Recycler-Only Operations Luminosity Projections

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INTRODUCTION

This note outlines a simple model for forecasting peak and integrated Tevatron luminosity for Recycler-Only operations.

LUMINOSITY LIFETIME

A fairly simple but accurate description for the evolution of Tevatron luminosity is given as[1]:

$$L(t) = \frac{L_o}{\left(1 + \frac{t}{\mu\tau}\right)^{\mu}} \tag{1}$$

where L_o is the initial luminosity, τ is the initial luminosity lifetime, and μ~5/3. This formula is valid only in the strong-strong case but works reasonably well in the weak-strong case for μ<5/3. Figure 1 shows the results of a fit for μ=1.6 for two sample stores during the month of October 2005.

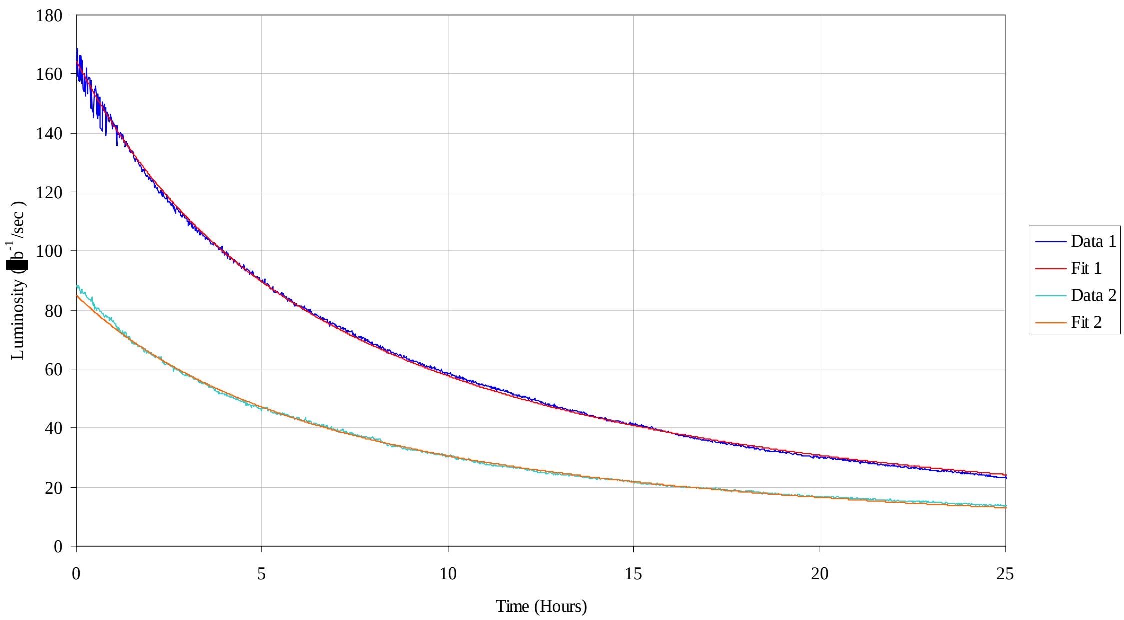

Figure 1. Luminosity Evolution for two stores during the month of October 2005. μ=1.6

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[1] "Tevatron Luminosity Evolution", V. Shiltsev, E. McCrory, Beams Document 1777

The integrated luminosity for a store is:

$$\int_0^{T_S} L dt = \left(\mu\tau - \frac{(\mu\tau)^{\mu}}{(\mu\tau + T_s)^{\mu-1}} \right) \frac{L_o}{\mu - 1} \tag{2}$$

where T_s is the length of the store. For modeling purposes, the luminosity lifetime is linearly interpolated as a function of initial luminosity:

$$\tau(L_o) = \frac{\tau_2 - \tau_1}{L_{o2} - L_{o1}} (L_o - L_{o1}) + \tau_1 \tag{3}$$

<u>INITIAL LUMINOSITY</u>

The initial luminosity is:

$$L_o = \frac{3\gamma}{\beta^*} B N_{\bar{p}} N_p \frac{F(\beta^*, \sigma^L_{p,\bar{p}}, \varepsilon_{p,\bar{p}})}{(\varepsilon_p + \varepsilon_{\bar{p}})} \tag{4}$$

which can be naively separated into the contributions due to protons and antiprotons:

$$L_o = \frac{N_p}{250x10^9} L_D(S_R) \tag{5}$$

Where L_D is called the luminosity density and is a function of the available antiproton stack S_R.

