Fermilab

Main Injector RF Power Requirement Calculations for the Proton Plan

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Introduction: Presently the Main Injector (MI) 53MHz RF system is used to slip-stack two Booster batches for pBar production and accelerate them along with another 5 Booster batches for NuMI. Intensities of $3.2x10^{13}$ protons have been achieved with the present system. The Proton Plan intends to implement multi-batch slip-stacking to increase the proton flux to NuMI. The Proton Plan design intensity is $5.5x10^{13}$ protons distributed within 5 double intensity and 1 single intensity Booster batches. This note presents the calculations of the expected RF system requirements for the Proton Plan design intensity. The calculation procedure is similar to that outlined in Ref. [1] except where noted. The calculations here include RF power amplifier (PA) operating conditions, series tube modulator (STM) dissipation, anode power supply (APS) requirements, and solid-state drive (SSD) power requirements.

RF System Station Configuration:

The MI h588 system consists of 18 cavity stations which are grouped as shown in Fig. 1. There are 3 Anode Power Supplies (APS) each of which powers 6 stations. The system is divided into two main groups: 9 Group A stations and 9 Group B stations. Each group is separately controlled in both amplitude and phase. This permits the creation of two accelerating voltage vectors at different frequencies for the slip-stacking process and allows 'paraphasing' (arbitrary vector addition) control of the vector sum.

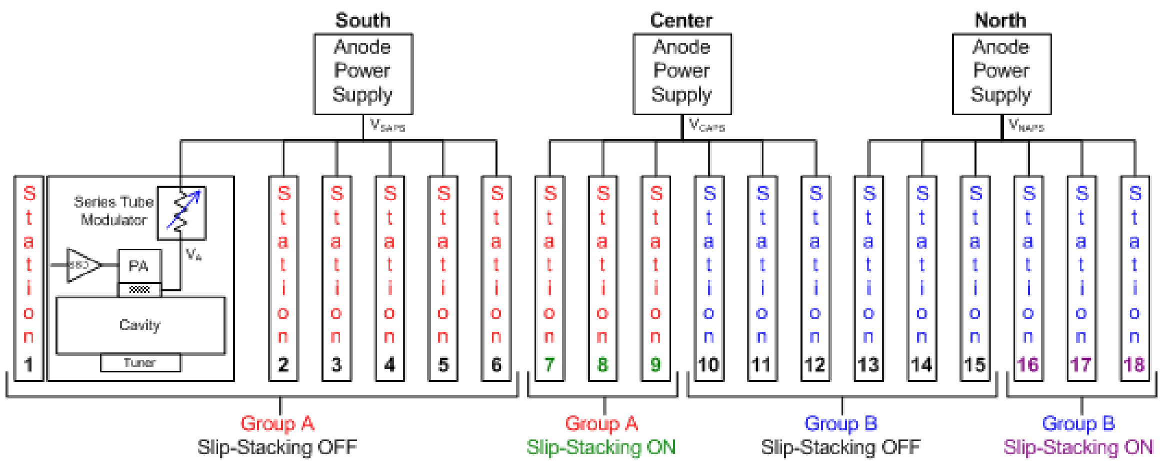


Figure 1: Present MI h588 RF system station configuration

Presently, within each group are 3 stations denoted as the 'Slip-Stacking ON' stations and 6 stations denoted as the 'Slip-Stacking OFF' stations. The Slip-Stacking ON stations are those stations which are used to produce the slip-stacking RF voltage for their respective group. The Slip-Stacking OFF stations are those stations whose LLRF drive is gated OFF in order not to contribute any RF voltage during slip-stacking; however, direct RF feedback and feed-forward beam loading compensation (BLC) is still active on these stations. This station on/off gating scheme is also used to create the necessary RF voltage for bunch rotation which is still needed for pBar production. It is the individual cavity minimum and maximum operating voltages in combination with the requested slip-stacking and bunch rotation voltages which have dictated the number of Slip-Stacking ON/OFF stations.

Main Injector Ramp:

The MI momentum ramp, bucket area, RF voltage, and synchronous phase angle for the multi-batch slip-stacking cycle are shown in figures 2-5 respectively.

