitude signals), and are terminated in 50 ohms at each end.

beam position detector, the plates have a subtended angle of 110° and a radius of 35mm. the ground plane outside the plates has a radius of 50 mm. As a voltage is induced on the pickup plates, they are not at ground potential. The ratio of induced voltage to induced current (on the outside of the plates) is determined by their characteristic impedance, about 50 ohms. Furthermore, the arc between the pickup plates is open (i.e no ground plane), allowing some field lines from the line charge to terminate on the ground plane at $r = 50$ mm.

If the pickup electrodes were at ground potential, then there would be no induced charge on the outside of the plates. Then we would expect the proper a, φ to be used in eqns 1 and 2 to be ~ 35mm and 110°. If the pickup plates had the same charge (but of opposite sign) induced on the outside and inside, then the plates would be at a potential (since they essentially follow the azimuthal equipotential lines) determined by setting $r = 35$ in the equation

$$V(r) = \frac{-Q}{2\pi\epsilon_0} \ln(r/a) = \frac{-Q}{2\pi\epsilon_0} \ln(35/50) \qquad (3)$$

In this case, we would expect the r_0, θ_0 dependence of the induced charge to be determined by a, φ approximately 50mm and 110°.

In the real case, since ½ the induced charge on the outside of the pickup electrode goes out the upstream port (also 50 ohms), the remaining induced charge on the outside of the pickup electrode is only ½ of the induced charge on the inside surface. With no further handwaving, the value of a in eqns 1 and 2 shall be set at the average of 35 and 50 mm, namely 42.5mm. It is possible that the arithmetic mean, 41.8mm, would have been a better choice.

The first indication that an intermediate value of a was required was that the observed sensitivity of the detector was about 0.65 db/mm at small distances. The sensitivity is defined as the db ratio (20 times the base 10 log) of the two output signals. In terms of eqns 1 and 2, this is

$$\left(\frac{Q_A}{Q_B}\right)_{db} \cong 20 \log_{10}\left[\frac{1+\frac{4}{\varphi}\left(\frac{r_0}{a}\right)\cos\theta_0 \sin\frac{\varphi}{2}}{1-\frac{4}{\varphi}\left(\frac{r_0}{a}\right)\cos\theta_0 \sin\frac{\varphi}{2}}\right] \tag{4}$$

$$\cong\left(\frac{20}{\ln 10}\right)\frac{8}{\varphi}\left(\frac{r_0}{a}\right)\cos\theta_0 \sin\frac{\varphi}{2} \quad \left(\text{since } \ln\frac{1+x}{1-x} = 2x+\frac{2}{3}x^3+\frac{2}{5}x^5+\cdots\right)$$

$$\cong \frac{160}{\ln 10}\,\frac{1}{\varphi}\,\frac{x}{a}\sin\frac{\varphi}{2}$$

The sensitivity is then

$$\frac{1}{x}\left(\frac{Q_A}{Q_B}\right)_{db} \sim \frac{69.5}{a\varphi}\sin\varphi/2 \quad (= \tag{5}$$

$\cong .85$ db/mm for $\varphi = 110°$, $a = 35$mm

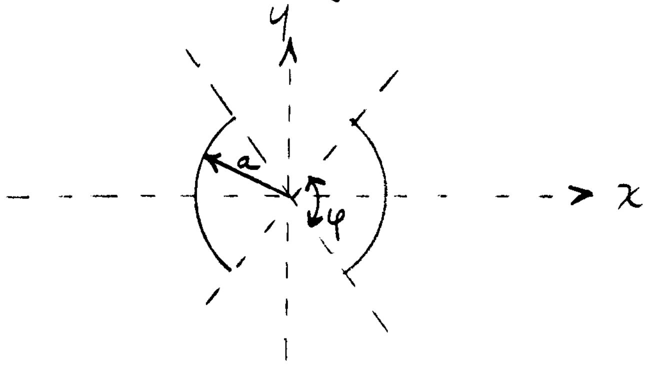
$\cong .70$ db/mm $\varphi = 110°$, $a = 42.5$mm

$\cong .59$ db/mm $\varphi = 110°$, $a = 50$mm

So we use $a = 42.5$mm throughout.