$$L_D = \kappa \frac{3\gamma}{\beta^*} 250x10^9 \frac{F(\beta^*, \sigma^L_{p,\bar{p}}, \varepsilon_{p,\bar{p}})}{(\varepsilon_p + \varepsilon_{\bar{p}})} m_e S_R \tag{6}$$

where κ is the antiproton transfer efficiency to collisions in the Tevatron and m_e is the Recycler mining efficiency from the Recycler. The Recycler mining efficiency is defined as the amount of antiprotons extracted from the Recycler divided by the total Recycler stash size. Figure 2 shows the luminosity density for the different modes of Tevatron operations during Run II. For modeling purposes, the luminosity density can be fitted to a quadratic:

$$L_D \approx \frac{dL_D}{dS_R} S_R \tag{7}$$

$$\frac{dL_D}{dS_R} \approx \frac{\left(L_{D2} \big/ S_{R2} \right) - \left(L_{D1} \big/ S_{R1} \right)}{S_{R2} - S_{R1}} (S_R - S_{R1}) + \left(L_{D1} \big/ S_{R1} \right) \tag{8}$$

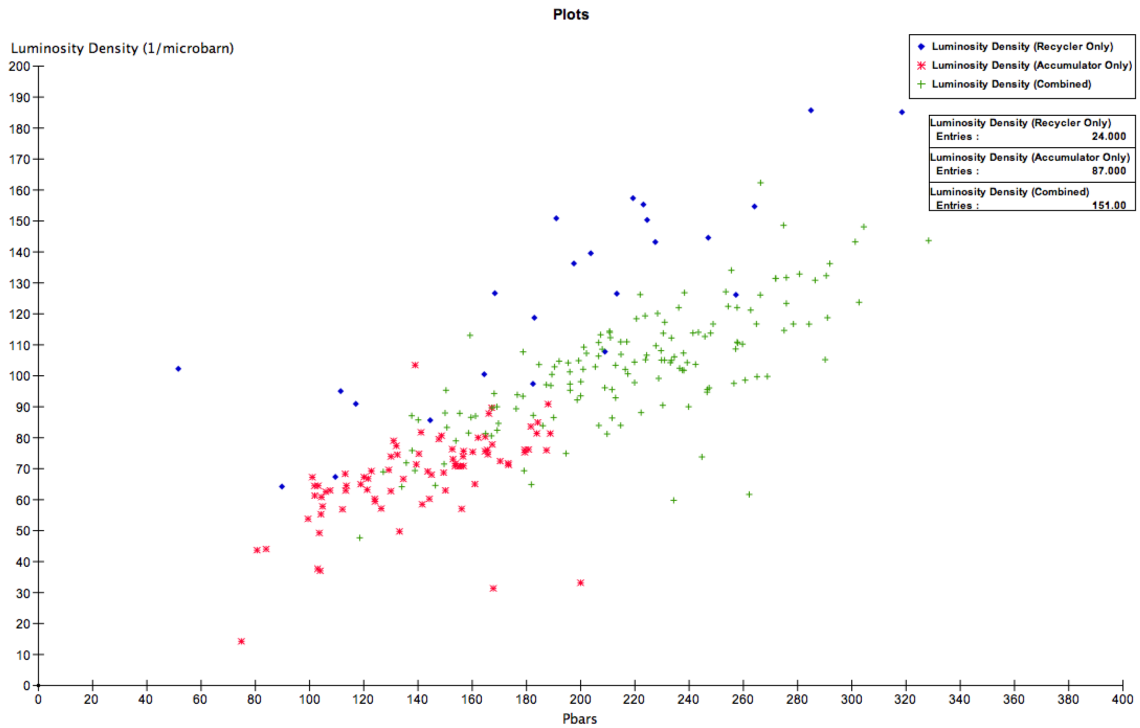

Figure 2. Luminosity density as a function of available antiproton stack. The x axis is in units of 10^{10}.