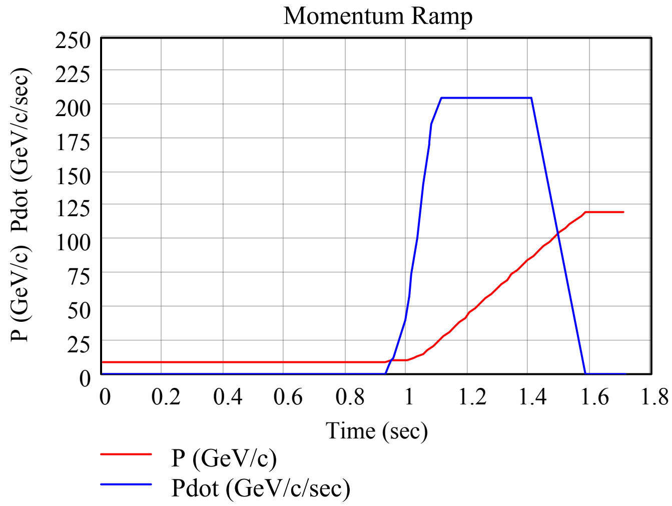


Figure 2: Multi-batch slip-stacking cycle momentum ramp

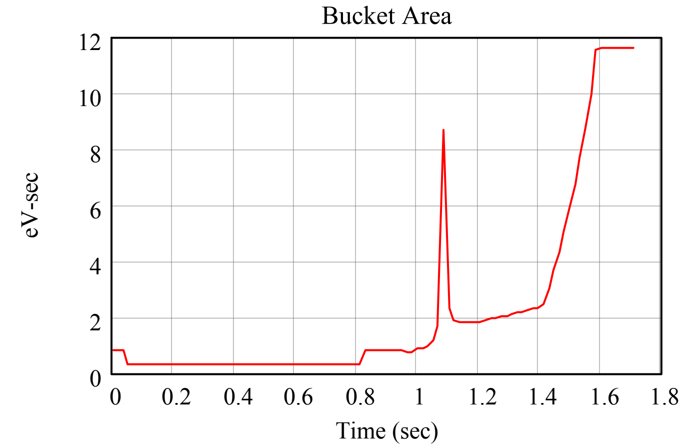


Figure 3: Multi-batch slip-stacking cycle bucket area

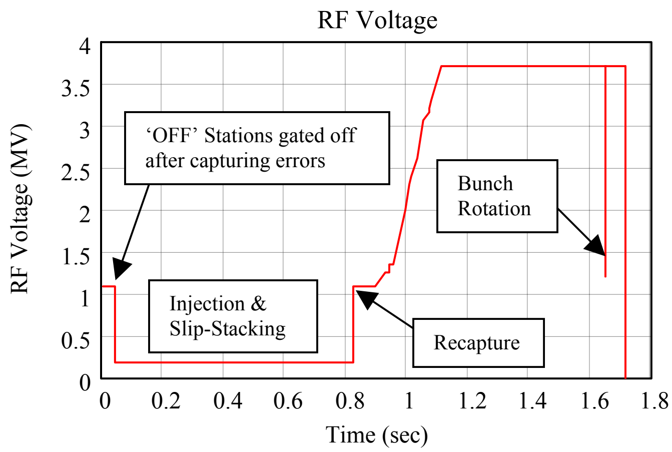


Figure 4: Multi-batch slip-stacking cycle RF Voltage profile.

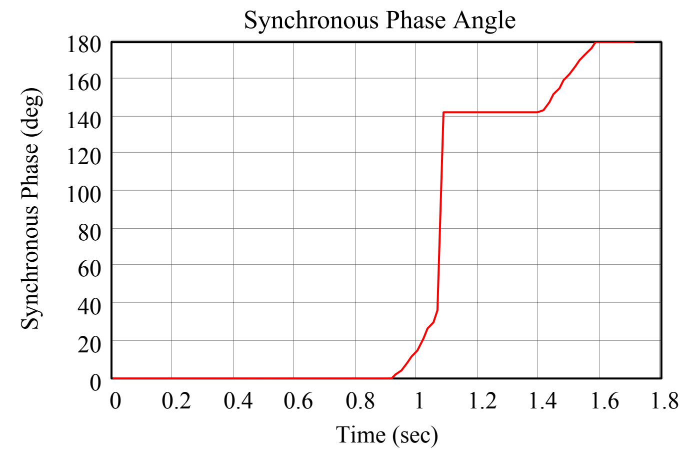


Figure 5: Beam synchronous phase angle; defined as the phase of the beam relative to the positive-slope zero crossing of the RF voltage.

RF Circuit Model:

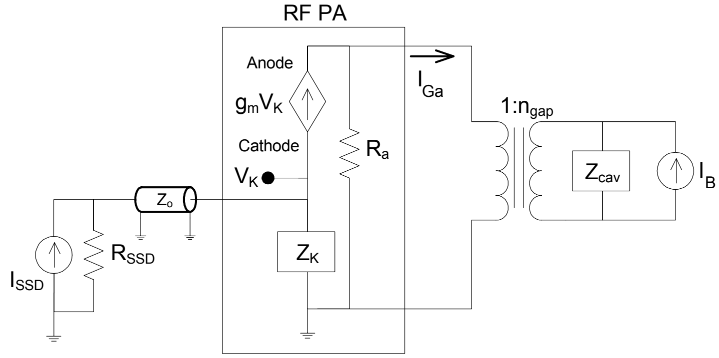


Figure 4: RF Circuit model for calculations.

The single station circuit of Fig. 6 is used as the model for estimating the RF PA operating conditions and associated power supply requirements. The beam image current, I_B, and the RF PA generated anode current, I_{Ga}, (transformed through a transformer with turns ratio, $n_{gap} \cong 12.25$, representing the gap:anode voltage step-up ratio) drive the cavity impedance, Z_{cav}. The RF PA is cathode driven; thus, the solid-state driver (SSD), represented as I_{SSD} with internal impedance $Z_{SSD} \cong 6.25\Omega$, must also provide the generated anode current. The SSD consists of eight 1KW amplifiers which are combined using two 4-way Wilkinson style combiners each of which is designed for a 12.5 Ohm output impedance. The four coaxial outputs of each combiner are combined for a total of eight equally driven 50Ω coax cables that drive the PA cathode circuit. The cathode circuit impedance, represented as Z_K, consists of a resistively tapped inductive stub and the PA tube cathode-to-grid capacitance. The resistively tapped inductive stub is designed to resonant with the cathode-to-grid capacitance and provide ~100Ohms of resistive loading when the tube is cut off.

Although the RF PA can be modeled as a voltage controlled current source with an appropriate output impedance R_a, it must provide the necessary anode current, I_{Ga}, to counteract the beam current and drive the cavity impedance. Thus, for the purposes of this note, R_a, is neglected and instead the necessary conditions to generate I_{Ga} are considered. Note however that R_a cannot be neglected when considering the dynamic response of the circuit.

The calculations in this note follow the convention of Refs. [2] and [3] such that the phasor diagram as shown in Fig. 7 is used:

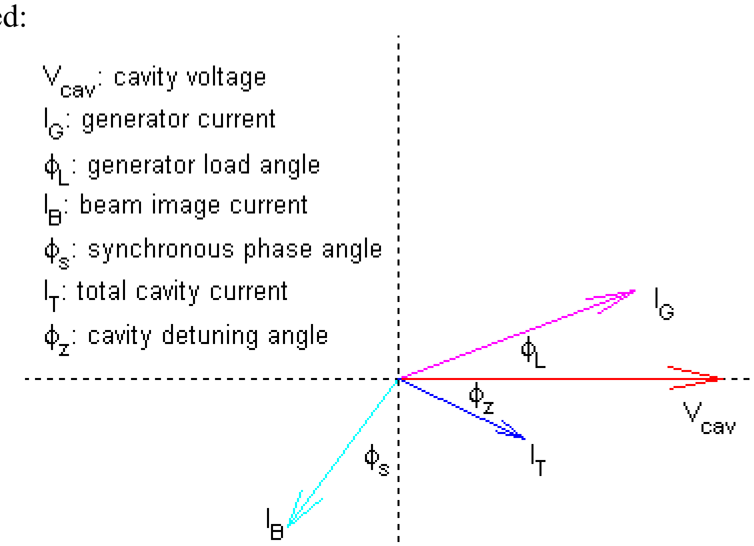


Figure 5: Cavity phasor diagram

The cavity voltage phasor is given as:

$$\hat{V}_{cav} = \hat{I}_T \cdot Z_{cav} \quad (1)$$

where $\hat{I}_T = \hat{I}_G + \hat{I}_B$ is the total current given by the vector addition of the generator (at the gap) and beam image currents. The cavity impedance is given as:

$$Z_{cav} = R_s \cos\phi_Z \cdot e^{j\phi_Z} \quad (2)$$

where R_s is the cavity shunt impedance and ϕ_z is the cavity tuning angle given by:

$$\tan\phi_z \cong 2Q\frac{\Delta f}{f_o} \quad (3)$$

with $\Delta f = f_o - f$ giving a measure of the cavity detuning as the difference between cavity resonant frequency f_o and the RF drive frequency f, and Q being the quality factor. For this note, it is assumed that the cavity $Q = 4000$ and $\frac{R_s}{Q} = 100$.

