Luminosity Evolution In Tevatron

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Abstract: In this paper we derive approximate form of a luminosity evolution in a high intensity hadron collider taking into account the most important phenomena of intrabeam scattering (IBS), beam burn-up due to luminosity and beam-beam effects. The approximation is compared with Tevatron luminosity dynamics.

Evolution of Luminosity: Weak-Strong and Strong-Strong Case

If one takes well known formula for luminosity in head-on collisions

$$L = \gamma f_B \frac{N_1 N_2}{4\pi\beta^* \varepsilon} H(\sigma_s / \beta^*) \qquad (1)$$

and leaves only time-dependent factors, then

$$L(t) = C \frac{N_1(t) N_2(t)}{\varepsilon(t)} H(t) \qquad (2)$$

(C is a constant, combination of machine parameters) from where the luminosity lifetime is

$$\tau_L^{-1} = \frac{dL(t)}{L(t)dt} = \tau_{N1}^{-1} + \tau_{N2}^{-1} - \tau_{\varepsilon}^{-1} + \tau_H^{-1} \qquad (3).$$

In the Eqs. (1) and (2) above, emittance is average of emittances of two round beams $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$, *H(x)* is "hourglass factor". In some cases, *L(t)* can be found analytically, e.g., for e+e- colliders when $N_1(t)=N_2(t)$, ε = const and luminosity burnup dominates luminosity lifetime [1]:

$$L(t) = \frac{L_0}{(1+at)^2} \qquad (4).$$

In this section, we consider a more complex example of a hadron collider where the IBS and beam-beam effects play significant role. If one takes a look into the Tevatron proton-antiproton collider operation in 2004, with range of the initial luminosities between 0.5 to 1.0e32 cm-2 s-1, then following observations are valid [2]:

a) proton bunch intensity is much higher than antiproton one, $N_p >> N_a$ (typical values at the beginning of the luminosity store 250e9 and 30e9)
b) the biggest contributor into the luminosity decay is growth of beam emittance with a typical time of τ_ε ~ 15-20 hours; the growth is dominated by IBS in the proton beam with small contribution from beam-beam effects;
c) antiproton lifetime of about τ_a ~ 20-25 hours by 80% is determined by the luminosity burn rate and by 20% by beam-beam effects
d) proton lifetime is mostly driven by beam-beam effects and varies in a wide range τ_p ~ 35-200 hours (the latter value is approximately equal to proton lifetime determined by inelastic interactions with antiprotons in collisions and with vacuum molecules)

e) bunch lengthening due to the IBS - again, mostly in proton bunches – results in the hourglass factor decay with $\tau_H \sim$ 70-80 hours
f) as the result, the luminosity lifetime was $\tau_L \sim$ 7.5-9 hours (all lifetime values are average for the first two hours of a store).

Based on that, the IBS-induced emittance growth and pbar burnup due to luminosity dominate the luminosity decay. For this two effects we can derive analytical formulae for *L(t)*. Theory and simulations of the IBS in the bunches with small longitudinal velocity spread compared to the transverse one predict growth of transverse and longitudinal emittances [3]:

$$\frac{d\varepsilon_T}{dt} = \frac{N_p C_T}{\varepsilon_T^{1.5}\varepsilon_L^{0.5}} \text{ and } \frac{d\varepsilon_L}{dt} = \frac{N_p C_L}{\varepsilon_T^{1.5}\varepsilon_L^{0.5}} \quad (5),$$

C_L and C_T are constants determined by machine parameters. From (5) one gets an asymptotics of $\varepsilon_T \propto \varepsilon_L \propto t^{1/3}$ if $t \rightarrow \infty$, and, approximately:

$$\varepsilon_T(t) \approx \varepsilon_{T0}(1+\frac{t}{\tau_T \beta})^{\beta} \text{ and } \varepsilon_L(t) \approx \varepsilon_{L0}(1+\frac{t}{\tau_L \beta})^{\beta} \text{ where } \beta \approx 1/3 \quad (6).$$

