Main Injector RF Requirements for a 1.0 Megawatt 120 GeV Proton Source

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INTRODUCTION

Beam loading will dominate the operation of the Main Injector RF system for 120 GeV beam powers greater than 700kW. This note outlines a simple calculation of the RF power requirements in the Main Injector for a 1.0 MW 120 GeV proton source described in Beams Document 1782.

MAIN INJECTOR RF RAMPS

To deliver 1.0MW at 120 GeV, eighteen Booster batches are combined by momentum stacking in the Accumulator and box-car stacking in the Recycler. To load eighteen batches into the Accumulator and Recycler at 15 Hz requires 1.2 seconds. However, the minimum cycle time of the Main Injector is limited to 1.33 seconds because of the ramp rate of the magnet power supplies. A cycle time of 1.33 seconds is equivalent to twenty Booster 15Hz cycles so the Booster would run at a 90% duty factor. The energy ramp used in this note is show in Figure 1. The ramp acceleration time is 0.6 seconds with a parabola time of 0.133 seconds. The maximum acceleration rate is 240 GeV per second.

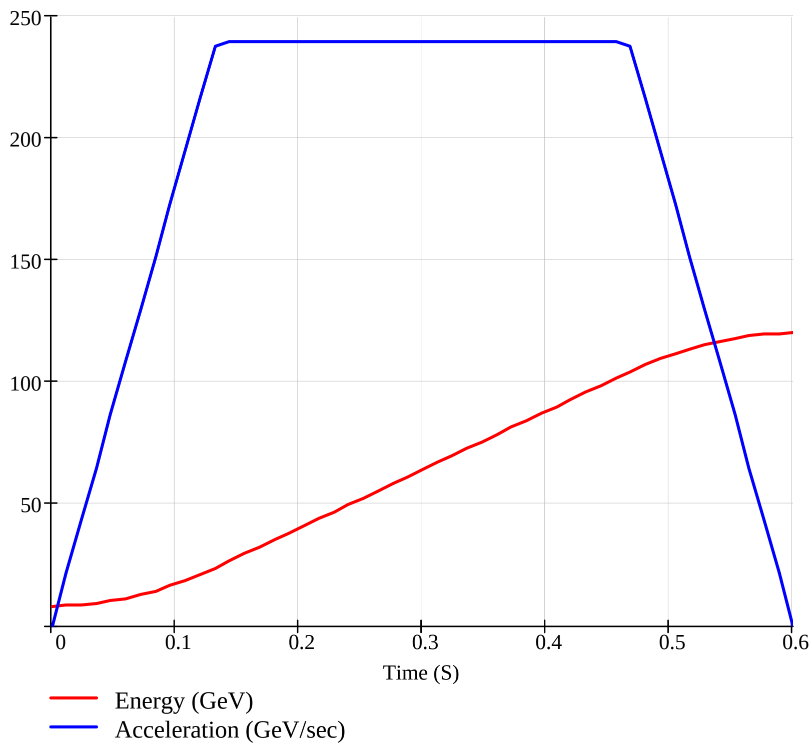

Figure 1. Main Injector Energy Ramp

The RF voltage curve is determined by the acceleration rate and the bucket area. The intensity of each Booster batch is $4\text{x}10^{12}$ protons resulting in a total intensity of $7.2\text{x}10^{13}$ protons injected into the Main Injector. The longitudinal emittance of the beam is set by the momentum stacking process in the Accumulator. The momentum stacking process dilutes the longitudinal emittance by about twenty percent. For a total of eighteen Booster batches loaded into the Main Injector, it would be necessary to stack three Booster batches into the Accumulator and six Accumulator batches into the Recycler. The longitudinal emittance of a 53MHz bunch in the Booster is 0.08eV-Sec. A twenty percent emittance dilution for stacking three Booster batches results in an equivalent emittance of 0.38eV-Sec for a 53 MHz RF bucket. It will be assumed that momentum dilution in the Recycler will not increase the longitudinal emittance above 0.5eV-sec per 53MHz bucket. A minimum RF bucket area of 1.0eV-sec in the Main Injector should be sufficient to contain the bunches. The RF voltage is shown in Figure 2. The bucket area shown is held constant at 1 eV-sec for most of the ramp except near transition and near extraction. To hold a 1eV-sec bucket area during these times would require the total RF voltage to drop unrealistically low. In this note, the RF voltage will be held constant though transition and will linearly decrease during the high energy parabola.

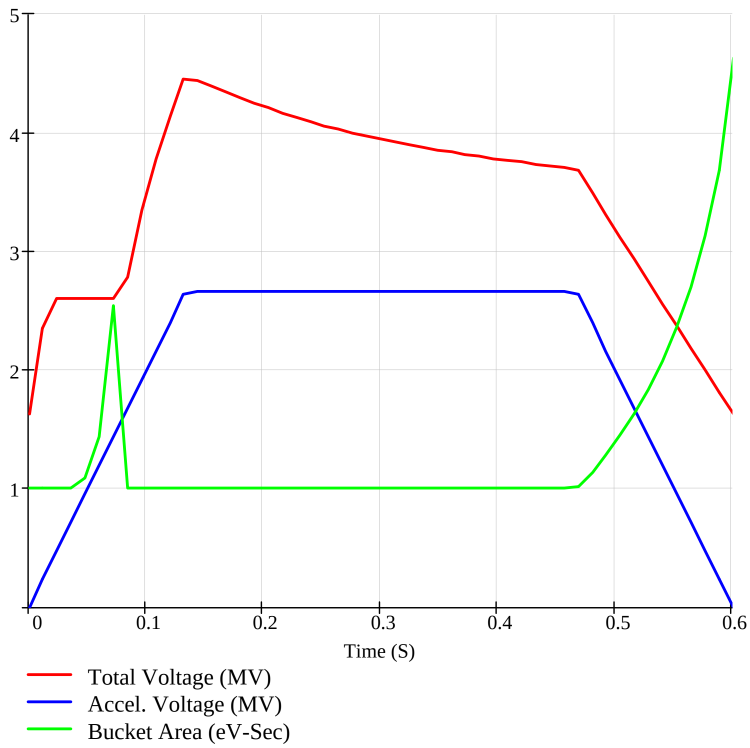

Figure 2. RF voltage curves for a 1.0 eV-Sec bucket area.

RF BEAM CURRENT.

The RF beam current is a function of the bunch length. If the bunches are long then the amount of beam current that the RF cavities see will be reduced. However because of the large synchronous phase angle in the Main Injector during acceleration, the bunch length is very short even for full RF buckets. Particles execute trajectories in phase space according to:

$$H = \frac{A}{2} y^2 + B\left(\phi \sin \phi_s \mp \left(\cos(\phi) - 1\right)\right) \tag{1}$$

where H is a constant for a given trajectory and the top sign is used above transition. The normalized particle energy deviation **y** is:

$$y = \frac{\Delta E}{\omega_{rf}} \tag{2}$$

which has units of eV-Sec. The coefficients A and B are given as:

$$A = \left(\frac{\omega_{rf}}{\beta}\right)^2 \frac{\eta}{E_s} \tag{3}$$

$$B = \frac{qV_{rf}}{2\pi h} \tag{4}$$

The bucket edge is given by the value of H that satisfies:

$$\begin{aligned} \phi &= \pm(\phi_s - \pi) \\ y &= 0 \end{aligned} \tag{5}$$

The voltage for a given bucket area reaches its largest value at the end of the low energy parabola as shown in Figure 2. The bucket shape at the end of the low energy parabola for a 1 eV-Sec bucket is shown as the blue trace in Figure 3.

