Coherent Instabilities in Fermilab Booster

Valeri Lebedev, Alexei Burov, William Pellico and Xi Yang,
FNAL

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Fermilab

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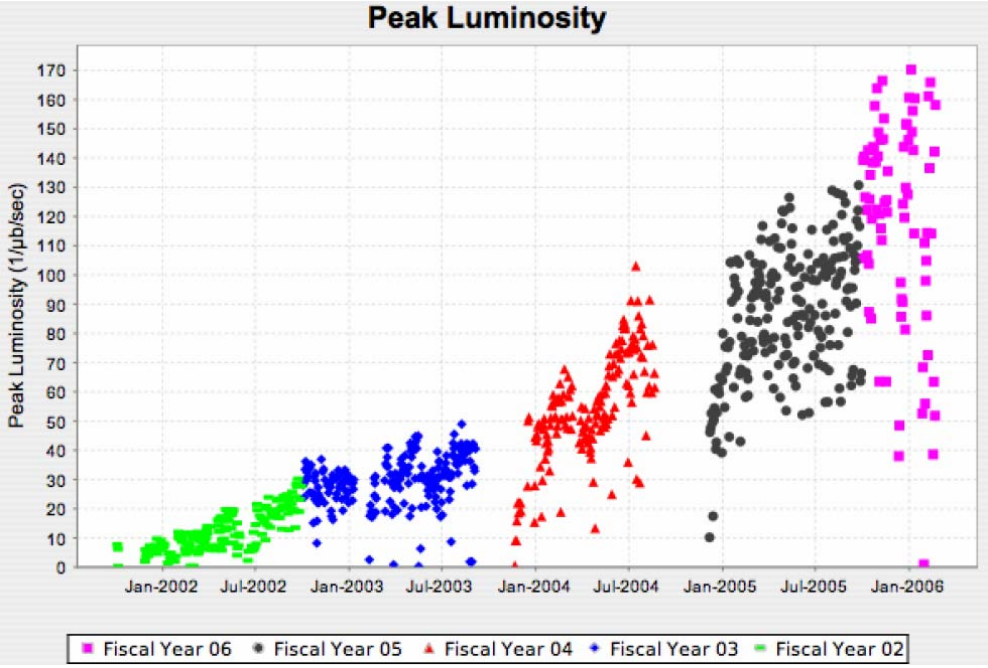
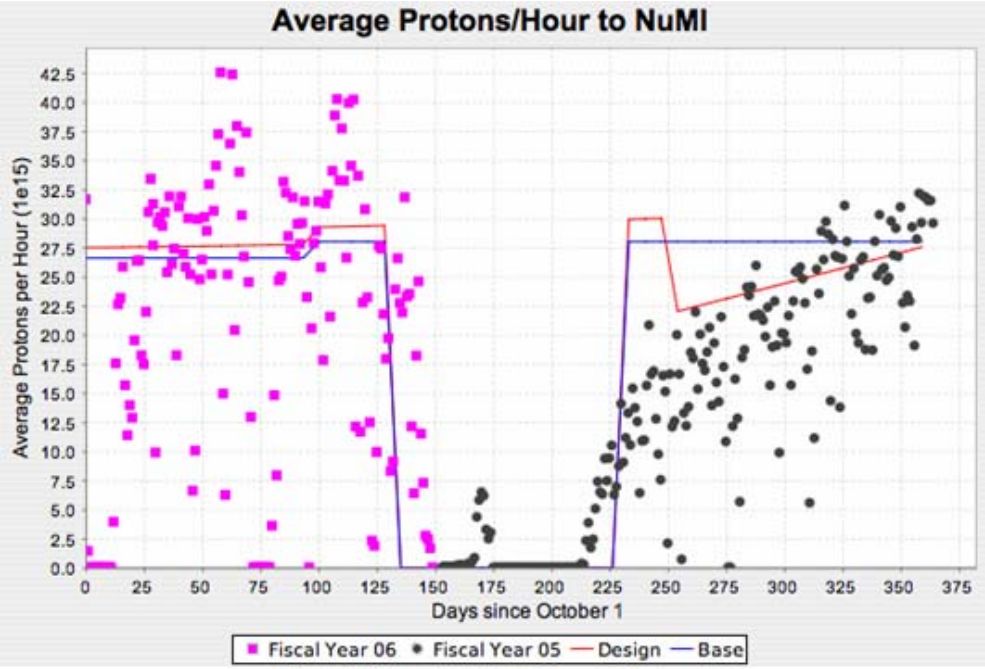
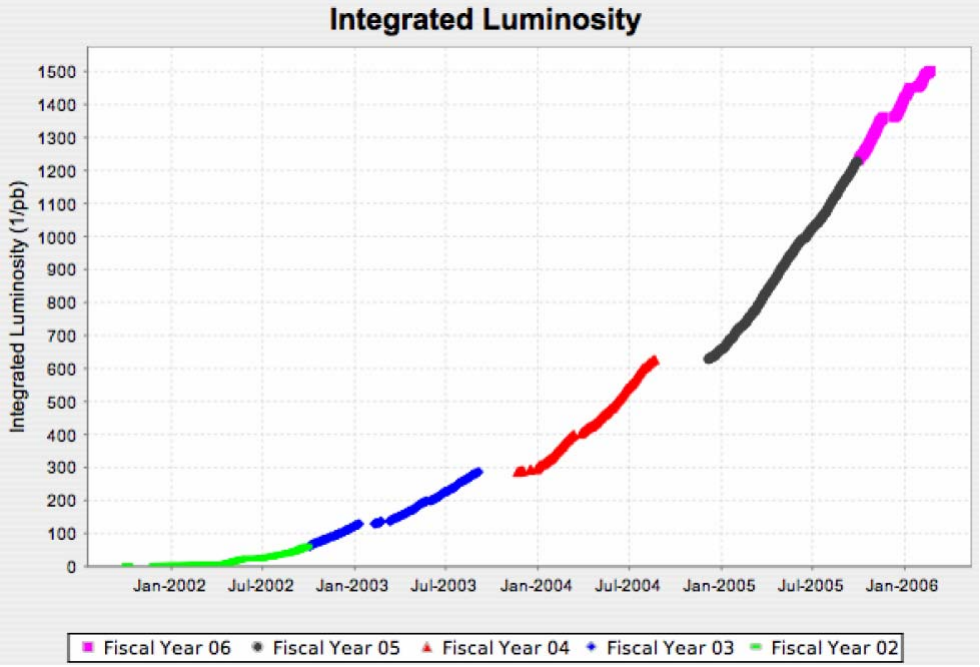
1. Introduction

Motivation

- Simultaneous run of 2 experiments
  - Tevatron Run II
  - NUMI (started March 2005)
- Thirsty for protons
  - Stress for Booster and MI
- Good understanding and characterization of the accelerator





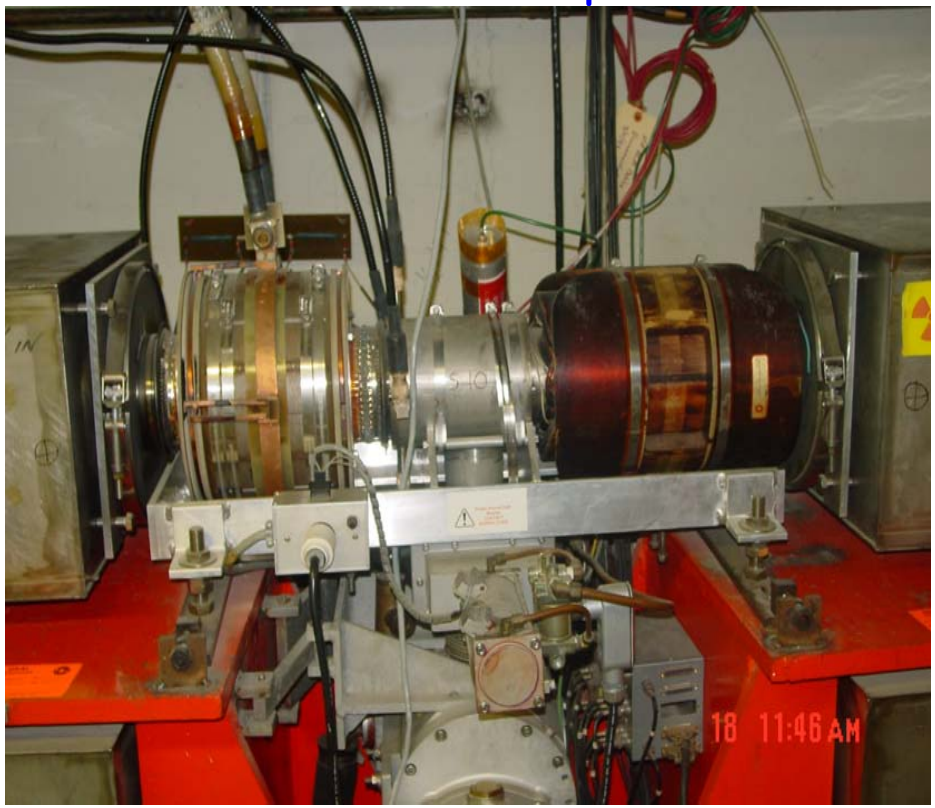


Problems aimed to be addressed

- Beam lifetime through the cycle
  - Reduction of chromaticity ??
    - Loss of transverse beam stability
    - Improvements of beam stability with transverse bunch-by-bunch damper
      - Previous attempt was unsuccessful
    - Impedance measurements and beam stability estimates
- Study of possibility of fast chromaticity measurements
- Transition crossing
  - Suppression of quadrupole oscillations
    - Quadrupole damper
    - Novel technique – RF voltage jump

Booster

- Machine is more than 30 years old
- Rapid cycling synchrotron - 15 Hz
- Combined function dipoles
- No formal vacuum chamber to avoid eddy currents
- Laminated steel vacuum chamber causes increased impedance



Booster parameters

Energy	0.4 - 8 GeV
Transition energy	5.1 GeV
Tot. number of particles	4.5e12
Circumference	474.2 m
Harmonic number, *q*	84
Betatron tunes, Q_x, Q_y	~6.75/6.75
RF voltage	0.7- 0.9 MV
Injection	H^-, 10 turns

- Naïve transv. impedance estimate
  - Magnetic permeability $Z \propto \sqrt{\mu}$
    - factor of **20** relative to stainless steel ($\mu \sim 2 \cdot 10^3$, $\sigma_{st}/\sigma_{ss} \sim 5$)
  - Laminations

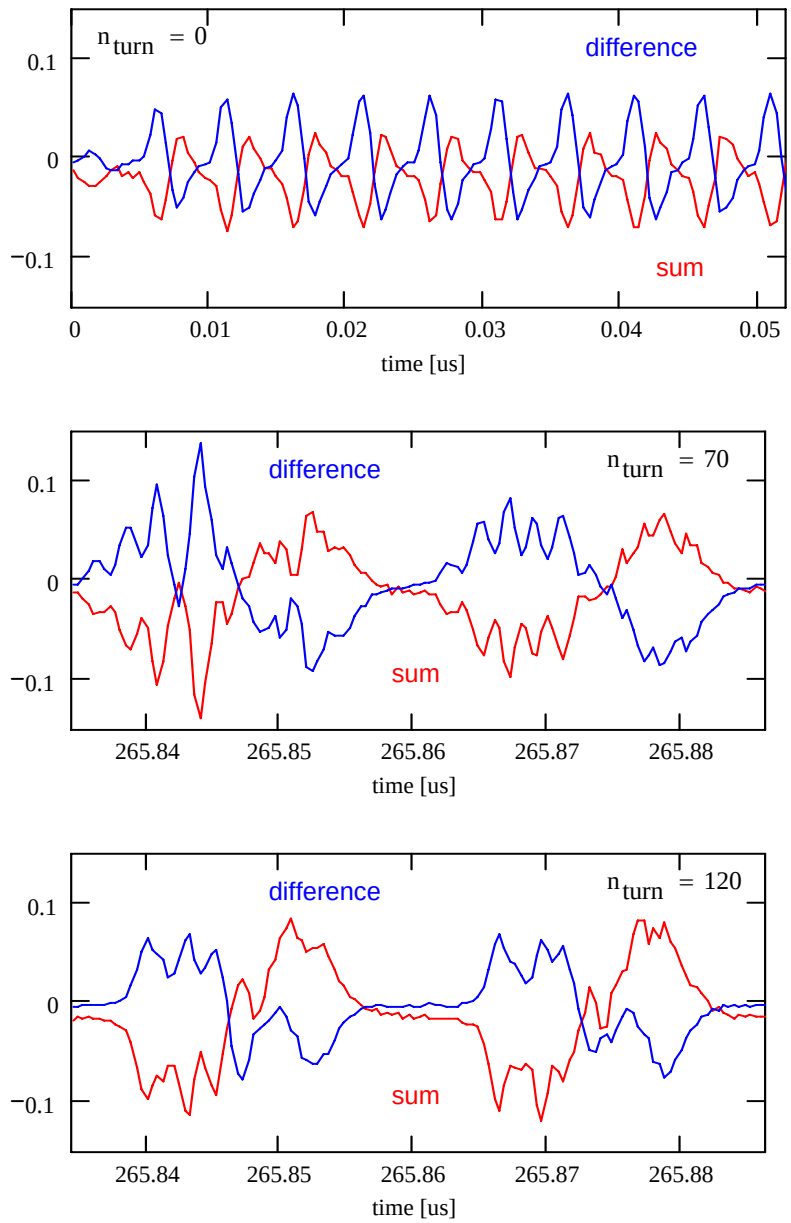
$$Z \propto Z_{smooth} \frac{2a}{d_{\text{laminations}}}$$