In the measured signal strength output of the detector measured in Industrial #1, the signal observed was about 1.2db ± 2db higher than calculated. This discrepancy can be eliminated by increasing φ by 1.2 db, from 110° to about 130°. the sensitivity (eqn 4) then becomes about .65 db/mm, in good agreement with the observed value. this increase in the apparent subtended angle over the geometric angle should be expected due to the fact that with the missing ground planes between the pickup electrodes, the corners of the pickup electrodes should attract additional field lines (Recall that the electrodes are at 1/2 the potential given by eqn 3)

Table I a and I b present calculations of $(Q_A/Q_B)_{db}$ and $\frac{1}{x}(Q_A/Q_B)_{db}$ as a function of x and y, using $a = 42.5$mm and $\varphi = 130°$. the coordinate system is:



We can now compare the predictions of this model to two other sets of data measured at Industrial #1. A y scan was made at both $x = 0$ and $x = 10.64$mm. these measurements, shown in Figures 2 and 3, show an apparent tilt of about 33.5 mrad, This tilt was included in the smooth curves. Both the absolute sensitivity in Figure 3 (6.95db calculated vs 6.85db observed) and the y dependence in Figures 2 and 3 seem to be

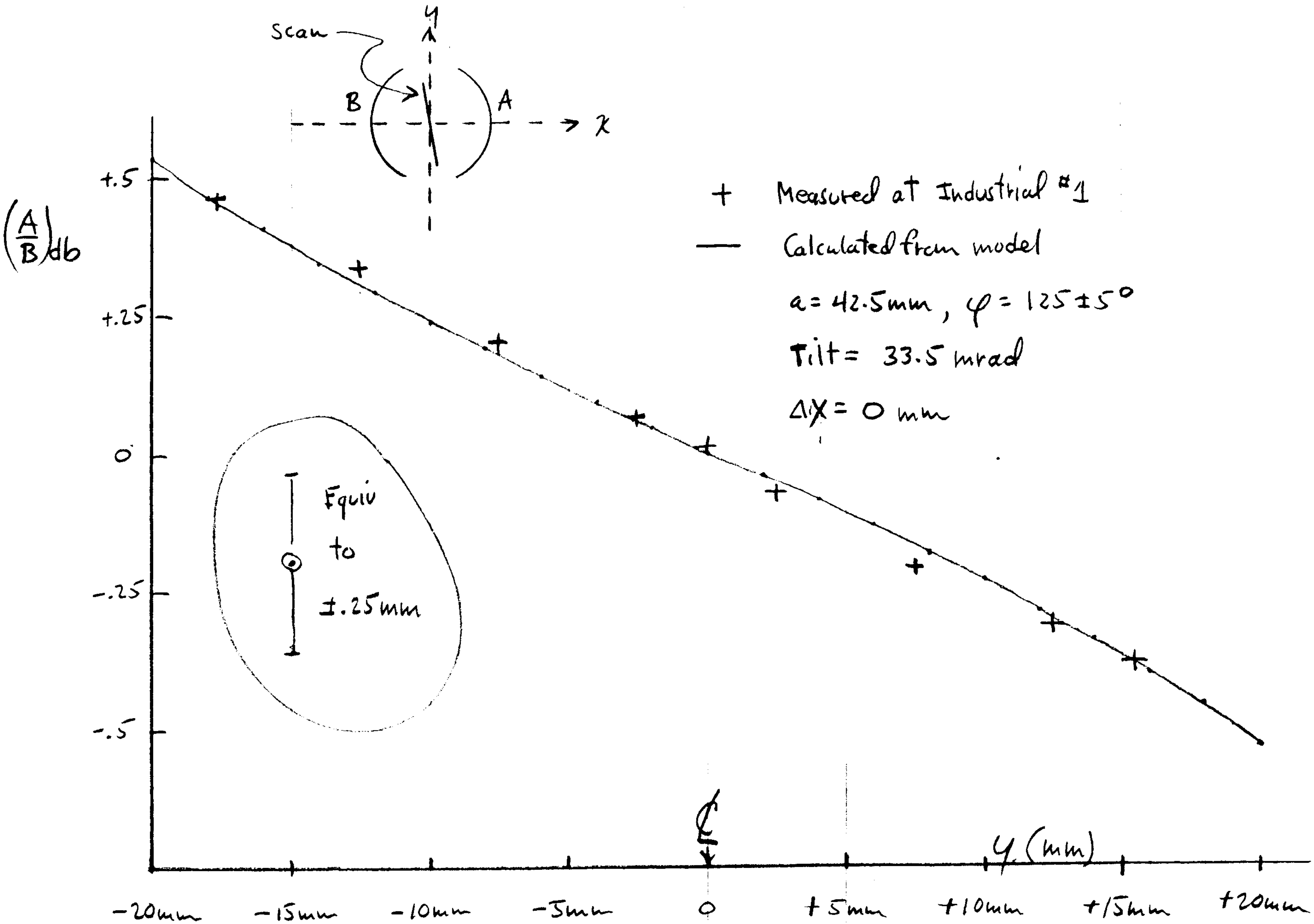
	x(mm)											
y(mm)	2	4	6	8	10	12	14	16	18	20	22	24
24.0	1.75	3.50	5.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22.0	1.67	3.35	5.05	6.76	8.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20.0	1.61	3.22	4.84	6.47	8.12	9.79	11.47	0.00	0.00	0.00	0.00	0.00
18.0	1.55	3.10	4.66	6.22	7.80	9.39	11.00	12.63	0.00	0.00	0.00	0.00
16.0	1.49	2.99	4.49	6.00	7.52	9.05	10.59	12.16	13.75	0.00	0.00	0.00
14.0	1.45	2.90	4.35	5.82	7.28	8.76	10.26	11.77	13.31	14.88	0.00	0.00
12.0	1.41	2.82	4.24	5.66	7.08	8.52	9.97	11.44	12.94	14.47	0.00	0.00
10.0	1.38	2.76	4.14	5.53	6.92	8.32	9.74	11.18	12.64	14.13	15.67	0.00
8.0	1.35	2.70	4.06	5.42	6.79	8.17	9.56	10.97	12.40	13.87	15.38	0.00
6.0	1.33	2.66	4.00	5.34	6.69	8.05	9.42	10.81	12.22	13.67	15.16	16.71
4.0	1.32	2.64	3.96	5.29	6.62	7.96	9.32	10.69	12.09	13.53	15.00	16.54
2.0	1.31	2.62	3.93	5.25	6.58	7.91	9.26	10.63	12.02	13.44	14.91	16.43
0.0	1.31	2.61	3.93	5.24	6.56	7.90	9.24	10.61	12.00	13.42	14.88	16.40

Table Ia: $\left(\frac{Q_A}{Q_B}\right)_{db}$ as a function of x and y for $a = 42.5$ mm, $\varphi = 130^{\circ}$