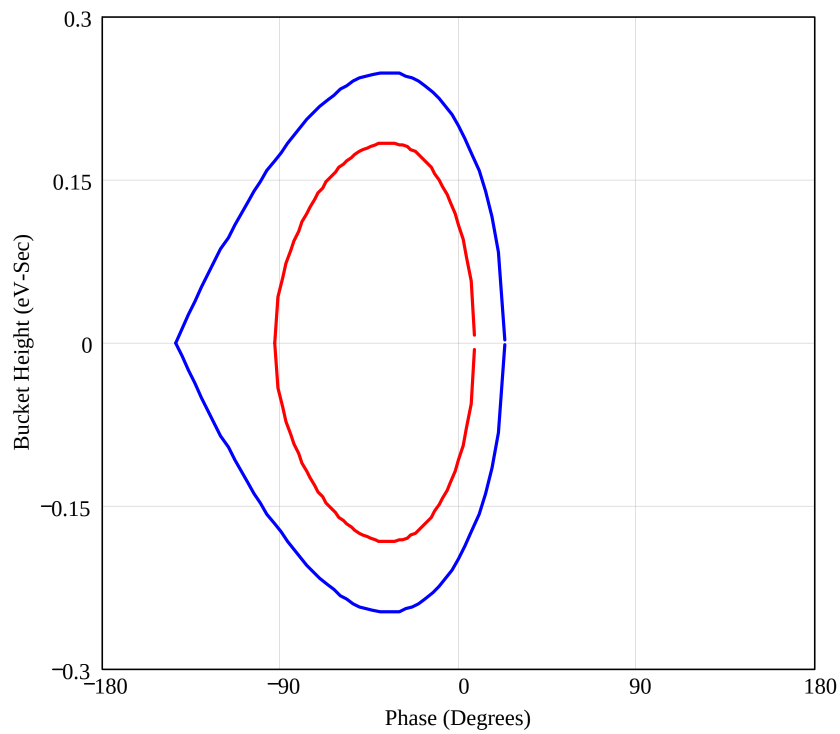

Figure 3. Phase space of a 1eV-sec bucket at the end of the low energy parabola is shown in blue. The beam edge for a longitudinal emittance of 0.5eV-Sec. is shown in red.

The beam edge for 0.5eV-Sec longitudinal emittance is shown as the red trace of Figure 3. Even though the bucket is rather full, the bunch only extends along 90 degrees of phase. If the phase space density is uniform, then the RF beam current is given as:

$$i_b = \frac{4I_{dc}}{f_R \varepsilon_L} \left| \int_{-\pi}^{\pi} y_e(\phi) e^{j\phi} d\phi \right| \quad (6)$$

where y_e is the edge of the beam phase space, I_{dc} is the DC beam current, ε_L is the longitudinal emittance, and f_R is the ring fill factor which is 6/7 for the Main Injector. Figure 4 shows the ratio of RF beam current to DC beam current for the voltage profile shown in Figure 3.

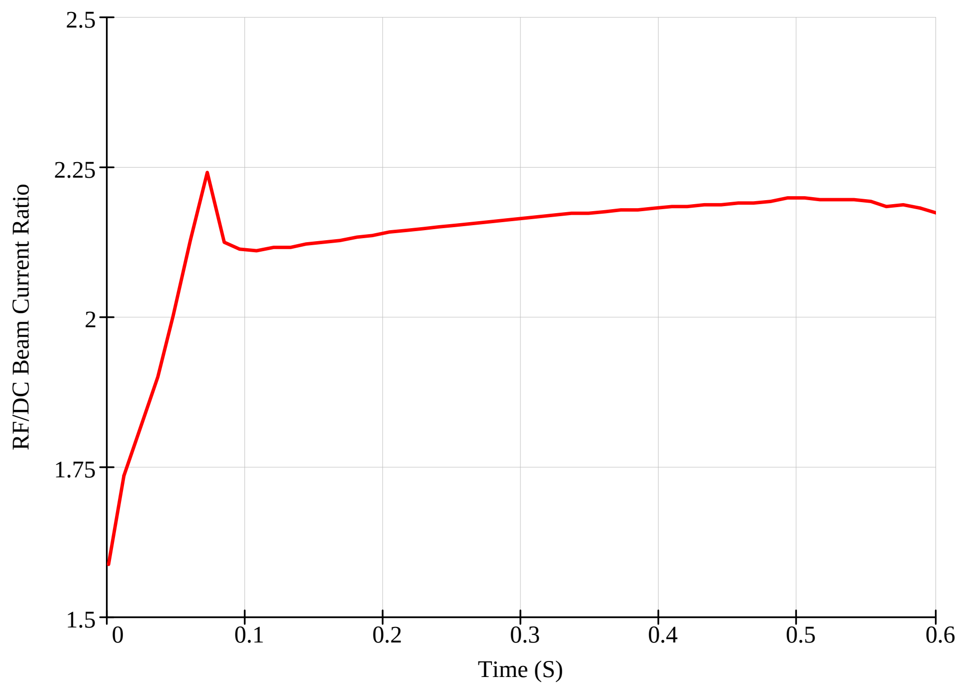

Figure 4. Ratio of RF beam current to DC beam current for a uniform density of 0.5eV-sec in a 1.0eV-sec bucket. The ring fill factor is 6/7.

Beam Loading

The equivalent circuit model for the cavity is shown in Figure 5. The impedance of the cavity is given as:

$$Z_c = R_c \cos(\phi_z) e^{\pm j\phi_z} \quad (7)$$

where R_c is the real part of the cavity impedance and ϕ_z is the cavity detuning angle. The power amplifier is modeled as a current source nI_g with internal impedance R_g/n^2. The coupling of the power amplifier to the cavity is modeled as a step-up transformer with **n** turns. For the Main Injector cavities, this step-up ratio is approximately 12~13. The ratio of the cavity resistance to the generator resistance (as seen by the cavity) is defined as the cavity coupling **r**.

$$R_c = rR_g \quad (8)$$

The ratio of the total resistance to the cavity resistance is given as:

$$R = \frac{1}{r+1} R_c \tag{9}$$

For a tetrode power amplifier, r<<1.

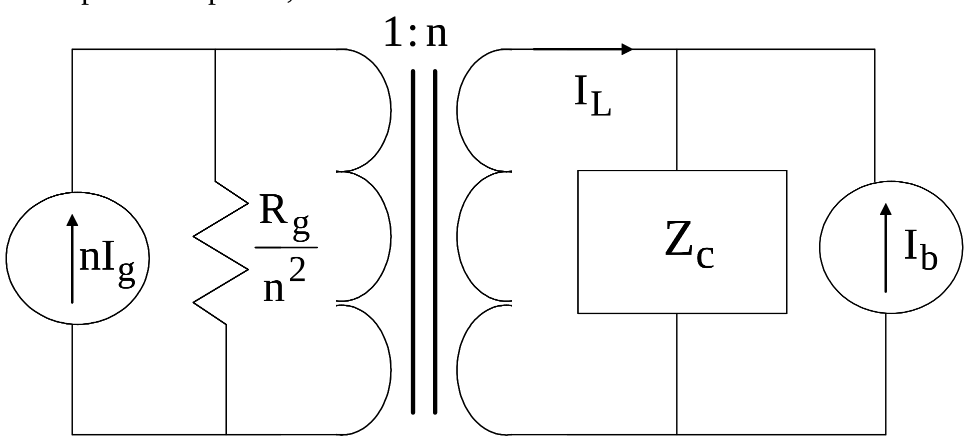

Figure 5. Circuit model for a cavity

The phasor diagram of the cavity circuit above transition is shown in Figure 6. The beam current is given as:

$$I_b = \mp j i_b e^{\mp j\phi_s} \tag{10}$$

The generator current is broken into a component that cancels the beam current (-x$\mathbf{I_b}$) and a component in phase with the cavity voltage (Δi_{gr}).

$$I_g = \Delta i_{gr} \pm jxi_b e^{\mp j\phi_s} \tag{11}$$

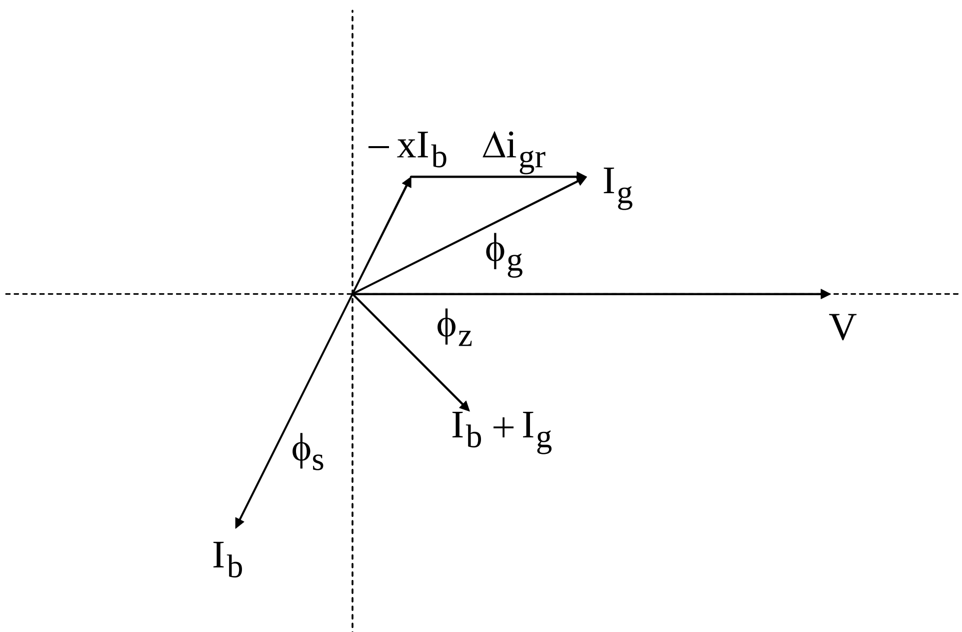

Figure 6. Cavity phasor diagram. The phasor rotates counter-clockwise with time.