    - another factor of ~**30** relative to a smooth vacuum chamber (a~2.5 cm, $d_{laminations}$ =0.63 mm)
  - Total: **factor of about 600 rel. to SS**

2. Beam Parameters and Beam Instability at Injection

Measurement technique

- Hybrid for making sum and difference
  - 4 channels: **S** and Δ for hor.&vert. BPMs
- Digital scope
  - Sampling rate - 0.4 ns
  - 2 millions points in each channel: 1.6ms
  - Total turns 722 = 678 with beam + 44 turns with no beam (pretriggering)
- Data analysis
  - Mark boundaries of each bucket
    - Frequency changes are fast,
      - 0.15% at 200 turns
  - Remove orbit offset from difference signal: offset depends on turn number
  - Integrate signals to get bunch density and dipole moment
- Slow decoherence of 200 MHz structure of linac injected beam

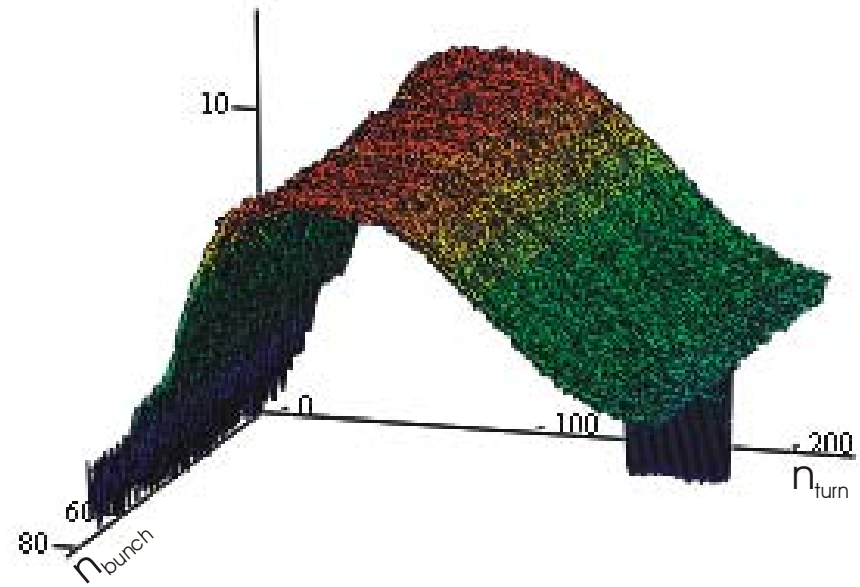


Raw BPM signals of H.BPM, 2 RF buckets

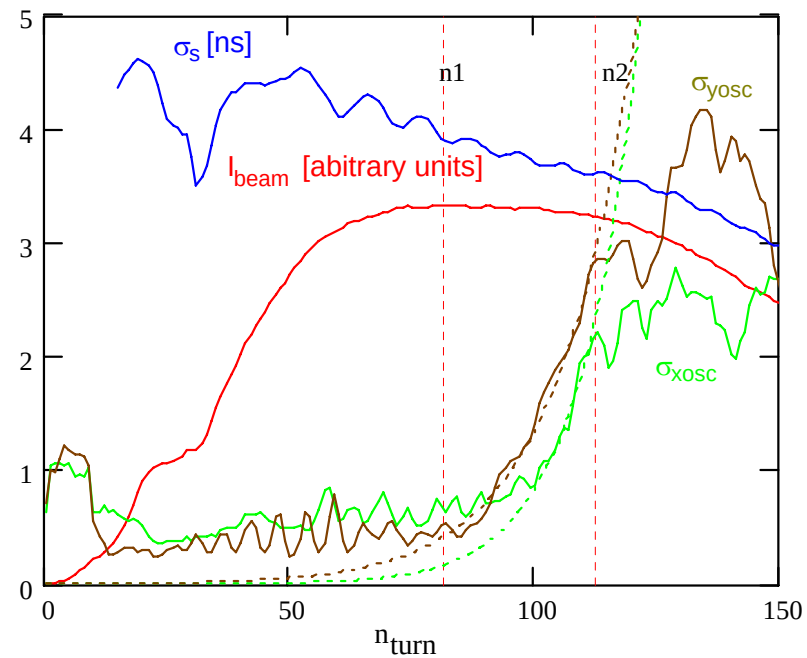
Instability development

- Both horizontal and vertical chromaticities were reduced to give the onset of instability
- That caused development of very fast head-tail instability in both planes
  - X-plane – 12 turns
  - Y-plane – 16 turns
  - ♦ instability in X-plane starts later and develops faster
  - ♦ instability starts (becomes visible) after beam is completely bunched bunch (turn 70)

$$D_{RMS} = \sqrt{\int D(s)^2 ds}$$



AC bunch intensity for first 220 turns



AC beam intensity, rms bunch length and rms transverse motions in X and Y planes

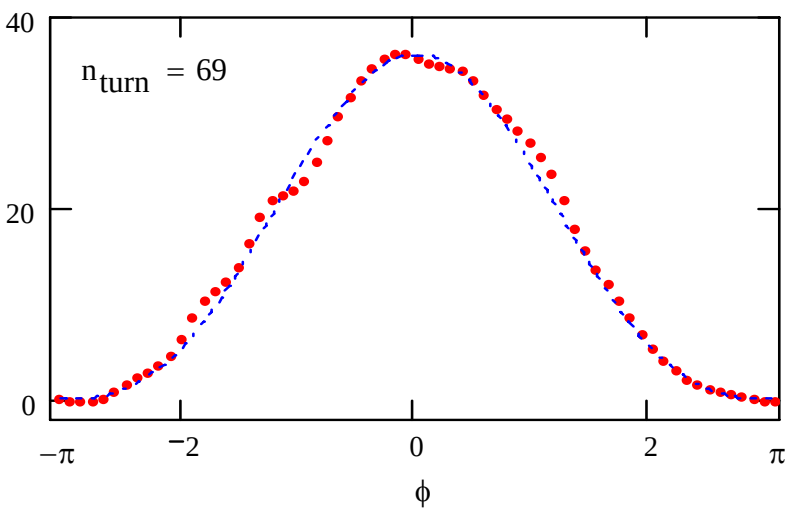
Longitudinal distribution

- When bunching is complete (turn 69) the longitudinal distribution is close to a linear function of energy
- That yields: $\varepsilon_{L100\%}$ = 0.06 eV·s

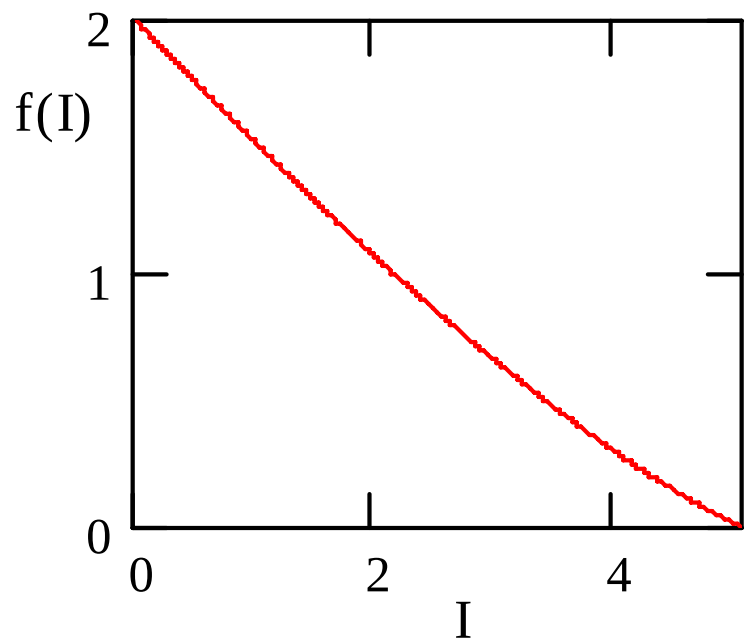
$$f(E) = \frac{2}{C}\left(1 - \frac{E}{E_{\max}}\right) \quad ,$$

$$E = \frac{\dot{\phi}^2}{2} + \Omega_s{}^2\left(1 - \cos\phi\right) \quad ,$$

$$E_{\max} = 2\Omega_s{}^2 \quad , \quad C = \int_0^{I(E_{\max})} f\left(E(I)\right) dI \quad .$$

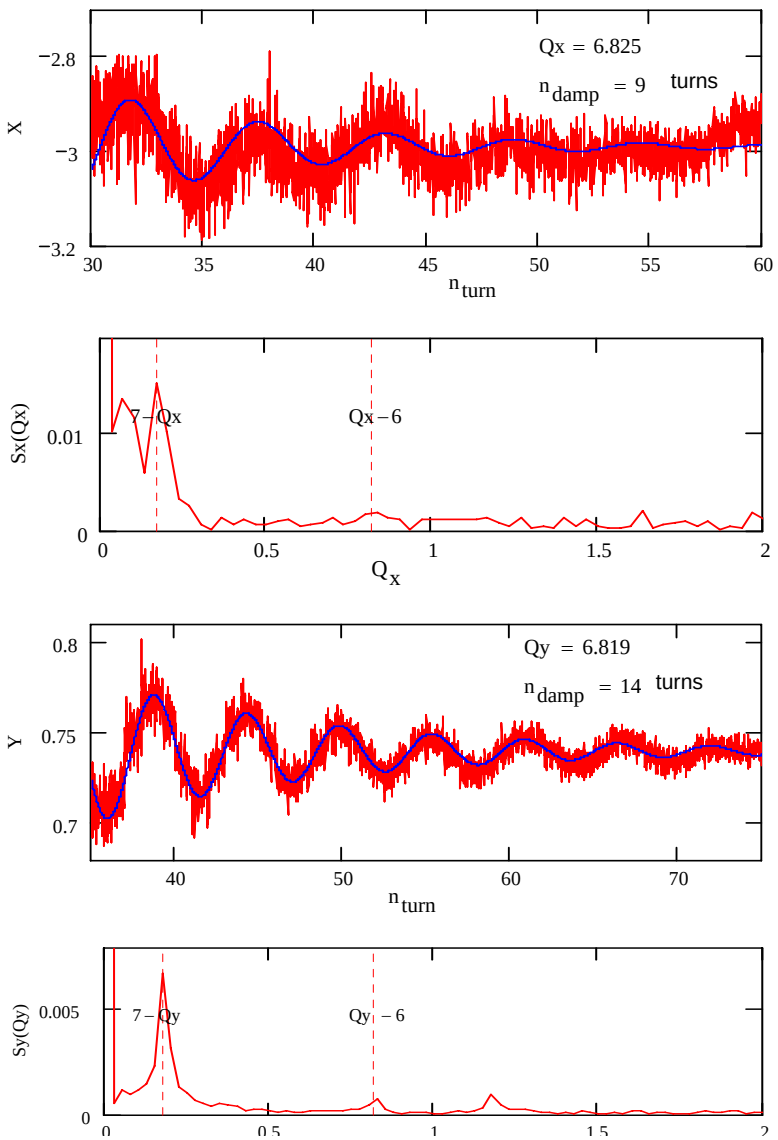


Measured and fitted longitudinal distributions

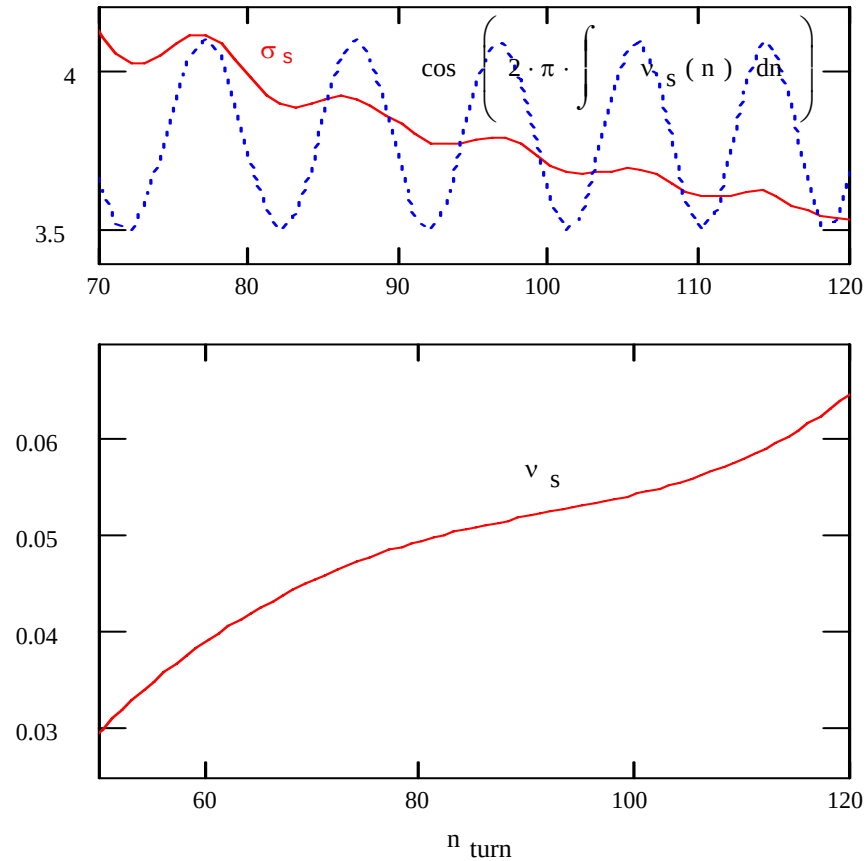


Betatron Tunes

Injection process leaves small betatron oscillations which has been used for tune measurements: Q_x=6.825, Q_y=6.819