Given that a) $d\varepsilon_y/dt \approx 0.5 d\varepsilon_x/dt$ for protons; b) both transverse and longitudinal emittance growth rates are smaller for antiprotons $d\varepsilon_a/dt << d\varepsilon_p/dt$; c) $\tau_H >> \tau_\varepsilon$ one gets:

$$\varepsilon_{eff}(t) = C\ \frac{N_p N_a}{L} = \frac{\varepsilon_p + \varepsilon_a}{2H} \approx \varepsilon_0(1+\frac{t}{\tau_\varepsilon \alpha})^{\alpha} \text{ where } \alpha \approx 1/3 \quad (7).$$

Fig.1 below demonstrates that such an approximation (7) satisfactorily describes emittance evolution in the Tevatron store 3655 (07/14/2004).

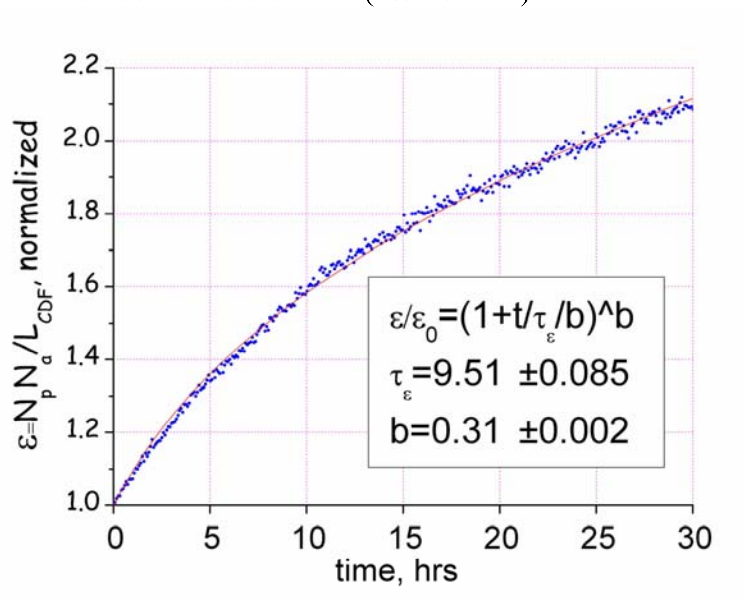


Fig.1: ε_{eff} vs time in store #3655 (blue dots – data, red line – fit curve).

Now, the beam burn up rate due to luminosity is

$$\frac{dN_a}{N_a dt} = -\frac{\sigma_{in} L}{N_a} = \frac{const}{\varepsilon_{eff} / N_p} \quad (8)$$

and, given that $\tau_{Np} >> \tau_{Na}$, one gets

$$\frac{-dN_a}{N_a dt} = \frac{C_a}{(1 + t/\tau_a/\omega)^{\omega}} \quad \text{where } \omega \approx \alpha \approx 1/3 \text{ and } \tau_a \approx \tau_\varepsilon \quad (9).$$

Solution of Eq.(9) is

$$\frac{N_a}{N_0} = \exp(-\frac{\omega \tau_a C_a}{1-\omega}\left[(1 + t/\tau_a/\omega)^{1-\omega} - 1\right]) \quad (10),$$

and combining Eqs(7) and (10), one has

$$\frac{L}{L_0} = \frac{\exp(-\frac{\omega \tau_a C_a}{1-\omega}\left[(1 + t/\tau_a/\omega)^{1-\omega} - 1\right])}{(1 + t/\tau/\omega)^{\omega}} \quad (11).$$

Eq.(11) gives approximate evolution of luminosity in the case when antiproton beam intensity is much smaller than the proton beam one.