The cavity voltage is:

$$V_c = i_b(1-x)R\frac{\cos(\phi_s)}{\tan(\phi_z)} \quad (12)$$

The generator current becomes:

$$I_g = \frac{V_c}{R} + i_b\sin(\phi_s) \pm jxi_b\cos(\phi_s) \quad (13)$$

The current provided by the power amplifier as seen by cavity becomes:

$$I_L = \frac{V_c}{R(r+1)} + i_b\sin(\phi_s) \pm jxi_b\cos(\phi_s)\frac{r+x}{r+1} \quad (14)$$

The power supplied by the power amplifier is:

$$P_L = \frac{1}{r+1}\frac{V_c^{\,2}}{2R} + \frac{V_c i_b}{2}\sin(\phi_s) \quad (15)$$

The power dissipated in the cavity is:

$$P_c = \frac{1}{r+1}\frac{V_c^{\,2}}{2R} \quad (16)$$

ROBINSON STABILITY

The high intensity Robinson threshold requires:

$$\frac{Ri_b}{V_c}(1-x)\frac{\sin(2\phi_z)}{\cos(\phi_s)} = \frac{2}{\sigma^2} \quad (17)$$

$$\sigma^2 > 1$$

Using the equations for the cavity voltage and Robinson threshold, the constraint on cavity de-tuning becomes:

$$\sin^2(\phi_z) = \frac{1}{\sigma^2}\cos^2(\phi_s) \quad (18)$$

The stability factor as a function of cavity voltage and beam current becomes:

$$\sigma^2 = \left(\frac{V_c}{i_b R(1-x)}\right)^2 + \cos^2(\phi_s) \quad (19)$$

There are two approaches to keep the beam stable for large beam currents. The simplest approach is to reduce the cavity resistance or the cavity Q with an external load. This approach permits the generator current to be in phase with the cavity voltage so that the power amplifier sees a real load which minimizes plate dissipation. However, the benefits of this approach are offset by the increased power requirements of having to generate more power just to develop the desired cavity voltage.

The other approach is to cancel some of the beam current with a generator current. This cancellation can be done with either a feed-forward system or RF feedback. The advantage to this approach is that there is no extra power wasted in an external load.

The disadvantages of the approach are the complexity of the feed-forward or feedback systems and the generator current is out of phase with the cavity voltage so that the power amplifier does not see a real load.

REQUIRED POWER AMPLIFIER POWER

The power required from the power amplifier will be calculated in this section. The parameters used in this calculation are shown in Table 1. The stability factor (σ^2) for 700kW and 1 MW of beam power is shown in Figure 7. The active beam loading curves, shown in red, use 90% beam loading compensation. Ninety percent compensation is equivalent to 19dB of loop gain of RF feedback. The stability factor resulting from resistively loading the cavities to a Q of 1000 (25% of nominal) is shown in the blue traces.

The delivered RF current from the power amplifier as seen by the cavity is shown in Figure 8. The delivered RF power from the power amplifier is shown in Figure 9. Even though resistive loading has less then a factor of two less stability as compared to active beam loading compensation, resistive loading requires almost double the RF current and power from the power amplifier.

Parameter	Value	Units
RF Frequency	53	MHz
Harmonic	588	
γ_t	13.2	
Ramp time	0.6	S
Cycle Time	1.33	S
Parabola Time	0.133	S
Number of Cavities	19	
R/Q	100	Ω
Q	4000	
Coupling	0.1	
Coupler Step-up Ratio	12.5	
Minimum Bucket Area	1	eV-S
Longitudinal Emittance	0.5	eV-S
Ring Filling Factor	0.86	
Beam Loading Compensation	90%	
Resistive Loading	25%	

Table 1. Main Injector RF Parameters

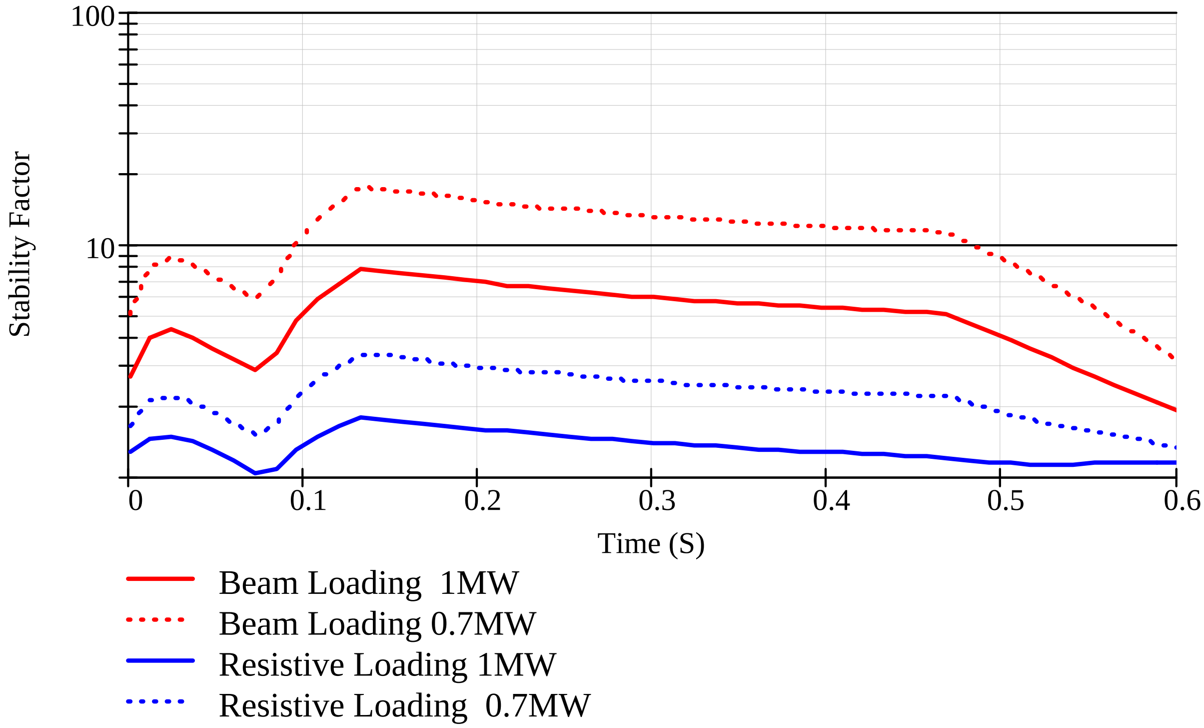

Figure 7. Stability factor (σ^2) for 90% active beam loading compensation (red traces) compared to 25% resistive cavity loading (effective Q=1000) (blue traces).

