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CHARACTERISTICS OF DIRECTIONAL COUPLER BEAM POSITION MONITORS

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1. Introduction

The purpose of this paper is to review some of the properties of directional coupler (stripline) beam position montiors. These devices are used extensively as non-intercepting beam position monitors (BPM's) around accelerators, and offer certain advantages over the more conventional electrostatic (split plate) type pickups. The general approach will be from the engineering rather than the physics point of view. Although starting from Maxwell's equations would be more precise, it is also more pedantic, and obscures some of the more important engineering features. Due to lack of space, detailed derivations are not included, but references are given where relevant.

The discussion here will be limited to cylindrical geometry couplers, primarily because the expressions are usually simpler than rectangular geometry couplers. In many cases, rectangular geometry would be preferred over cylindrical, but the basic engineering features remain the same.

The text is divided into the following sections:

2. Basic operation in the time domain
3. Basic operation in the frequency domain
4. Response to displaced beam
5. Response to RF modulated beam
6. Directivity
7. High frequency limits
8. Methods of signal processing
9. Ultimate position resolution limits
10. Longitudinal coupling impedance
11. Transverse coupling impedance
12. Comparison to electrostatic pickups

2. Basic Operation in the Time Domain

Consider a pair of stripline pickups exposed to a short beam bunch as shown in Figure 1. Each stripline may be considered as a section of transmission line with a characteristic impedance $Z_0 = \sqrt{L/C}$ where L and C are the inductance and capacitance per unit length. Any signal between the stripline and ground plane will propagate at a velocity $\beta_s c = 1/\sqrt{LC} = c/\sqrt{\mu\epsilon}$ where c is the speed of light and μ and ϵ are the effective permeability and permittivity. At each end the stripline is attached via a port to a transmission line of the same characteristic impedance. Hence any signal induced on the stripline in either direction will propagate through the ports and onto the transmission lines without reflection. Signals traveling in one direction are independent of and do not interfere with signals traveling in the other. The transmission lines should be terminated with (or replaced by) resistive terminations of the proper characteristic impedance to minimize reflections. We are not considering here the possibility that if more than one stripline is present there will be additional coupling between them.

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When a beam bunch of time distribution I(t) and velocity $\beta_b c$ travels along the axis of the beam pipe, it is accompanied by image currents on the walls of the beam pipe. For a centered beam, the azimuthal distribution is uniform. Furthermore, the longitudinal distribution for relativistic beams is the same as and coincident with the beam bunch itself (see Section 7). If each stripline subtends an azimuthal angle ϕ_0, than a fraction $\phi_0/2\pi$ of the total image current will flow on its inner surface. As the stripline is not grounded, however, the net induced charge on it is zero. As the image currents pass the upstream gap between the stripline and the ground plane, they flow across a gap of impedance $Z_0/2$, representing the parallel impedance of the stripline and the transmission line at the upstream port. Equal amplitude signals travel downstream along the outside of the stripline and out the upstream port. The signal in the transmission line travels at a velocity determined by its permeability and permittivity. This signal has the same time distribution as the beam bunch and half the charge of the image current on the inside surface of the electrode. The beam bunch, when it passes the downstream gap, induces similar signals, which again are equal amplitude signals, but of opposite polarity to the signals induced at the upstream port. They propagate out the downstream port, and upstream along the outer surface of the stripline. If the stripline has a length ℓ then the voltage seen at the upstream port is

$$V(t) = \frac{Z_0}{2}\left(\frac{\phi_0}{2\pi}\right)\left[I(t) - I\left(t - \frac{\ell}{\beta_b c} - \frac{\ell}{\beta_s c}\right)\right] \qquad 2.1$$

and at the downstream port is

$$V(t) = \frac{Z_0}{2}\left(\frac{\phi_0}{2\pi}\right)\left[I\left(t - \frac{\ell}{\beta_s c}\right) - I\left(t - \frac{\ell}{\beta_b c}\right)\right] \qquad 2.2$$

The net signal seen at each port is a bipolar doublet with each lobe having essentially the same time distribution as the beam bunch itself. Note in particular that there is complete cancellation at the downstream port if the beam velocity and the signal velocity are the same. For this reason, the upstream port is the signal port. It is possible to defeat this directivity without affecting other properties of the stripline pickups (see Section 6).

