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On the Longitudinal Coupling Impedance of the Energy Doubler Beam Position Detectors

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The purpose of this Note is to calculate the longitudinal impedance of the Energy Doubler beam position pickups. Although this subject has been discussed previously (refs 1,2), the detailed structure of the frequency dependence has not been adequately reviewed. This Note will show that the pickups look inductive only below about 400 MHz. At higher frequencies, the imaginary component of the longitudinal impedance oscillates between capacitive and inductive.

The approach used in this calculation is somewhat unorthodox. First, the signal response for a typical stripline detector is discussed. As the signal output of a pickup can produce a voltage across a terminating resistor and hence represents real power dissipation, this power must be expressable in terms of a real component of the longitudinal impedance. Due to the analytic nature of impedances, the imaginary component can then be expressed in terms of a dispersion relation involving the real component. The results thus obtained will then be compared to the results of refs 1 and 2.

In Figure 1, a typical stripline pickup electrode with characteristic impedance Z_0, length ℓ, and azimuthal half-angle ϕ_0 is shown. The characteristic impedance is determined by the inductance and the capacitance per unit length, as is the propagation velocity. We will assume signals and the beam both propagate at the velocity of light.

Consider a beam current of amplitude $I(t)$ circulating in the accelerator. If $\omega = n\omega_0 = 2\pi n f_0$ where f_0 is the beam revolution frequency, then $I(t)$ may be expanded in terms of harmonics of ω_0 :

$$I(t) = \sum_n{}' I_n(t) = \sum_n{}' I_{n0} \sin n\omega_0 t \tag{1}$$

We first want to consider the response of the electrode to a single harmonic component n. We will assume that the characteristic impedance of the electrode is the same as the terminations (or the coaxial transmission lines in the actual case). Both the upstream port and the downstream port are thus terminated. The beam current induces image currents of equal magnitude and opposite sign in the walls of the beam pipe. For a beam centered in a circular pipe, the azimuthal distribution is uniform. If the pickup electrode has an azimuthal half angle ϕ_o then a fraction (ϕ_o/π) of the wall image currents appear on the inside surface of the electrode. As there is no net charge induced on the electrode, when the beam approaches the upstream end of the electrode, currents are also induced on the outside of the electrode and in the termination. As the impedances of the electrode and the termination are equal so are the currents. Hence the current through the termination is:

$$\frac{1}{2}\left(\frac{\phi_o}{\pi}\right) I_n(t) \tag{2}$$

When the beam arrives at the downstream end, additional currents are induced, but of opposite polarity. The current induced across the downstream termination will be exactly cancelled by the current induced on the outside surface of the electrode at the upstream end, as it reaches the downstream termination at the same time as the signal induced at the downstream end. The signal induced on the outside surface of the electrode, however, travels backwards along the electrode and into the upstream termination, but is retarded in time as well as being of opposite polarity, so cancellation cannot occur. If the electrical length of the electrode is $t_o = \ell/c$, where c is the velocity of light, then the retardation is $2t_o$. The total current induced in the upstream termination for the n^{th} harmonic is then (ref 3):

$$\begin{aligned} I_{nT}(t) &= \frac{1}{2}\left(\frac{\phi_o}{\pi}\right)\left[I_n(t) - I_n(t-2t_o)\right] \\ &= \left(\frac{\phi_o}{\pi}\right) I_{no} \sin(n\omega_o t_o) \cos n\omega_o(t-t_o) \end{aligned} \tag{3}$$

The voltage induced on the terminator is then:

$$V_{nT}(t) = Z_o I_{nT}(t) = Z_o\left(\frac{\phi_o}{\pi}\right) I_{no} \sin(n\omega_o t_o) \cos n\omega_o(t-t_o) \tag{4}$$

and the average power dissipation is then:

$$\langle P_1 \rangle = Z_o \langle I_{nT}^2(t) \rangle = Z_o\left(\frac{\phi_o}{\pi}\right)^2 \sin^2(n\omega_o t_o) \langle I_n^2(t) \rangle \tag{5}$$