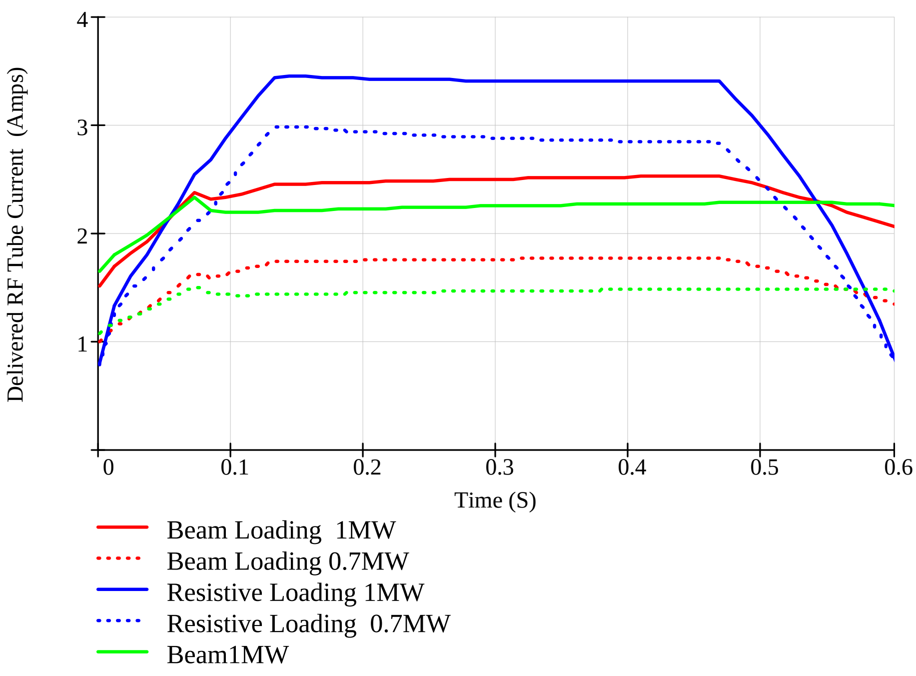

Figure 8. Delivered RF current from the power amplifier as seen by the cavity for 90% beam loading compensation (red traces) and 25% resistive cavity loading (effective Q=1000) (blue traces). The green traces are the beam current as seen by the cavity.

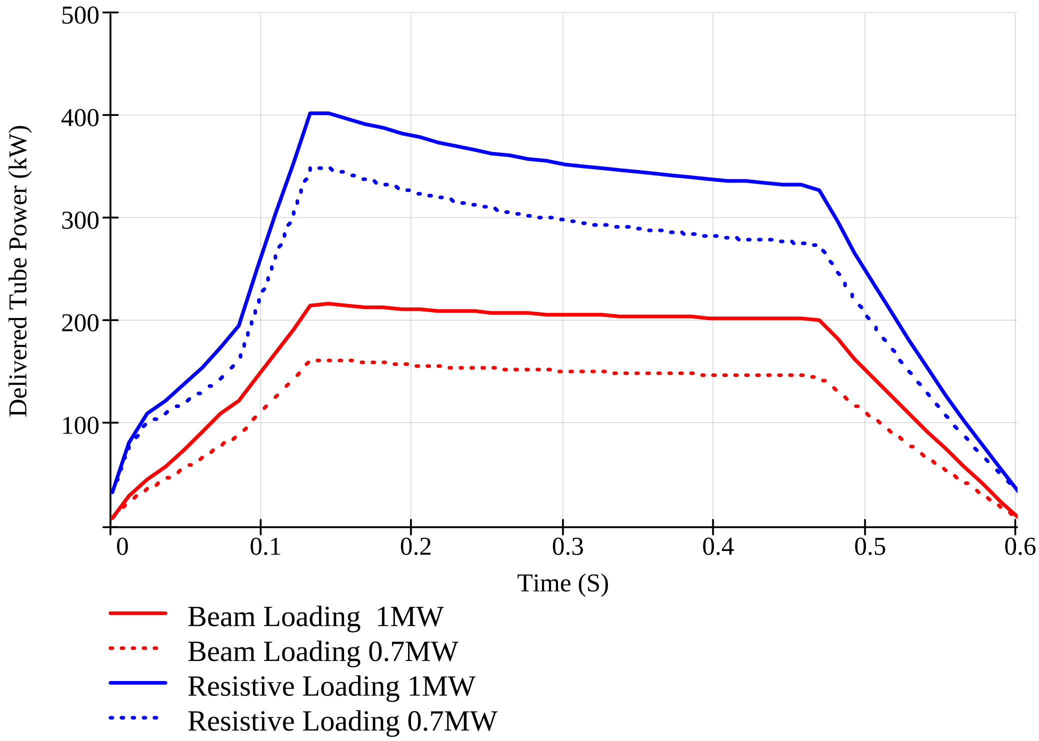

Figure 9. Delivered RF power by the power amplifier for 90% beam loading compensation (red traces) and 25% resistive cavity loading (effective Q=1000) (blue traces).

POWER AMPLIFIER CHARACTERISTICS

A Main Injector Cavity is powered single ended by a single Eimac Y-567B tetrode mounted directly on the cavity. The cavity has an additional port to mount another tube so that cavity can be driven in the push-pull mode. The ratings of the tetrode in Class C operation are shown in Table 2. The constant anode current curves for the tube are shown in Figure 10.

Because the RF anode current as a function of RF anode voltage has been calculated above, it is easier to compute the tube operating points if the tube curves are given in the constant grid voltage form. To perform this transformation, the anode current, for anode voltages above 4kV, is fitted to the following function:

$$I_a = \left[K_a \left(V_g + \frac{V_a}{\mu_a} + \frac{V_s}{\mu_s} \right) \right]^m \tag{20}$$

Figure 11 shows the fit compared to the data at two separate anode voltages for the following parameters:

$$I_a = \left[0.012 \left(V_g + \frac{V_a}{700} + \frac{1250V}{3.8} \right) \right]^{2.732} \tag{21}$$

For a constant screen current, the relationship between grid voltage and anode voltage was fit to the following form:

$$V_g = -v_0(I_s)e^{-\frac{V_a}{v_1(I_s)}} + v_2(I_s) \qquad (22)$$

Where v_0,v_1,v_2 where empirically determined from the curves in Figure 10 and are listed in Table 3. The constant current curves developed from the models in Eqn. 21-22 are shown in Figure 12

	Maximum	Typical	
Plate Voltage	22	20	kV
Screen voltage	2500	1500	V
Grid voltage	-1500	-800	V
Plate current	20	15.2	Adc
Plate Dissapation	150	84	kW
Screen Dissapation	1750	850	W
Grid Dissapation	500	100	W
Output Power		220	kW

Table 2. Ratings for the Eimac Y-567B Tetrode operating as a Class C radio frequency power amplifier

Screen Current	0.2	0.5	1	2	4.1	6	10	14	A
v_0	495	739	725	711	726	1749	1196	848	V
v_1	1959	9007	3845	2842	3058	10537	7010	6694	V
v_2	29	474	354	341	457	1590	1145	919	V

Table 3. Coefficients for screen current fit.