Synchrotron tune



- Bunching is not perfectly adiabatic
- It excites longitudinal quadrupole oscillations
- Observed Q_s is about 10% less than the small amplitude synch. motion

Chromaticity estimate

- Beam decoheres during turns 30 to 60
  - Presuming that damping time is set by chromaticity $n_{decoh} \approx \frac{1}{\left|\xi\, \sigma_{\Delta p/p}\right|}$

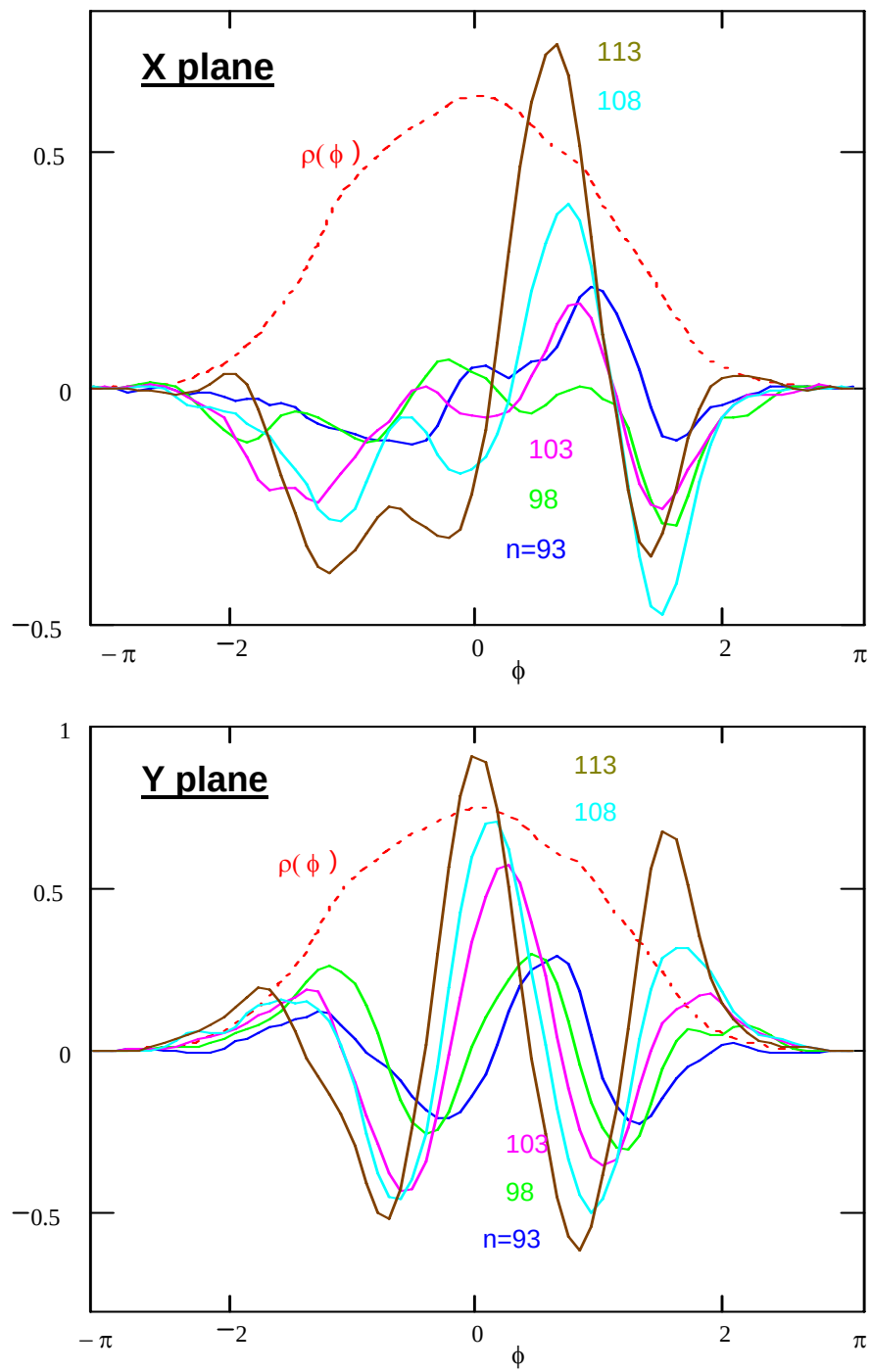
    $\Rightarrow$ $|\xi_x| \geq 85$, $|\xi_y| \geq 70$

    - where $\sigma_{\Delta p/p} = 8 \cdot 10^{-4}$ was calculated from long. dynamics
  - More accurate estimate using the dispersion equation for continuous beam and computed RW impedance yields slightly smaller value $|\xi_{x,y}| \geq 60$

Structure of head-tail modes

- Changes of dipole moment along the bunch with turn number show almost no signs of betatron motion
- The shape of dipole moment distribution is the same for all bunches of given turn and changes little with turn number. The main change is the growth of amplitude
- There is big asymmetry between head and tail for X-plane data

  $\Rightarrow$ Strong head tail effect
- There is only moderate asymmetry for Y-plane data, but in distinguish from X-plane the wave structure is slowly moving from head to tail
  - Similar behavior in X-plane for half intensity
- Weak head-tail? Growth rates are much smaller than the synchrotron tune
  - $\lambda \sim 0.07$ turn^{-1}
  - $2\pi Q_s \sim 0.3$ turn^{-1}



Np=4.5e12, average of 84 bunches

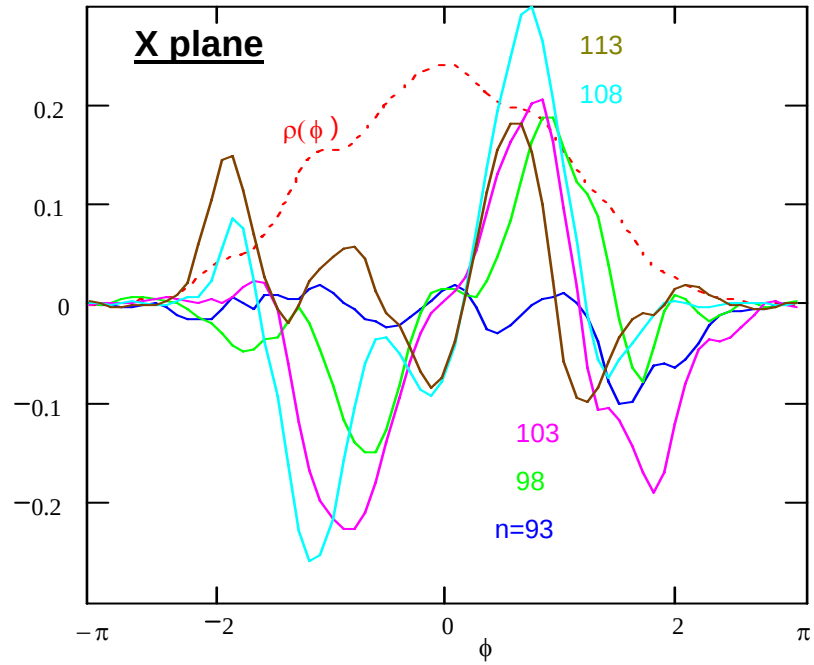
Measurement results for half of nominal beam intensity ($N_{tot}=2.3\cdot10^{12}$)

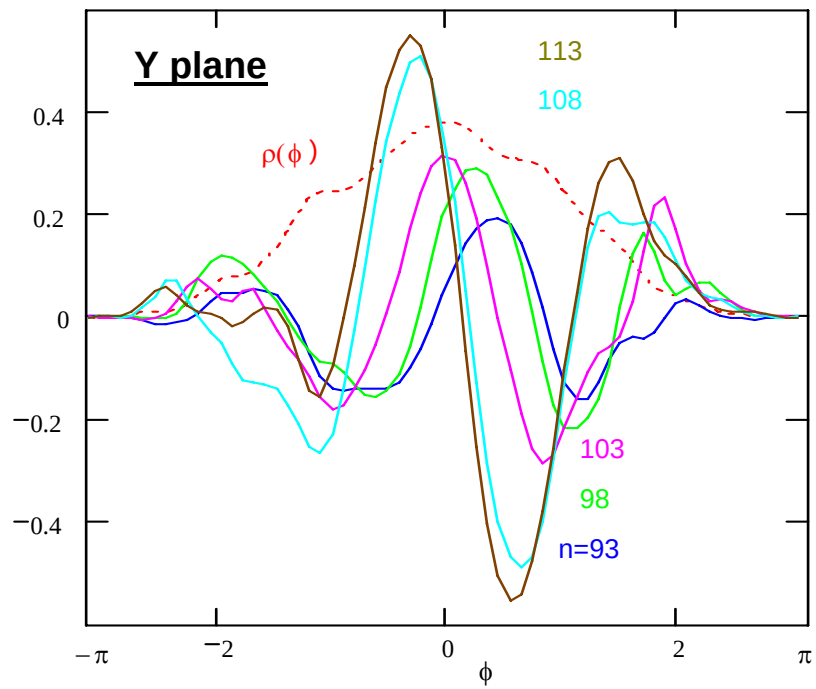
Tunes, decoherence times and growth times for full and half beam intensity

$N_{particle}$	$2.3\cdot10^{12}$	$4.5\cdot10^{12}$
Q_x	6.830	6.825
Q_y	6.831	6.819
Q_s at turn 100	0.047	0.057
$\Delta Q_x=7-Q_x-3Q_s$	0.029	0.004
$\Delta Q_y=7-Q_y-3Q_s$	0.028	0.010
X-plane decoh., turns	15	13
Y-plane decoh., turns	18	16
X inst. growth time, turns	18	12
Y inst. growth time, turns	14	14

Unperturbed tunes: $Q_x=6.835$

$Q_y=6.843$





Np=2.3e12, average of 84 bunches

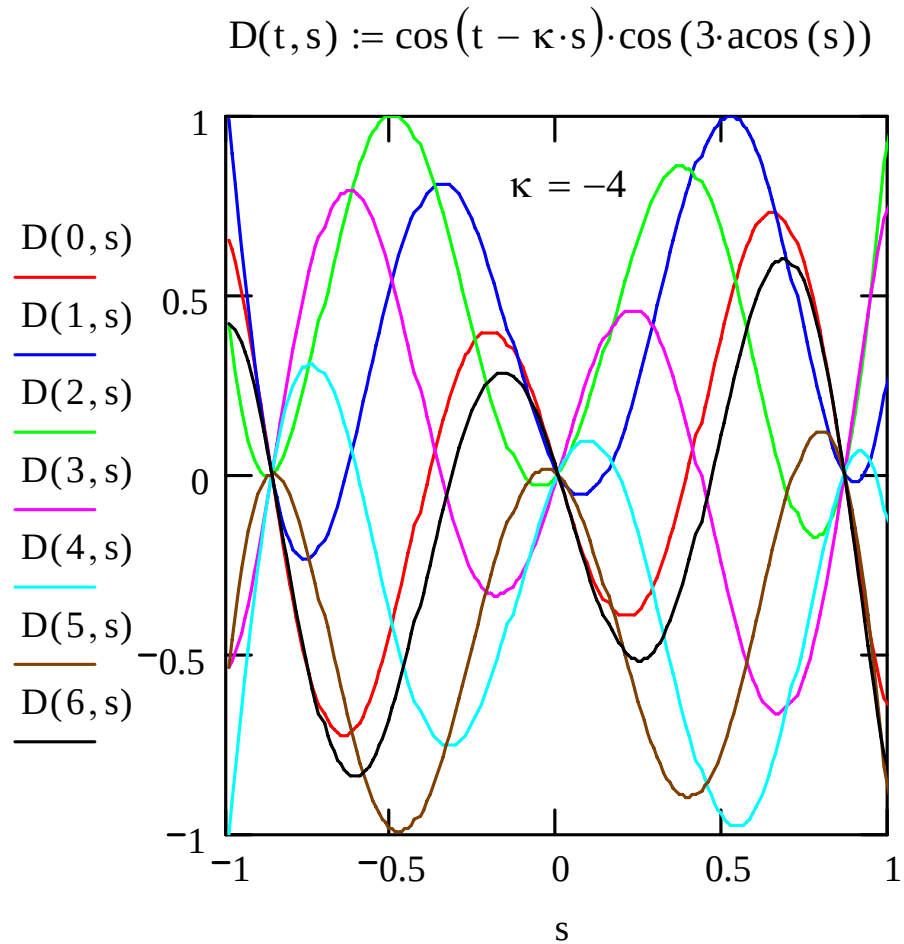
Betatron and Synchro-betatron Tunes

- For weak head-tail the "air-bag" model predicts the following dependence of dipole moment along the bunch on turn number

$$D(n,s)_m = D_0 \cos\left(2\pi\left(Q_b + mQ_s\right)n - \kappa\frac{s}{L_b} + \psi\right)\cos\left(m\,\mathrm{acos}\frac{s}{L_b}\right)$$

where $\kappa = \frac{\xi L_b}{\eta R}$ is the head-tail phase

- To make betatron motion invisible the tune of corresponding synchro-betatron mode needs to be close to integer $\left(Q_b + mQ_s\right) - k \approx 0$ Q
  - $Q_x + 3\,Q_s \sim 6.996$ ($Q_s \sim 0.057$)
  - $Q_y + 3\,Q_s \sim 6.990$
- For X-plane the distance from the resonance ($2\pi\Delta Q_x \sim 0.004$) is smaller than the instability growth rate (0.083 turn^{-1})