RECYCLER STASH

Antiprotons are loaded into the Recycler from the Accumulator periodically while the store collides in the Tevatron. A simple approximation to the Recycler Stash size would be:

$$S_R = n_T x_T S_A (1 - p_L)(2 - m_e) \tag{9}$$

where n_T is the number of Accumulator to Recycler transfers during the store, x_T is the transfer efficiency, S_A is the Accumulator stack size, p_L is the fraction of beam left over in the Accumulator after an Accumulator to Recycler transfer, and m_e is the Recycler mining efficiency. However, two intensity dependent effects should also be considered. First, because of the long Tevatron stores, the beam lifetime in the Recycler should be taken into account. Second, although the transfer efficiency from the Accumulator to the Recycler is independent of Recycler stash size, the efficiency of the Recycler to merge a new batch of antiprotons from the Accumulator is a function of Recycler stash size. The Accumulator to Recycler transfer efficiency and the merging efficiency will be grouped into a transfer efficiency that is a function of Recycler stash size:

$$x_T(S_R) = \frac{x_{T_2} - x_{T_1}}{S_{R_2} - S_{R_1}}\left(S_R - S_{R_1}\right) + x_{T_1} \tag{10}$$

After the Tevatron has been loaded for a store, the Recycler Stash is at a minimum. If the Recycler was completely empty the amount of beam in the Recycler after the first transfer is:

$$S_{R_1} = x_T(0) S_A (1 - p_L) \tag{11}$$

For subsequent transfers to the Recycler, the Recycler stash size grows to:

$$S_{R_n} = \left[S_{R_{n-1}} e^{-\frac{T_s}{n_T \tau_{rec}}} + S_A (1 - p_L) x_T \left(S_{R_{n-1}} e^{-\frac{T_s}{n_T \tau_{rec}}} \right) \right] \tag{12}$$

Before transfer to the Tevatron, the beam is cooled in the Recycler. The beam available to the Tevatron becomes:

$$S_R = (2 - m_e) S_{R_n} e^{-\frac{T_s}{n_T \tau_{rec}}} \tag{13}$$

Although it would be more correct to include the term due to the mining efficiency in the transfer efficiencies used in Eqns. 11 and 12, the final Recycler stash size is easier to calculate if the term is added in at the end of the algorithm.

ACCUMULATOR STACK

The stacking rate in the Accumulator is a function of stack size[2]. A quadratic approximation to the stacking rate as a function of stack size is:

$$\frac{dS}{dt} = R(S) = R_o \frac{S_m - S}{S_m} + R_\Delta \left[1 - \left(\frac{S_m - 2S}{S_m} \right)^2 \right] \tag{14}$$

where R_o is the stack rate when the stack size is zero and S_m is the stack size when the stack rate is zero. The quadratic term R_Δ is a function of the how the Main Injector cycle time is increased as the stack size grows:

$$R_\Delta = \frac{R_o}{2} \frac{1 - 2\frac{S_m - S_h}{S_m}}{1 - \left(\frac{S_m - 2S_h}{S_m} \right)^2} \tag{15}$$

where S_h is the stack size when the stacking rate is $R_o/2$.

The time available for stacking between Accumulator to Recycler transfers is:

$$T_{stack} = \frac{T_s}{n_T} - \Delta T_{AR} \tag{16}$$

where ΔT_{AR} is the time it takes to transfer the stack from the Accumulator to the Recycler.

[2] "Antiproton and Neutrino Production Accelerator Timeline Issues", D. McGinnis, Beams Document 1941

The stack in the Accumulator is found from the integral equation:

$$\kappa_{tL} T_{stack} = \int_{(1-p_L)S_A}^{S_A} \frac{d\sigma}{R(\sigma)} \qquad (17)$$

where κ_{tL} is the fraction of the timeline that is used for stacking. Equation 17 can be easily solved numerically using a lookup table if Equation 17 is re-written as:

$$\kappa_{tL} T_{stack} = \int_{0}^{S_A} \frac{d\sigma}{R(\sigma)} - \int_{0}^{(1-p_L)S_A} \frac{d\sigma}{R(\sigma)}$$