For the *"strong-strong"* case, when intensities of both beams are comparable $N_1(t) \approx N_2(t) = N(t)$, (situation anticipated later in the Tevatron Run II, when after various upgrades antiproton bunch intensity will reach half of the proton one), Eq. (7) is still valid, and if we introduce normalized intensity $F = N(t)/N(t=0)$, then luminosity evolution is given by

$$\frac{L}{L_0} = \frac{F^2}{(1 + t/\tau_\varepsilon/\alpha)^{\alpha}} \quad (12),$$

and, therefore,

$$\frac{dF}{dt} = -\frac{\sigma_{in} L}{N_0} = -\frac{C_0 F^2}{(1 + t/\tau_\varepsilon/\alpha)^{\alpha}} \quad (13).$$

Solutions of Eqs.(12) and (13) are:

$$F = \frac{N}{N_0} = \frac{1}{1 + A\left[(1 + t/\tau_\varepsilon/\alpha)^{1-\alpha} - 1\right]}; \quad A = C_0 \tau_\varepsilon \alpha/(1-\alpha)$$

$$L = \frac{L_0}{\left[1 + A\left[(1 + t/\tau_\varepsilon/\alpha)^{1-\alpha} - 1\right]\right]^2 (1 + t/\tau_\varepsilon/\alpha)^{\alpha}} \quad (14).$$

These equations can be further approximated by formulae having correct asymptotics at $t >> \tau_\varepsilon$:

$$N \approx \frac{N_0}{\left[1 + t/\tau_N/\alpha\right]^{\alpha}}; \quad L \approx \frac{L_0}{\left[1 + t/\tau_L/\mu\right]^{\mu}}; \quad \alpha \approx 1/3; \mu \approx 2 - \alpha = 5/3 \quad (15).$$

Interestingly, such simple form of approximation as in Eq.(15) - rational of fractional power law - satisfactorily works for the "weak-strong" case considered above in Eqs.(10,11) when the power coefficient is reduced μ<5/3, see Fig.2.

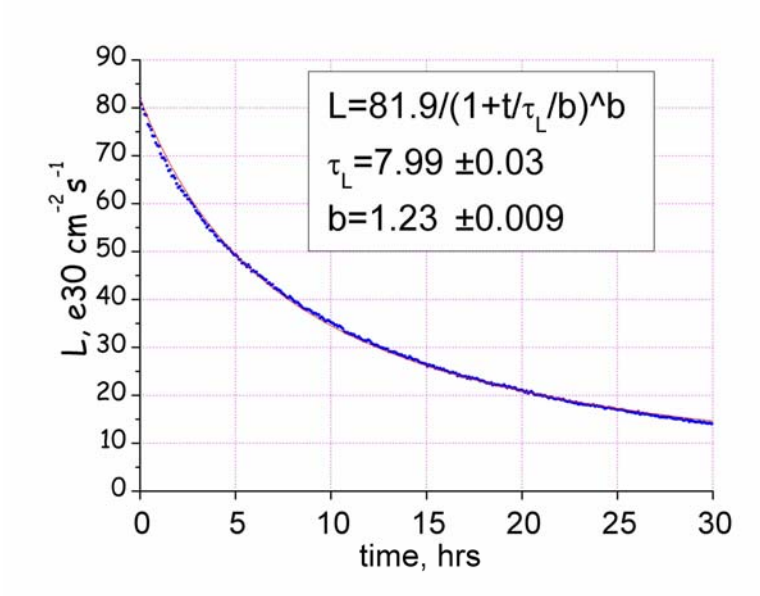


Fig.2: L vs time in store #3655 (blue dots – data, red line – fit curve).

Further below, we will apply the above conclusions and rational power law fits like in Eq.(15) for analysis of luminosity evolution in the Tevatron Run II stores.