This power must be expressable in terms of a real component of the longitudinal impedance. If the longitudinal impedance for M such electrodes is of the form:

$$Z_L(\omega) = \mathrm{Re}\, Z_L(\omega) + j\, \mathrm{Im}\, Z_L(\omega) \tag{6}$$

then

$$\begin{aligned} \langle P_M \rangle &= \mathrm{Re}\, Z_L \langle I_n^2(t) \rangle \\ &= M \langle P_1 \rangle = M Z_o\left(\frac{\phi_o}{\pi}\right)^2 \sin^2(n\omega_o t_o) \langle I_n^2(t) \rangle \end{aligned} \tag{7}$$

hence

$$\mathrm{Re}\, Z_L(n\omega_0) = M Z_0 \left(\frac{\phi_0}{\pi}\right)^2 \sin^2(n\omega_0 t_0) \tag{8}$$

The real and imaginary components of $Z_L(\omega)$ must satisfy dispersion relations due to the causal nature of impedances (ref 4). Specifically

$$\mathrm{Im}\, Z_L(n\omega_0) = \frac{2n\omega_0}{\pi}\int_0^\infty \frac{\mathrm{Re}\, Z_L(\omega') - \mathrm{Re}\, Z_L(n\omega_0)}{(\omega')^2 - (n\omega_0)^2}\, d\omega' \tag{9}$$

$$= \frac{2n\omega_0 M Z_0}{\pi}\left(\frac{\phi_0}{\pi}\right)^2 \int_0^\infty \frac{\sin^2(\omega' t_0)}{(\omega')^2 - (n\omega_0)^2}\, d\omega' \tag{10}$$

This definite integral can be evaluated (ref 5), resulting in a longitudinal impedance:

$$Z_L(n\omega_0) = M Z_0 \left(\frac{\phi_0}{\pi}\right)^2 \left[\sin^2(n\omega_0 t_0) + j \sin(n\omega_0 t_0)\cos(n\omega_0 t_0)\right] \tag{11}$$

which when divided by the harmonic number yields:

$$\left(\frac{Z_L}{n}\right)_n = \frac{M Z_0}{n}\left(\frac{\phi_0}{\pi}\right)^2 \left[\sin^2(n\omega_0 t_0) + j \sin(n\omega_0 t_0)\cos(n\omega_0 t_0)\right] \tag{12}$$

Using R as the machine radius, the low frequency limit for the imaginary component is:

$$j\, \mathrm{Im}\left(\frac{Z_L}{n}\right) = +j\, \frac{M Z_0 \ell}{R}\left(\frac{\phi_0}{\pi}\right)^2 \tag{13}$$

Using values of $Z_0 = 50\Omega$, $\ell = .18\,m$, $(\phi_0/\pi) = .3$, $R = 1000\,m$, and $M = 432$, the real and imaginary components of (Z_L/n) are plotted against ω in Figure 2. It is seen that for very low frequencies the imaginary component approaches the DC value $+j\,.35\Omega$, but becomes capacitive at about 420 MHz.

Using certain assumptions, eqn (1) and (2) of ref 2 may be written as:

$$\left(\frac{Z_L}{n}\right)_n = \frac{2M Z_0}{n}\left(\frac{\phi_0}{\pi}\right)^2 \left[\sin^2(n\omega_0 t_0) + j \sin(n\omega_0 t_0)\cos(n\omega_0 t_0)\right] \tag{14}$$

demonstrating the same frequency dependence as eqn (12) above, but with the wrong coefficient. The low frequency value given in ref 1, however, is in agreement with the above derivation, provided it is recognized that each beam position detector has two electrodes. It should be noted, however, that the formulas in refs 1 and 2 are based on the derivation in ref 6, which assumes only one termination per electrode, rather than two. As there is no voltage induced on the downstream termination of a stripline pickup, however, perhaps the difference is academic.

It perhaps is worthwhile pointing out at this time that as the beam is displaced from the center, the azimuthal distribution of induced wall currents becomes non-uniform. In the case of resistive wall power losses this radial displacement dependence was shown to increase the power loss by a factor (ref 7):

$$\left[\frac{2}{1-(r_0/a)^2} - 1 \right] \tag{15}$$

where a is the beam pipe (or pickup) radius, and r_0 is the displacement from the center. Very likely the longitudinal impedance in eqn (12) should scale similarly.