3. Basic Operation in the Frequency Domain

The beam current is sometimes more easily expressed in the frequency domain. In addition, some signal processing calculations are also more easily evaluated in the frequency domain. Suppose the beam has a component

$$I_b(\omega) = I_{b0}(\omega)\sin \omega t \qquad 3.1$$

where the beam phase is measured at the midpoint of the electrode. The frequency ω may be a harmonic of the bunching (acceleration) frequency, or a harmonic of the revolution frequency, if the beam is accelerated in a circular accelerator. The signal seen at the upstream port is then

$$V(\omega) = V_p(\omega)\sin\left(\omega t + \frac{\pi}{2} - \omega\tau\right)$$

$$V_p(\omega) = Z_0\left(\frac{\phi_0}{2\pi}\right)\sin\left[\frac{\omega\ell}{2c}\left(\frac{1}{\beta_b} + \frac{1}{\beta_s}\right)\right] I_{b0}(\omega) \qquad 3.2$$

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where $\pi/2$ represents a 90° phase shift due in essence to the differentiation of the signal by the two pickup gaps, and τ represents the group delay measured from the reference plane at the midpoint of the stripline. When both the beam and signal velocity are nearly the speed of light, the output power per electrode for a centered beam is

$$P(\omega) = \frac{1}{2}\frac{V_p^2(\omega)}{Z_0} = \frac{Z_0}{2}\left(\frac{\phi_0}{2\pi}\right)^2 \sin^2\left(\frac{\omega \ell}{c}\right) I_{b0}^2(\omega) \qquad 3.3$$

Note that the power output peaks for stripline lengths corresponding to $\ell=\lambda/4$. This corresponds to the two lobes of the bipolar doublet being a half wavelength apart. For this reason this type pickup is sometimes referred to as a quarter wave loop.

4. Response to Displaced Beam

If the beam is displaced from the axis of the beam pipe, the azimuthal image current distribution on the walls of the beam pipe is (see Figure 2):

$$i(r_0,\theta_0,\phi) = \frac{-I_b}{2\pi b}\left[1 + 2\sum_{n=1}^{\infty}\left(\frac{r_0}{b}\right)^n \cos n(\phi-\theta_0)\right] \qquad 4.1$$

If the two striplines each subtend an angle ϕ_0 as shown in Figure 2, then the induced image currents on the two striplines are

$$I_A = \frac{-I_b\phi_0}{2\pi}\left[1 + \frac{4}{\phi_0}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r_0}{b}\right)^n \cos n\theta_0 \sin\left(\frac{n\phi_0}{2}\right)\right] \qquad 4.2$$

and

$$I_B = \frac{-I_b\phi_0}{2\pi}\left[1 + \frac{4}{\phi_0}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r_0}{b}\right)^n \cos n\theta_0 \sin\left[n\left(\pi+\frac{\phi_0}{2}\right)\right]\right] \qquad 4.3$$

Signal processing can either measure the difference over the sum: $x=r_0\cos\theta_0;\ y=r_0\sin\theta_0$

$$\frac{I_A-I_B}{I_A+I_B} = \frac{4\sin(\phi_0/2)}{\phi_0}\frac{x}{b} + O\left(\frac{x^3}{b^3}\right) + O\left(\frac{xy^2}{b^3}\right) \qquad 4.4$$

or the amplitude ratio (actually the ratio expressed in db):

$$20\log_{10}\left(\frac{I_A}{I_B}\right) = \frac{160}{\ln 10}\frac{\sin(\phi_0/2)}{\phi_0}\frac{x}{b} + O\left(\frac{x^3}{b^3}\right) + O\left(\frac{xy^2}{b^3}\right) \qquad 4.5$$

Both methods have advantages and disadvantages (see Section 8). From the above expressions some nonlinearity would be expected at large displacements. The nonlinear response vs displacement is not large, however, and can usually be corrected in software.

