IPM measurements at the Fermilab Booster

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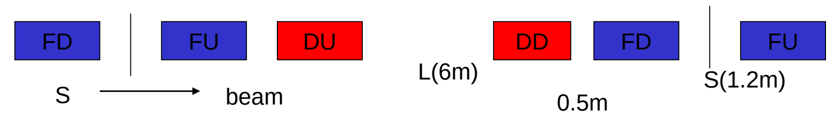
Since 1996, the Fermilab Booster is equipped with turn-by-turn beam profile measurements. This is the first attempt to systematically measure the evolution of the beam emittances during the acceleration cycle, and their dependence on the beam intensity. This effort is important because the demand of neutrino-experiments: The proton demand on the Booster has increased from 7E15 to as much as 1.8E17 protons/hr because of the MiniBooNE and NuMI projects.

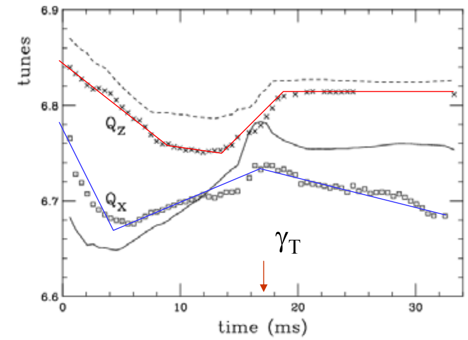
1. Introduction
2. Measurements and Modeling in emittance reduction
3. Modeling by numerical simulations
4. Conclusions

An Overview of Booster's lattice



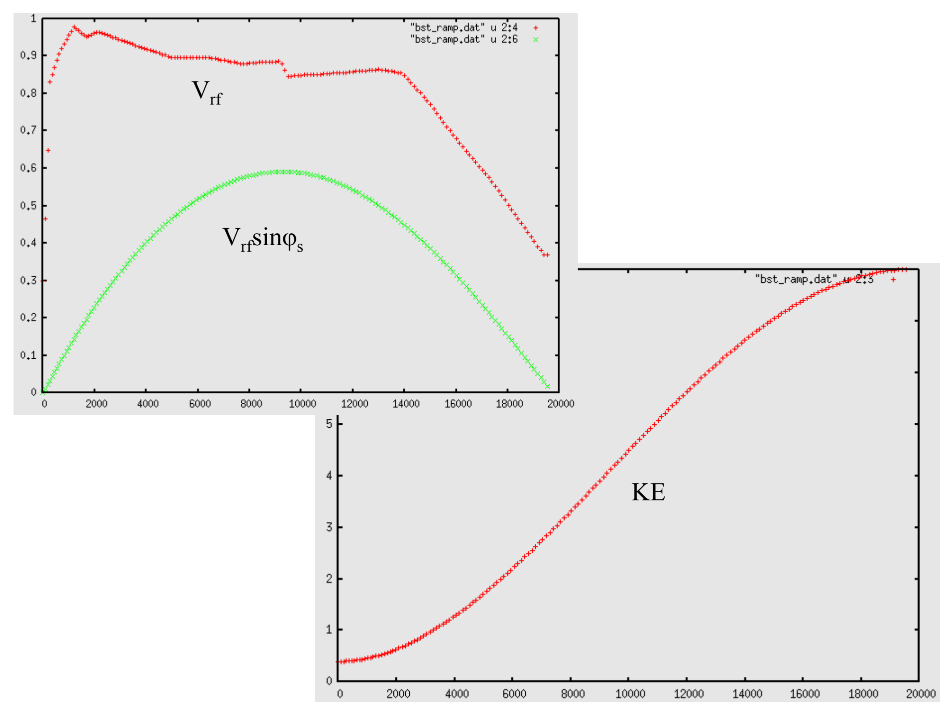
$\beta_{x,\max} = 33.7\,\mathrm{m}$ $\beta_{z,\max} = 20.4\mathrm{m}$ $D_{\max} = 3.2\mathrm{m}$

- Superperiod=24
- In each period, four Combined-function magnets. Length 2.889 m. Gradients are $K_1(F)=0.0542\ \mathrm{m}^{-2}$ and $K_1(D)=-0.0577\ \mathrm{m}^{-2}$. The integrated focusing strengths are 0.313 m^{-1} and $-0.333\ \mathrm{m}^{-1}$.
- Horizontal tune 6.7, vertical tune 6.8
- Extraction doglegs in L03 and L13 perturb the lattice functions

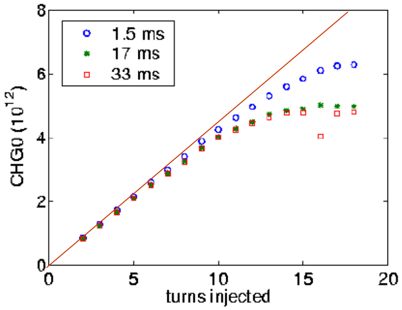


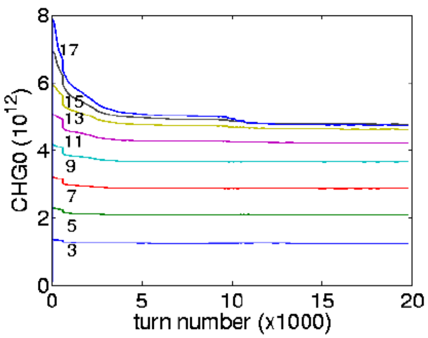
The Fermilab Booster

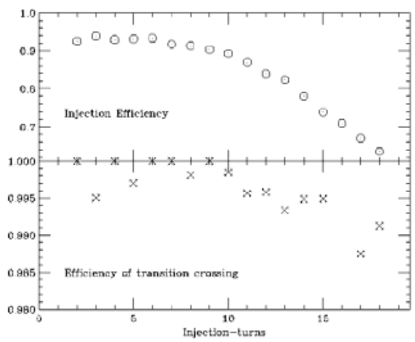
Circumference(m)	474.2
Inj/ext energy (GeV)	0.4/8.0
Cycling rate (Hz)	15
Hori/vert betatron tunes	6.7/6.8
Superperiod	24
Transition gamma	5.446
Harmonic number	84
Protons per pulse	5.0E12
Longitudinal 95% emittance (eV-s)	0.10 at injection
Transverse 95% emittance (mm-mr)	12π (normalized) at injection



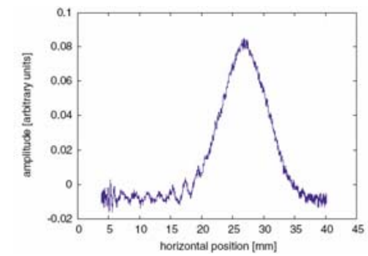
Typical Injection scheme at the Fermilab Booster: H- at 400 MeV from Linac is about 30 mA. Strip injection into the Booster gives about 4.2×10^{11} protons per injection turn. At 400 MeV, the revolution frequency is 4.51×10^{5} Hz. The Normal operation has 12 injection turns, and the total intensity is about 5×10^{12} ppp at 15 Hz rep-rate. In fact, when the injection turn is larger than 12, the Booster loss problem becomes severe.