Ref 1. A. G. Ruggerio, UPC 72 (1/8/79)
Ref 2. A. G. Ruggerio, UPC 92 (1/79). We have replaced $+i$ with $-j$ to give engineering units.
Ref 3. F. Mills, Internal memo (4/15/78)
Ref 4. H. W. Bode, "Network Analysis and Feedback Amplifier Design", (Van Norstrand, 1945), See especially chapter 14 and the tabl on page 335.
Ref 5. Gradshteyn and Ryzhik, "Table of Integrals, Series, and Products" (Academic Press) 1980.
Ref 6. L. J. Laslett, Int. Symp. on Electron and Positron Storage Rings, (Saclay) 1966, page IV-5-1.
Ref 7. R. Shafer, UPC 90 (3/27/79)

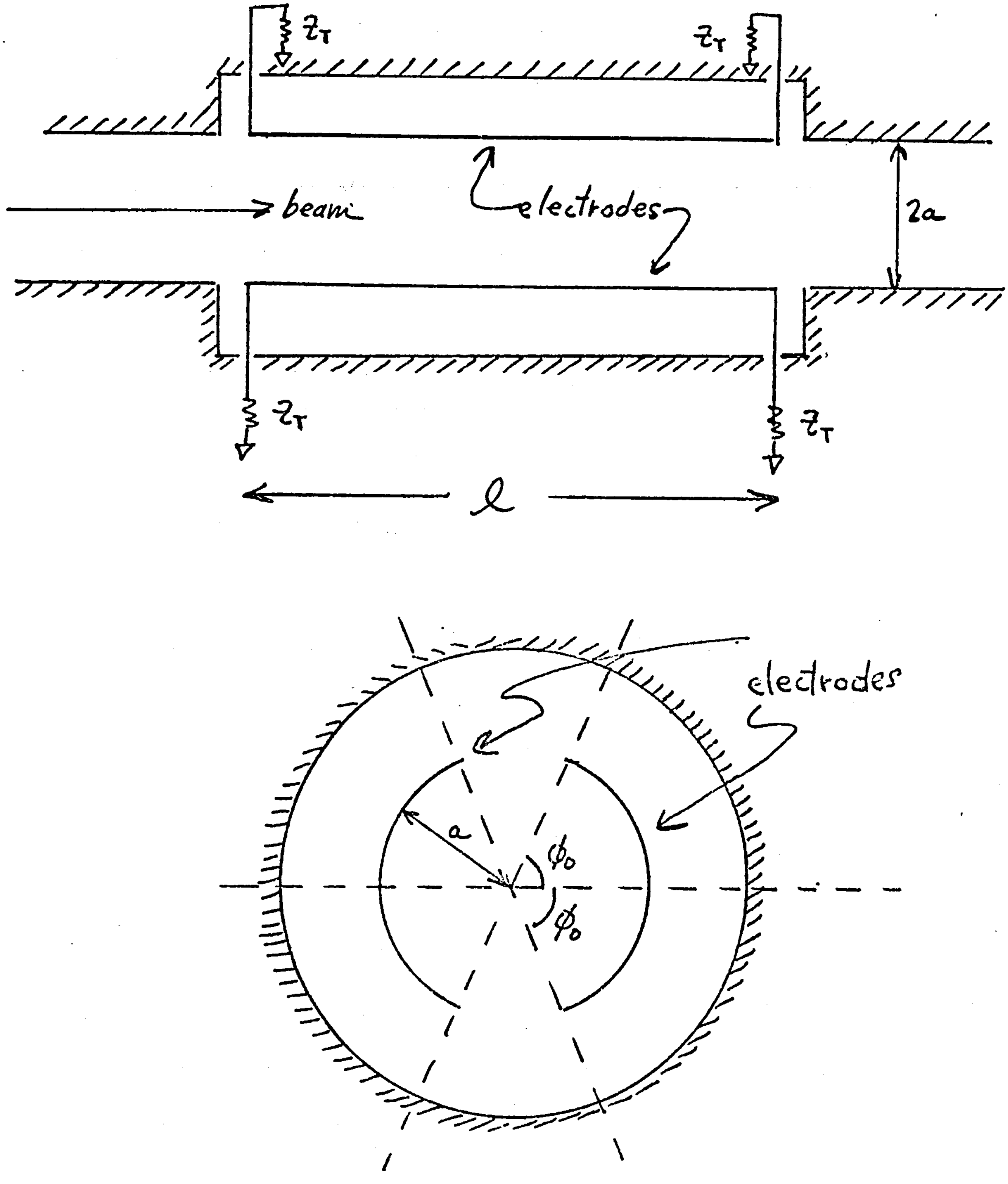

Figure 1 Beam Pickup Electrode Geometry.