	x(mm)											
y(mm)	2	4	6	8	10	12	14	16	18	20	22	24
24.0	.873	.875	.879	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22.0	.837	.839	.841	.845	.849	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20.0	.804	.805	.807	.809	.812	.815	.819	0.000	0.000	0.000	0.000	0.000
18.0	.774	.775	.776	.778	.780	.782	.786	.789	0.000	0.000	0.000	0.000
16.0	.747	.748	.749	.750	.752	.754	.757	.760	.764	0.000	0.000	0.000
14.0	.725	.725	.726	.727	.728	.730	.733	.735	.739	.744	0.000	0.000
12.0	.705	.705	.706	.707	.708	.710	.712	.715	.719	.723	0.000	0.000
10.0	.689	.689	.690	.691	.692	.694	.696	.699	.702	.707	.712	0.000
8.0	.676	.676	.677	.678	.679	.681	.683	.685	.689	.693	.699	0.000
6.0	.666	.666	.667	.668	.669	.671	.673	.675	.679	.683	.689	.696
4.0	.659	.659	.660	.661	.662	.664	.666	.668	.672	.676	.682	.689
2.0	.655	.655	.656	.656	.658	.659	.662	.664	.668	.672	.678	.685
0.0	.653	.654	.654	.655	.656	.658	.660	.663	.666	.671	.676	.683

Table I b: $\frac{1}{x}\left(\frac{Q_A}{Q_B}\right)_{db}$ as a function of x and y for $a = 42.5$ mm, $\varphi = 130^{\circ}$