As an example eqn 4.5 yields 0.85 db per mm for the Tevatron striplines ($\phi_0=110°$, b=35 mm). The actual measured response is about 0.7 db/mm over about 70% of the aperture. The deviation of the measured response from the calculated value is believed to be due to the interelectrode capacitance and the deviation of the actual electrode geometry from the model.

5. RF Modulated Beam

Suppose the beam is bunched into Gaussian bunches of rms width σ sec and bunch spacing (period) of $T=2\pi/\omega_0$. Then if there are N particles per bunch, the current in the time domain is:

$$I_b(t) = \frac{eN}{\sqrt{2\pi}\,\sigma}\exp\left[\frac{-t^2}{2\sigma^2}\right] \qquad 5.1$$

We may transform this into a Fourier series of harmonics of the RF frequency:

$$I_b(t) = \frac{eN\omega_0}{2\pi} + \sum_{m=1}^{\infty} I_m \cos(m\omega_0 t) \qquad 5.2$$

where the Fourier coefficients are:

$$I_m = \frac{eN\omega_0}{\pi}\exp\left[\frac{-m^2\omega_0^2\sigma^2}{2}\right]; \quad m \geq 1 \qquad 5.3$$

Then the peak voltage output of a single stripline into a matched termination is:

$$V_m = \frac{eN\omega_0 Z_0}{\pi}\left(\frac{\phi_0}{2\pi}\right)\sin\left(\frac{m\omega_0\ell}{c}\right)\exp\left[\frac{-m^2\omega_0^2\sigma^2}{2}\right] \qquad 5.4$$

A specific example is the stripline pickup used in the Tevatron[1]:

Z_0=50 ohms	ℓ=19 cm	V_1=0.511 volts
$N=10^{10}$	$c=3\times10^{10}$ cm/sec	P_1=2.6mW
$\omega_0=3.33\times10^8$/sec	σ=1 nsec	=+4 dbm
ϕ_0=110°	m=1	

Figure 3 is a plot of the voltage output as a function of bunch width for several harmonics of the RF frequency. As the fundamental (m=1) harmonic is the least sensitive to the bunch shape, the electronics is designed to operate at this frequency. It should be noted that the stripline is much shorter than quarter wave at this frequency. The electronics could operate down to about -40 dbm which corresponds to about 10^8 particles per bunch, after allowing for cable attenuation etc.

6. Directivity

Directivity is the ratio of signal power at the upstream and downstream ports in response to a beam current. Directional couplers used for detection of beams can typically achieve anywhere from 20 to 35 db of directivity depending on many factors. Maintaining the high degree of directivity these pickups are capable of requires that:

A) the velocity of the beam and the signal be matched fairly well (see eqn. 2.2). For highly relativistic beams this requires a minimum amount of dielectric material in the vicinity of the stripline.

B) a matching of the stripline impedance to the transmission line or termination impedance at both ends. An impedance mismatch of 10% will reflect 0.25% of the power to the wrong port. This would limit the directivity to 26 db.

C) minimization of the coupling between the striplines. If the interelectrode capacitance per unit length is too high, then one stripline can induce signals in the other.

The directional coupler striplines in the Tevatron achieve about 24 db directivity at 53 MHz for a centered beam. This corresponds to a 250:1 forward to reverse power ratio.

It is possible to defeat the directivity without otherwise destroying the signal response of the striplines. When one end is either shorted to ground or left unterminated, all the signal power incident on this end is reflected to the terminated port. In the case of the unterminated end, the signal induced by the beam is twice the amplitude and the opposite polarity of the reflected signal which is not inverted. When the end is shorted, the beam induces no signal but the reflected signal is inverted. The polarity of the bipolar doublet depends on whether the signal port is at the upstream or downstream end of the stripline.