AN EXAMPLE

Parameter	Value	Units
Number of Protons per bunch	231	$x10^{9}$
Luminosity Density @100x10^{10}	82	μb^{-1}/sec
Luminosity Density @ 300x10^{10}	189	μb^{-1}/sec
Init Tevatron Lifetime @ 80μb^{-1}/sec	7.1	Hours
Init Tevatron Lifetime @ 160μb^{-1}/sec	6.9	Hours
HEP Store Hours per week	110	Hours
Acc-Rec Transfer Efficiency @ 0x10^{10}	93	%
Acc-Rec Transfer Efficiency @ 300x10^{10}	80	%
Acc-Rec Transfer Time	0.75	Hours
Recyler Lifetime	500	Hours
Recycler Mining Efficiency	95	%
Zero Stack Stack Rate	14	x10^{10}/Hour
Half Rate Stack Size	225	x10^{10}
Maximum Stack size	350	x10^{10}
Timeline Utilization Factor	80	%
Accumulator Leftover Factor	5	%

Table 1. Typical Parameters for October 2005

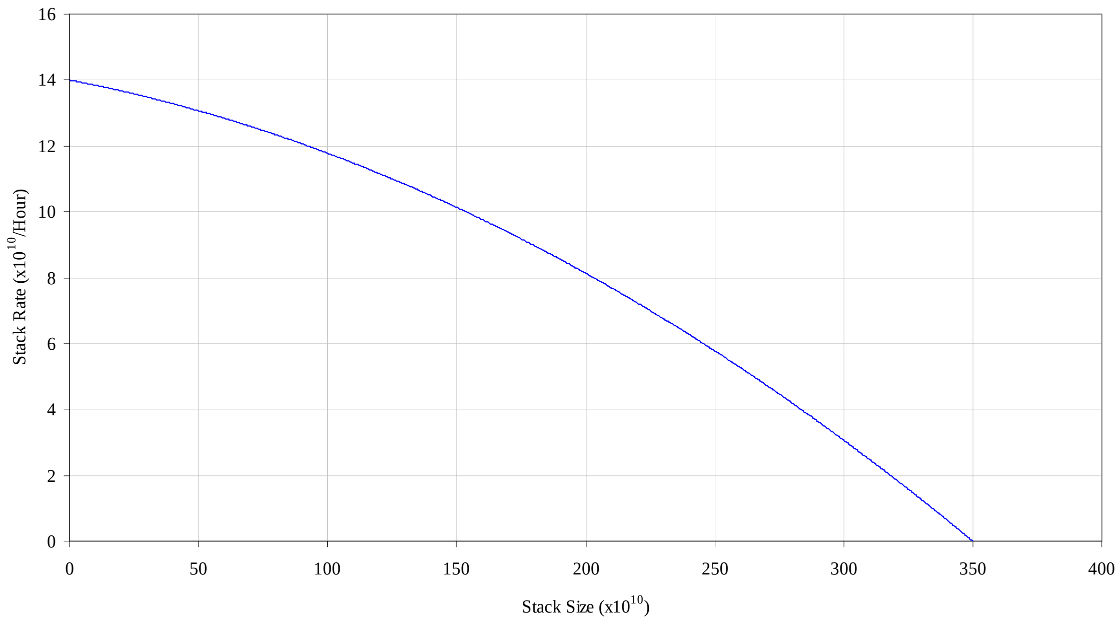

Figure 3. Stack Rate vs. Stack Size

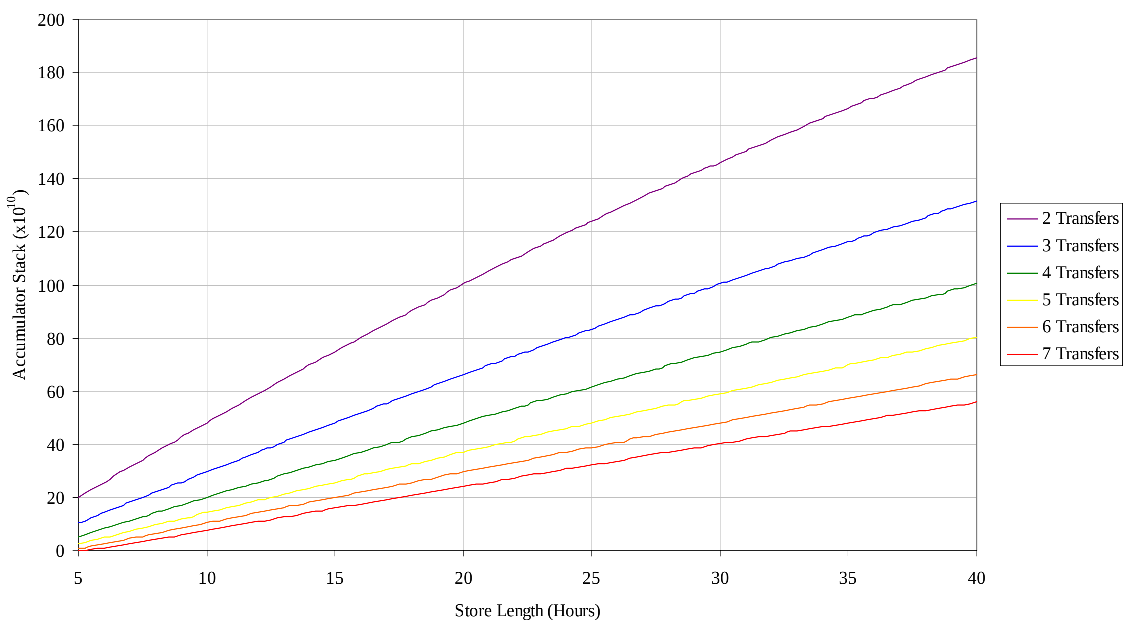

Figure 4. Accumulator Stack Size vs. Store Length for various numbers of Accumulator to Recycler Transfers.

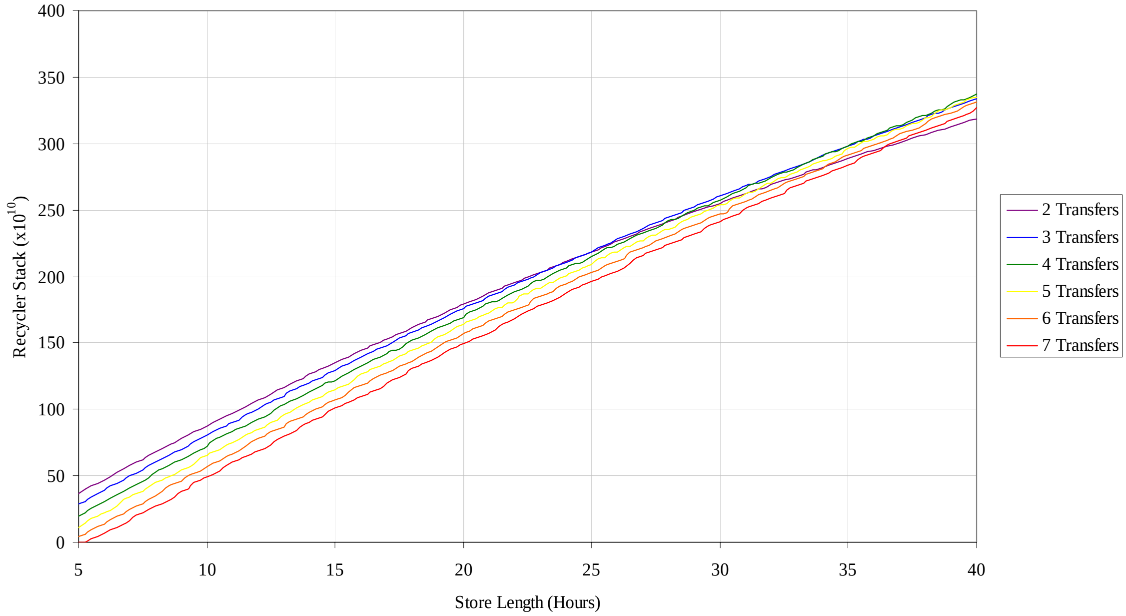

Figure 5. Recycler Stash size vs. Store Length for various numbers of Accumulator to Recycler Transfers.