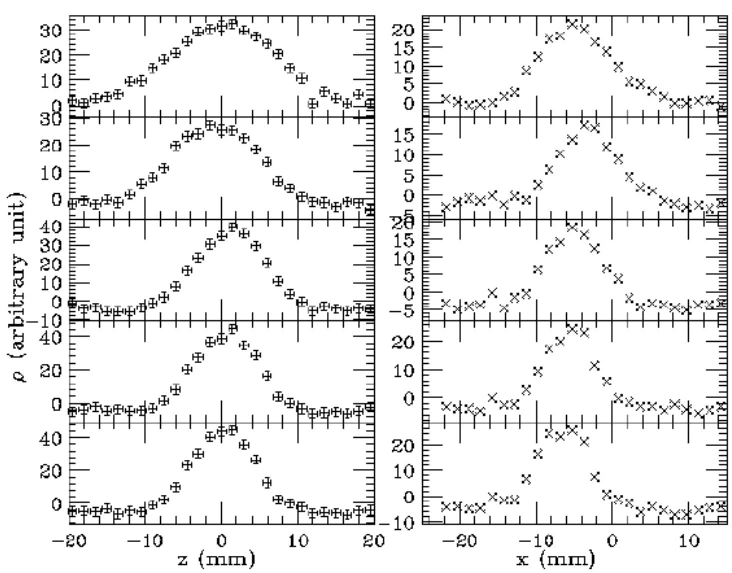




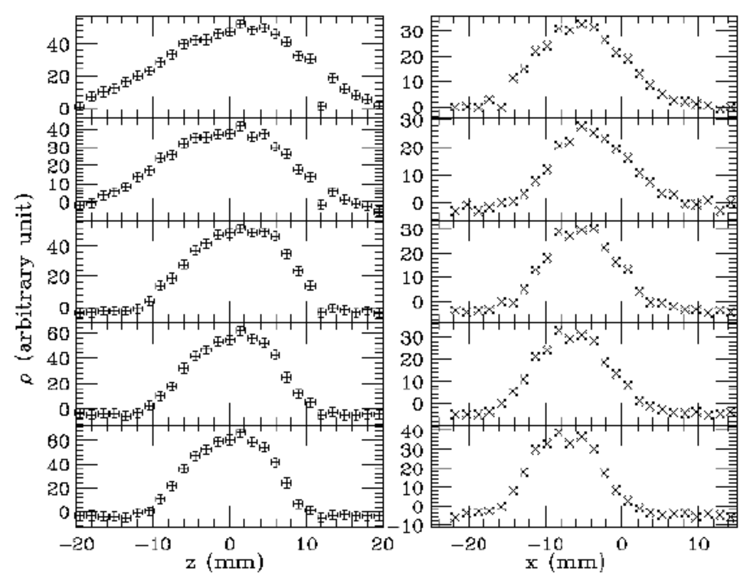
Ionization beam profile monitors (IPM's) measure the beam profile by collecting electrons from background gas ionization. Measurement accuracy will depend on the number of electrons collected. The number of electrons collected depends on pressure, bunch intensity, and data collection mode. The center-to-center distance between microchannels is of the order of 0.2mm. Thus one can measure the beam width to within 0.5mm. For the IPM at Fermilab Booster, the space charge correction has been shown to be important [J. Amundsen et al PRSTAB **6** 102801 (2003)]



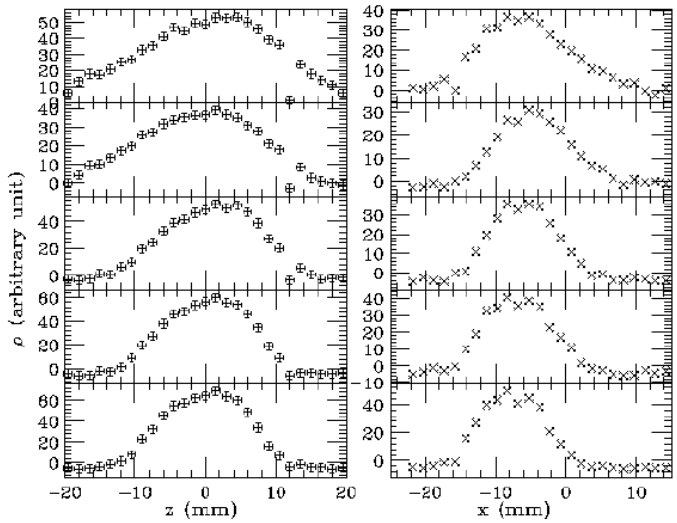
$$\varepsilon = \frac{\sigma^2}{\beta}$$



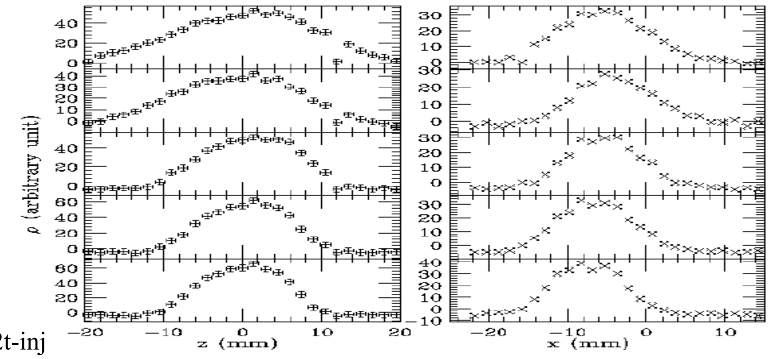
6t-inj



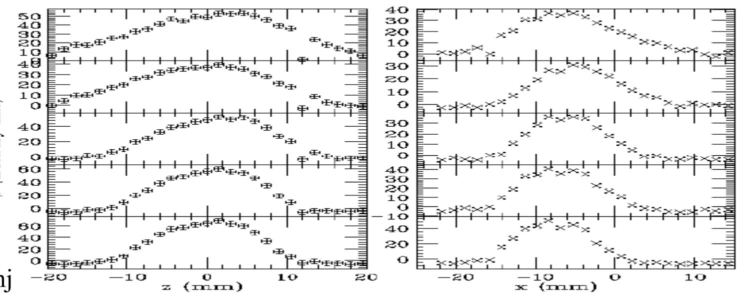
12t-inj



14t-inj



12t-inj



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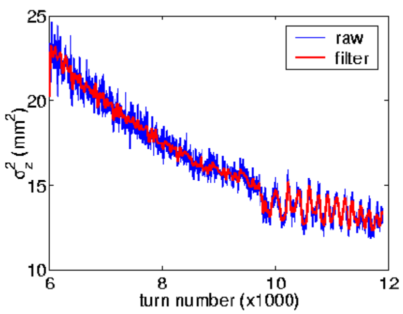
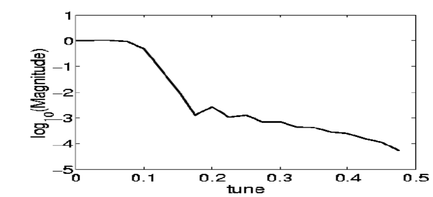
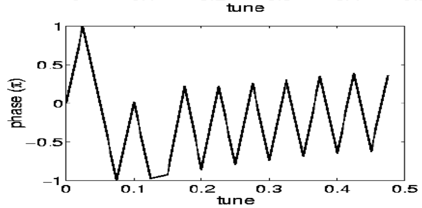
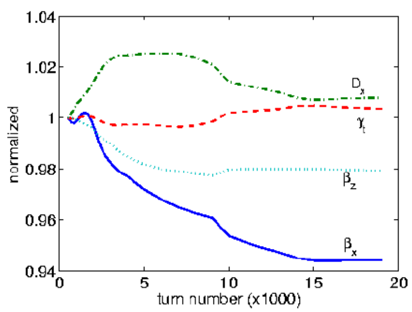


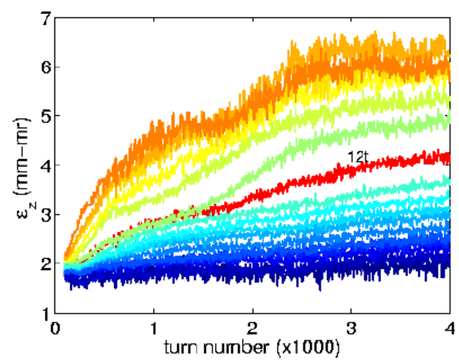


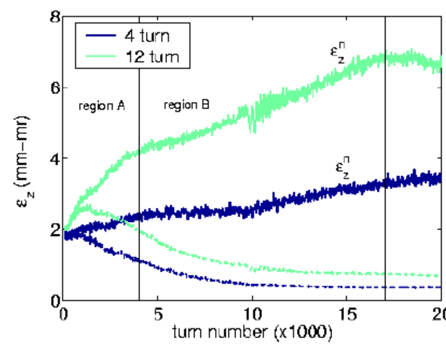




Machine modeling:
βx=6.3, βz=21.4, Dx=2.54,
at the IPM location with
Qx=6.7, Qz=6.8

Space charge effects: Linac delivers about 30 mA beam current to the Fermilab Booster, i.e. about 4.2×10^{11} particles in one injection turn.





$$\varepsilon = a_0 + b_1 t + b_2 \int_0^t K_{sc}\, dt$$

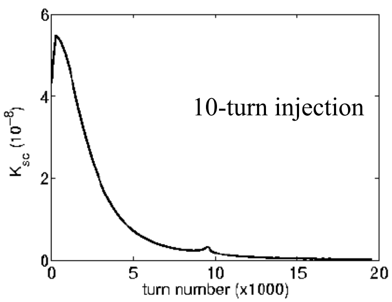
$$K_{sc} = \frac{2Nr_0}{\beta^2\gamma^3}$$

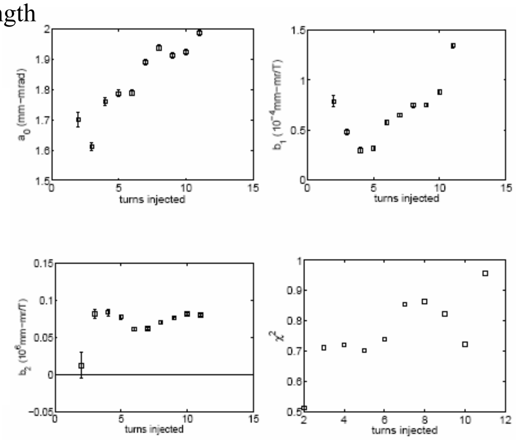
The space charge perveance can be calculated from the beam parameters, and there are three fitting parameters, a_0, b_1, b_2.