Comparizon of observed head-tail motion and simple model

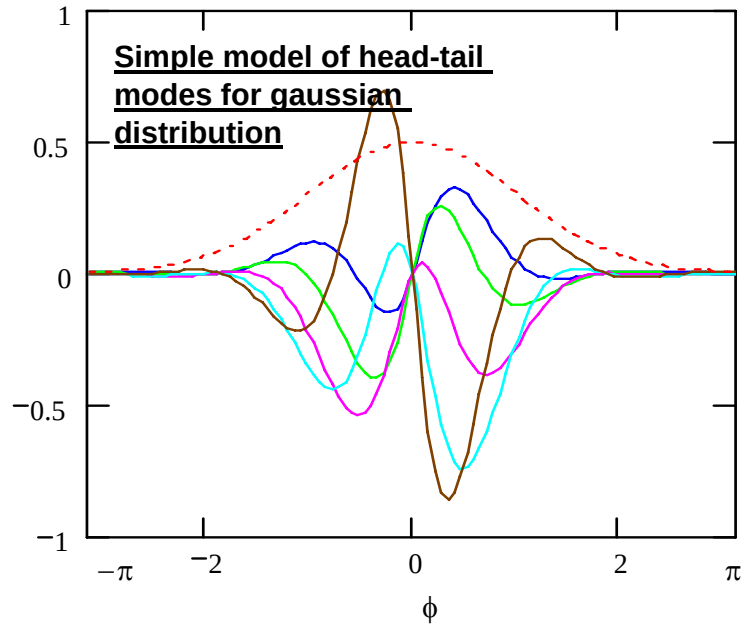
- Presume
  - all radial modes are phased
  - Transverse amplitude is proportional to long. action
  - Gaussian distribution.
- => the dipole moment distribution is

$$D(n,s)_m \propto \cos\left(2\pi(Q_b + mQ_s)n - \hat{\kappa}\frac{s}{\sigma_s} + \psi\right) \times$$

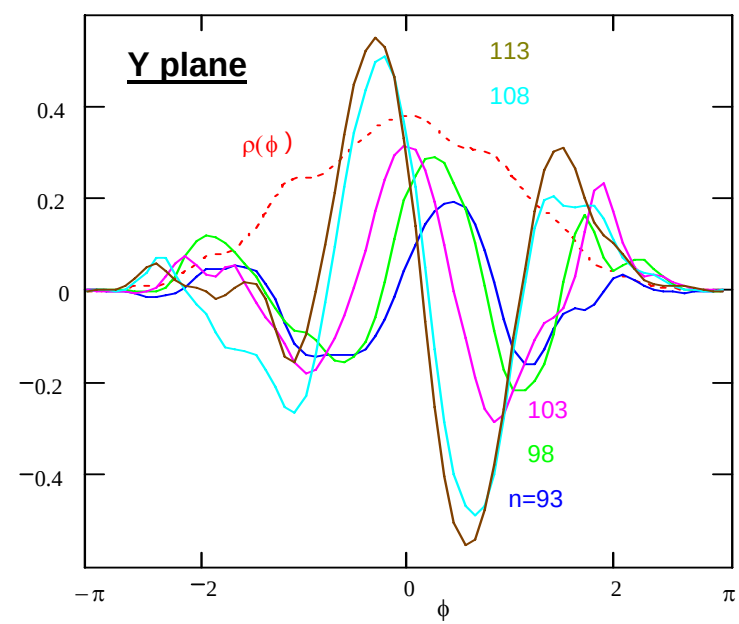
$$\int_0^\infty \exp\left(-\frac{s^2+x^2}{2\sigma_s^2}\right)\left(\frac{s^2+x^2}{2\sigma_s^2}\right)\cos\left(m\,\mathrm{acos}\frac{s}{\sqrt{s^2+x^2}}\right)dx$$

where $\hat{\kappa} = \frac{\xi\sigma_s}{\eta R}$

- Comparison with measurements verify
  - values of detuning from resonance $\Delta\nu$
  - Chromaticities



ξ_y = -70



N_p=2.3e12, average of 84 bunches

Another Chromaticity estimate

- Wave propagation inside the bunch gives independent measurement and additionally allows to determine the sign
  - For "air-bag" model

$$D(n,s)_m = D_0 \cos\left(2\pi\left(Q_b + mQ_s\right)n - \kappa\frac{s}{L} + \psi\right)\cos\left(m\,\mathrm{acos}\frac{s}{L_b}\right) \Rightarrow \kappa = \frac{2\pi\left(Q_b + mQ_s - p\right)}{\Delta s / L_b}$$

where n is the turn number

$$\Delta Q \equiv Q_b + mQ_s - p \sim -0.03\,, \quad (p = 7)$$

$\Rightarrow$ the head-tail phases are positive and roughly are

for X-plane $\Delta s / L_b \sim -0.022$ per turn => $\kappa_x \approx 8$

for Y-plane $\Delta s / L_b \sim -0.025$ per turn => $\kappa_y \approx 7$

$\Rightarrow$ chromaticities are negative $\Rightarrow$ $\xi_x \approx -85$, $\xi_y \approx -70$

Tune dependence on intensity and estimate of Im($Z_\perp$)

- Tune shift for 4.5e12 particles:
  - ΔQ_x~0.010, ΔQ_y~0.024
- Tunes are affected by
  - Imaginary part of $Z_\perp$
  - Combination of space charge incoherent tune shift and momentum spread
  - Detuning wake
- Leaving only impedance contribution one can estimate imaginary part of the impedance from tunes measured at the cycle beginning (turns 30 - to 60)

$$\Delta Q = \frac{I_0 R}{4\pi\left(mc^2/e\right)\gamma\beta^2 Q}\left(-\operatorname{Im}\left(Z(\omega_n)\right) + i\operatorname{Re}\left(Z(\omega_n)\right)\right) \quad ,$$

$$\omega_n = \omega_0\left(Q+n\right)$$

  - For frequency of 76 kHz ($(7-Q)f_0$) that yields:
    - $\Rightarrow$ Im(Z_x) $\approx$ 24 MΩ/m
    - $\Rightarrow$ Im(Z_y) $\approx$ 58 MΩ/m
- Earlier measurements (Huang Xiaobiao) with beam was excitation by kicker and beam motion recorded by regular booster BPMs showed close results
  - Many modes are excited
    - frequency is not well determined
  - Only vertical tune shifts

Instability growth rate and estimate of $Re(Z_\perp)$

- For bunched beam the "Air-bag" model in the case of week head-tail yields the instability growth rate per turn:

$$\lambda_m = -2\pi\,\mathrm{Im}(\Delta Q) = -\frac{I_0 R}{2\left(mc^2/e\right)\gamma\beta^2 Q}\sum_{n=-\infty}^{n=\infty}\mathrm{Re}\left(Z_\perp(\omega_n)\right)\left(\mathrm{J}_m\left(\frac{\omega_n L_b}{\mathrm{v}_0}-\kappa\right)\right)^2,$$

$$\omega_n = \omega_0\left(p+Q+mQ_s+nN_b\right)$$

where L_b – bunch length

N_b – number of bunches ($N_b = q$)

- If we presume that the addend with smallest frequency makes major contribution one obtains

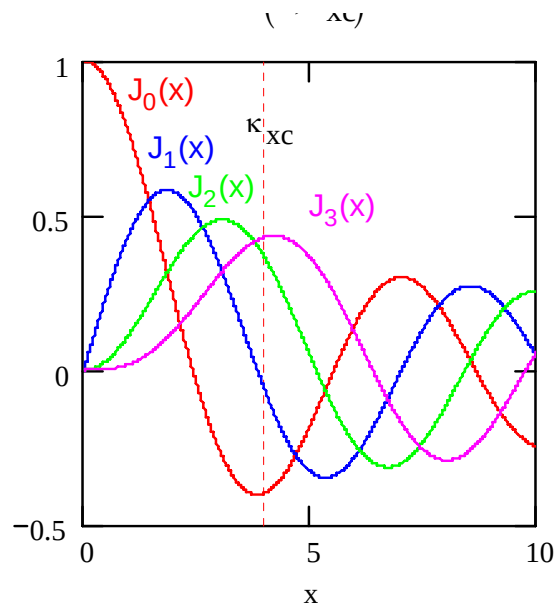
$$\lambda_m \approx -\frac{I_0 R}{2\left(mc^2/e\right)\gamma\beta^2 Q}\mathrm{Re}\left(Z_\perp(\Delta\omega)\right)\mathrm{J}_m{}^2(\kappa), \qquad \Delta\omega = \omega_0\Delta Q = \omega_0\left(Q_{x,y}+mQ_s-7\right), \quad m=3$$

- That for $\kappa = 4$ that yields impedance estimate

  $\Rightarrow$ $Re(Z_x) \approx 42*5.4 \approx 230$ MΩ/m at 12 kHz

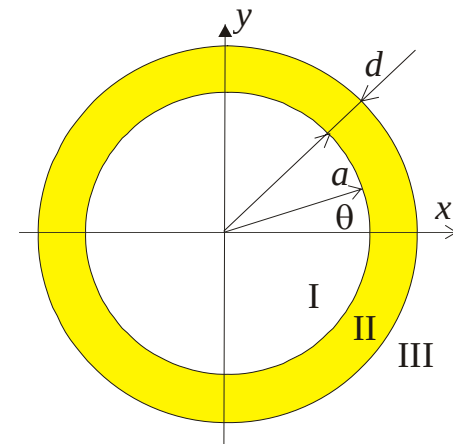
  $\Rightarrow$ $Re(Z_y) \approx 55*5.4 \approx 300$ MΩ/m at 12.5kHz

  - Data at $2.3\cdot10^{12}$ is used where head-tail is weak
  - while real and imaginary parts of tune shifts are close the suppression due to head tail boosts real part of impedance by factor of 5 ($J_3(\kappa)^{-2} \approx 5.4$)
    - Note difference in frequency



3. Estimate of Transverse Booster Impedance

Impedance of stainless steel round vacuum chamber with thin walls



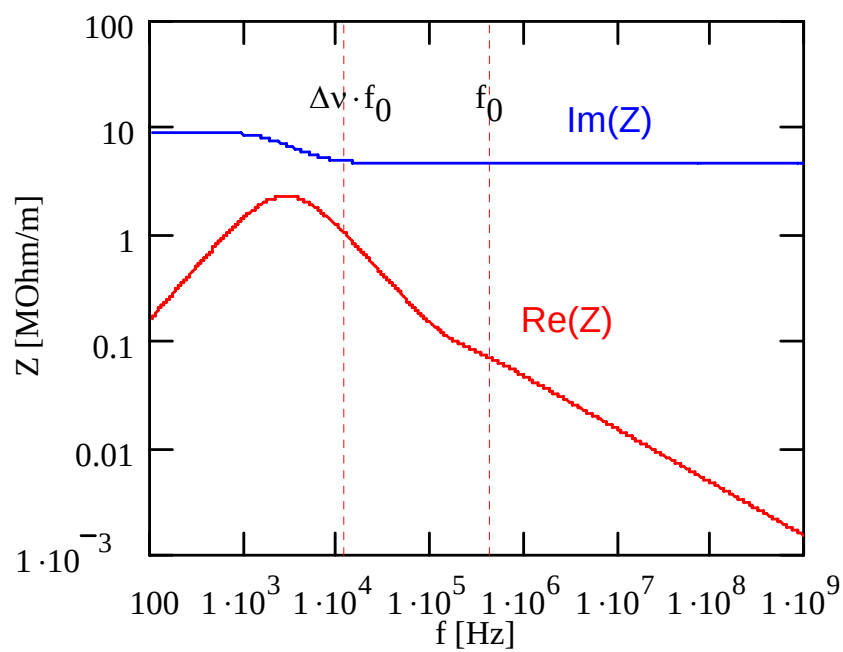
Impedance per unit length ($\omega > 0$) is:

$$Z_{L\perp} = i\frac{Z_0}{2\pi a^2\beta}\left[\frac{2\beta^2\left(e^{kd}(1+kb)-e^{-kd}(1-kb)\right)}{e^{kd}(1+ka)(1+kb)-e^{-kd}(1-ka)(1-kb)}+1-\beta^2\right]$$

$$k=\frac{1+i}{\delta},\quad \delta=\frac{c}{\sqrt{2\pi\sigma\omega}}\quad,\quad Z_0=\frac{4\pi}{c}\approx 377\,\Omega$$

with asymptotic behavior as

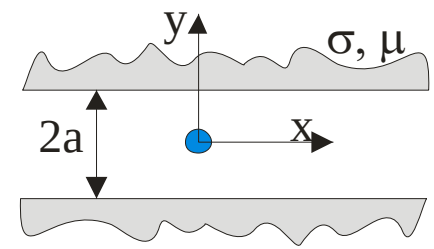
$$Z_{L\perp}\approx\frac{Z_0}{2\pi a^2\beta}\begin{cases}\dfrac{i}{\gamma^2}+\dfrac{c(1+i)\beta^2}{a\sqrt{2\pi\sigma_R\omega}} & ,\quad \delta\le d\\[2ex] \dfrac{c^2\beta^2}{2\pi\sigma_R\omega a d}+i(\ldots) & ,\quad \sqrt{ad}\ge\delta\ge d\\[2ex] \dfrac{2\pi a\beta^2\sigma_R\omega d}{c^2}+i & ,\quad \delta\ge\sqrt{ad}\end{cases}$$



Contribution to Booster impedance from stainless still vacuum chamber: L=200 m, a=4.3 cm, d=1.6 mm