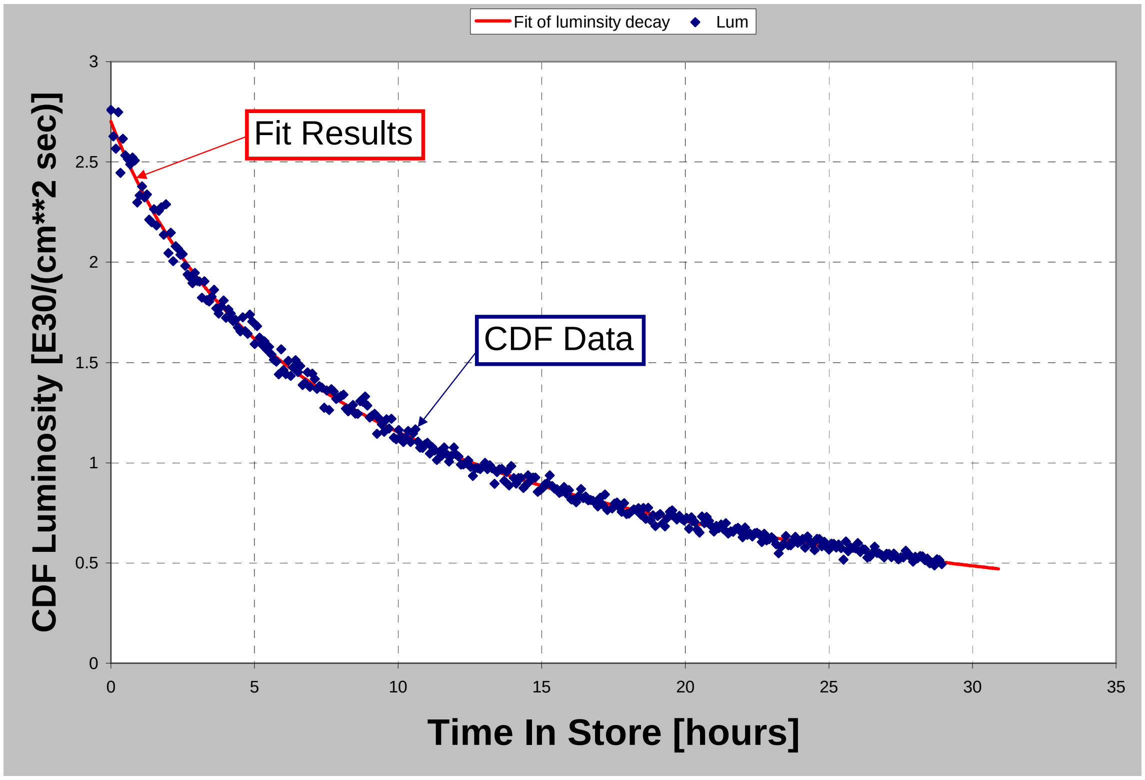
[Elliott's part starts here]
Luminosity Burn Rate for Store 3739, Bunch 13

Background

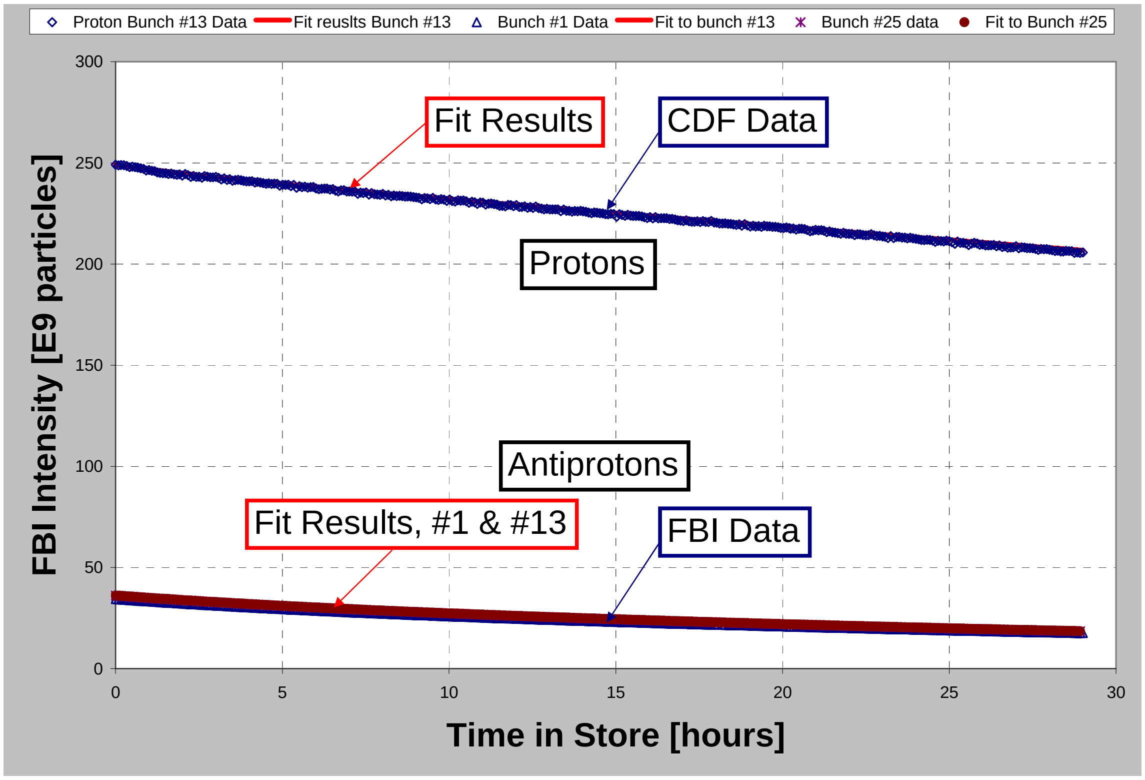
Working from previous paper ("Luminosity Burn Rate," September 20, 2004, BeamDocs-1353), I have applied the formulae from that paper to store 3739, bunch number 13.