Quantities can be transformed from the cavity (or gap) side to the anode through the simple ideal transformer relationships. Thus, the necessary generator current, $\hat{I}_{Ga}$, that the RF PA must develop for a given cavity voltage is determined by simple vector addition once the beam current and cavity detuning angle are known.

Once $\hat{I}_{Ga}$ is known, the required cathode drive voltage phasor, $\hat{V}_K$, to generate this current can be determined from the tube constant current curves. For this note, the PA tube current curves are approximated by the functional fits found in [1]. The corresponding PA tube DC anode current can be used to estimate the STM power dissipation and APS supply requirements. Finally, once $\hat{V}_K$ is known, the required SSD forward power, $P_{SSD\ fwd}$, can be determined from:

$$P_{SSD\ fwd} = \frac{P_L}{1 - \left|\Gamma_L\right|^2} \quad (4)$$

where $P_L \cong \frac{1}{2}\frac{\left|\hat{V}_K\right|^2}{R_k} + \frac{1}{2}\hat{V}_K \hat{I}_{Ga}^*$ is the power delivered to the cathode circuit load $R_k = 100\Omega$ and to the anode circuit through the cathode, and $\Gamma_L = \frac{Z_{K\ tot} - Z_o}{Z_{K\ tot} + Z_o}$ is the load reflection coefficient that the total cathode circuit impedance $Z_{K\ tot}$ presents to the SSD cabling. The total cathode circuit impedance can be found from $Z_{K\ tot} = \frac{R_k \cdot Z_{K\ tube}}{R_k + Z_{K\ tube}}$ with $Z_{K\ tube} \cong \frac{\hat{V}_K}{\hat{I}_{Ga}}$.

The Beam Image Current and Bunch Form Factor:

The beam image current of a batch can be represented by a pulse-wave amplitude-modulated bunch pulse train as shown in Fig. 8.

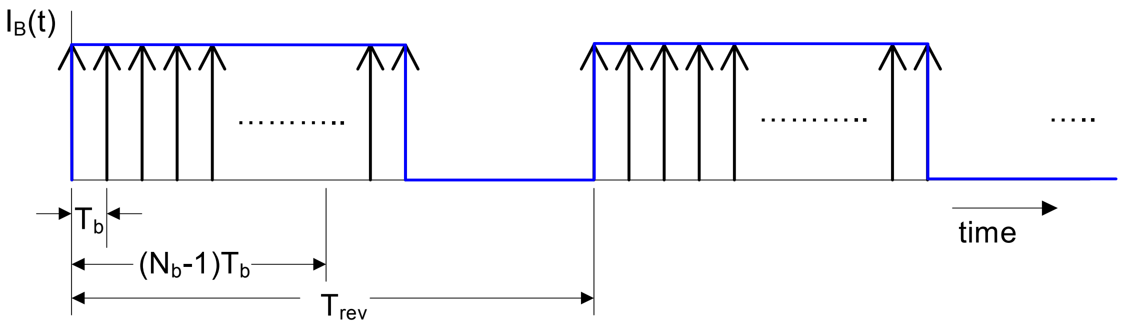


Figure 6: Beam image current representation as a pulse-wave amplitude modulated current-pulse train.

Mathematically, the beam image current can be represented as the multiplication of two Fourier Series;

$$I_B(t) = \left[I_{dc} + \sum_{k=1}^{\infty} I_k \cos(k\,2\pi\, f_{RF}\, t - \phi_k) \right] \cdot \left[A_{dc} + \sum_{m=1}^{\infty} A_m \cos(m\,2\pi\, f_{rev}\, t - \theta_m) \right] \quad (5)$$

with the first series representing the bunch current pulse train with a period equal to the bunch spacing, $T_b = \frac{1}{f_{RF}}$, and the second series representing the pulse-wave with a period equal to the revolution period, $T_{rev} = \frac{1}{f_{rev}}$. The duty factor for the pulse-wave can be adjusted according to the ring filling factor and the representation adjusted to include both single and double intensity batches. For simulating the response at the fundamental mode impedance of the RF cavity, it suffices to consider only the fundamental RF component, k=1, of the first series:

$$I_B(t) \cong \left[I_{RF} \cos(2\pi\, f_{RF}\, t - \phi_k) \right] \cdot \left[A_{dc} + \sum_{m=1}^{\infty} A_m \cos(m\,2\pi\, f_{rev}\, t - \theta_m) \right] \quad (6)$$

Thus, the beam image current can be considered to be a pulse-wave amplitude-modulated fundamental RF signal. The bunch form factor, F_b, has historically been defined as

$$2F_b = \left| \frac{I_{RF}}{I_{dc}} \right| \quad (7)$$

and can be found from the bunch shape. This note has used the estimate technique of Ref. [1] to determine the bunch shape and hence the bunch form factor. The technique assumes that in longitudinal phase space the particles are uniformly distributed within a contour that encloses the longitudinal emittance as shown in the example of Fig. 9.