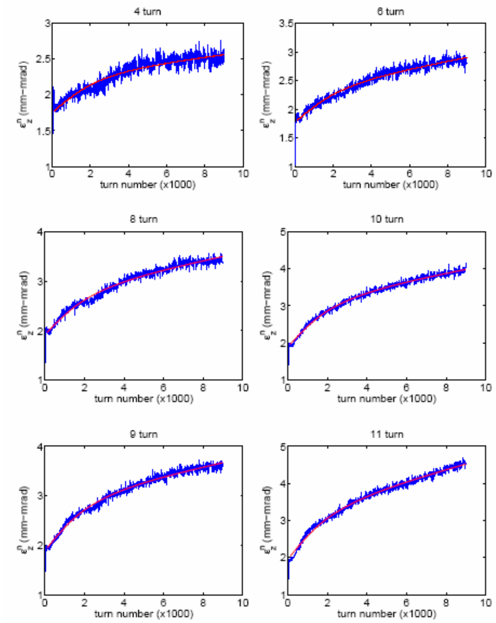
$$K_{sc} = \frac{2Nr_0}{\beta^2\gamma^3}$$

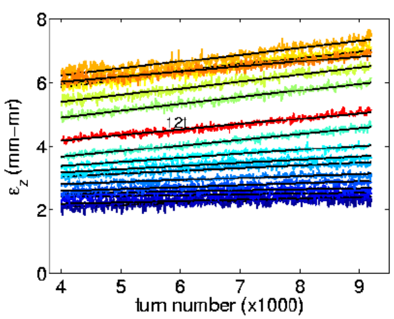
$$\varepsilon = a_0 + b_1 t + b_2 \int_0^t K_{sc}\, dt$$

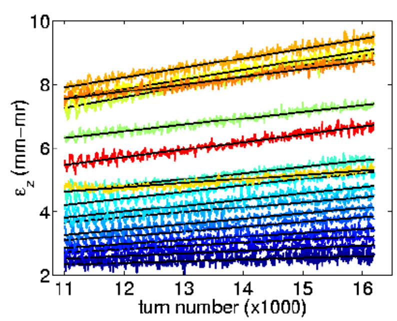
N= number of particles per unit length

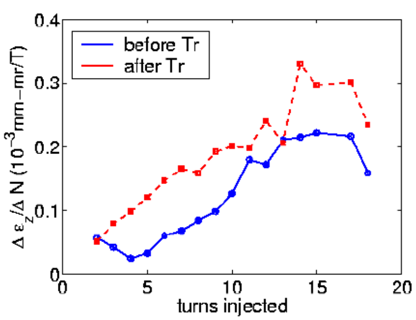








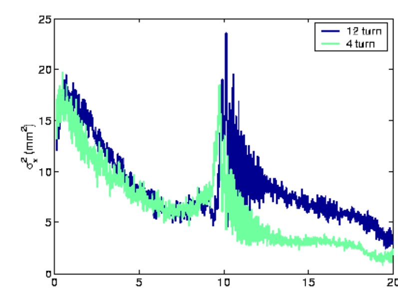


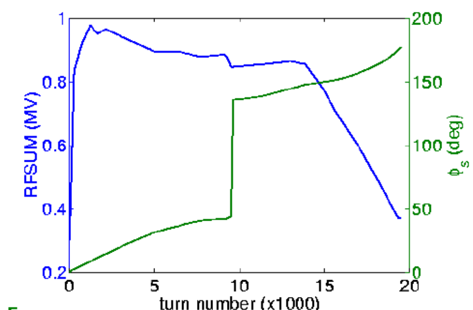


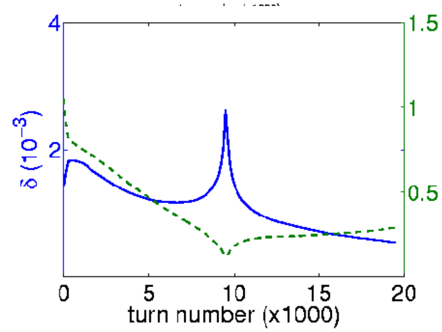
Summary:

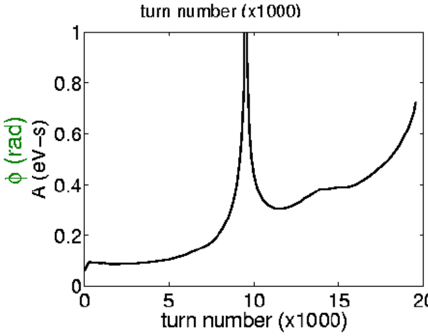
1. $d\varepsilon_z/dt \sim K_{sc}$
2. ε_z increases linearly with t.

Horizontal IPM measurements







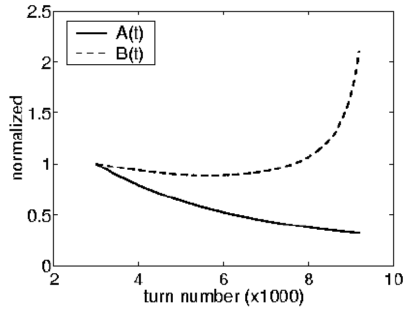


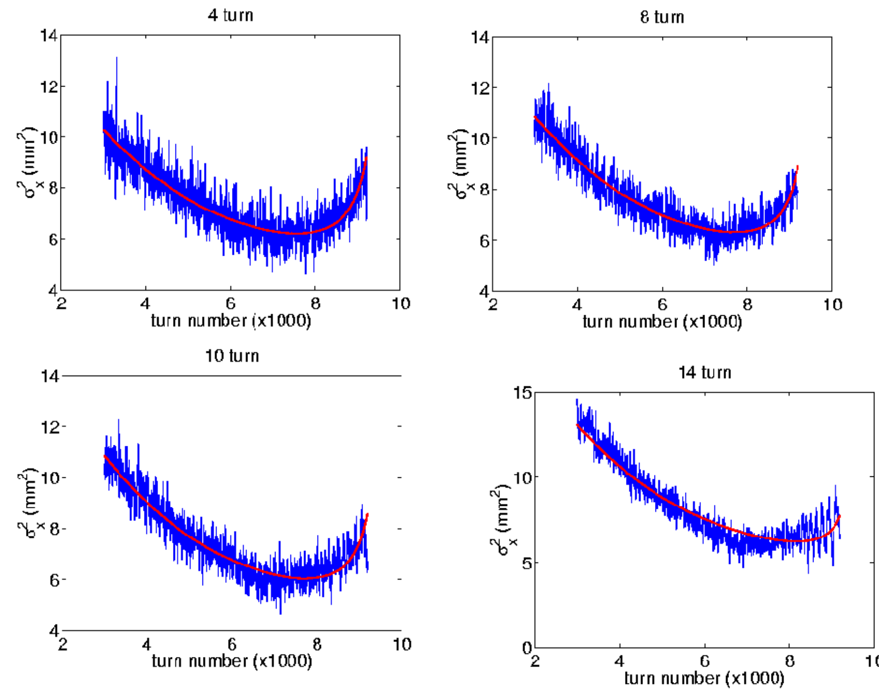
$$\sigma_x^2 = \beta_x \epsilon_{\rm rms} + D^2\sigma_\delta^2 = \beta_x \frac{\epsilon_{\rm rms}^n}{\beta\gamma}\frac{\beta_{x0}(\beta\gamma)_0}{\beta_{x0}(\beta\gamma)_0} + D^2\sigma_{\delta 0}^2 \frac{\sigma_\delta^2}{\sigma_{\delta 0}^2} = aA(t) + bB(t)$$