Figure 2. Y scan thru center of detector, with apparent tilt



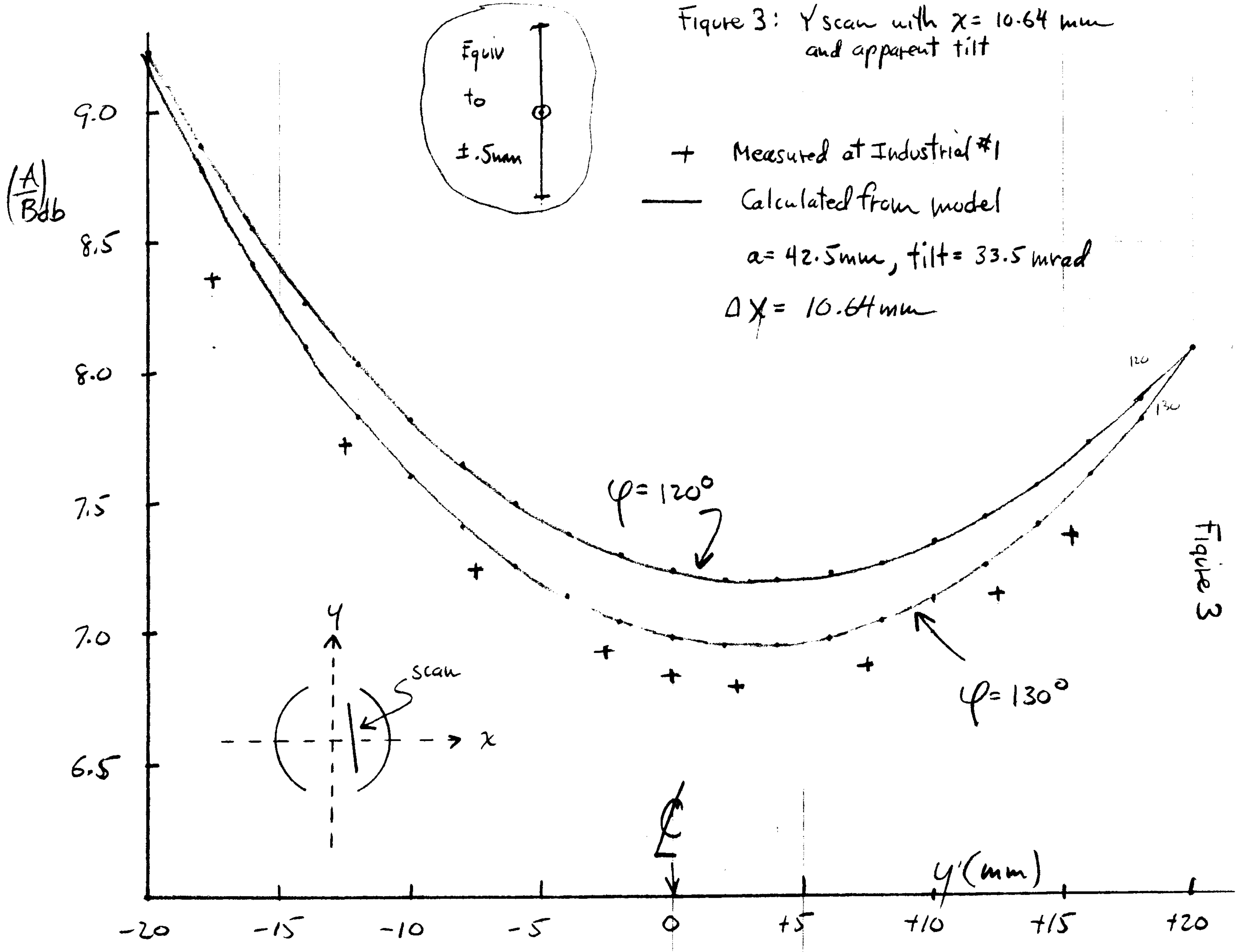


Figure 3: Y scan with X = 10.64 mm
and apparent tilt
Equiv
to
±.5mm
+ Measured at Industrial #1
— Calculated from model
a = 42.5mm, tilt = 33.5 mrad
ΔX = 10.64mm
(A/B)db
9.0
8.5
8.0
7.5
7.0
6.5
φ = 120°
φ = 130°
120
130
y
x
scan
℄
y'(mm)
-20
-15
-10
-5
0
+5
+10
+15
+20

Figure 3

in good agreement.

In addition, the sum of the two signals vs x was measured in an x scan. We expect the sum signal amplitude to rise from $-\frac{4}{\pi}Q$ to $-Q$ as the wire position moves toward either plate from the center. In Figure 4, this effect is calculated for both 110° and 130° pickup electrodes (the curves are normalized to 1 at the center). The crosses show the measured values.