This is factor of 300 below the value determined from instability growth rate

Impedance of steel flat vacuum chamber



Impedance per unit length ($\omega > 0$) is:

$$Z_{Lx} = i\frac{Z_0}{2\pi a^2\beta}\left[\frac{\pi^2}{24} + \beta^2\int_0^\infty \frac{(u - ka/\mu)\exp(-u)u\,du}{k\cosh(u) + u\sinh(u)}\right]$$

$$Z_{Ly} = i\frac{Z_0}{2\pi a^2\beta}\left[\frac{\pi^2}{12} + \beta^2\int_0^\infty \frac{(u - ka/\mu)\exp(-u)u\,du}{k\sinh(u) + u\cosh(u)}\right]$$

$$k = \frac{1+i}{\delta}, \quad \delta = \frac{c}{\sqrt{2\pi\sigma\omega}} \quad , \quad Z_0 = \frac{4\pi}{c} \approx 377\,\Omega$$

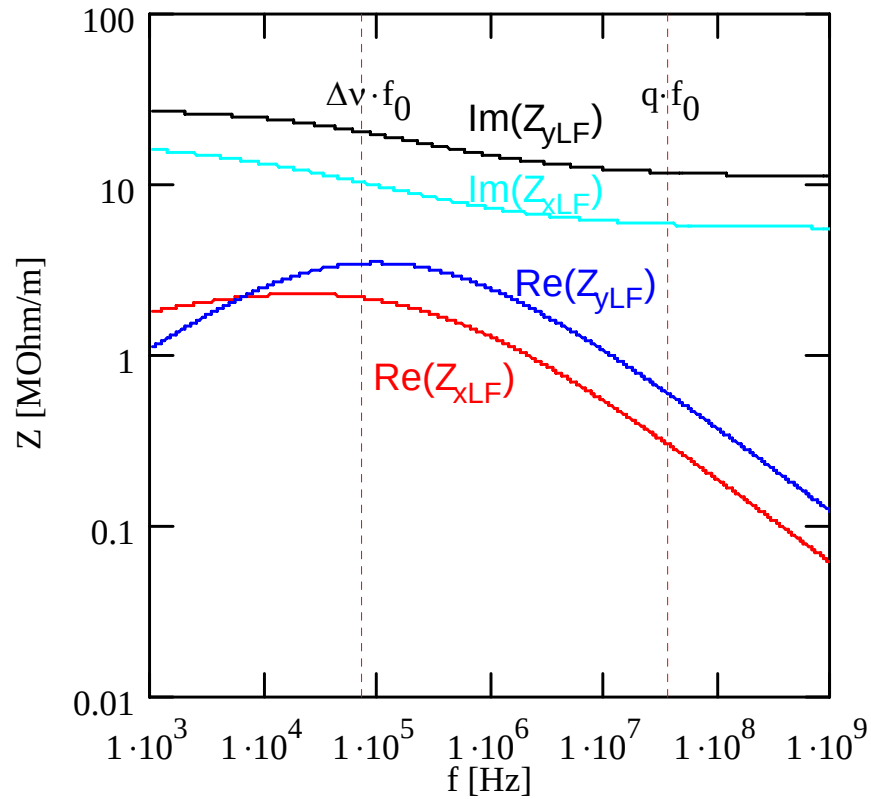
At high frequencies ($\delta << a/\mu$) asymptotic is

$$Z_{Ly} \approx \frac{Z_0}{2\pi a^2\beta}\left(\frac{\pi^2}{12}\right)\left(\frac{i}{\gamma^2} + \frac{c(1+i)\beta^2}{a\sqrt{2\pi\sigma_R\omega}}\right)$$

$$Z_{Lx} \approx Z_{Ly}/2$$

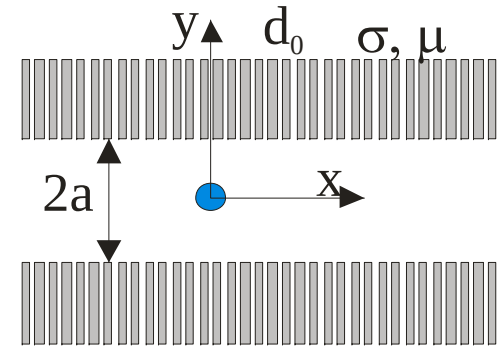
Real part achieves maximum at freq. where $\delta \approx a/\mu$ and beam magnetic field is changed so that $B_{||}$ at surface is decreased

- Real part of impedance is still too small at frequencies of interest: $f_0 \Delta Q$ ~ 12 kHz, qf_0 ~ 38 MHz



Contribution to Booster impedance if vacuum would be manufactured from non-laminated steel (μ=1000)
(F dipoles) L=140 m, a=2.08 cm,

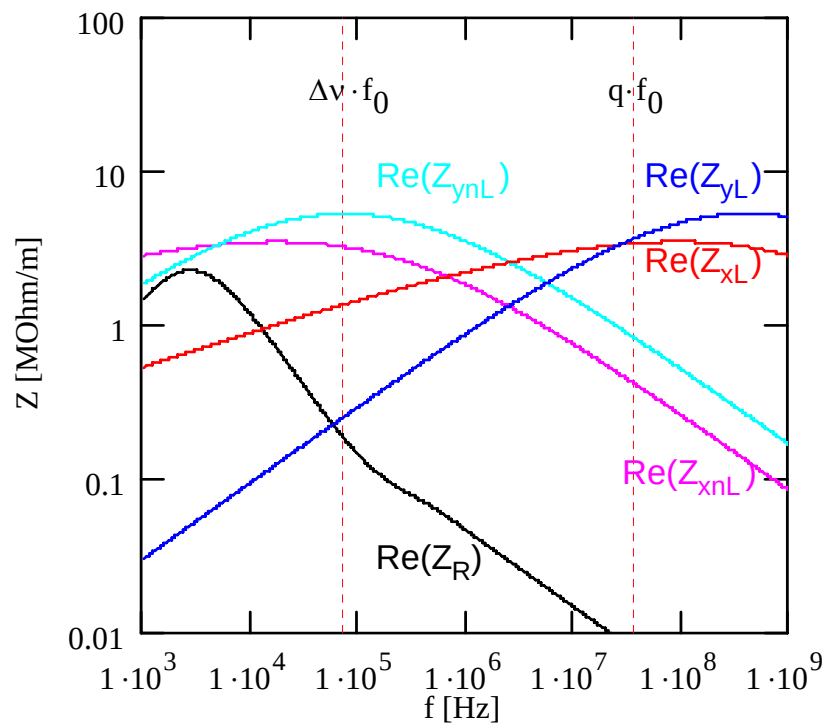
Effect of laminations



- Laminations force the beam current to flow not only on the inner surface but also deep inside laminations (~a)
- That greatly increases the impedance.
  - For high frequencies it was found by Burov and Danilov $Z_\perp = (2a/d_{lam}) Z_{\perp no\,lam}$
    - d_{lam} = 1.6 mm => amplification ~ 30
  - Laminations increase resistance and magnetic conductivity per unit length by $(2a/d_{lam})$
    - Recipe: $ka/\mu \Rightarrow k\, d_{lam}/2\mu$
    - Coincides with B-D
    - Does not exceed the "Careless" limit (A. Chao)

$$|Z_{L\perp}(\omega)| \le Z_m \equiv \frac{Z_0}{2\pi a^2 \beta} L_{tot}$$

- Exact knowledge of $Z_\perp$ below the first few harmonics of qf_0 is not required
  - no resonant harmonics for f < qf_0
  - small contribution for f < (2-4) qf_0



Contribution to Booster impedance from laminated (left curves) and non-laminated (right curves) dipoles (μ = 1000, $q = N_b$)
and SS round vac.chamber
(F dipoles) L=140 m, a=2.08 cm,
+ (D dipoles) L=140 m, a=2.85 cm

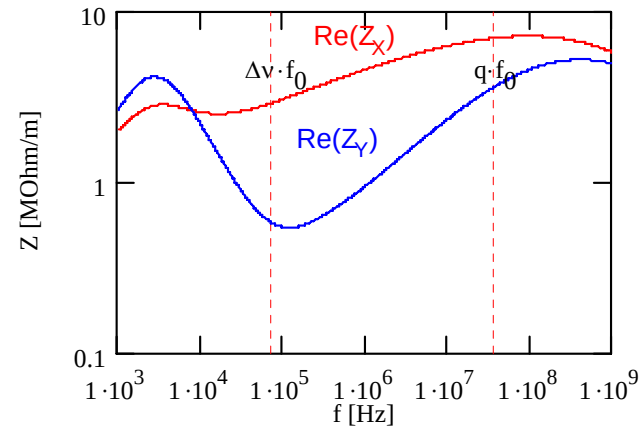
Main contributors to Booster impedance

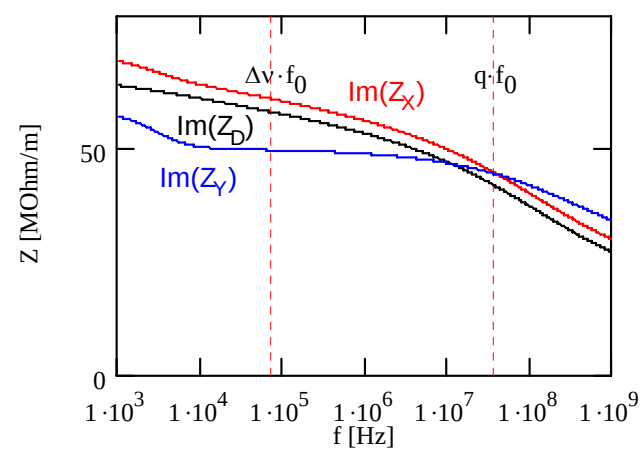
	F dipoles	D dipoles	Other vacuum chamber
Geometry	Flat	Flat	Round
Total length, m	138.7	138.7	196.7
Material	Steel	Steel	Stainless steel
Half-gap or radius, mm	20.8	28.5	42.8
Thickness of laminations, mm	0.63	0.63	N/A
Wall thickness, mm	N/A	N/A	1.6

- The following details are neglected in the model
  - All other imperfections
  - Angles of poles to horizontal plane (~5 deg)
- Averaging over beta-functions has been taken into account in computing $Z_\perp$

$$Z_{\perp eff}(\omega) = \frac{Q}{R}\oint Z_{L\perp}(\omega, s)\beta(s)ds$$

  - It resulted the following correction factors
    - Focusing dipoles: Kx=2.64, Ky=0.65
    - Defocusing dipoles: Kx=1.00, Ky=1.62
    - Round vacuum chamber: Kx=0.64, Ky=1.82





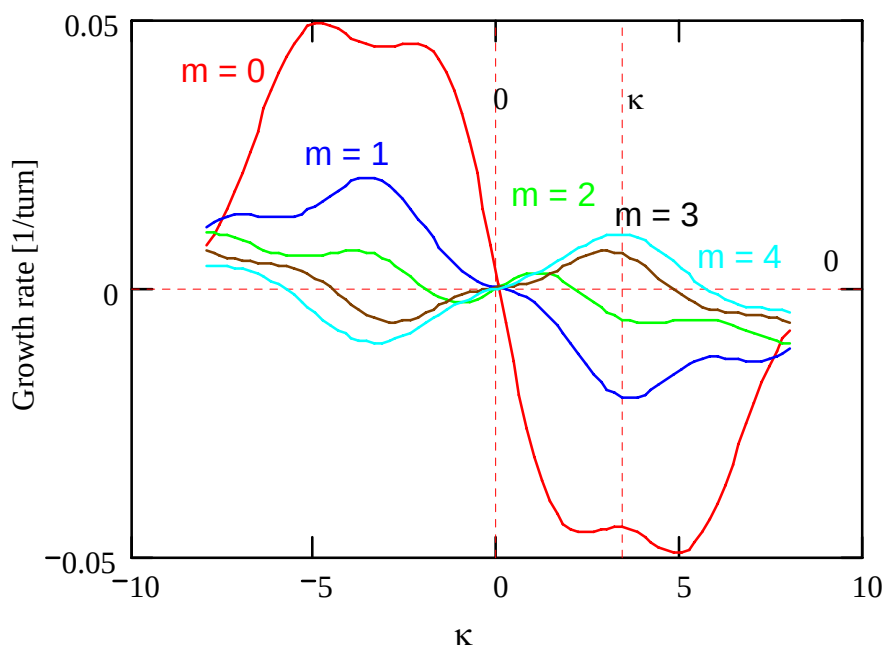
Total impedances

Estimate of instability growth rate

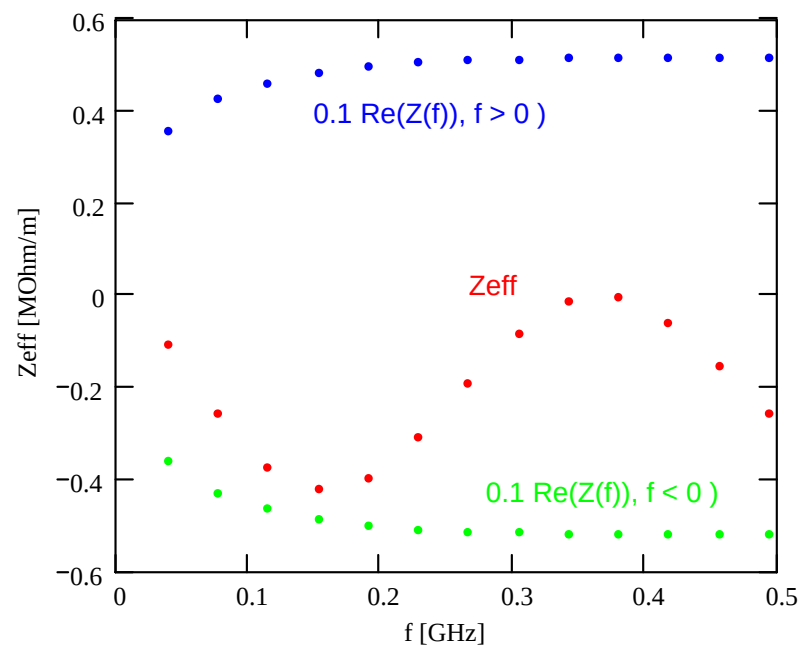
$$\lambda_m = -\frac{I_0 R}{2\left(mc^2/e\right)\gamma\beta^2 Q}\sum_{n=-\infty}^{n=\infty}\mathrm{Re}\left(Z_{\perp eff}(\omega_n)\right)\left(\mathrm{J}_m\left(\frac{\omega_n L_b}{\mathrm{v}_0}-\kappa\right)\right)^2 \xrightarrow{p+Q+mQ_s\equiv\Delta Q<<1}$$

$$-\frac{I_0 R}{2\left(mc^2/e\right)\gamma\beta^2 Q}\left(\mathrm{Re}\left(Z_{\perp rff}(\Delta Q\,\omega_0)\right)\mathrm{J}_m(\kappa)+\sum_{n=-\infty}^{n=\infty}\mathrm{Re}\left(Z_{\perp eff}(\omega_n)\right)\left(\left(\mathrm{J}_m\left(\frac{\tilde{\omega}_n L_b}{\mathrm{v}_0}-\kappa\right)\right)^2-\left(\mathrm{J}_m\left(\frac{\tilde{\omega}_n L_b}{\mathrm{v}_0}-\kappa\right)\right)^2\right)\right)$$

$$\omega_n = \omega_0\left(p+Q+mQ_s+nN_b\right) \quad\Rightarrow\quad \tilde{\omega}_n = nN_b\omega_0$$