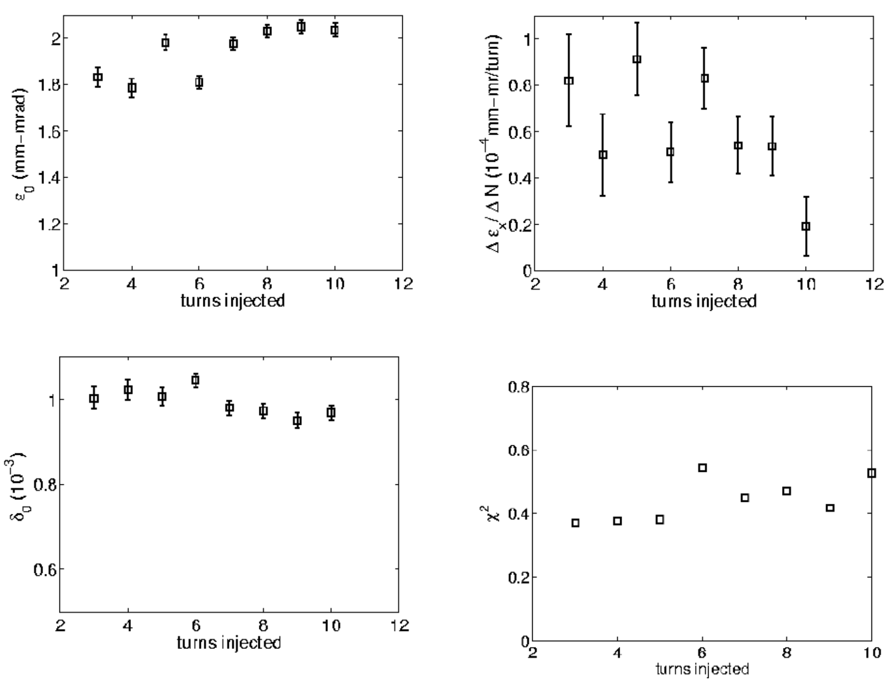
$$a = \epsilon_{\rm rms}^n \frac{\beta_{x0}}{\beta_0\gamma_0}, \quad A(t) = \frac{\beta_x}{\beta_{x0}}\frac{\beta_0\gamma_0}{\beta\gamma}$$

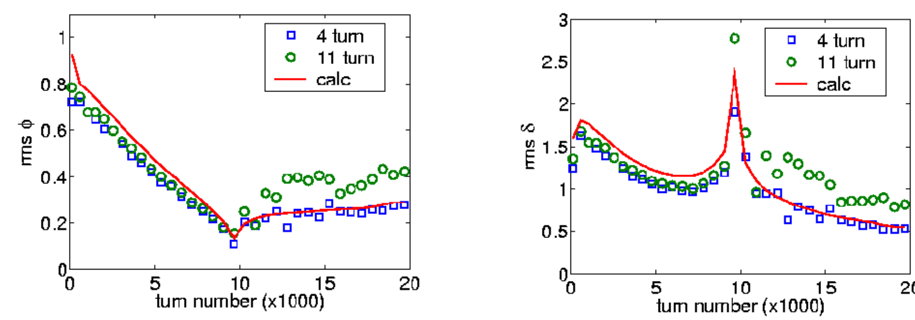
$$b = D^2\sigma_{\delta 0}^2, \quad B(t) = \frac{\gamma_0\sqrt{\gamma_0|\eta_0|/V_0|\cos\phi_{s0}|}}{\gamma\sqrt{\gamma|\eta|/V|\cos\phi_s|}},$$

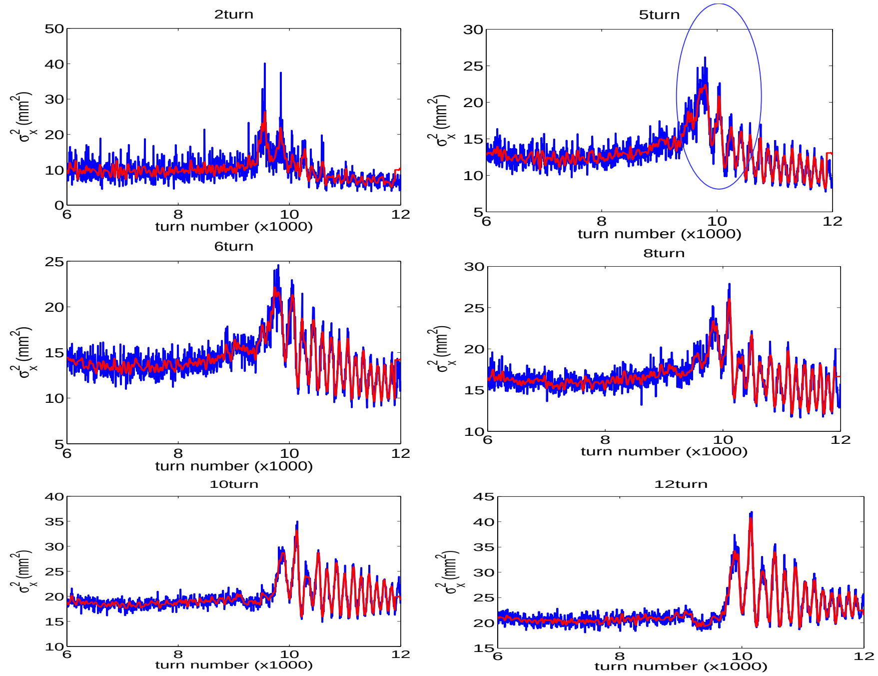
$$\sigma_x^2 = (a_0 + a_1 t)A(t) + b_0 B(t).$$











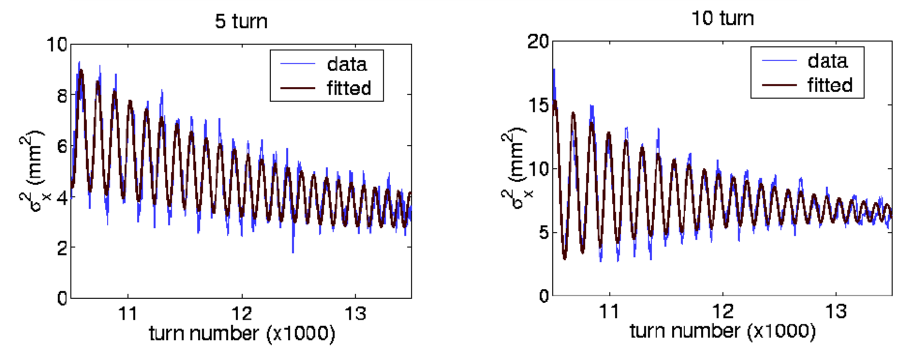
Post Transition Horizontal beam size oscillation

$$\sigma_x^2(t) = a + bt + ct^2 + A\exp(-\alpha t)\cos[2\pi(f_1 t + f_2 t^2) + \chi],$$

$$2A = \sigma_{x,\max}^2 - \sigma_{x,\min}^2.$$

$$\sigma_{x,\text{static}}^2 = a + bt + ct^2$$

$$\sigma_{x,\text{static}}^2 \;=\; \beta_x\epsilon_{x,\text{rms}} + D^2\bar{\delta}^2 = \frac{\beta_x}{\beta\gamma}\epsilon_{x,\text{rms}}^n + D^2\bar{\delta}^2 = \frac{\beta_x}{\beta\gamma}\epsilon_{x0}^n(1+\alpha_x t) + D^2\bar{\delta}^2,$$



$$\sigma_x^2 = \beta_x\varepsilon_{\text{rms}} + D^2\delta^2$$
$$= a + bt + ct^2 + Ae^{-\alpha t}\cos[2\pi(f_1 t + f_2 t^2) + \varphi]$$

$$\delta = (\Delta p/p)_{\text{rms}}$$

$$a + bt + ct^2 = \beta_x\varepsilon_{\text{rms}} + D^2\bar{\delta}^2$$

$$f_{\text{syn}}(t) = f_1 + 2f_2 t$$

$$2A = D^2(\delta_1^2 - \delta_2^2)$$

$$\delta_1 = \frac{\gamma_T}{3^{1/6}\beta\tau_{\text{ad}}\Gamma(\frac{2}{3})}\left(\frac{2A_{t\Delta E}}{3mc^2\dot{\gamma}}\right)^{1/2}$$