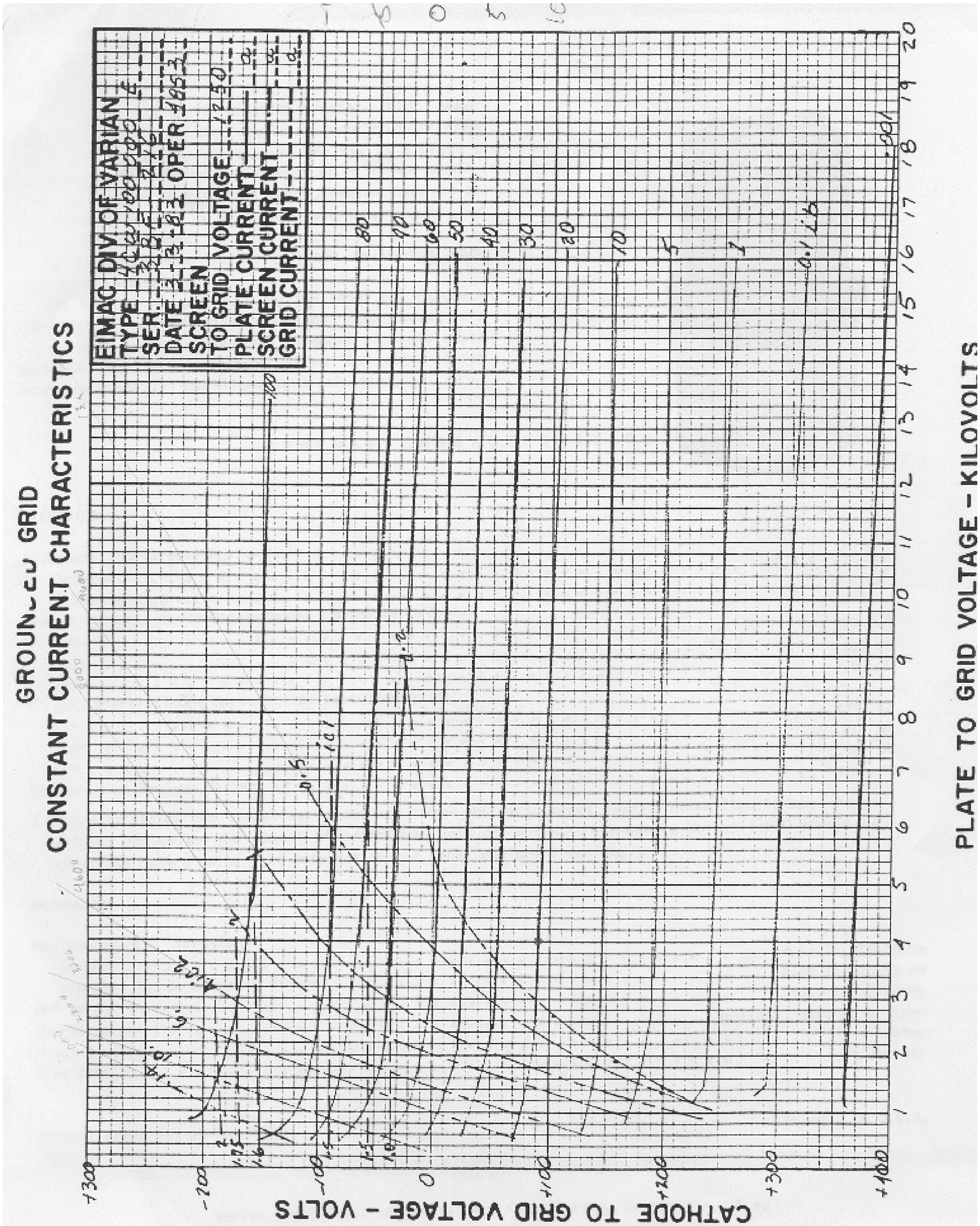

Figure 10. Constant anode current curves for the Y-567B tetrode.

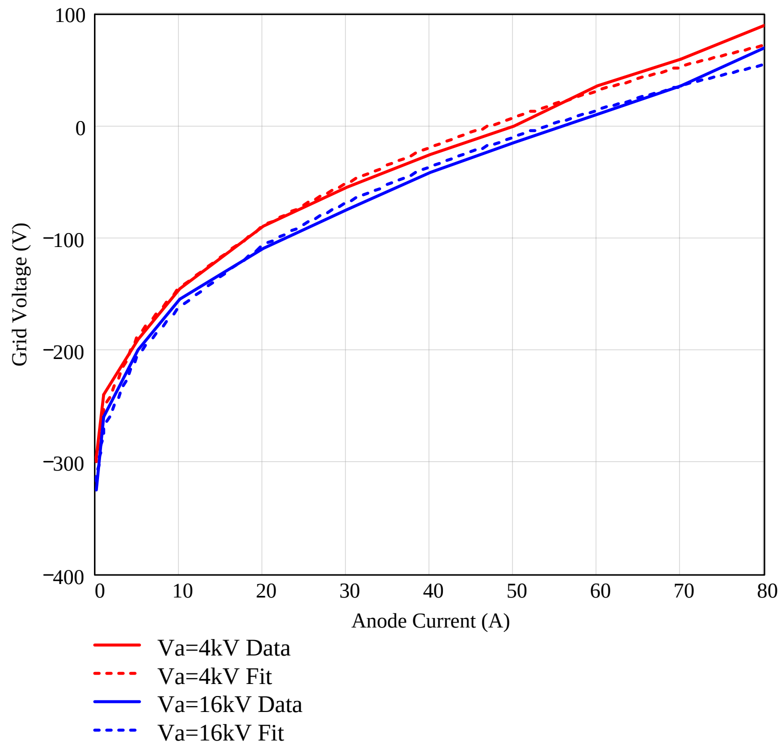

Figure 11. Anode current fit vs. grid voltage for an anode voltage of 4kV (red curves) and 16kV (blue curves)

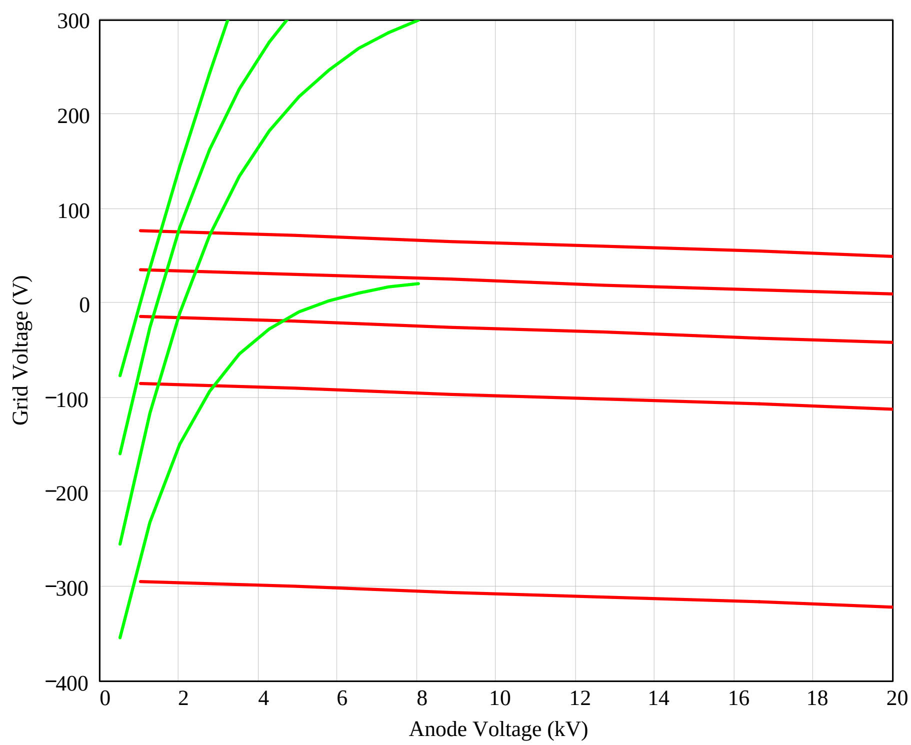

Figure 12. Constant current curves fits for the Y-567B tetrode. The red traces are for constant anode current of 0.1A, 20A, 40A, 60A, and 80A. The green traces are constant screen current of 0.2A, 2A, 4A, and 6A.

POWER AMPLIFIER OPERATING POINT

. The rest of the note will assume that the RF system is using 90% of active beam loading compensation. The anode voltage is given as:

$$V_a(t) = \frac{V_{tot}}{nN_{cav}}\left(\cos(\omega_{rf} t) + 1\right) + V_{a\,min} \quad (23)$$

where **n** is the coupler step-up ratio, V_{tot} is the total RF voltage applied to the beam, and N_{cav} is the number of cavities, The minimum anode voltage V_{amin} will be chosen to be 3kV in this note so that the tube does not draw excessive screen current. The modulator will have to provide a DC anode bias:

$$V_{aDC} = \frac{V_{tot}}{nN_{cav}} + V_{a\,min} \quad (24)$$

The grid voltage has the form

$$V_g(t) = V_{gAC}\cos(\omega_{rf} t + \phi_g) + V_{gDC} \quad (25)$$

The grid voltage phase and amplitude must be adjusted to provide the desired anode current for a given anode voltage so that

$$\frac{V_{tot}}{N_{cav}R_c} + i_b \sin(\phi_s) = \frac{n_{tube}}{n} 2f_{rf} \int_{-\frac{1}{2f_{rf}}}^{\frac{1}{2f_{rf}}} I_a\left(V_g(t), V_a(t)\right)\cos(\omega_{rf} t)dt \quad (26)$$

$$\frac{r+x}{r+1} i_b \cos(\phi_s) = -\frac{n_{tube}}{n} 2f_{rf} \int_{-\frac{1}{2f_{rf}}}^{\frac{1}{2f_{rf}}} I_a\left(V_g(t), V_a(t)\right)\sin(\omega_{rf} t)dt \quad (27)$$

where n_{tube} is the number of tetrodes mounted to each cavity.