Dependence of vert. growth rate per turn on head-tail phase ($L_b=\lambda_{RF}/4$, marker at ξ=-70)



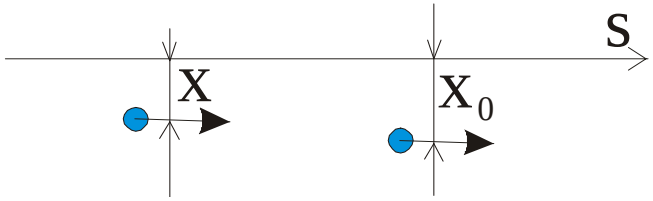
Contribution of differ. harmonics to the sum

Estimate of tune shifts from $Z_\perp$ (continuous beam, turns 30-60)

- Flat vacuum chamber
  - Both normal wake and detuning wake need to be taken into account

$$\int_L F_x ds' = e^2\left(x_0 W_x(s) - xD(s)\right)$$

$$\int_L F_y ds' = e^2\left(y_0 W_y(s) + yD(s)\right)$$



  - For round vacuum chamber: $D(s) = 0$
  - For flat vacuum chamber: $W_x(s) = D_x(s)$
- Making transition from two particles to continuous beam we obtain

$$\begin{bmatrix} \Delta Q_x \\ \\ \Delta Q_y \end{bmatrix} = -\frac{I_0 R}{4\pi\left(mc^2/e\right)\gamma\beta^2 Q}\begin{bmatrix} \mathrm{Im}\left(Z_x(\omega_n)\right) - \mathrm{Im}\left(Z_D(0)\right) \\ \\ \mathrm{Im}\left(Z_y(\omega_n)\right) + \mathrm{Im}\left(Z_D(0)\right) \end{bmatrix} = -\frac{I_0 R}{4\pi\left(mc^2/e\right)\gamma\beta^2 Q}\begin{bmatrix} \mathrm{Im}\left(Z_{yeff}(\omega_n)\right) \\ \\ \mathrm{Im}\left(Z_{yeff}(\omega_n)\right) \end{bmatrix}$$

- For vert. plane eff. impedance that yields $\mathrm{Im}(Z_{yeff})$ = 85 MΩ/m at 86 kHz
  - Decent agreement with measured values of 58 MΩ/m
- For horizontal plane:
  - Computations: 26 MΩ/m - Measurements: 24 MΩ/m
- Good agreement for such simple model
  - No space charge, no momentum spread, no accurate model for laminated magnets, no partial bunching

Discussion

- Tune shifts: Im($Z_\perp$) was measured at the lowest betatron sideband, 76 kHz
  - Computation yields: major contribution comes from Lasslett tune shift, effect of laminations on Im($Z_\perp$) is small
  - Computed Z_x and Z_y coincides comparatively well with measurements
- Growth rates: Instability is driven by Re($Z_\perp$) at many frequencies
  - f = ~ 12 kHz, $\pm k \cdot 38$ MHz
  - Weighting impedances over beta-function makes horizontal impedance larger than vertical one - the same as in the measurements
  - High frequency part of the impedance (80 – 300 MHz) makes major contribution to the instability development
    - Summing includes only f < 500 MHz

      $\Rightarrow$ Plate-to-plate capacitance should reduces $Z_\perp$ above 100-200 MHz
  - Estimated increments are about factor of 6 smaller than measured

    $\Rightarrow$ μ = 1000 is used.
      - It is OK for low frequencies
      - Questionable on high frequencies, δ_{skin} = 0.5 μm at 100 MHz, while domain size ~10 μm
  - Better understanding of laminated dipole impedance is required before we can make further conclusions

Could storing of electrons explain the instability?*

- Time of space charge compensation is about 0.1 second for $P=10^{-7}$ Torr
  - Multipactor production?
- Maximum transverse impedance related to electrons is set by beam radius and is 10-20 times larger
  - At free space

$$Z_{\perp e} \approx \frac{iZ_0}{2\pi a_{beam}{}^2 \beta}\left(1-\frac{\omega_p{}^2}{\omega_p{}^2-\omega^2+i\lambda\omega}\right) \quad , \qquad \omega_p \sim 100 \text{ MHz}$$

- It is quite probable!!!
  - More experimental observations are needed to prove it

Study proposal

- Perform measurements at smaller chromaticities and larger range of intensities (1, 2,4 and 8 turn injection)
  - multipactor has a threshold above 1 turn injection
  - Smaller chromaticity should suppress 3-rd synchro-betatron mode

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* Vadim Dudnikov

Backup transparencies

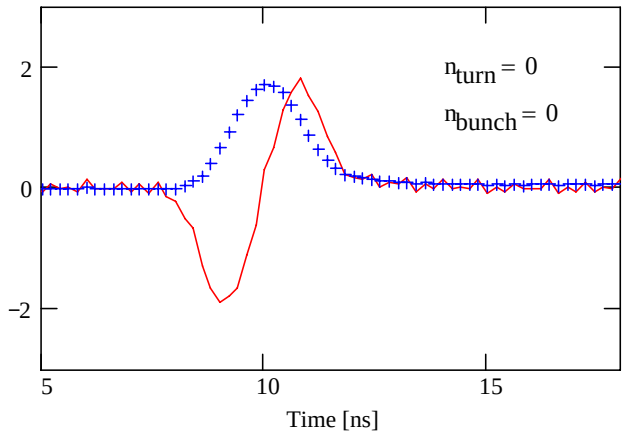
- **Problems aimed to be addressed**

- Beam lifetime through the cycle
  - Reduction of chromaticity
    - Loss of transverse beam stability – reason is still unknown
    - Improvements of beam stability with transverse bunch-by-bunch damper ??? (>200 MHz}
      - Previous attempt was unsuccessful
    - Impedance measurements and beam stability estimates – much better clarity
- Study of possibility of fast chromaticity measurements – very difficult at full intensity because of strong head-tail; possible at small intensity
- Transition crossing - do not need before transverse beam stability is addressed
  - Suppression of quadrupole oscillations
    - Quadrupole damper
    - Novel technique – RF voltage jump

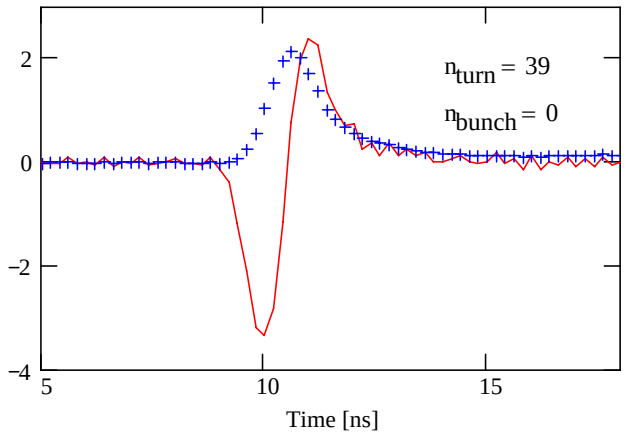
4. Transition crossing with RF jump technique

Experimental observations

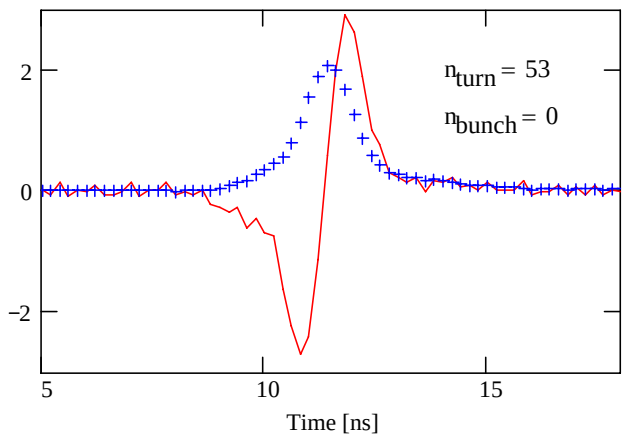
- Signal is taken from 15 cm BPM and is digitized by the scope at 5 Gsamples/s
- Beam current is proportional to the integral of BPM signal



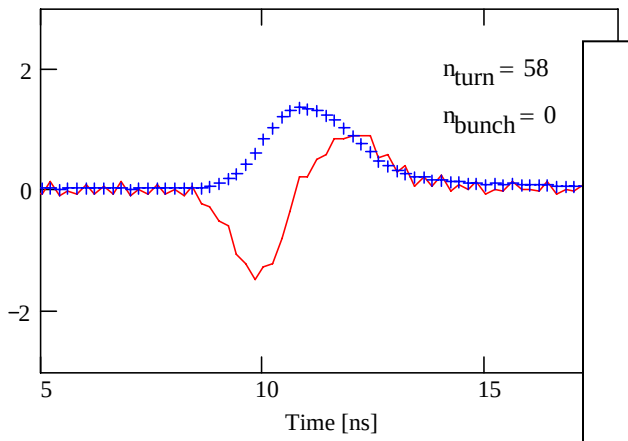
18.0 ms (before transition)



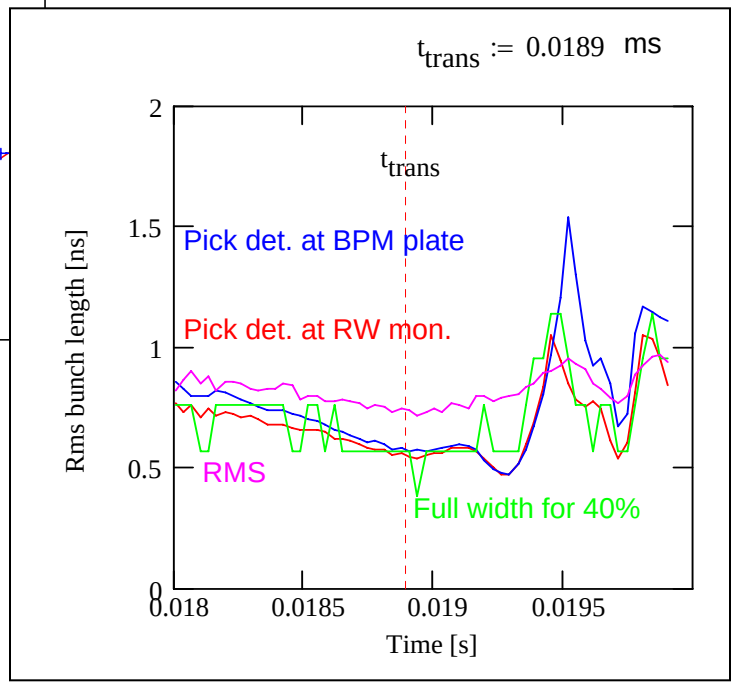
19.26 ms (1-st minimum in length)



19.71 ms (2-nd minimum in length)



19.87 ms (2-nd maximum)