7. High Frequency Limits

For a highly relativistic beam, the wall current longitudinal distribution, and therefore the current flowing across the gaps at the ends of the striplines, is the same as the charge density distribution of the beam. However, for less relativistic beams the wall current is spread out longitudinally, and the peak wall current is less than the beam. This is best understood by considering a point charge at rest on the axis of a beam pipe of radius b. The wall charge is spread over about ±0.3b longitudinally. When a transformation is made to a moving frame along the beam pipe axis, the wall charge is contracted to a length ±0.3b/γ. In the relativisitic limit the wall current is coincident with the beam current. The net result is that for the current component at frequency ω the wall current is related to the beam current by[2]

$$I_{wall}(\omega) = \frac{I_b(\omega)}{I_0\left(\frac{\omega b}{\beta\gamma c}\right)} \qquad 7.1$$

where b is the beam pipe radius, γ the relativistic contraction, c is the speed of light, and I_0 is the modified Bessel function of order 0. The Bessel function has value 1 for argument 0, and increases monotonically for increasing argument. The 3 db point occurs when

$$\omega = \frac{1.2\beta\gamma c}{b} \qquad 7.2$$

In addition there is the gap factor due to the transit time of the current across the gap of length g. Hence signals from stripline pickups should include the additional factor for a centered beam:

$$H(\omega) = \frac{\sin(\omega g/2\beta c)}{(\omega g/2\beta c)} \, \frac{1}{I_0\left(\frac{\omega b}{\beta\gamma c}\right)} \qquad 7.3$$

When the Bessel function factor is important, the circular functions in eqn 4.2 should be replaced with Bessel functions (see Cuperus eqn 15). Another high frequency limit observed in electrode tests on microwave pickups is an apparent bandstop when the perimeter of the pickup electrode is a full wavelength. This effect is not understood.

Also not included (or fully understood) is what happens when the microwave cutoff frequency of the beam pipe is reached. The lowest order TE and TM modes occur at:

$$\omega_c = \frac{1.841c}{b}\,(TE_{11}) \quad \text{and} \quad \frac{2.405c}{b}\,(TM_{01}) \qquad 7.5$$

In comparing these thresholds to eqn 7.2, it is seen that at about βγ=1.5, beam coupling to the environment extends beyond the microwave cutoff frequencies.

8. Methods of Signal Processing

The two general methods of signal processing are to measure the difference over sum, or to measure the amplitude ratio. Their sensitivities to beam displacement are given in Section 4.

The difference over sum method is normally done by separately digitizing the two electrode signals, and then digitally calculating the difference over sum ratio. The signal capture can be done in the time domain for single bipolar doublets by gating and holding peak voltages, or in the in the frequency domain for bunched beams by using a bandpass filter to select a single frequency component for measurement. The difference over sum method is simple to implement, but has a limited amplitude dynamic range due to the ADC granularity. For example, to measure position to 1% of the aperture over a 100:1 dynamic range requires at least 12 bits.

The amplitude ratio measurement can be done digitally after digitizing the individual signals, or by using amplitude to phase (AM/PM) conversion[3] and detecting the phase difference. The digital method would require subsequent calculation of the db ratio to obtain position information, but the AM/PM method produces a signal proportional to the arctangent of the ratio minus π/4, and yields an analog signal proportional to displacement and independent of beam intensity. This latter feature allows operation over a very large amplitude dynamic range without the limitation of ADC granularity. In the Tevatron, AM/PM conversion provides a resolution of about 0.3% of the aperture with a 300:1 dynamic range in 200 nsec.

One limitation in the AM/PM method is that it must be done in the frequency domain. For bunched beams, a bandpass filter is used to select a single frequency component. For single beam bunches, the bipolar doublet impulse causes the bandpass filter to ring at the center frequency for a few cycles, depending on bandwidth. This ringing allows sufficient time to perform the AM/PM conversion. The analog position signal risetime is not limited by the width of the bandpass filter, and can be as fast as 5 to 10 RF cycles. However, the bandpass filters must be very well matched in both attenuation and center frequency. In the Tevatron system for example, the 5 MHz bandpass filters had to be matched in attenuation to about 0.2 db and in center frequency to about 0.1%. However, additional filtering may be done on the analog position signal. In one case, low pass filters were used on the position signal to obtain approximately 3 micron resolution. This was needed to locate the source of a 25 micron ripple on the beam which was 100% modulating the slow extraction.