The DC grid bias determines how efficient the amplifier will operate. A large negative grid bias pushes the amplifier into a more efficient Class C operation. However, the smaller conduction angle requires a larger peak anode current to provide the desired RF anode current. Figure 13 shows the anode dissipation (red trace) and the peak anode current (green trace) as a function of the grid bias at the end of the low energy parabola for a single tube mounted to each cavity. The anode dissipation quickly increases for grid biases above -300V. In this note a grid bias of -300V will be chosen.

Figure 14 shows tube waveforms at the end of the low energy parabola for a single tube mounted to each cavity. The grid voltage and anode current as a function of anode voltage for various beam powers and number of tubes mounted to the cavity is shown in Figure 15 and Figure 16. The DC anode current is calculated as:

$$I_{aDC} = f_{rf} \int_{-\frac{1}{2f_{rf}}}^{\frac{1}{2f_{rf}}} I_a\left(V_g(t), V_a(t)\right) dt \quad (28)$$

The DC anode current per tube throughout the acceleration cycle is shown in Figure 17. The DC current for single tube at 1 MW of beam power is below the maximum rated value as listed in Table 2.

The anode dissipation per tube is the DC tube power minus the AC power delivered to the load:

$$P_{dis} = V_{aDC} I_{aDC} - \frac{1}{2n_{tube}} \left(\frac{1}{R_c} \left(\frac{V_{tot}}{N_{cav}} \right)^2 + \frac{V_{tot}}{N_{cav}} i_b \sin(\phi_s) \right) \quad (29)$$

The anode dissipation though the cycle is shown in Figure 18. For a single tube at 1MW of beam power, the peak anode dissipation is slightly above the maximum rated value for the Y567B tetrodes as given in Table 2. Since the RF is on for 0.6 seconds out of 1.33 seconds (45%), the average anode dissipation for a single tube operating at 1 MW of beam power is well below the maximum rated value for the tube. It should also be noted that at 700kW of beam power, a cavity driven by a single tube has ~70% of the RF power and the anode dissipation as that of a single tube at 1 MW of beam power. The two tubes per cavity configuration is well within the maximum tube specification at 1 MW of beam power.

The anode dissipation shown in Figure 18 was calculated for an anode voltage bias the changed throughout the acceleration cycle as given by Equation 24. Figure 19 shows the anode dissipation for the case of a constant anode voltage bias. In this case, the anode dissipation at injection and extraction is large so it would seem that a modulated anode voltage bias is necessary.

The peak anode current is shown in Figure 20. A single tube configuration at 1 MW of beam power pushes the current to ~70A in which the tube can provide as shown in Figure 10. The peak grid and screen currents though the acceleration cycle are shown in Figure 21 and Figure 22. The DC values of the grid and screen current are well within the tube specifications for all configurations.

SUMMARY

Active beam loading compensation can achieve larger Robinson stability margins for less RF power than externally loading the RF cavities. It is possible to accelerate 1 MW of beam power with the current Main Injector RF system driven by a single tube and stay within the maximum rated specifications of the current power tetrode if active beam loading compensation is implemented and the power tetrodes are operated in Class C. It should be noted that the requirements for 700kW of beam power are not substantially different than the 1MW requirements. The two tubes per cavity configuration provides substantial margin in operating at or above 1 MW of beam power.

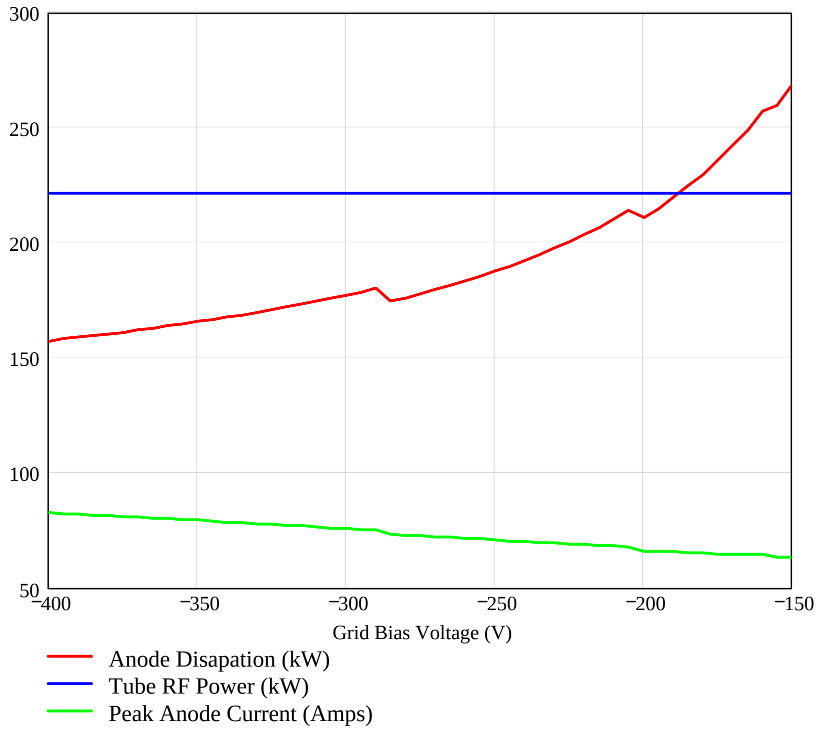

Figure 13. Anode dissipation and peak anode current at the end of the low energy parabola as a function of the grid bias for a single tube mounted to each cavity using 90% of active beam loading compensation.

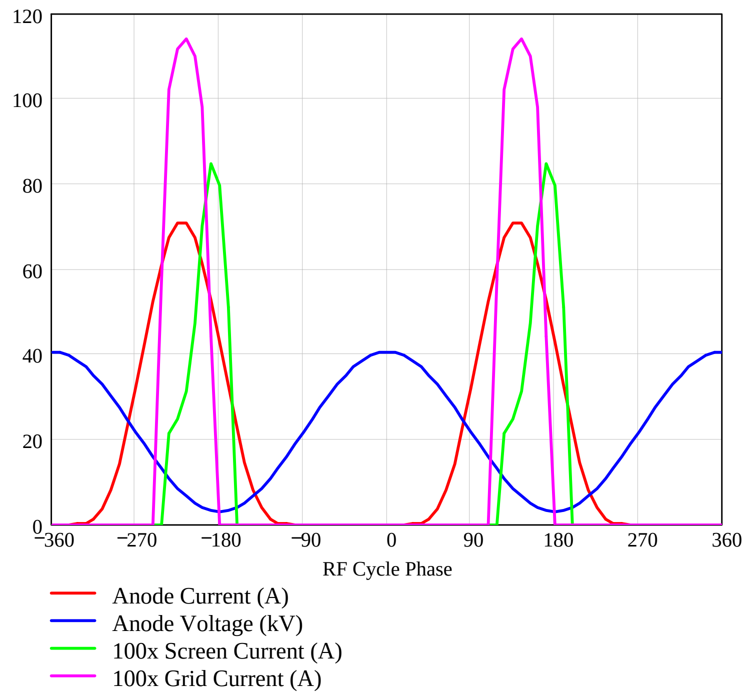

Figure 14. Tube waveforms at the end of the low energy parabola for a single tube mounted to each cavity. The screen (green trace) and grid (magenta trace) currents are multiplied by 100 to fit on the vertical scale.