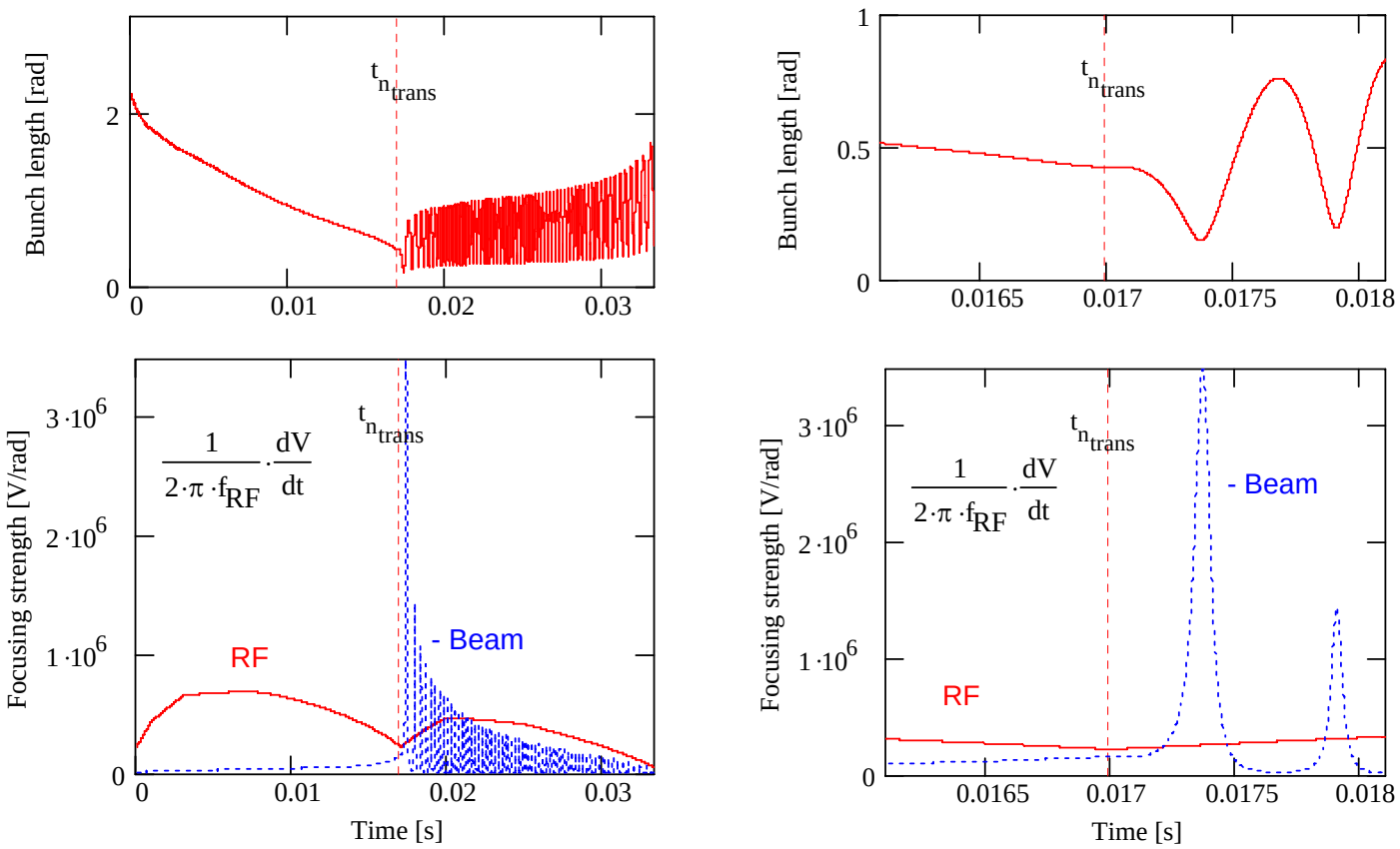
- After transition Coulomb repulsive force causes particle attraction

Linear model

- Both beam field and RF voltage are linearized

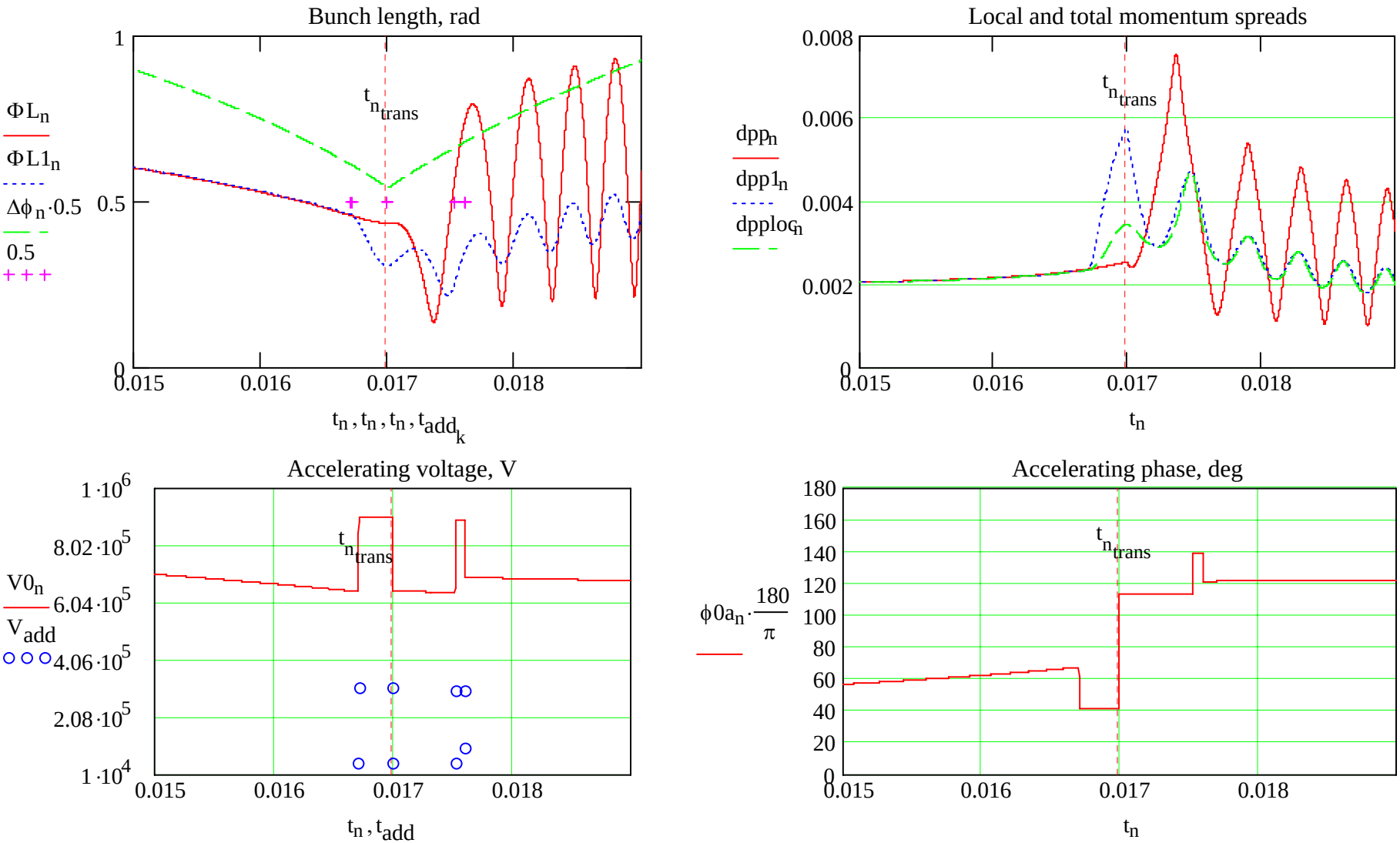
$$V(s) = V_0 \sin\left(\frac{2\pi}{\lambda}(s - s_0)\right) + V_{beam}(s) \Rightarrow \left[V_0 \cos\left(\frac{2\pi}{\lambda} s_0\right) + \frac{dV_{beam}}{ds}\right](s - s_0)$$

- Parabolic distribution of bunch charge
- Exact solution of longitudinal bunch envelope equations
  - motion is non-adiabatic near transition



Nominal transition crossing

Suppression of the bunch self-focusing (actually overfocusing)

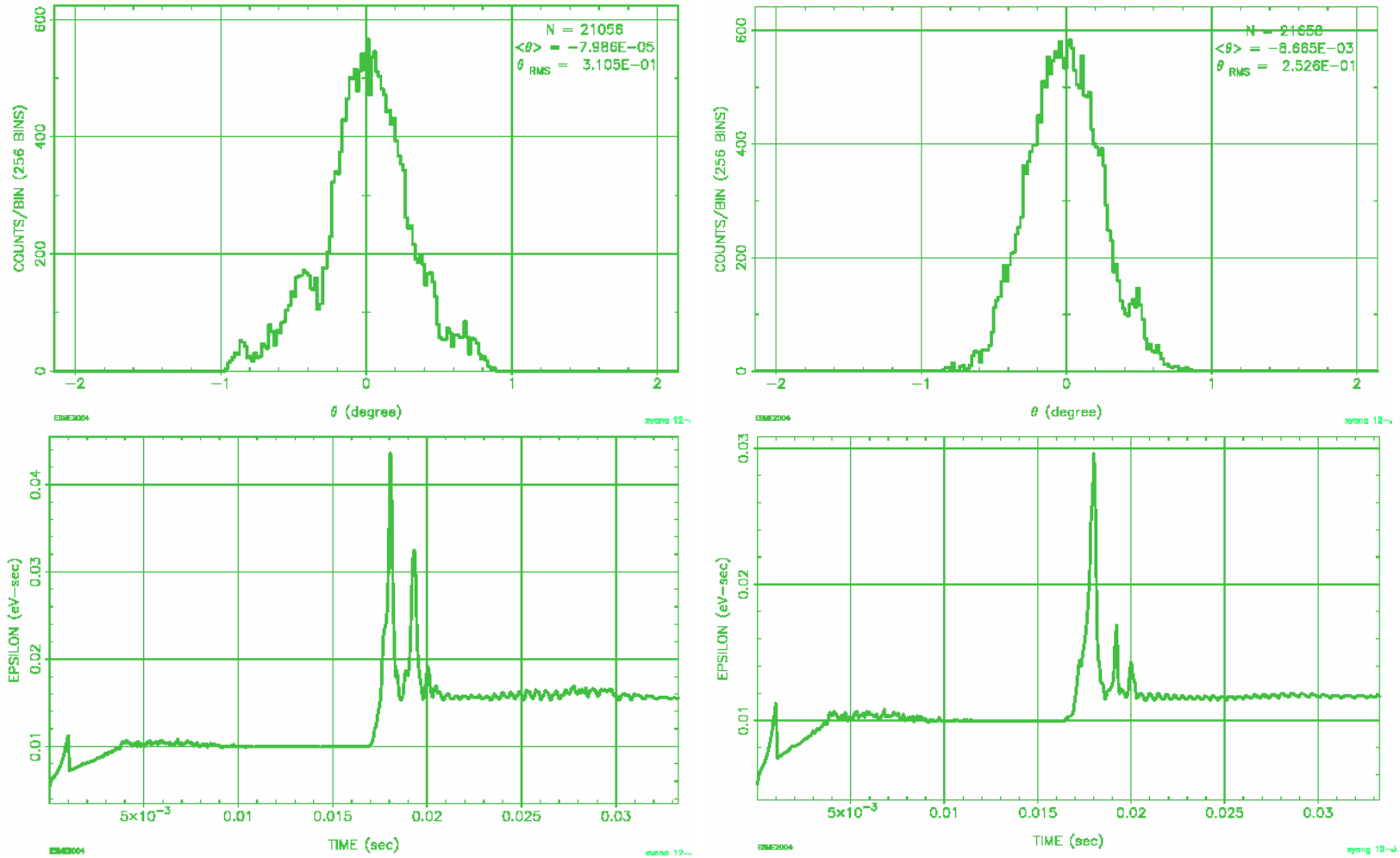


Two pulses. Length and time of the second pulse depends on intensity; for 5e12

- First pulse - duration of ~300 μs just before transition
- Second pulse – duration of ~80 μs, 530 μs after transition

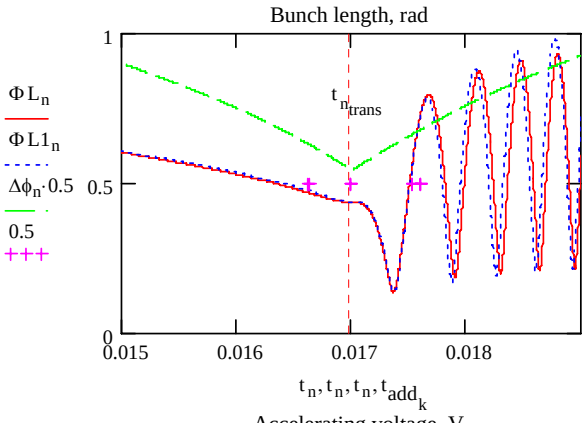
Numerical simulations

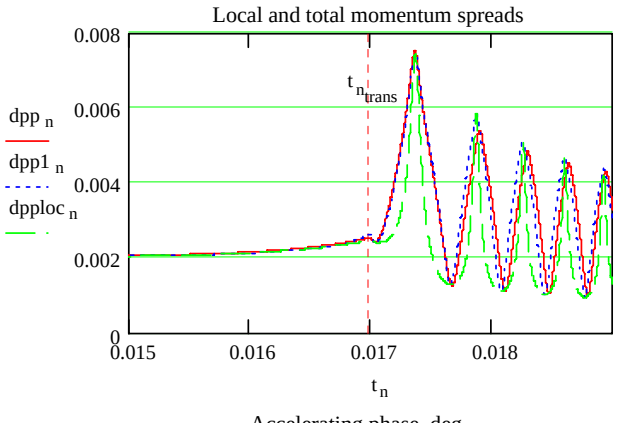
RF manipulation or not	charge at injection	charge at extraction	95% emittance at 8GeV (eVs)	rms deltaP (MeV)
yes	$5*10^{12}$	$4.55*10^{12}$	0.07052	3.357
no	$5*10^{12}$	$4.42*10^{12}$	0.09315	3.608