Fits of Luminosity and Intensities

This figure shows the luminosity profile of bunch 13, store 3739, and a fit to the decay of the luminosity.



The next figure shows the proton and the antiproton intensities during this store for this bunch, with an analytic fit to these data. The data are for proton bunch #13 and for pbar bunch #1 (which collides with proton bunch #13 at CDF) and #25 (for D0).



The fits of these three sets of data use the functional form:

$$F(t) = F(0) \,/\, (\, 1 + t \,/\, (\tau_0\, c_1)\,)^{\,x}$$
where, $x = c_1 + c_2\, t$

Fits are performed on the CDF data using Minuit. The results are as follows:

Function	*F(0)*	*Units*	τ_0	c_1	c_2
Luminosity	2.7017	E30/(cm**2 sec)	4.919	0.822	0.0045
Proton Intensity	247.89	E9 particles	0.213	0.0449	0.0017
Antiproton #1	34.413	E9 particles	5.961	0.458	0.0040
Antiproton #13	36.22	E9 particles	5.258	0.428	0.0043

Note: The proton bunches for this store are all a little crazy. Many of the bunches have a distinct “knee” at about 10 hours, although this bunch does not show it quite so clearly. Although the fit quality is very good, the numbers are a bit hard to understand. Notice that the antiproton numbers are cleaner. Note, furthermore, that the two antiproton bunches on this particular store, for this proton bunch, are quite similar. For this analysis, we will use only one of the antiproton bunches (#1) and multiply by 2, as appropriate. (The analysis conducted here is all within a spreadsheet. It is anticipated that a fuller analysis of all the bunches for many stores will use the appropriate two antiproton bunches in the correct manner.)

General Calculation Techniques

It is not appropriate to use the raw CDF numbers to attempt to calculate the luminosity burn-off: There is too much variation in the data. So, we will henceforth use only the fitted functions for the quantities for this store.

As an example, an "effective emittance" ($\varepsilon_{eff}(t)$) is calculated from these fit results in this manner:

$$\varepsilon_{eff}(t) = N_P(t)\, N_A(t) \,/\, L(t)$$

In other words, the effective emittance at any time *t* is obtained by calculating the quotient of the fitted number of protons, times the fitted number of antiprotons with the fitted luminosity, at that time. The antiproton bunch that collides with proton bunch #13 at CDF is #1. The result is presented in this figure:

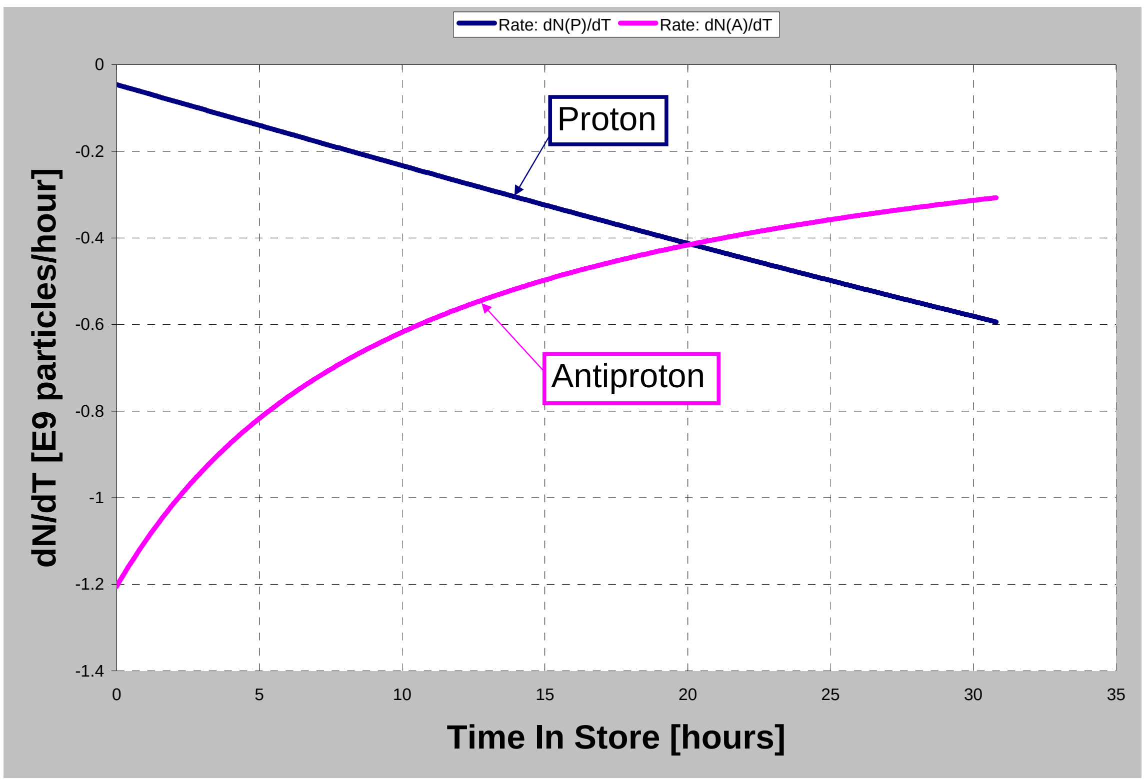
Other quantities will be calculated in this manner in the remainder of this document.

Calculating the Luminosity Burn-Off

From the previous document, BeamDocs-1353, the luminosity burn-off rate $R(t)$ is:

$R_L(t) = \mathrm{d}N/\mathrm{d}t|_L = \sigma\ (\ \Sigma\ L_n(t))$ is the particle loss rate,
where $L_n(t)$ is the luminosity at experiment "n" and
σ is 70 millibarns, the proton-antiproton total cross section.

The particle loss rates for the protons and antiprotons in this store are shown in the next figure:



Note again that the proton intensity numbers are a bit screwy! I do not think that the weirdness of the proton data affects any of the results presented here.

Armed with the dN/dt data and the proton and antiproton intensity data, we have:

$$\delta L\ /\ L\ = \delta\ N_P\ /\ N_P + \delta\ N_A\ /\ N_A + \delta\ \varepsilon\ /\ \varepsilon + \textit{other terms}$$