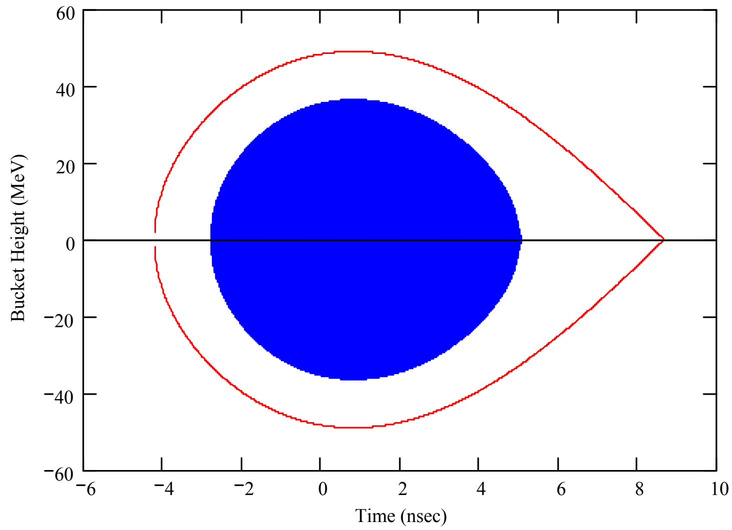


Figure 7: Example of the estimated bunch shape for calculating the bunch form factor.

From the expression for particle motion in $\Delta E\text{–}\phi$ phase space [4],

$$\Delta E^2 + \frac{q\beta^2 E_s V}{\eta h}\left[\cos(\phi) + \phi \sin(\phi_s)\right] = K\,, \quad (8)$$

the edge of the blue shaded area is determined by finding the value of the constant, K, that results in the contour given by (4) enclosing the beam longitudinal emittance. In (8) ΔE is the energy difference of the particle with respect to the ideal particle, E_s is the energy of the ideal synchronous particle, $\beta = \frac{v}{c}$ is the velocity factor, η is the slip-factor, h is the harmonic number, V is the RF voltage, ϕ_s is the synchronous phase angle, and $\phi = \omega_{RF}\, t$ is the particle phase.

The bunch form factor was solved for the MI ramp conditions shown in Figs. 2-5 assuming that the beam longitudinal emittance was 0.12 eV-sec from injection to recapture, 0.3 eV-sec from recapture to transition, and 0.5 eV-sec after transition. The result is shown in Fig. 9 below:

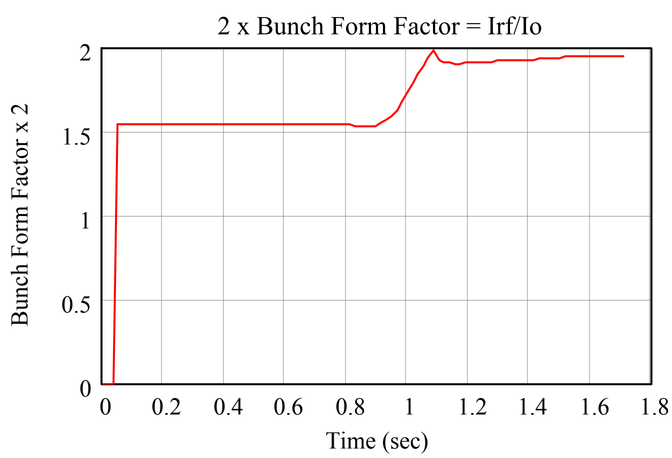


Figure 8: Calculated I_{RF}/I_o for the MI ramp assuming a longitudinal emittance of 0.12 eV-sec before recapture, 0.3 eV-sec from recapture to transition, and 0.5 eV-sec after transition

Calculation Procedure:

To estimate the RF system requirements throughout the entire cycle, the following was assumed:

- The final ring filling is: (1) 5/7 full of double intensity batches, (2) 1/7 full of a single intensity batch, and (3) 1/7 empty (beam gap). Thus, the RF requirements for these three 'instantaneous' conditions were separately solved and a 'moving average' was formulated from the appropriate weighted average of the three conditions. During the slipping portion of the cycle, the 'moving average' was taken to be equal to the single intensity batch. This is a good first order estimate since the slipping beams are at different revolution frequencies and hence the second batch beam current phasor is in effect rotating around the first batch beam current phasor at the beat frequency. An example of the cavity phasor diagram for these three 'instantaneous' conditions is shown in Figs. 11-13.

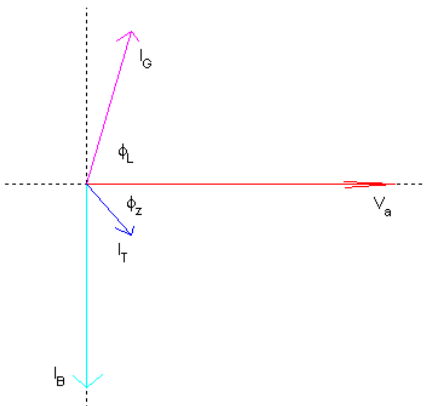


Figure 9: Double Intensity batch 'instantaneous' phasor diagram.

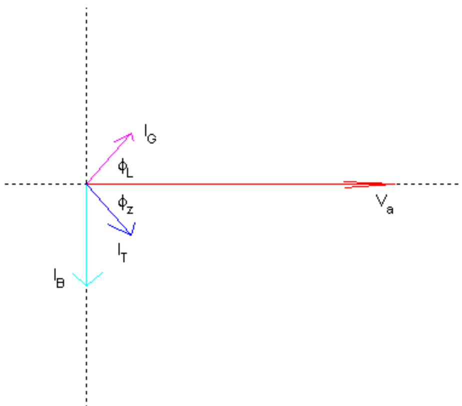


Figure 10: Single Intensity batch 'instantaneous' phasor diagram.

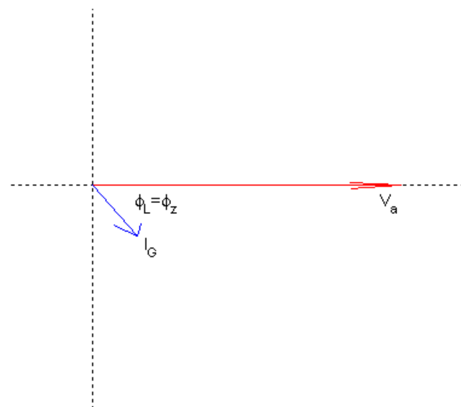


Figure 11: Beam Gap 'instantaneous' phasor diagram.

- The cavity detuning was dynamically adjusted as follows:
  - During Slipping:
    - The 'ON' stations were half detuned for the single intensity batch condition. This is a first attempt to reduce the peak installed generator current (see Ref. [3]).
    - The detuning of the 'OFF' stations was set to zero; however no matter what detuning is chosen, the 'OFF' stations have to provide an equal and opposite current to cancel the beam loading. In terms of estimating the station power requirements, these stations cannot benefit from detuning since there is no cavity voltage developed.
  - Remainder of the Cycle:
    - Both the 'ON' stations and the 'OFF' stations were detuned to present a real load impedance to the RF PA ($\phi_L = 0$) for the double intensity batch condition. This was a first order attempt to reduce the 'moving average' results since this is the condition with a weighting of 5/7.