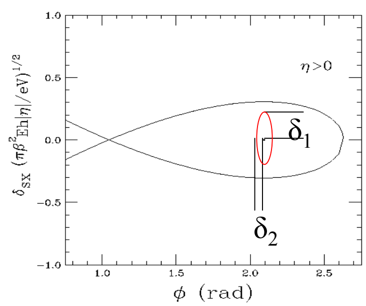
$$\delta_2 = \frac{\nu_s A_{\phi\delta}}{h\eta\pi\delta_1}$$

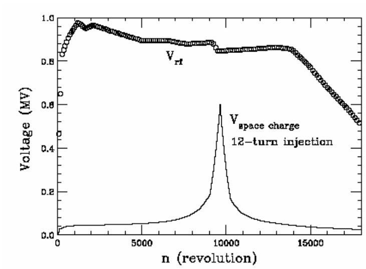
$$A_{\phi\delta} = \frac{h\omega_0}{\beta^2\gamma_T mc^2}A_{t\Delta E}$$

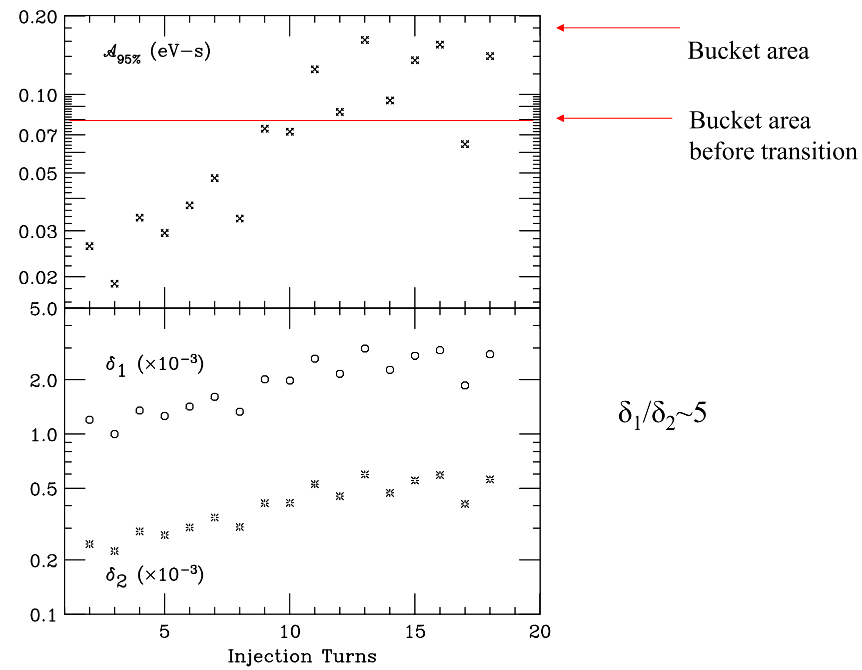
$$\tau_{\text{ad}} = \left(\frac{\pi\beta^2\gamma_T^4 mc^2}{\dot{\gamma}\omega_0^2 heV\,|\cos\phi_s\,|}\right)^{1/3}$$

$$\tau_{\text{ad}} = \left(\frac{\pi\beta^2 mc^2\gamma_T^4}{\dot{\gamma}\omega_0^2 heV\,|\cos\phi_s|}\right)^{1/3} \approx 0.20 \text{ ms},$$

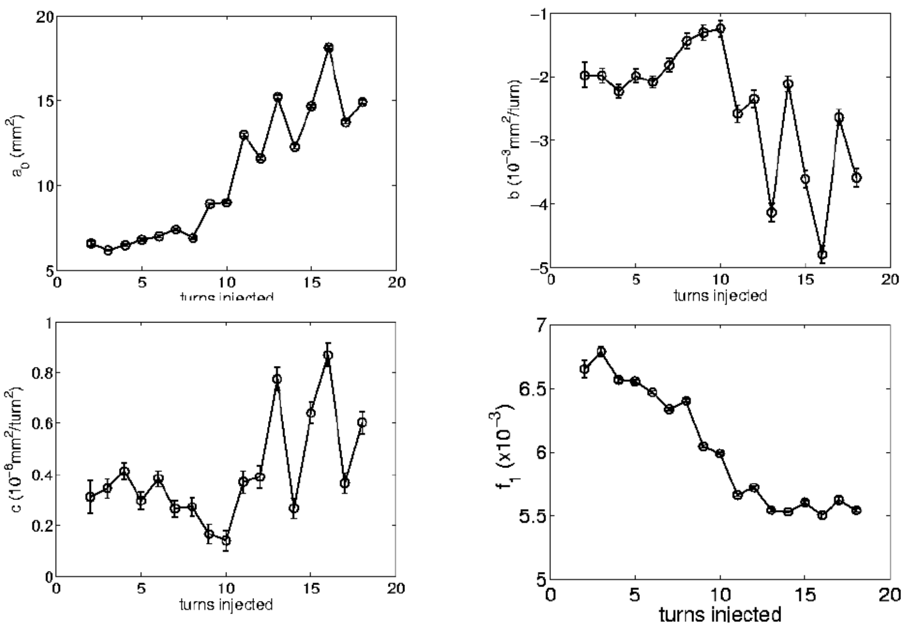
$$\tau_{\text{nl}} = \gamma_T^3\frac{\eta_1}{2\dot{\gamma}}\hat{\delta} = \gamma_T\frac{\frac{3}{2}\beta_0^2 + \gamma_T^2\alpha_1}{2\dot{\gamma}}\hat{\delta} \approx 0.07 \text{ ms},$$

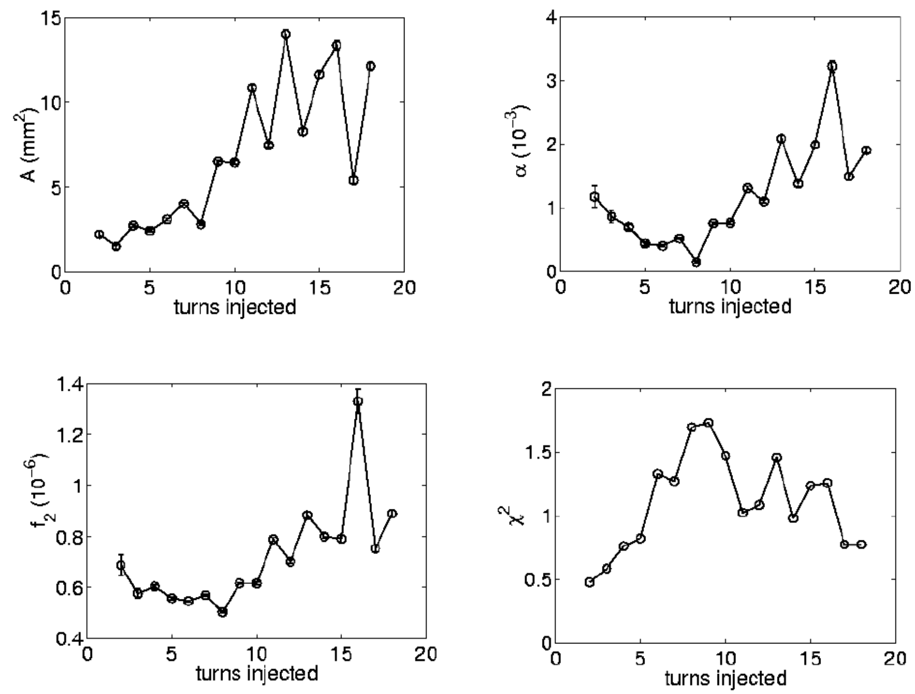






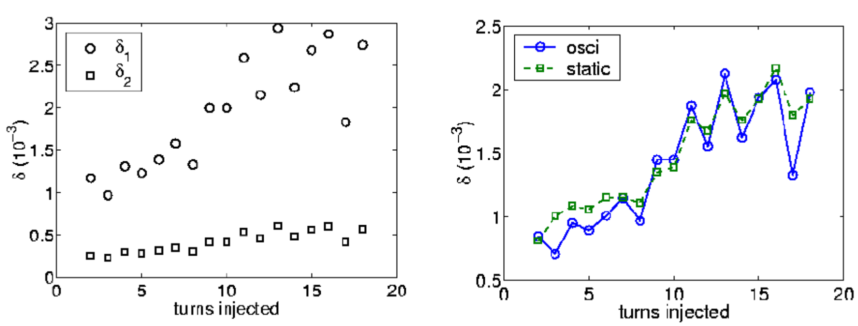
$\delta_1/\delta_2 \sim 5$





$$\bar{\delta}_{\rm static} = \frac{\sqrt{\sigma_x^2 - \beta_x \epsilon_{x,\rm rms}}}{D},$$

$$\bar{\delta}_{\rm osci} = \sqrt{\frac{\delta_1^2 + \delta_2^2}{2}},$$



Summary:

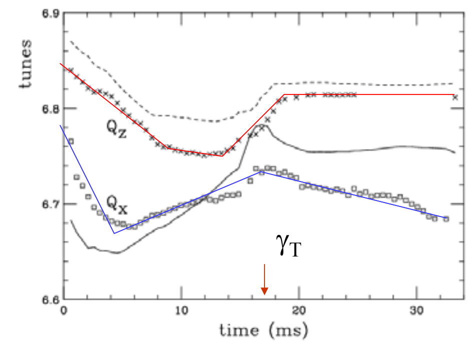
1. $d\varepsilon_z/dt \sim K_{sc}$
2. ε_z increases linearly with t at about 1 π-mm-mrad in 10^4 revolutions.
3. The horizontal emittance and the off-momentum spread can be separated by using different scaling law (energy dependence).
4. The horizontal emittance is less affected by the space charge force! Why
5. The slow linear growth of the horizontal emittance is the same as that of the vertical plane!
6. The post-transition horizontal bunch-width oscillation is induced essentially by the longitudinal mis-match of the bunch shape with rf potential well. Using the bunch shaoe mis-match, one can deduce the phase space area.

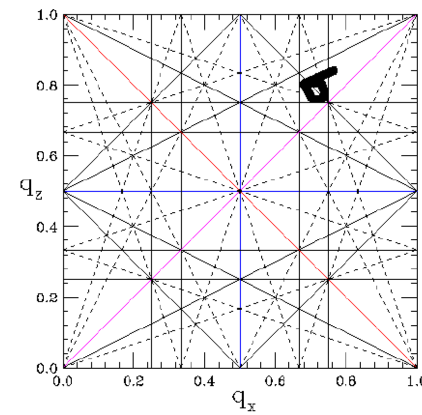
Modeling algorithm:

We consider N-particle in Gaussian distribution and construct a model with 24 Superperiod FODO cells

$$M_{\mathrm{D}\to\mathrm{F}} = \begin{pmatrix} \sqrt{\frac{\beta_{x,\mathrm{F}}}{\beta_{x,\mathrm{D}}}}\cos\psi_x & \sqrt{\beta_{x,\mathrm{F}}\beta_{x,\mathrm{D}}}\sin\psi_x & 0 & 0 \\ -\frac{1}{\sqrt{\beta_{x,\mathrm{F}}\beta_{x,\mathrm{D}}}}\sin\psi_x & \sqrt{\frac{\beta_{x,\mathrm{D}}}{\beta_{x,\mathrm{F}}}}\cos\psi_x & 0 & 0 \\ 0 & 0 & \sqrt{\frac{\beta_{z,\mathrm{F}}}{\beta_{z,\mathrm{D}}}}\cos\psi_z & \sqrt{\beta_{z,\mathrm{F}}\beta_{z,\mathrm{D}}}\sin\psi_z \\ 0 & 0 & -\frac{1}{\sqrt{\beta_{z,\mathrm{F}}\beta_{z,\mathrm{D}}}}\sin\psi_z & \sqrt{\frac{\beta_{z,\mathrm{D}}}{\beta_{z,\mathrm{F}}}}\cos\psi_z \end{pmatrix}$$

$$M_{\mathrm{F}\to\mathrm{D}} = \begin{pmatrix} \sqrt{\frac{\beta_{x,\mathrm{D}}}{\beta_{x,\mathrm{F}}}}\cos\psi_x & \sqrt{\beta_{x,\mathrm{F}}\beta_{x,\mathrm{D}}}\sin\psi_x & 0 & 0 \\ -\frac{1}{\sqrt{\beta_{x,\mathrm{F}}\beta_{x,\mathrm{D}}}}\sin\psi_x & \sqrt{\frac{\beta_{x,\mathrm{F}}}{\beta_{x,\mathrm{D}}}}\cos\psi_x & 0 & 0 \\ 0 & 0 & \sqrt{\frac{\beta_{z,\mathrm{D}}}{\beta_{z,\mathrm{F}}}}\cos\psi_z & \sqrt{\beta_{z,\mathrm{F}}\beta_{z,\mathrm{D}}}\sin\psi_z \\ 0 & 0 & -\frac{1}{\sqrt{\beta_{z,\mathrm{F}}\beta_{z,\mathrm{D}}}}\sin\psi_z & \sqrt{\frac{\beta_{z,\mathrm{F}}}{\beta_{z,\mathrm{D}}}}\cos\psi_z \end{pmatrix}$$





Space charge force is a local kick on every half cell:

$$\rho(x,z) = \frac{Ne}{2\pi\sigma_x\sigma_z}\exp\{-\frac{x^2}{2\sigma_x^2} - \frac{z^2}{2\sigma_z^2}\},$$

$$\begin{aligned} V(x,z) &= \frac{Nr_0}{\beta^2\gamma^3}\int_0^\infty \frac{1-\exp\{-\frac{x^2}{2\sigma_x^2+t} - \frac{z^2}{2\sigma_z^2+t}\}}{\sqrt{(2\sigma_x^2+t)(2\sigma_z^2+t)}}dt \\ &\approx \frac{Nr_0}{\beta^2\gamma^3}\left(\frac{x^2}{\sigma_x(\sigma_x+\sigma_z)} + \frac{z^2}{\sigma_z(\sigma_x+\sigma_z)}\right) \\ &\quad -\frac{Nr_0}{4\beta^2\gamma^3\sigma_x^2(\sigma_x+\sigma_z)^2}\left(\frac{2+R}{3}x^4 + \frac{2}{R}x^2z^2 + \frac{1+2R}{3R^3}z^4\right) + \cdots \end{aligned}$$

$$\begin{aligned} \Delta x' &= -\frac{\partial V}{\partial x}\ell \approx \frac{2Nr_0\ell}{\beta^2\gamma^3\sigma_x(\sigma_x+\sigma_z)}x\exp\{-\frac{x^2+z^2}{(\sigma+\sigma_z)^2}\}, \\ \Delta z' &= -\frac{\partial V}{\partial z}\ell \approx \frac{2Nr_0\ell}{\beta^2\gamma^3\sigma_z(\sigma_x+\sigma_z)}z\exp\{-\frac{x^2+z^2}{(\sigma+\sigma_z)^2}\}, \end{aligned}$$

- Sextupole nonlinearity on each half cell for nonlinearity in dipoles
- Linear coupling,
- Random quadrupoles with zero tune shifts
- Random closed orbit error
- Dynamical aperture of 80 by 50 pi-mm-mrad

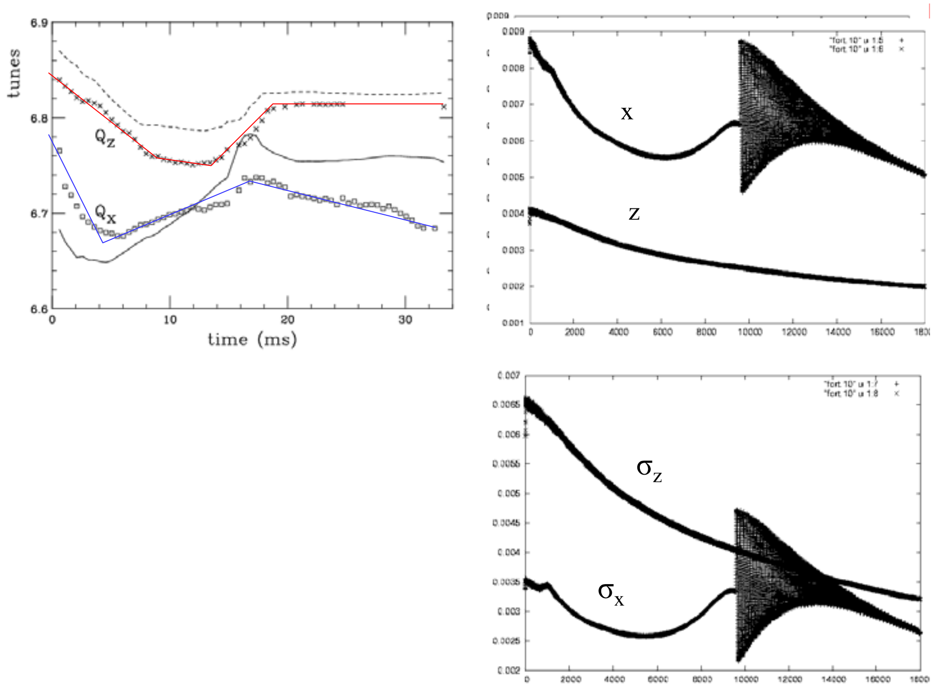
$$x'' + K_x(s)x = \frac{b_0(s)}{\rho} + \frac{b_1(s)}{\rho}x + \frac{a_1(s)}{\rho}z + \frac{1}{2}\frac{b_2(s)}{\rho}(x^2 - z^2)$$