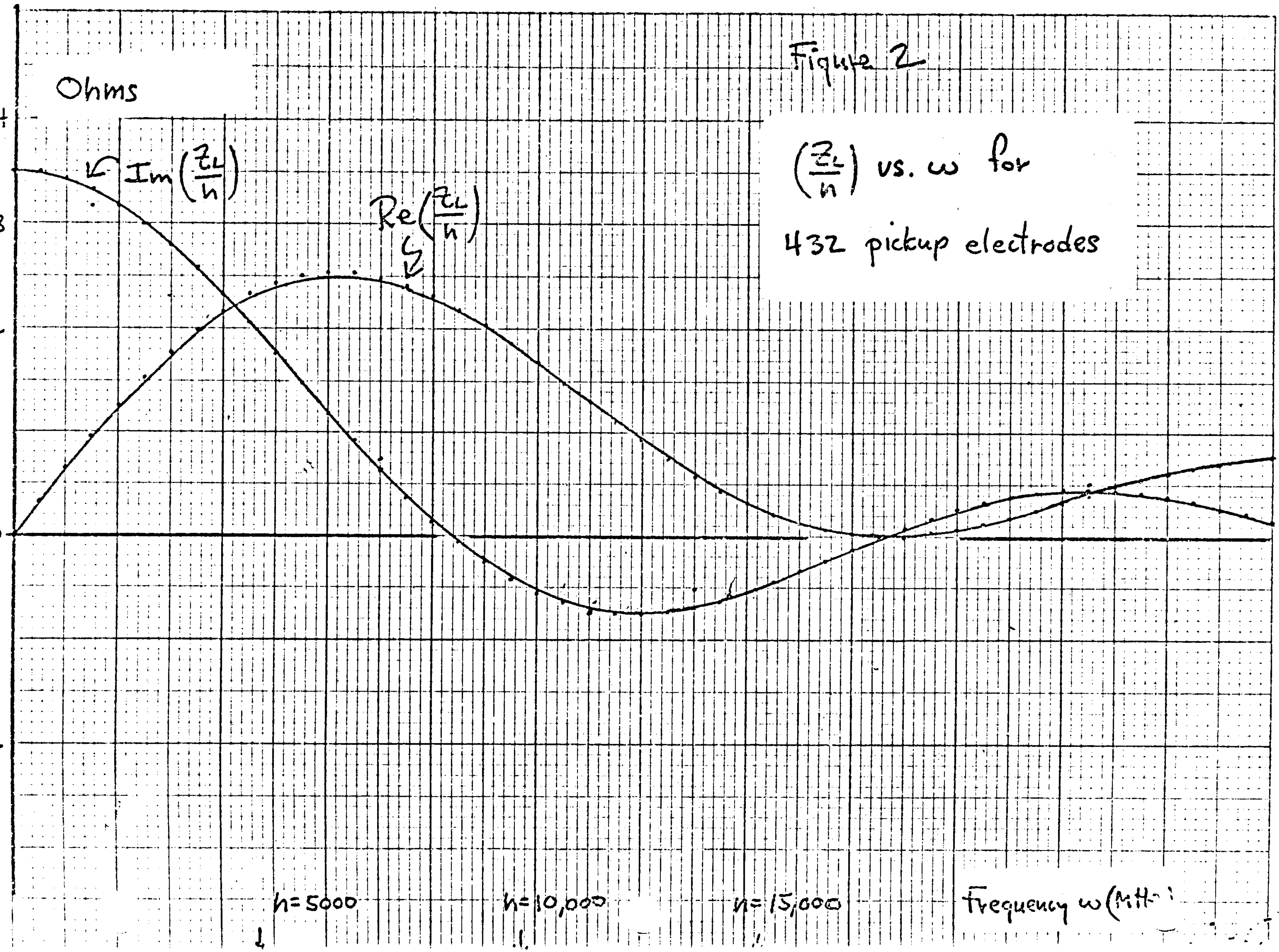
Figure 2
$\left(\frac{Z_L}{n}\right)$ vs. ω for
432 pickup electrodes
Ohms
$Im\left(\frac{Z_L}{n}\right)$
$Re\left(\frac{Z_L}{n}\right)$
.4
.3
.2
.1
0
.1
.2
n=5000
n=10,000
n=15,000
Frequency ω (MH