- The dynamic RF PA grid biasing was assumed to be equivalent to what is presently used during a mixed mode cycle (NuMI plus slip-stacking for pBar production). The present curve as adopted for the multi-batch cycle is shown in Fig. 14.

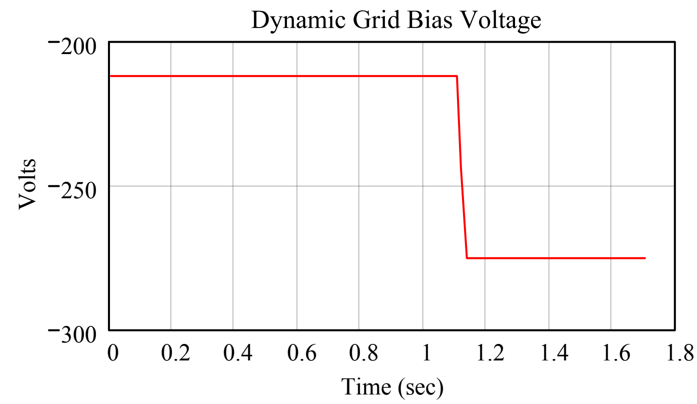


Figure 12: Assumed dynamic RF PA grid biasing for the multi-batch cycle.

- The RF PA anode biasing was adjusted as follows:
  - During Slipping:
    - The anode bias voltage was kept equal to 6.3 kV which is the present minimum anode voltage bias value. This assumes that the Mid-Level RF (MRF) system is installed and screen current regulation is not being employed. The previous system (pre MRF) would make the anode bias 3.5 kV during slipping. This would result in too high STM dissipation.
  - During the Remainder of the Cycle:
    - The anode bias voltage was adjusted to keep it's minimum instantaneous value equal to 2kV so as to not draw excessive screen current.
  - The dynamic RF PA anode biasing is shown in Fig. 15.

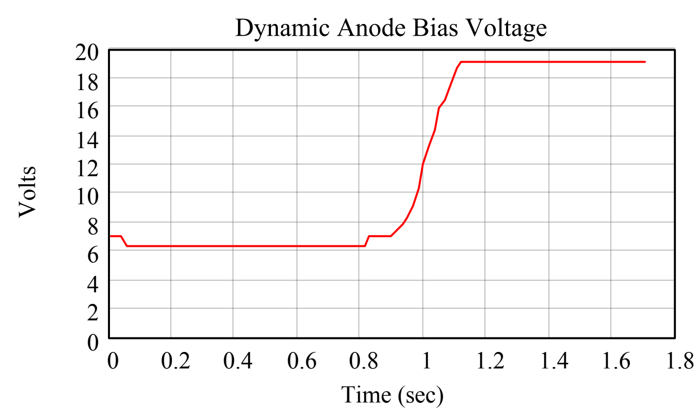


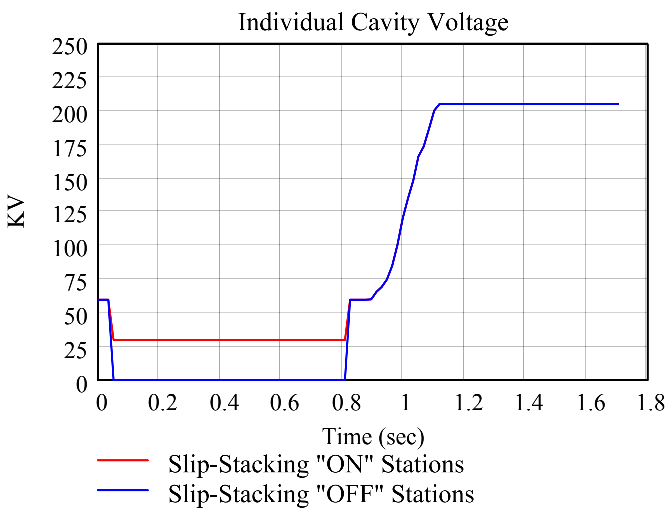
Figure 13: Dynamic RF PA anode biasing for the multi-batch cycle

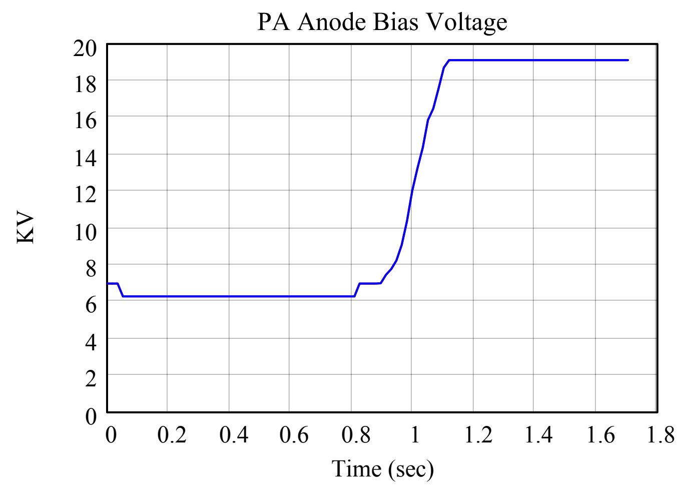
- The calculations were performed as follows:

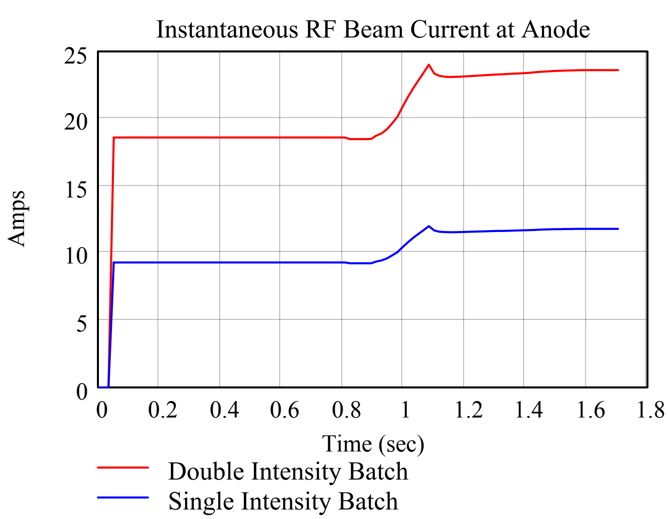
  1. The MI ramp of Figs. 2-5 and the bunch form factor of Fig.10 were used to calculate the beam currents for the three 'instantaneous' conditions. The single intensity batch was assumed to be $5x10^{12}$ particles per batch of 84 bunches; the double intensity batch, $1x10^{13}$ particles per batch of 84 bunches.
  2. Using the cavity detuning determined as described above, the single station required generator current at the PA anode was calculated using the PA biasing conditions and the desired cavity gap voltage for the three 'instantaneous' conditions.
  3. Using the PA tube current curve fits from [1], the cathode drive voltage necessary to produce the required generator current was found for each of the three 'instantaneous' conditions.
  4. Now that the tube conditions (biasing and dynamic cathode and anode swings) were completely described, the PA tube DC anode current for the three 'instantaneous' conditions could be found.
  5. Furthermore, since the cathode drive conditions at this point are known, the SSD requirements could be determined using Eq. (4) for the three 'instantaneous' conditions.
  6. Knowing the results for the three 'instantaneous' conditions, the 'moving average' could be calculated as described above to determine the STM dissipation and APS supply requirements. The South APS was assumed to supply 6 'OFF' statioins, while both the Center APS and North APS were each assumed to be supplying 3 'ON' stations and 3 'OFF' stations as shown in Fig. 1. In addition to the 'moving average', a cycle average was determined for the APS's assuming that the cycle repetition was every 2 sec.

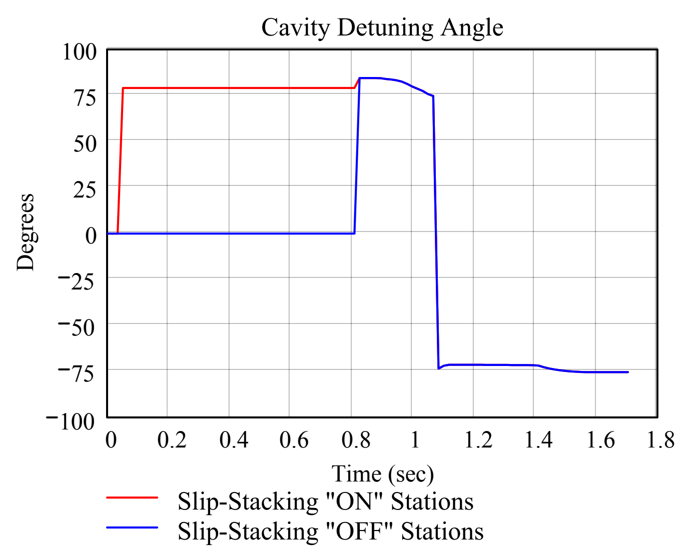
Results:

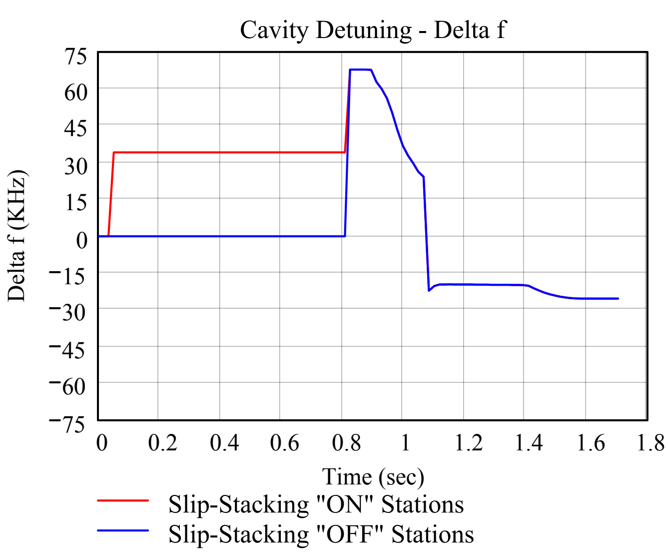
The results of the calculations are shown on the following pages.

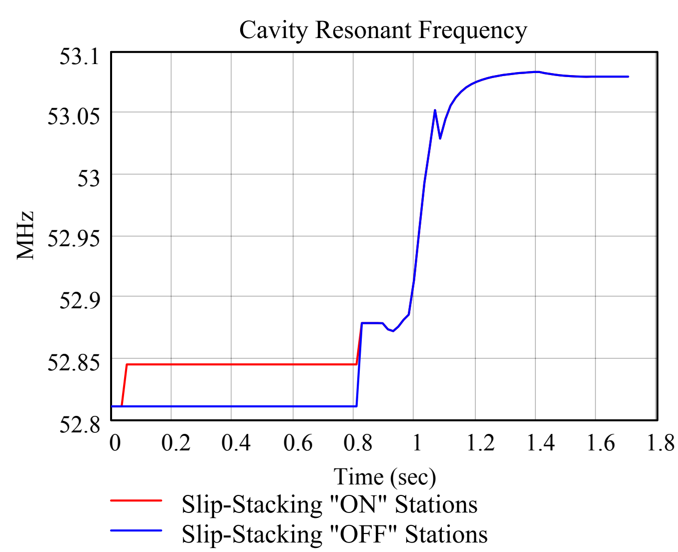

Individual Cavity Voltage
KV
Time (sec)
Slip-Stacking "ON" Stations
Slip-Stacking "OFF" Stations