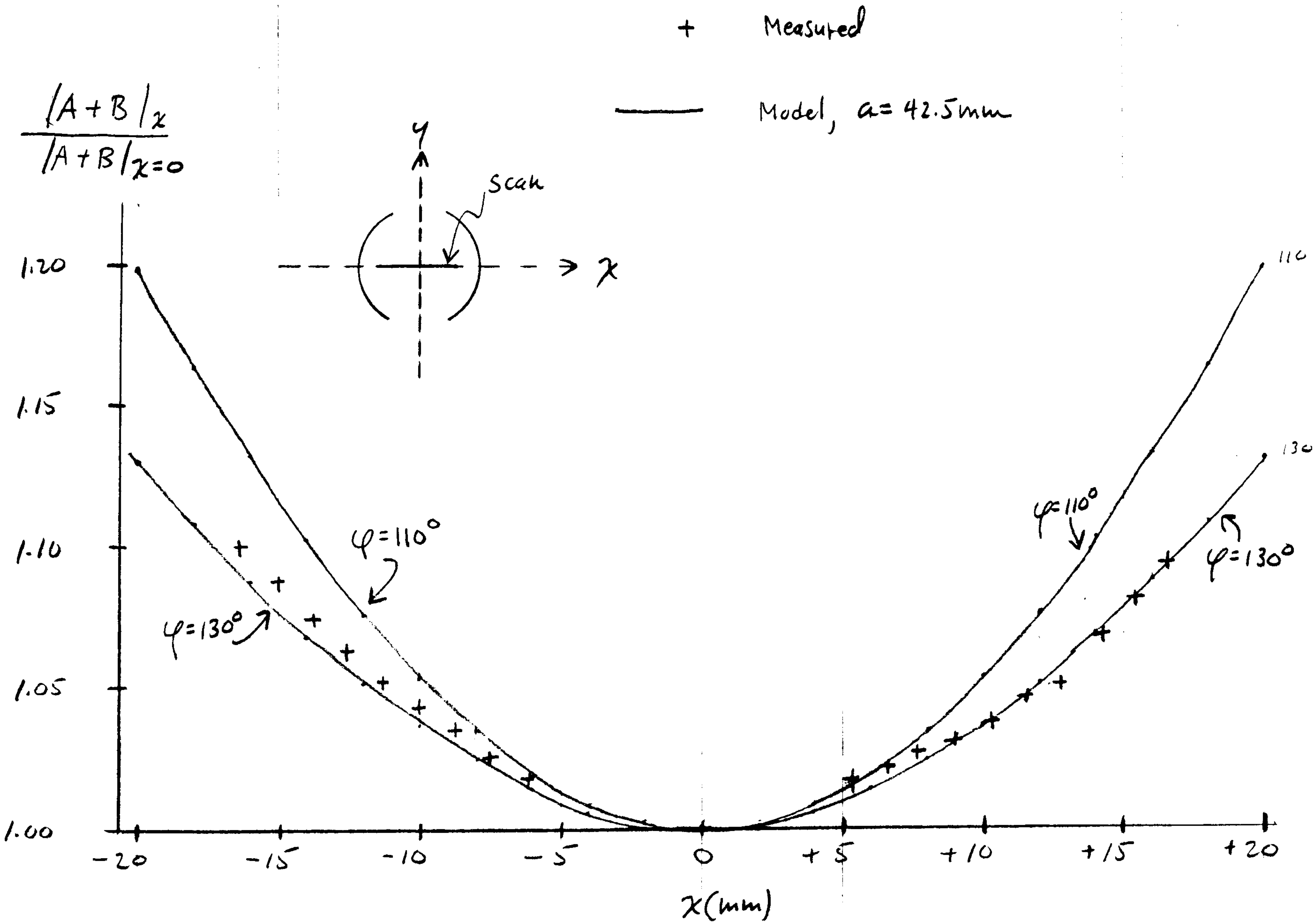
In addition, early wire measurements showed the x sensitivity to be about 0.67 db/mm at $y=0$, and 0.71 db/mm at $y = 12$ mm for scans going out to ± 25 mm. In the Industrial #1 measurements, these became .65 db/mm and .70 db/mm for ± 14 mm scans. The sensitivities given in table I b seems to reproduce this effect well (the sensitivity increases slightly for large x).

For all intents and purposes, we will adopt the sensitivity as given in table I as the expected response, and use the simplified algorithm for the db ratio of signal strengths A and B:

$$\left(\frac{A}{B}\right)_{db} = .67\left(1 + \frac{y^2}{1800}\right)x \sim \frac{.67x}{\left(1 - \frac{y^2}{1800}\right)} \qquad (6)$$

In Figure 5 the expected peak output voltage for the 53 MHz component of a circulating beam of 1×10^{10} ppb and a Gaussian bunch of rms width σ is plotted. See the article in IEEE NS-28 #3 page 2291 (1981) for details. A good average value is +4dbm (500 mV peak voltage) for a centered beam.

Figure 4. χ scan of sum of A and B signal amplitudes.