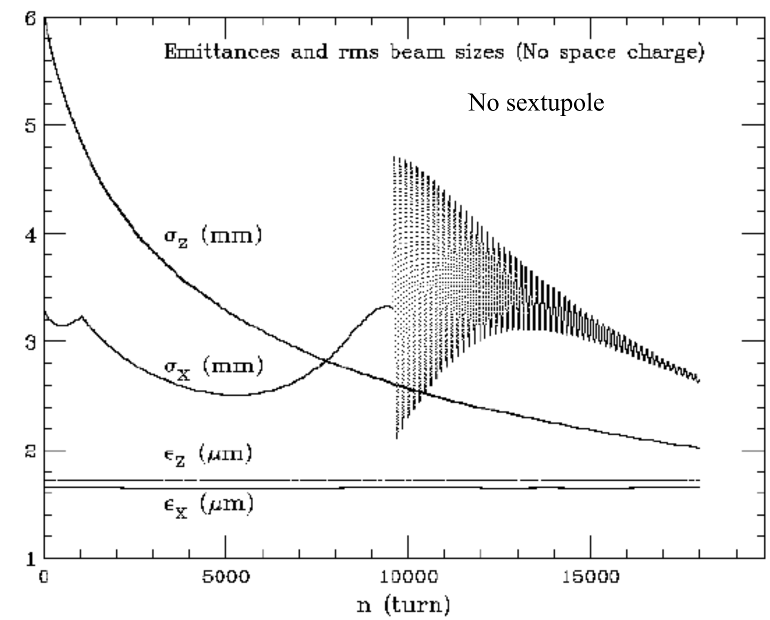
$$z'' + K_z(s)z = -\frac{a_0(s)}{\rho} - \frac{b_1(s)}{\rho}z + \frac{a_1(s)}{\rho}x - \frac{b_2(s)}{\rho}xz$$

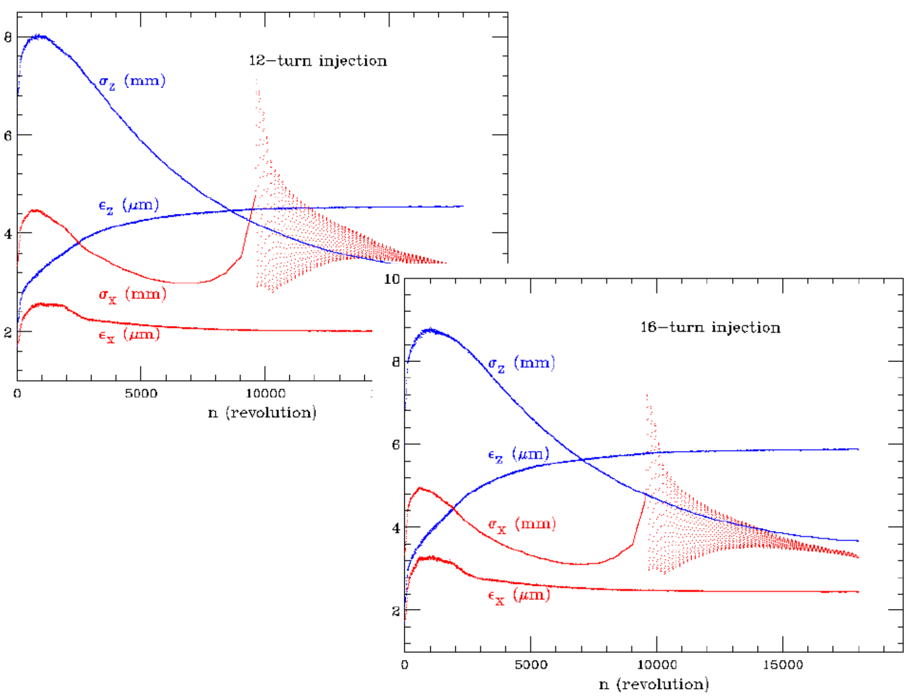
Random number generators are used to generate b0, a0, b1, and a1. The quadrupole error is subject to a constraint with zero tune shift. The integrated sextupole strengths are set to the systematic values: -0.0173 m^{-2} and -0.263 m^{-2} for focusing and defocusing dipoles respectively.

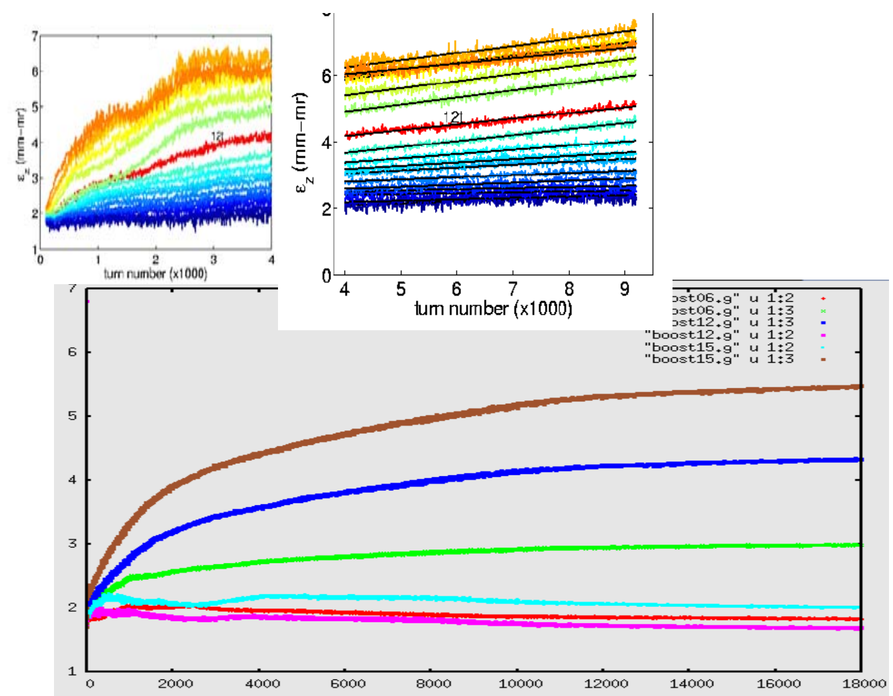
$$\begin{aligned} \delta_{\mathrm{rms}}(n) &= \delta_{\mathrm{rms}}(1)B_f(n)\,(1+(G_\delta-1)(1-\exp(-\alpha_g(n-n_t))) \\ &\quad [1+A_\delta\exp(-\alpha_g(n-n_t))\sin(2\pi(n-n_t)f)]\,, \end{aligned}$$

(For $n>n_t=9600$) $G_\delta=2$, $A_\delta=0.5$, $f=1/150$, $\alpha_\delta=1/(15*150)$


tunes
Q_z
Q_x
time (ms)
x
z
σ_z
σ_x

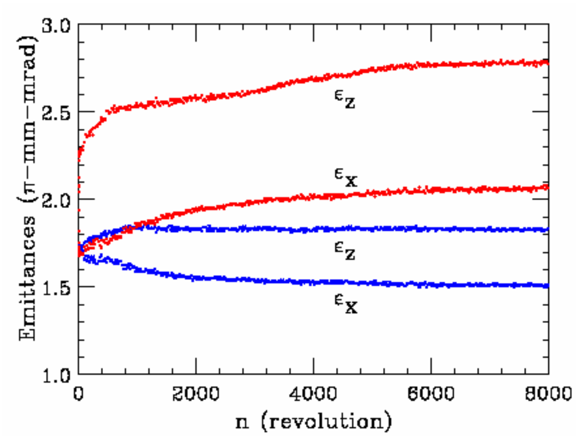

Emittances and rms beam sizes (No space charge)
No sextupole
σ_z (mm)
σ_x (mm)
ϵ_z (μm)
ϵ_x (μm)
n (turn)