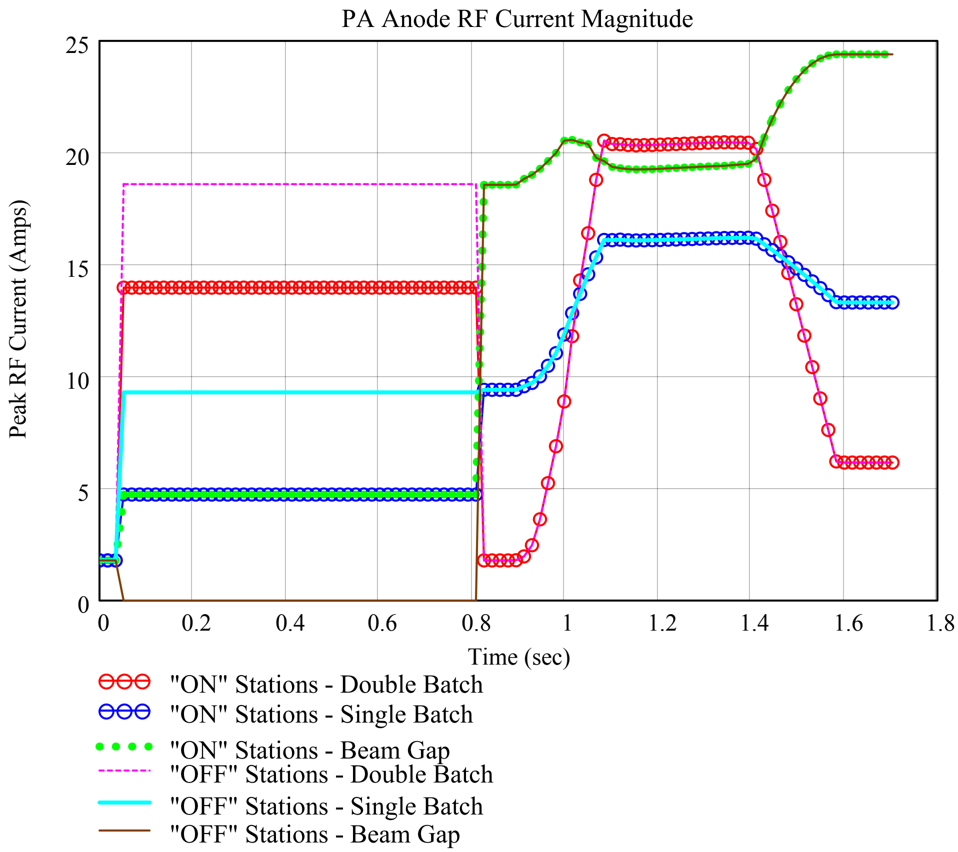

PA Anode Bias Voltage
KV
Time (sec)

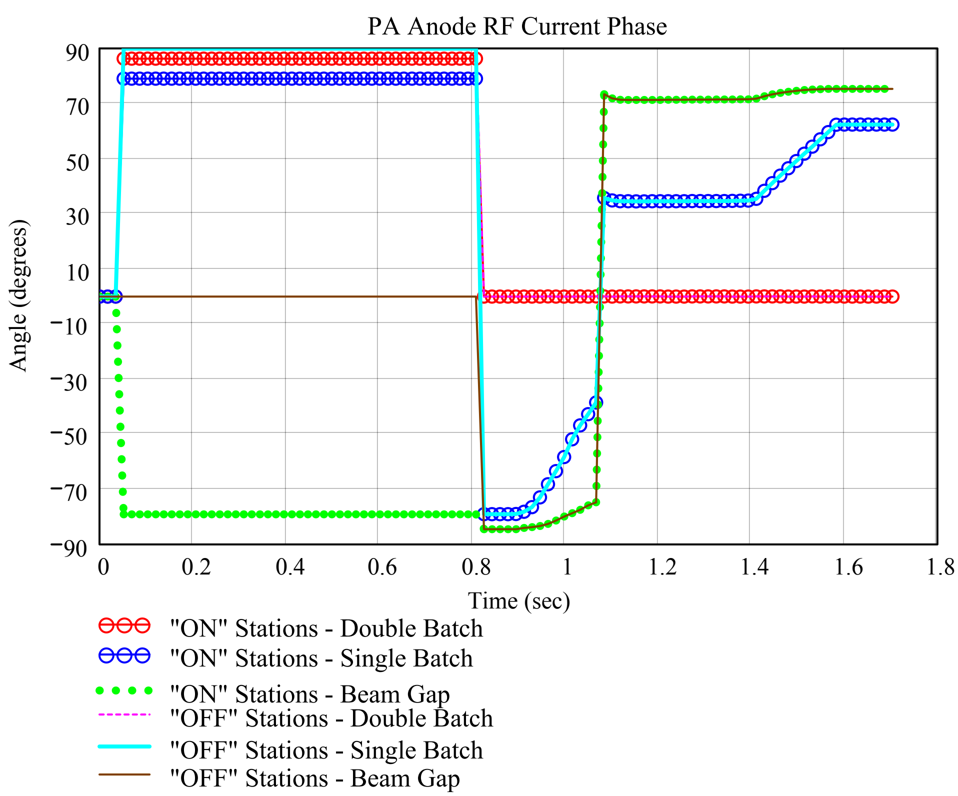

Instantaneous RF Beam Current at Anode
Amps
Time (sec)
Double Intensity Batch
Single Intensity Batch

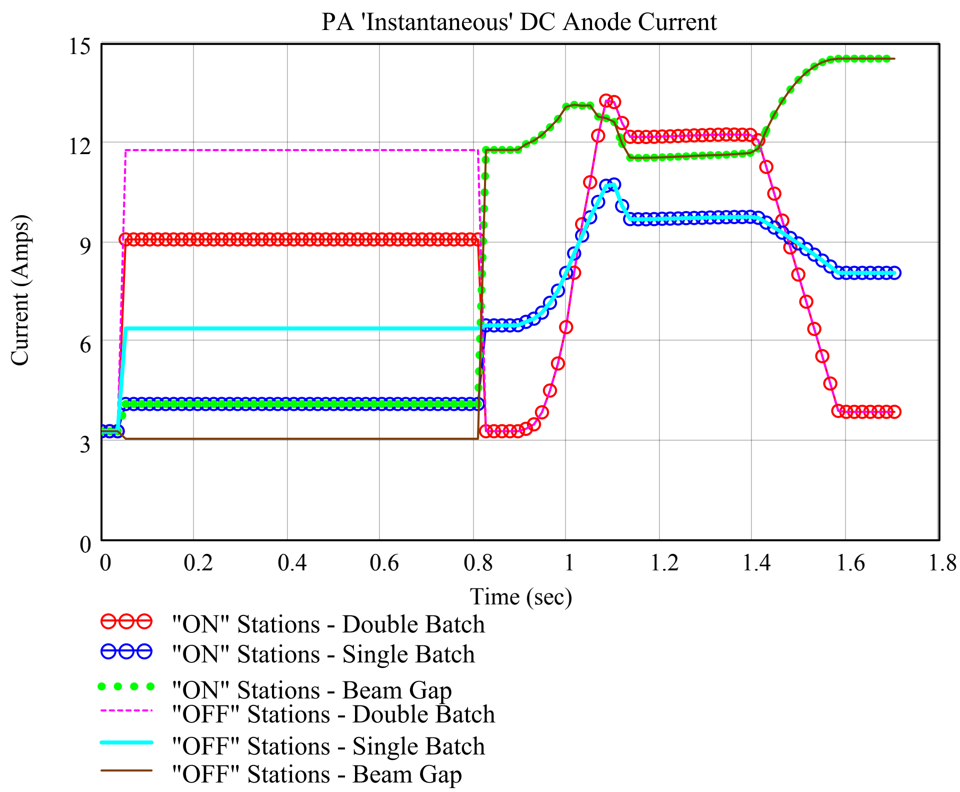

Cavity Detuning Angle
Degrees
Time (sec)
Slip-Stacking "ON" Stations
Slip-Stacking "OFF" Stations