9. Position Resolution Limits

There are two limits to the position resolution, depending on whether one is referring to absolute or relative beam position. Absolute beam position measurement requires a high precision of alignment and complete understanding and correction of attenuation unbalances, circuit offsets and drifts, gain variations etc.

Perhaps more important is the ability to measure small transverse motions of the beam. Transverse motion can be caused by magnet power supply ripple, component vibration, coherent instabilities etc. The ultimate position resolution of a pair of striplines of angular width ϕ_0 and aperture 2b is

$$\frac{\delta x}{b} = \frac{1}{4\sqrt{2}}\,\frac{\phi_0}{\sin(\phi_0/2)}\,\frac{\delta V}{V} \qquad 9.1$$

where V is the rms signal voltage and δV is the noise voltage. This applies to both the sum over difference and the amplitude ratio measurement techniques.

Assuming electromagnetic noise pickup can be eliminated, δV then is limited by thermal and amplifier noise. Thermal noise at room temperature is about -114 dbm/MHz (i.e. about 4×10^{-12} milliwatts per MHz bandwidth. This should be increased about 3 db for amplifier noise. Hence in the Tevatron system with a 5 MHz bandwidth, the noise floor is probably about -100 dbm. As the signal power is about 0 dbm (see Section 5), one should expect ultimate resolution of about 0.2 microns for a 70 mm aperture. About 30 microns has been observed in repeated measurements on the same segment of beam on a turn-by-turn basis, but this may include actual coherent betatron motion as well. Sub-micron resolution would be required for example in a feedback system to be used during collider operation in order not to cause emittance growth.

In referring to Section 3, it should be noted that the signal power scales linearly with the characteristic impedance (or shunt impedance) of the pickup, while the noise power does not. Hence a high impedance position pickup will provide better resolution. Also optimizing other features such as length, aperture and width of the electrodes would be useful. Specifically, using the results of Section 5 for a bunched beam:

$$\frac{\delta x}{b} = \frac{\pi^2}{2eN\omega_0}\sqrt{\frac{kTB}{Z_0}}\;\frac{\exp\left[\frac{+m^2\omega_0^2\sigma^2}{2}\right]}{\sin(\phi_0/2)\left|\sin\left(\frac{m\omega_0\ell}{c}\right)\right|} \qquad 9.2$$

where kTB is the thermal noise power in bandwidth B. No allowance is included for amplifier noise or circuit losses.

10. Longitudinal Coupling Impedance

The longitudinal impedance in an accelerator is the longitudinal voltage per turn seen by a particle, divided by the beam current:

$$Z_{\parallel}(\omega) = \frac{1}{I_b(\omega)}\int_0^{2\pi R} E_z(\omega)\,dz \qquad 10.1$$

As $E_z(\omega)$ is not necessarily in phase with $I_b(\omega)$, $Z_{\parallel}(\omega)$ can be complex. Since the power absorbed by the striplines represents the real part of $Z_{\parallel}(\omega)$, we can write for P pair of striplines for position measurement in the x plane (see eqn 3.3):

$$\frac{1}{2}\,\mathrm{Re}\left[Z_{\parallel}(\omega)\right] I_{b0}^2(\omega) = P Z_0\left(\frac{\phi_0}{2\pi}\right)^2 G(x,y)\sin^2\left(\frac{\omega\ell}{c}\right) I_{b0}^2(\omega) \qquad 10.2$$

Hence

$$\mathrm{Re}\left[Z_{\parallel}(\omega)\right] = 2PZ_0\left(\frac{\phi_0}{2\pi}\right)^2 G(x,y)\sin^2\left(\frac{\omega\ell}{c}\right) \qquad 10.3$$

where

$$G(x,y) = 1 + \left[\left(\frac{4}{\phi_0}\right)^2\sin^2(\phi_0/2) + \left(\frac{4}{\phi_0}\right)\sin\phi_0\right]\frac{x^2}{b^2} - \left[\left(\frac{4}{\phi_0}\right)\sin\phi_0\right]\frac{y^2}{b^2} \qquad 10.4$$