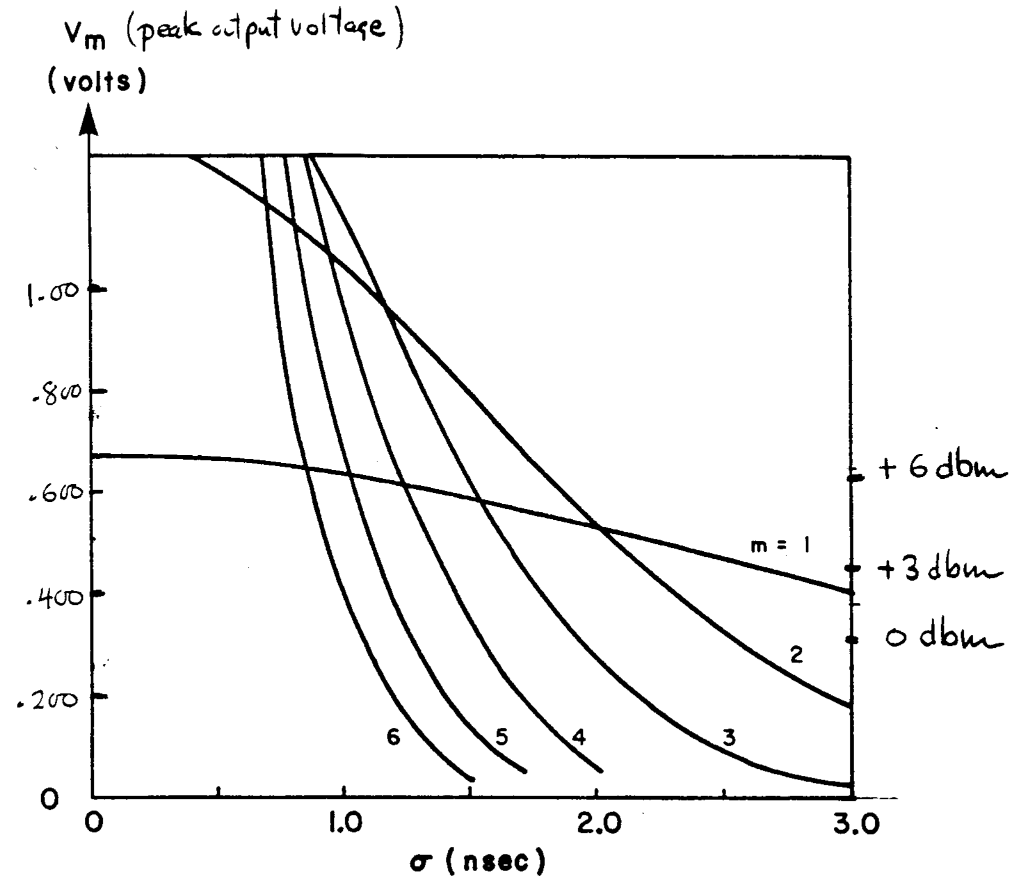


Figure 5. Expected peak voltage output for a centered beam in a $\varphi = 130^\circ$ pickup electrode for the various harmonics (m) of 53 MHz, for a beam of 1×10^{10} protons/bunch and a Gaussian bunch shape of rms width σ (nsec).

Compile with FTN5!

```
r,*
   7 FILE(S) PROCESSED.
/copy,detdb
      PROGRAM DETDB(INPUT,OUTPUT)
      IMPLICIT REAL (N)
      DIMENSION DBMM(25)
      PI=2.*ASIN(1.0)
      A=42.5
      PRINT 100
100   FORMAT(" ENTER PHI (DEGREES)")
      READ *,PHI
      PHI=PI*PHI/180.
      Y=26.0
      DO 10 I=1,13
      Y=Y-2.
      X=0.
      DO 20 J=1,12
      X=X+2.
      R=SQRT(X**2+Y**2)
      IF(R.GT.25)THEN
      DBMM(J)=0.
      GO TO 20
      END IF
      TH=ATAN(Y/X)
      SUM1=SUM2=N=0
      DO 30 K=1,40
      N=N+1.
      CTH=COS(N*TH)
      SPHI1=SIN(N*PHI/2.)
      SPHI2=SIN(N*(PI+PHI/2.))
      SUM1=SUM1+(4.*CTH*SPHI1)/(N*PHI)*(R/A)**N
      SUM2=SUM2+(4.*CTH*SPHI2)/(N*PHI)*(R/A)**N
30    CONTINUE
      DB=(20./ALOG(10.))*ALOG((1.+SUM1)/(1.+SUM2))
      DBMM(J)=DB/X
20    CONTINUE
      PRINT 500,Y,(DBMM(J),J=1,12)
500   FORMAT(1X,F4.1,2X,12F6.3)
10    CONTINUE
      STOP
      END
 EOI ENCOUNTERED.
/
```

this program calculates table I b.