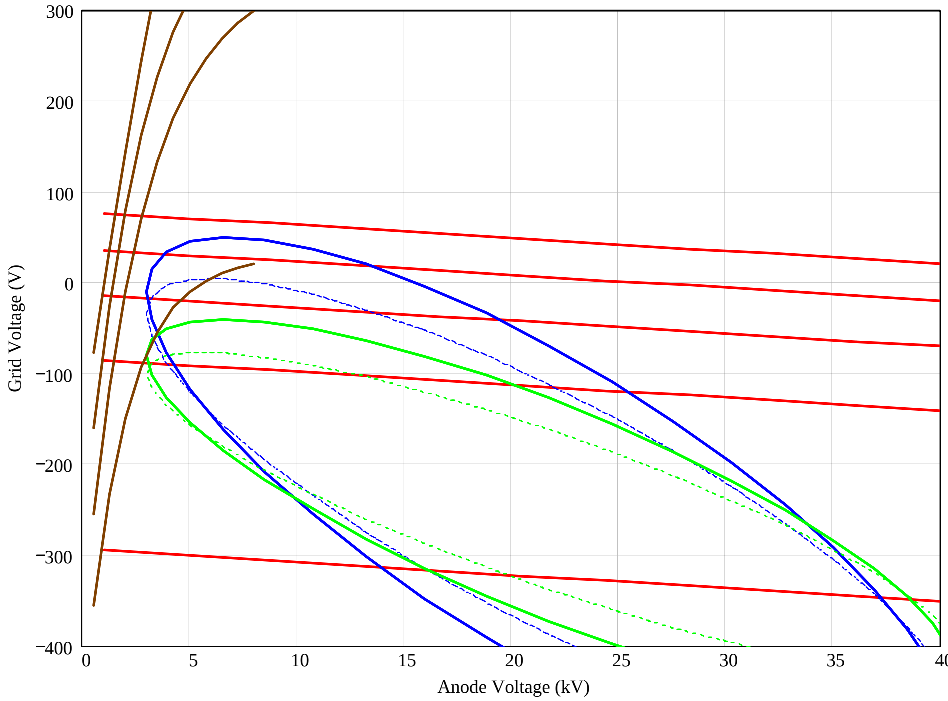

Figure 15. Grid voltage vs. anode voltage at the end of the low energy parabola. The red and brown traces are constant anode and screen current curves as shown earlier in Figure 12. The trajectories for a single tube mounted to each cavity are shown in blue and for two tubes mounted to each cavity are shown in green. The solid traces are for 1 MW of beam power and the dotted traces are for 700kW of beam power.

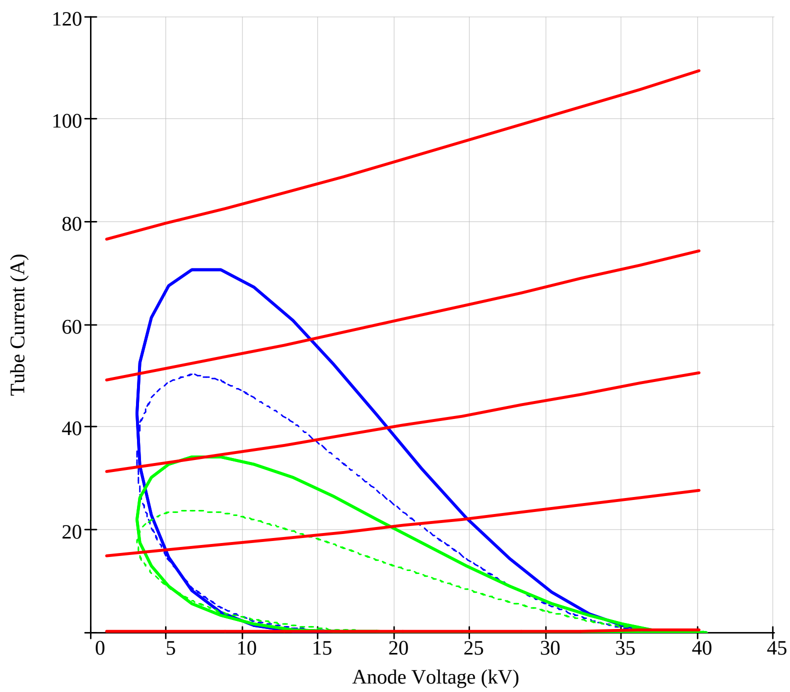

Figure 16. Anode current vs. anode voltage at the end of the low energy parabola. The red traces are constant grid voltages-325V, -110V, -42V, 10V, 70V.

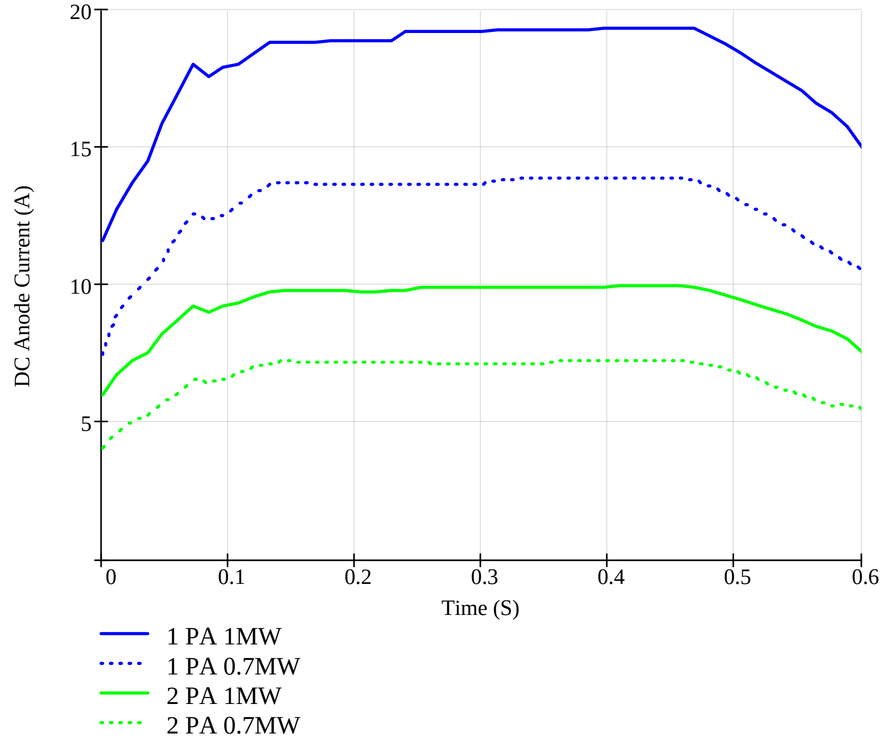

Figure 17. DC anode current per tube throughout the acceleration cycle.

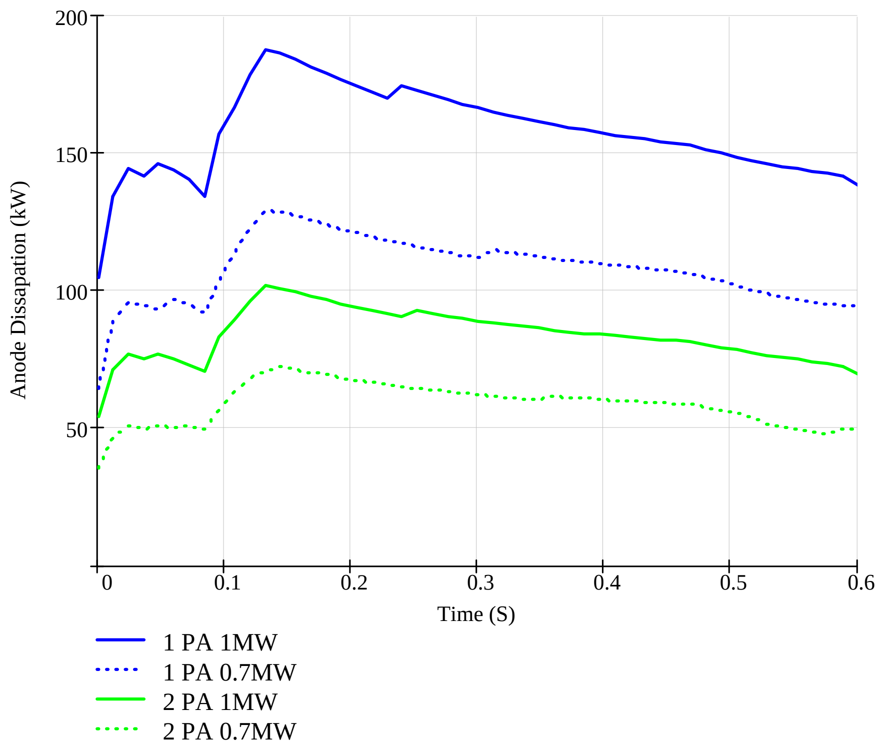

Figure 18. Anode dissipation thought the acceleration cycle.