The factor G(x,y) represents the additional power absorbed by the striplines as the beam is moved off-axis. Higher order terms are not shown. Note that as the beam is displaced vertically the coupling decreases. We may complete the impedance function using the Kramers Kronig relations[4]:

$$Z_{\parallel}(\omega) = 2PZ_0\left(\frac{\phi_0}{2\pi}\right)^2 G(x,y)\left[\sin^2\left(\frac{\omega\ell}{c}\right) + j\sin\left(\frac{\omega\ell}{c}\right)\cos\left(\frac{\omega\ell}{c}\right)\right] \qquad 10.5$$

The longitudinal coupling impedance is the reactive component divided by n , the revolution harmonic number:

$$\frac{Z_{\parallel}}{n} = \frac{2PZ_0}{n}\left(\frac{\phi_0}{2\pi}\right)^2\sin\left(\frac{\omega\ell}{c}\right)\cos\left(\frac{\omega\ell}{c}\right) \approx 2PZ_0\left(\frac{\phi_0}{2\pi}\right)^2\frac{\ell}{R}\ \text{ohms} \qquad 10.6$$

For the Tevatron, P=216, Z_0=50 ohms, ϕ_0=110 degrees, ℓ=19 cm, and R=1000 m, yielding

$$\frac{Z_{\parallel}}{n} = 0.35\ \text{ohms} \qquad 10.7$$

As the frequency increases, the coupling alternates between inductive and capacitive. For this reason, a more accurate value of the coupling is obtained by weighting the coupling impedance with the power density spectrum of the bunched beam. Not included here are the high frequency effects discussed in Section 7.

11. Transverse Coupling Impedance

The transverse impedance is the lateral electromagnetic field seen by the particle, integrated around the ring (hence a voltage), and divided by the beam current (hence an impedance), and divided by the lateral displacement. By convention, it is also divided by $-j\beta$:

$$Z_T(\omega) = \frac{j}{\beta I_b(\omega)\Delta}\int_0^{2\pi R} E_T(\omega) + \left[v\times B(\omega)\right]_T dz \qquad 11.1$$

The important part of the transverse impedance is that part which is in phase with the displacement, as it can cause a shift in the betatron sideband frequencies.

Following the prescription of Sacherer,[5] the transverse impedance for a horizontal pair of striplines is at low frequencies:

$$Z_x = \frac{+R}{b^2}\left[\left(\frac{4}{\phi_0}\right)^2\sin^2\left(\frac{\phi_0}{2}\right) + \left(\frac{4}{\phi_0}\right)\sin\phi_0\right]\frac{Z_{\parallel}}{n}\ \text{ohms/m} \qquad 11.2$$

and

$$Z_y = \frac{-R}{b^2}\left[\left(\frac{4}{\phi_0}\right)\sin\phi_0\right]\frac{Z_{\parallel}}{n}\ \text{ohms/m} \qquad 11.3$$

Because position monitoring electrodes are traditionally placed at locations where the betatron amplitude functions are a maximum, values of Z_T for a typical ring may be as much as twice as large as the "canonical" resistive wall value:

$$Z_T = \frac{2R}{b^2}\,\frac{Z_{\parallel}}{n}\ \text{ohms/m (resistive wall)} \qquad 11.4$$

12. Comparison to Electrostatic Pickup Electrodes

Traditionally, electrostatic pickup structures have been used in many accelerators. One primary reason is to select a geometry which makes the difference over sum response linear in displacement in one coordinate, and independent of displacement in the other. Another reason is that it is possible to obtain a higher shunt impedance at the measuring frequency and therefore more signal power and better signal to noise ratios for low current beams.

One major drawback in high current rings is that the beam sees the equivalent of a transmission line essentially unterminated at both ends and hence can be resonant with a fairly high Q at certain frequencies. This could contribute to beam instabilities. In addition, the bipolar doublet response to a single beam bunch would be very poor. Directional properties would be non existant. If the electrode structure is a diagonally cut geometry, signal phase differences must be accounted for if AM/PM signal processing is used. If the transmission line used to bring out the signal needs to be back terminated, it must be done at or near the electrode, unlike the stripline which can be terminated any distance from the downstream port.