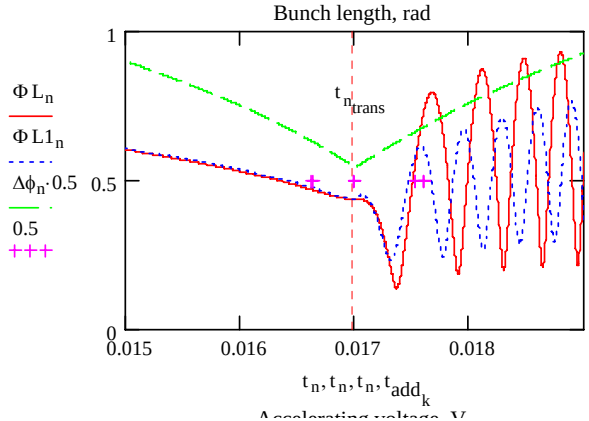
γt-jump

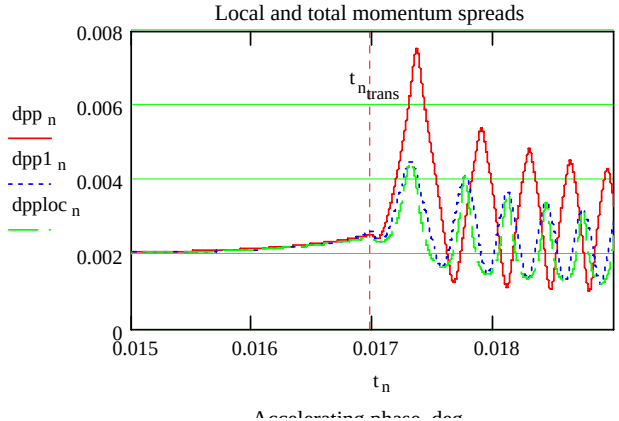
$\Delta\gamma_t = 0$



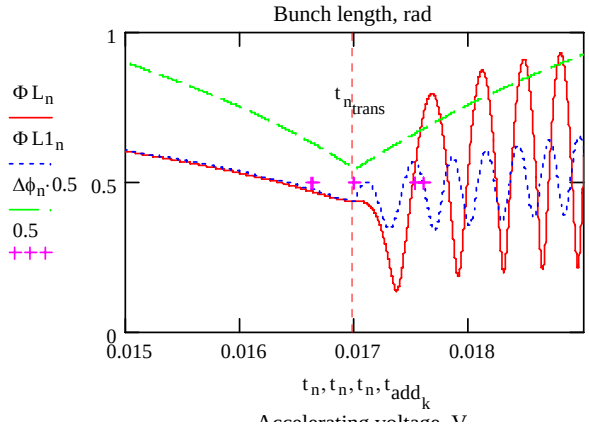


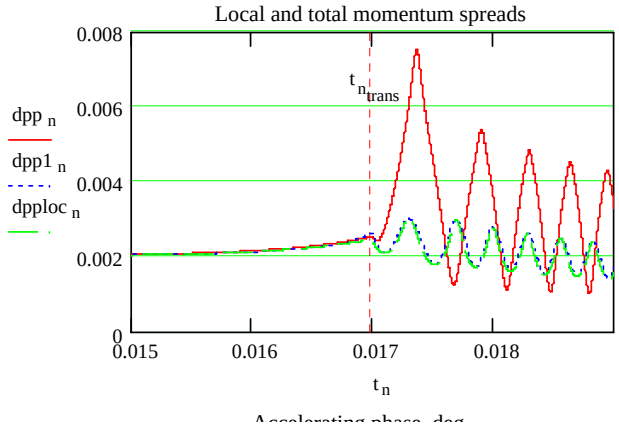
$\Delta\gamma_t = -0.2$





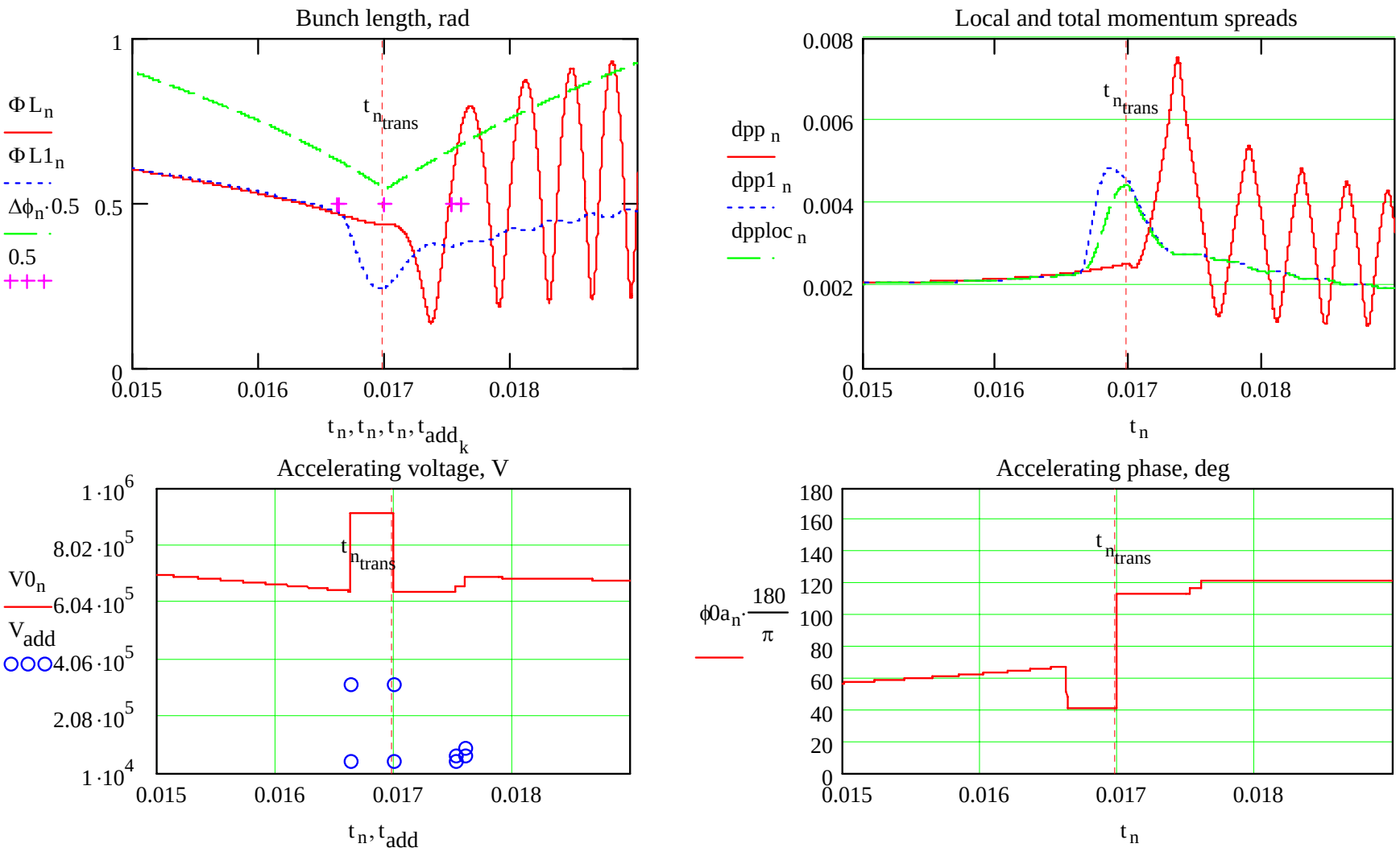
$\Delta\gamma_t = -0.5$





γt-jump and RF voltage jump

$\Delta\gamma_t = -0.2$



Experimental results

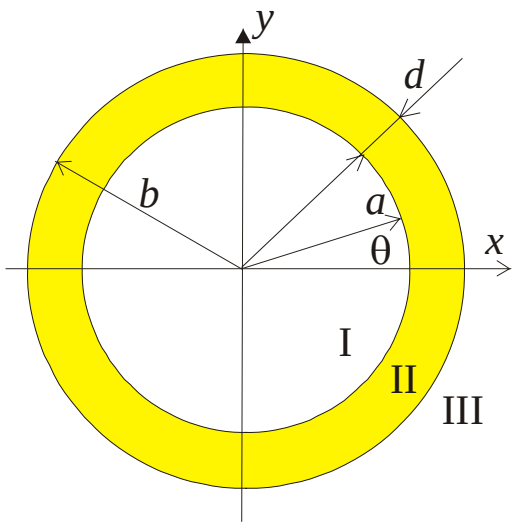
- The method was tested in Booster
  - Demonstrated suppression of the emittance growth with single voltage jump located before transition
  - 3 shifts of real operation
- Problems/concerns
  - Sensitive to loosing RF station (insufficient voltage)
  - Present low level RF hardware has insufficient time resolution preventing good tuning
  - Brings additional complications in operations
- Quadrupole longitudinal damper is presently used as main remedy preventing longitudinal emittance growth after transition

Transverse impedance for round vacuum chamber with thin walls

Impedance calculation for round vacuum chamber

$$k = \frac{1+i}{\delta} \quad , \quad \delta = \frac{c}{\sqrt{2\pi\sigma\omega}} \quad ,$$

$$Z_0 = \frac{4\pi}{c} \approx 377\,\Omega \quad .$$



I_0 – the beam current

x_0 – transv. beam motion amp.

Solutions for the vector potential in different regions

$$A_z(r,\theta,t) = \begin{bmatrix} A_I \\ A_{II} \\ A_{III} \end{bmatrix} = \begin{bmatrix} \dfrac{2I_0 x_0}{cr} + C_1 r \\ C_2 e^{k(r-a)} + C_3 e^{-k(r-a)} \\ \dfrac{C_4}{r} \end{bmatrix} e^{i\omega t} \cos\theta$$

Matching solutions at boundaries:

$$\left.\frac{dA}{d\theta}\right|_r = \left.\frac{dA}{d\theta}\right|_{r+0} \; ; \quad \left.\frac{dA}{dr}\right|_r = \left.\frac{dA}{dr}\right|_{r+0}$$

yields four linear equations for coefficients C_i. The solution is:

$$\begin{cases} C_1 = \dfrac{2I_0 x_0}{ca^2}\left[2\dfrac{e^{kd}(bk+1)+e^{-kd}(bk-1)}{e^{kd}(ak+1)(bk+1)-e^{-kd}(ak-1)(bk-1)} - 1\right] , \\ C_2 = \dfrac{4I_0 x_0}{ca}\dfrac{e^{-kd}(bk-1)}{e^{kd}(ak+1)(bk+1)-e^{-kd}(ak-1)(bk-1)} , \\ C_3 = \dfrac{4I_0 x_0}{ca}\dfrac{e^{kd}(bk+1)}{e^{kd}(ak+1)(bk+1)-e^{-kd}(ak-1)(bk-1)} , \\ C_4 = \dfrac{8I_0 x_0}{ca}\dfrac{b^2 k}{e^{kd}(ak+1)(bk+1)-e^{-kd}(ak-1)(bk-1)} . \end{cases}$$

Taking into account contribution from electric field of the beam we obtain an expression for the vacuum chamber transverse impedance ($\omega > 0$):

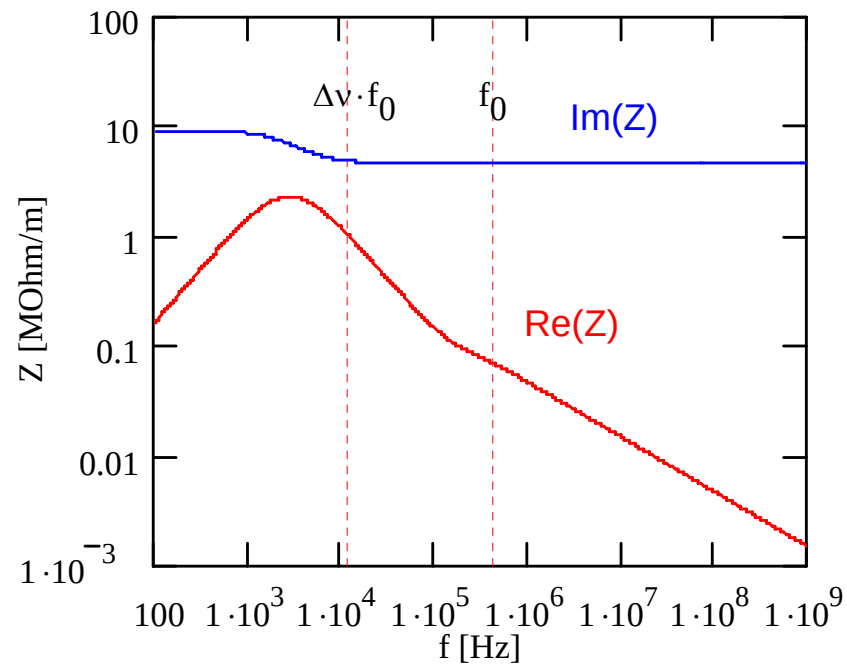
$$Z_\perp = i\left(\frac{C_1\beta}{I_0 x_0} + \frac{2}{ca^2\beta}\right) = i\frac{Z_0}{2\pi a^2\beta}\left[\frac{2\beta^2\left(e^{kd}(1+kb) - e^{-kd}(1-kb)\right)}{e^{kd}(1+ka)(1+kb) - e^{-kd}(1-ka)(1-kb)} + 1 - \beta^2\right]$$

$$\mathrm{Re}(Z_\perp(-\omega)) = -\mathrm{Re}(Z_\perp(\omega))$$

$$\mathrm{Im}(Z_\perp(-\omega)) = \mathrm{Im}(Z_\perp(\omega))$$

There are following asymptotes for transverse impedance:

$$Z_\perp \approx \frac{Z_0}{2\pi a^2\beta}\begin{cases} \dfrac{i}{\gamma^2} + \dfrac{c(1+i)\beta^2}{a\sqrt{2\pi\sigma_R\omega}} & , \quad \delta \le d \\ \dfrac{c^2\beta^2}{2\pi\sigma_R\omega a d} + i(\ldots) & , \quad \sqrt{ad} \ge \delta \ge \\ \dfrac{2\pi a\beta^2\sigma_R\omega d}{c^2} + i & , \quad \delta \ge \sqrt{ad} \end{cases}$$



Contribution to Booster impedance from stainless still vacuum chamber: L=200 m, a=4.3 cm, d=1.6 mm