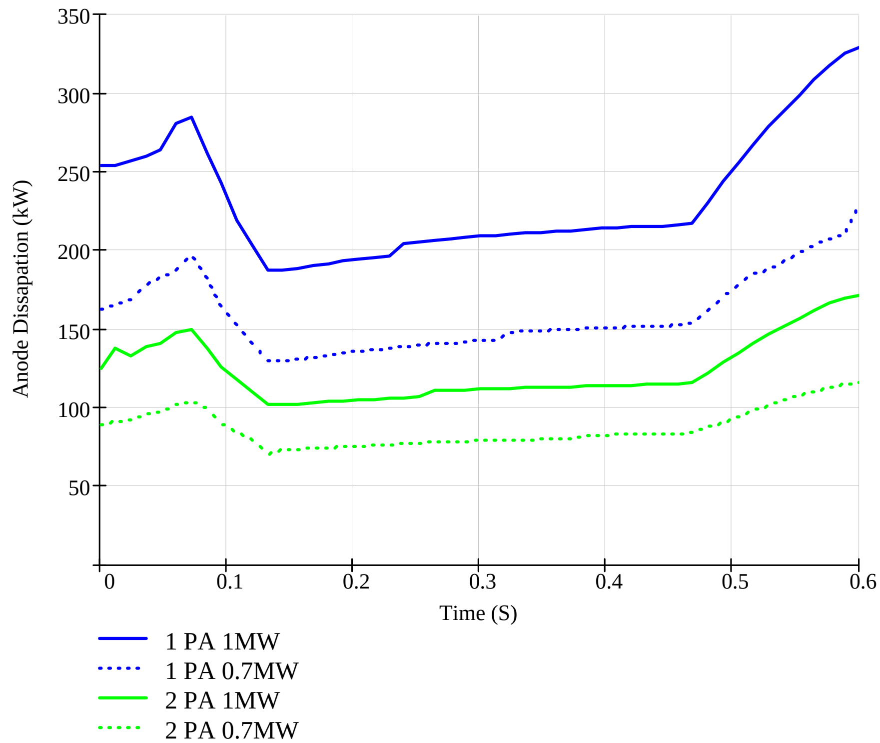

Figure 19. Anode dissipation for a constant anode voltage bias.

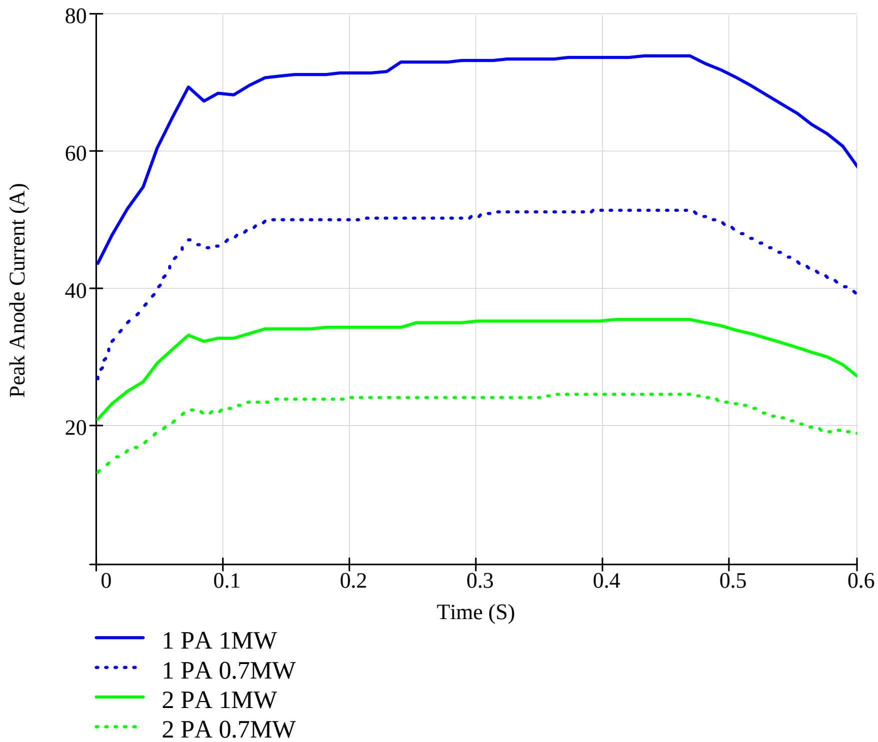

Figure 20. Peak anode current throughout the acceleration cycle.

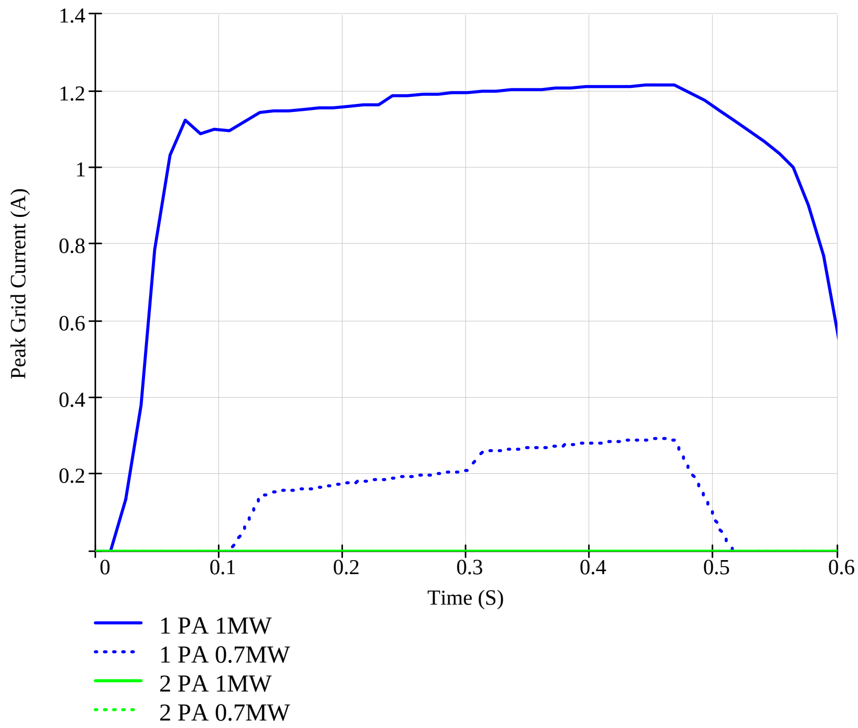

Figure 21. Peak grid current throughout the acceleration cycle.

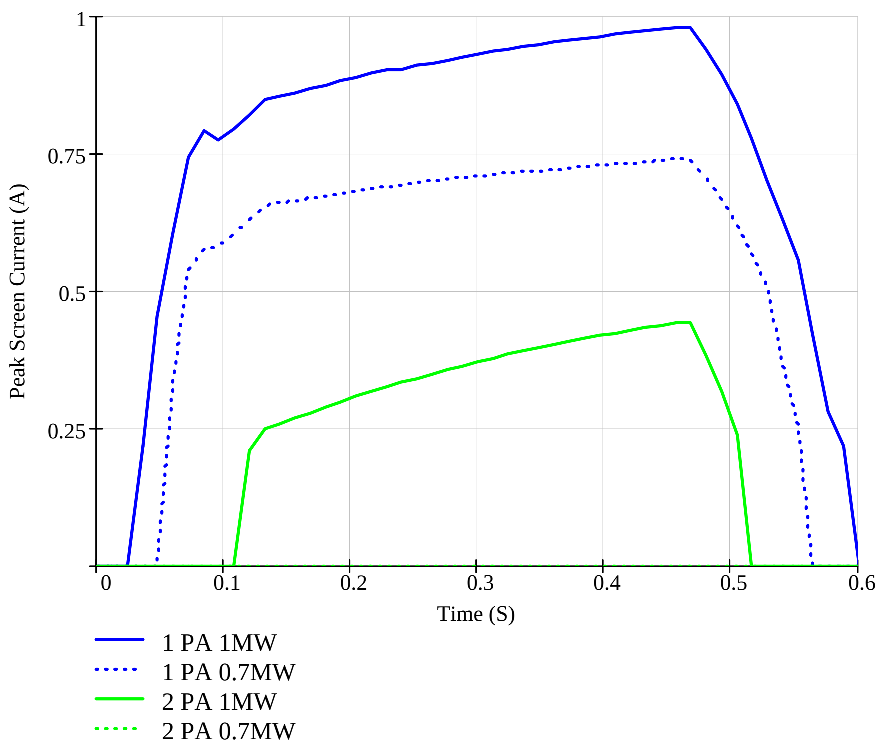

Figure 22. Peak screen current throughout the acceleration cycle.