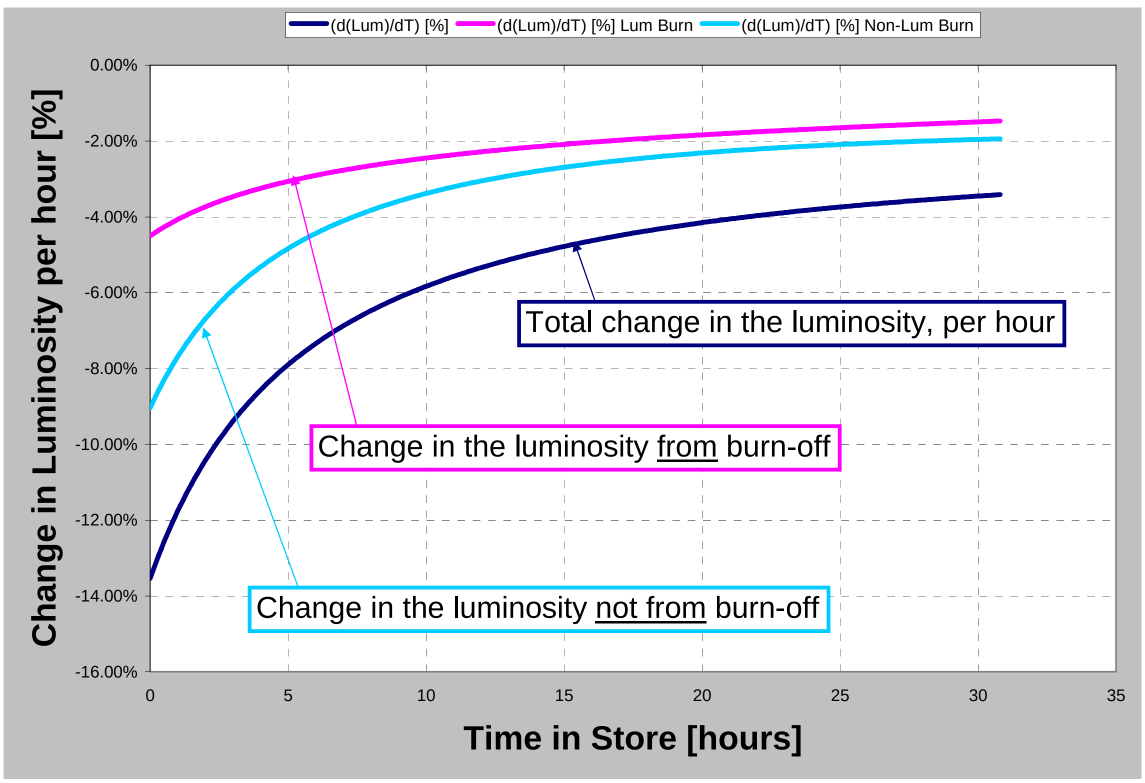
which implies:

$$(\mathrm{d}\ L\ /\ \mathrm{d}t\ )\ /\ L\ =\ (\mathrm{d}\ N_P\ /\ \mathrm{d}t\ )\ /\ N_P + (\mathrm{d}\ N_A\ /\ \mathrm{d}t\ )\ /\ N_A + (\mathrm{d}\ \varepsilon\ /\ \mathrm{d}t\ )/\ \varepsilon$$

Some points need to be made about this equation.