12−turn injection
σ_z (mm)
ϵ_z (μm)
σ_x (mm)
ϵ_x (μm)
n (revolution)
16−turn injection
σ_z (mm)
ϵ_z (μm)
σ_x (mm)
ϵ_x (μm)
n (revolution)

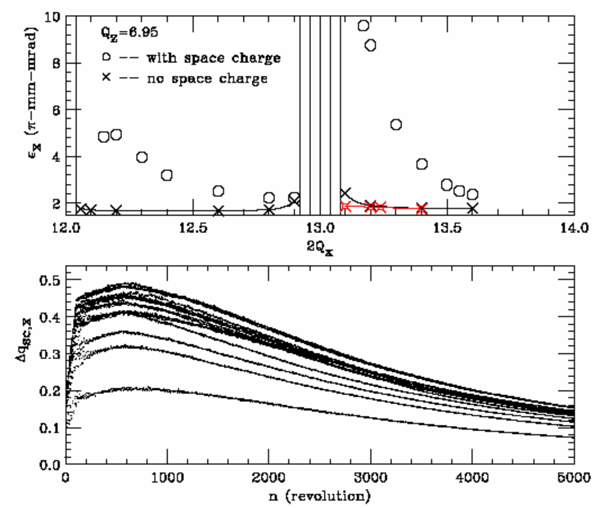

ϵ_z (mm−mr)
turn number (x1000)
ϵ_z (mm−mr)
turn number (x1000)
"boost12.g" u 1:2
"boost12.g" u 1:3
"boost15.g" u 1:2
"boost15.g" u 1:3

Effect of dipole field errors:

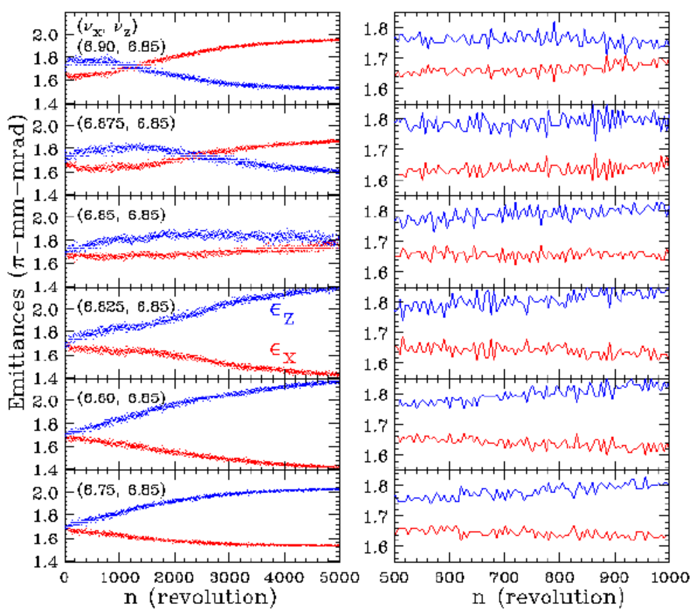
Red curves: Δx’=0.000020 rad, Δz’=0.000075 rad
Blue curves: No dipole error
Betatron tunes are regular operation tunes



Effect of random quadrupole field error: 40e-4 m-1, while changing the horizontal betatron tune from 6 to 7.



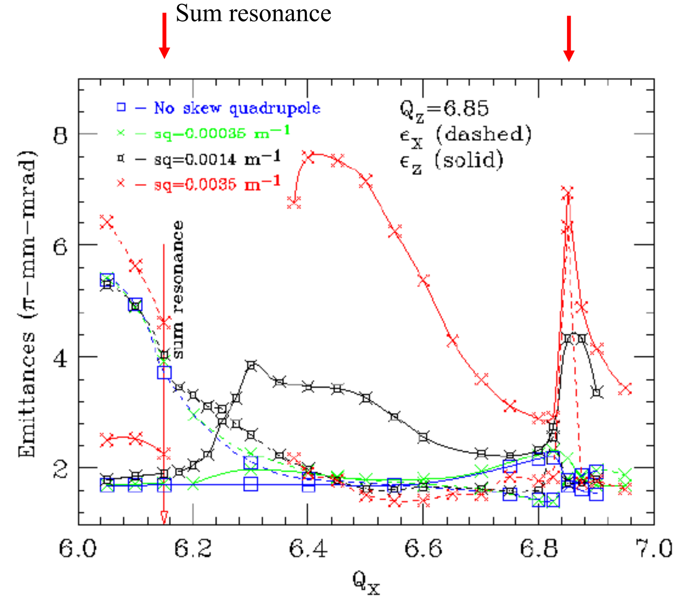
Montague Resonance: (No linear errors)

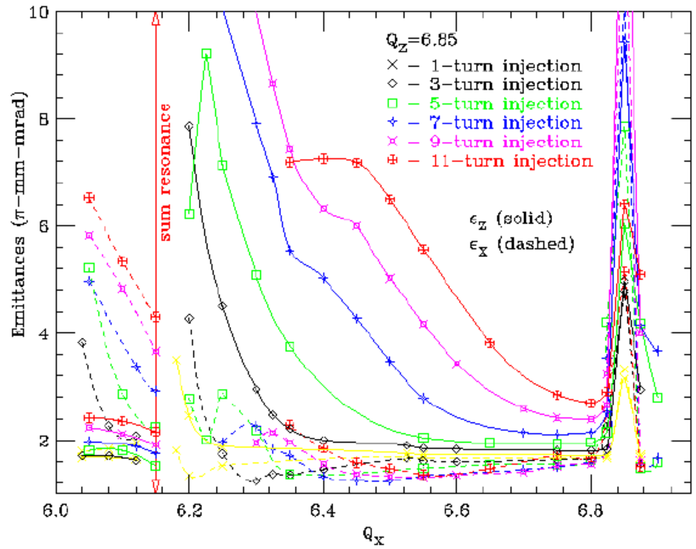


Effects of linear skew quadrupoles

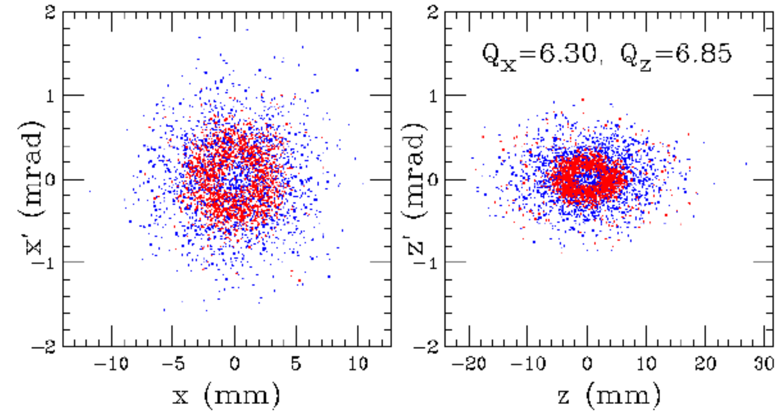
Different resonance

Sum resonance





Effect of a sum resonance!



Conclusion:

1. We are able to fit experimental measured IPM data (vertical and horizontal) to deduce the betatron emittances of the beam.
2. We note that the vertical emittance increases rapidly in the initial stage of acceleration. The horizontal emittance seems to be less affected by the space charge at the initial stage.
3. The bunch width oscillation after the transition crossing has been used to deduce the bunch-mismatch factor, and the result is consistent with the rf program in the Booster operation.
4. We use the rms space charge force model to carry out numerical simulations. We are able to fit the observed data of emittance growth. The emittance growth in the vertical plane has resulted mainly from the skew quadrupoles, that induce a sum resonance at Q_x+Q_z=integer. This induces emittance growth mainly in the vertical plane. The reason is that the Montague resonance suppresses the growth of the horizontal emittance, and enhance the vertical emittance by about 25%. The random dipole field error also generates about 25% vertical emittance growth.
5. **Need more work to understand more detailed mechanism of skew quadrupoles in the presence of Montague Resonance!**