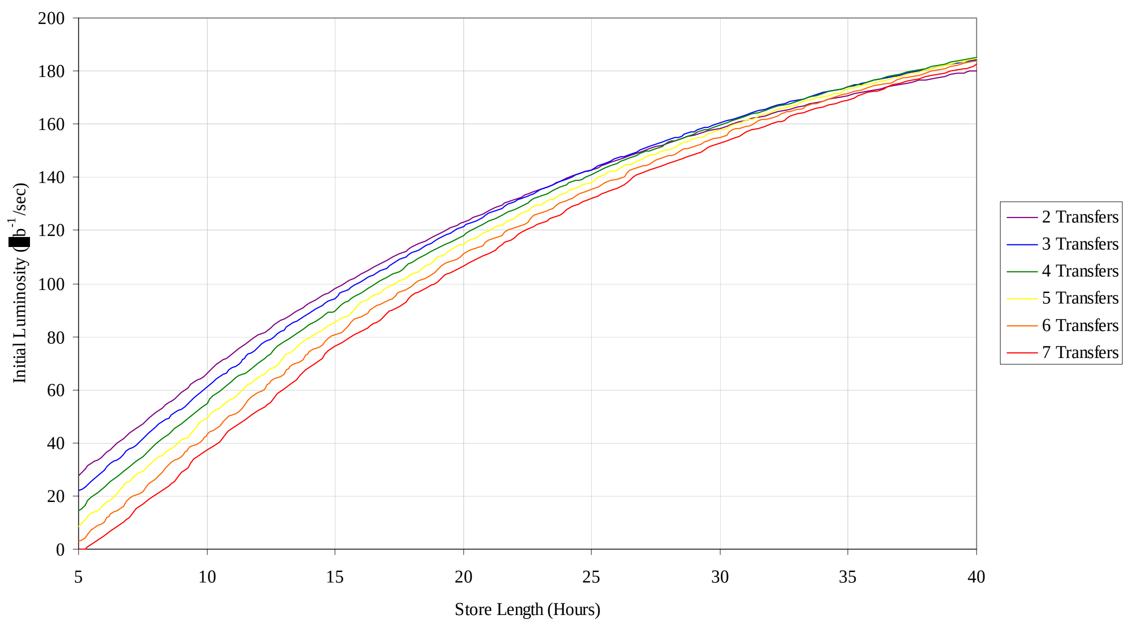

Figure 6. Initial Luminosity vs. Store Length for various numbers of Accumulator to Recycler Transfers.

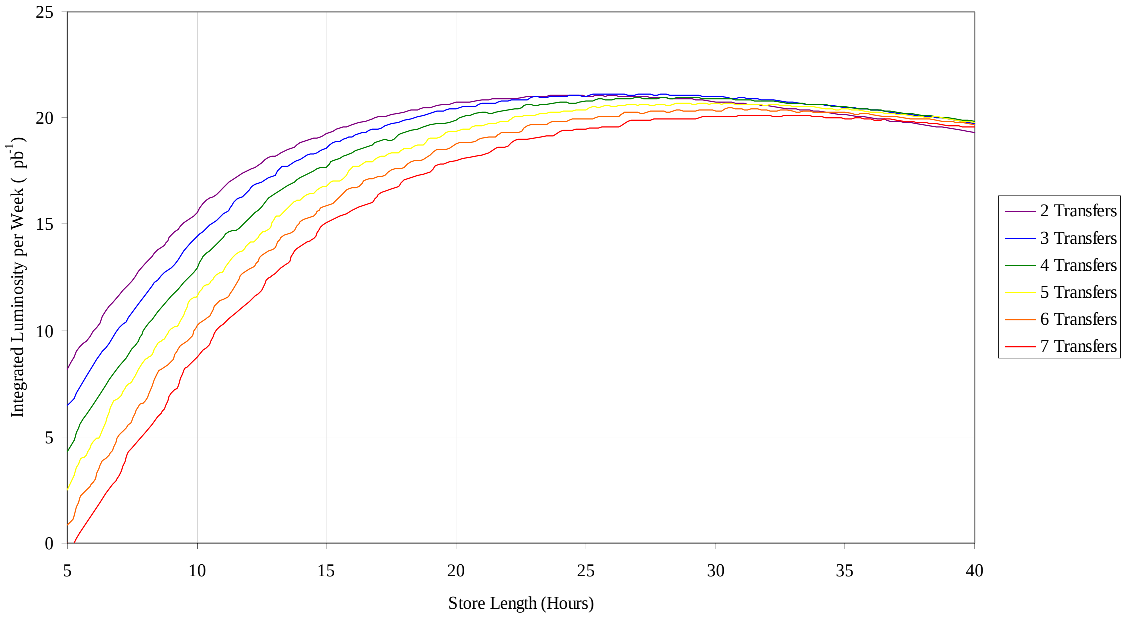

Figure 7. Weekly Integrated Luminosity vs. Store Length for various numbers of Accumulator to Recycler Transfers.