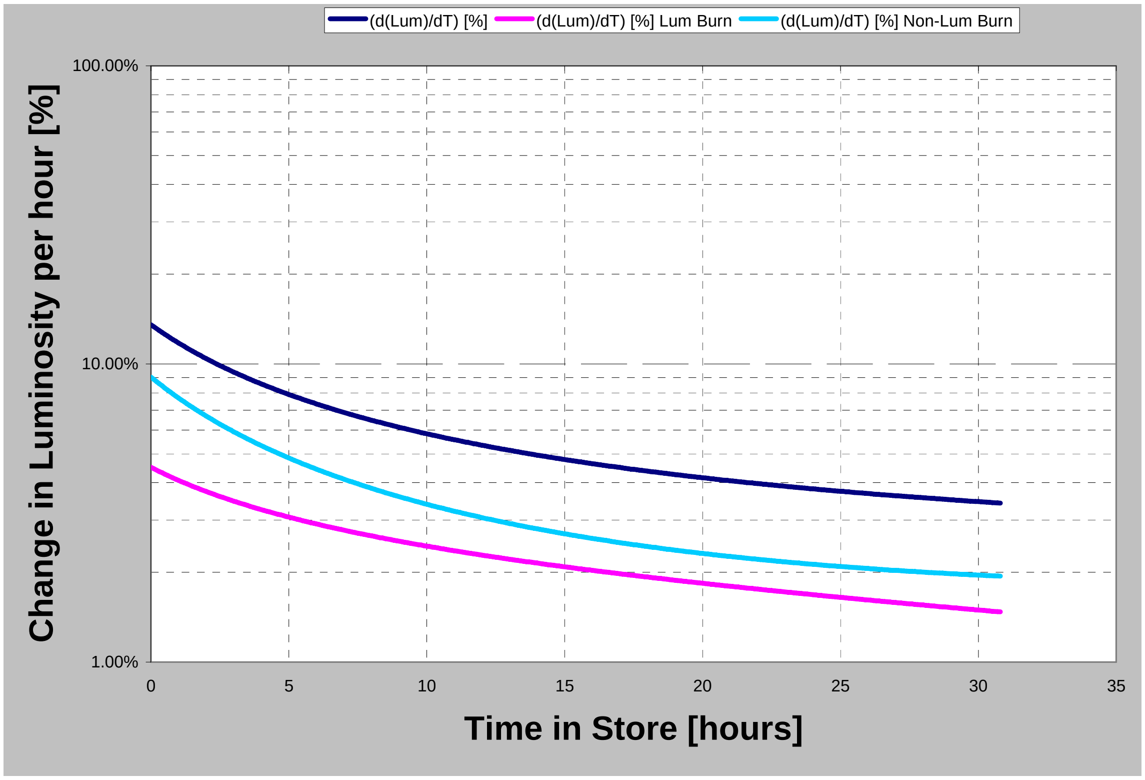
1. The “other terms,” which contain things like the hour glass factor, are (safely?) ignored in this analysis
2. Since the fractional luminous losses for the proton beam is less that that of the antiproton beam, ignoring the “weirdness” of the proton decay should be valid.
3. The fractional change in the emittance cannot (easily) be broken down into “luminous” and “non-luminous” components. Firstly, we have no direct and consistent time-dependent measurement of the emittance (maybe, someday, the synchrotron light monitors will provide this, but it is not as yet available). Secondly, the luminosity burn-off equations do not directly predict any change in the emittance. Therefore, the contribution of the emittance change will not be considered here.

Substituting the data for the luminosity burn-off of the protons and the antiprotons, we obtain the following chart for the percentage change in the luminosity due to burn-off.



The dark blue line is the change in the luminosity per hour, directly calculated from the fit of the CDF luminosity data for this store and bunch. The cyan line is the difference between the total luminosity change and the luminosity change that comes from only the luminosity burn-off.

It is interesting to look at these data differently: Invert the numbers (making them all positive) and then plot on a log scale:



The magenta curve, representing the luminosity burn-off, is almost completely exponential after about 15 hours. Moreover, the cyan curve, representing the non-luminous burn-off, has a very non-linear, non-exponential character throughout the entire store.

Conclusions

The purpose of this document is to demonstrate a technique for determining the non-luminous decay of the luminosity in a Tevatron bunch throughout a store. It is anticipated that this analysis will lead to a study of all the bunches in many stores.

The character of the non-luminous luminosity decay, presented most prominently in the last figure, shows some interesting behavior. Some believe that an understanding of these non-luminous decays is a highly desirable goal. It is hoped that this analysis will lead to a better understanding of non-luminous luminosity decay.

References:

[1] Handbook of Accelerator Physics and Engineering, A.W.Chao, M.Tigner, eds, World Scientific, (1998), p.217.
[2] V.Shiltsev, beams-doc-1357, see also in Proc. EPAC 2004 (Lucern).
[3] D.Finley, FNAL-TM-1646 (1989); V.Lebedev, in Proc. IEEE PAC 2003 (Portland).
[4] S.Mishra, private communication.