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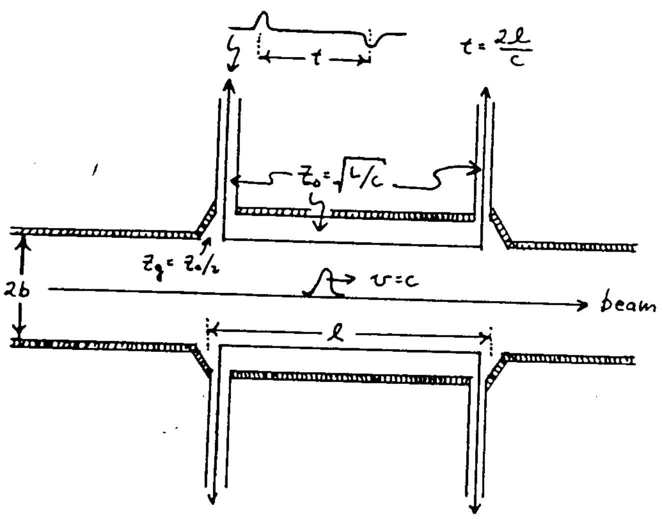

Figure 1. Geometry for calculations in Sections 2 and 3.

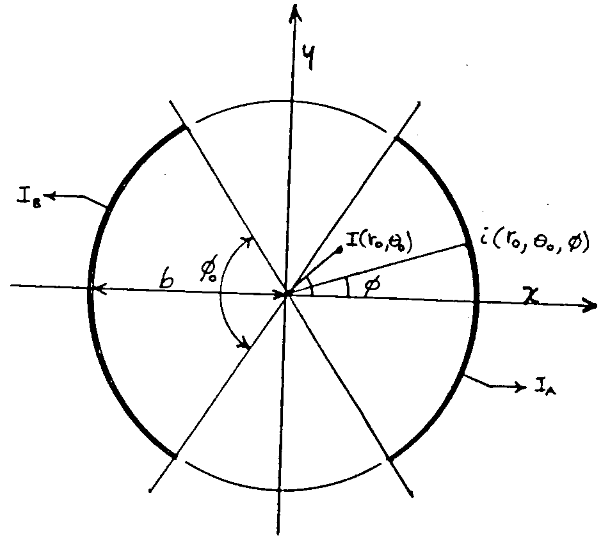

Figure 2. Geometry for calculations in Section 4.

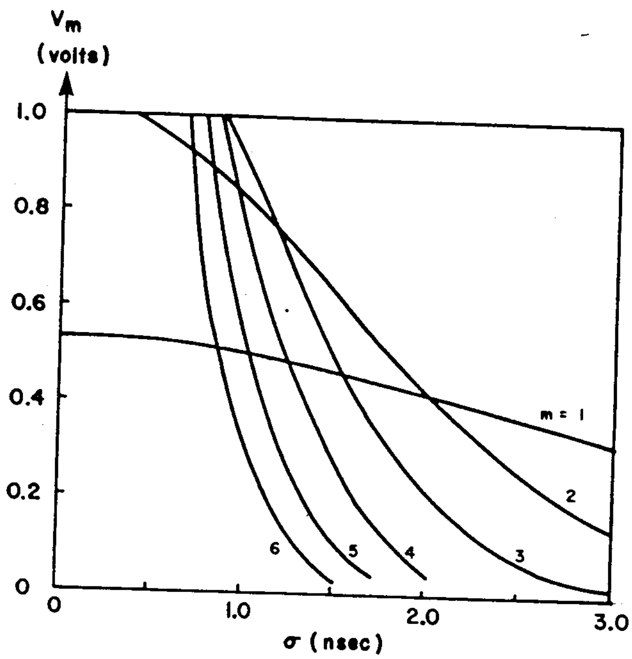

Figure 3 Peak output voltage for various harmonics of the RF bunching frequency in the Tevatron directional couplers for 10^{10} protons per bunch, vs rms bunch width σ. See equation 5.4