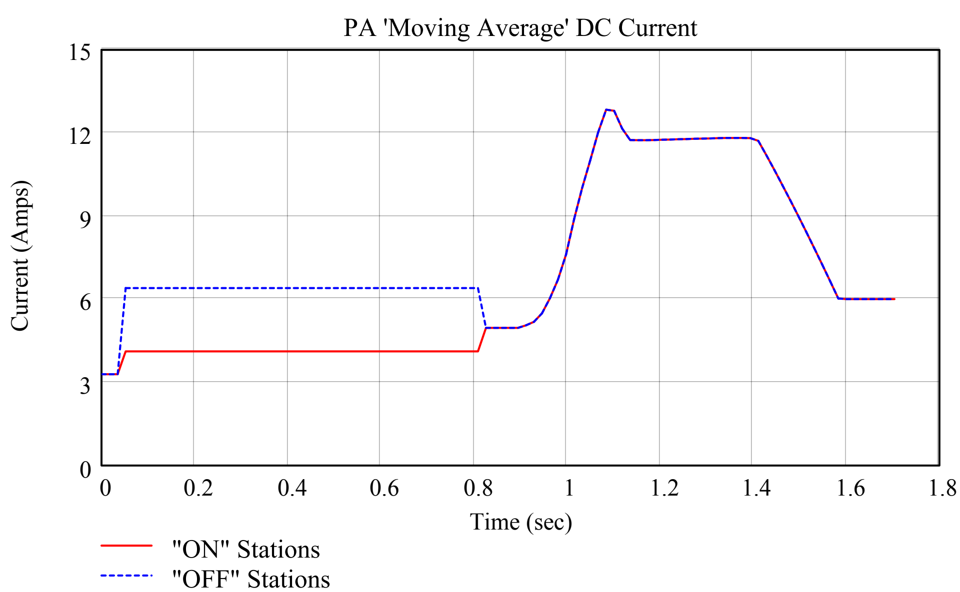

Cavity Detuning - Delta f
Delta f (KHz)
Time (sec)
Slip-Stacking "ON" Stations
Slip-Stacking "OFF" Stations

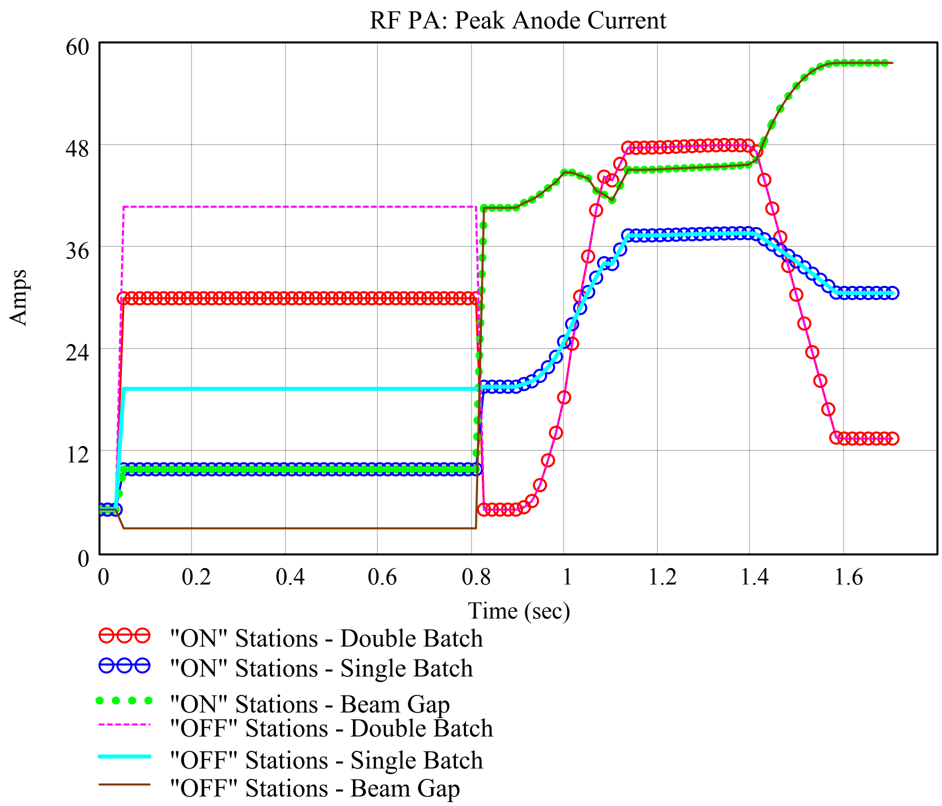

Cavity Resonant Frequency
MHz
Time (sec)
Slip-Stacking "ON" Stations
Slip-Stacking "OFF" Stations


PA Anode RF Current Magnitude
Peak RF Current (Amps)
Time (sec)
0
5
10
15
20
25
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
"ON" Stations - Double Batch
"ON" Stations - Single Batch
"ON" Stations - Beam Gap
"OFF" Stations - Double Batch
"OFF" Stations - Single Batch
"OFF" Stations - Beam Gap


PA Anode RF Current Phase
Angle (degrees)
Time (sec)
90
70
50
30
10
−10
−30
−50
−70
−90
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
"ON" Stations - Double Batch
"ON" Stations - Single Batch
"ON" Stations - Beam Gap
"OFF" Stations - Double Batch
"OFF" Stations - Single Batch
"OFF" Stations - Beam Gap

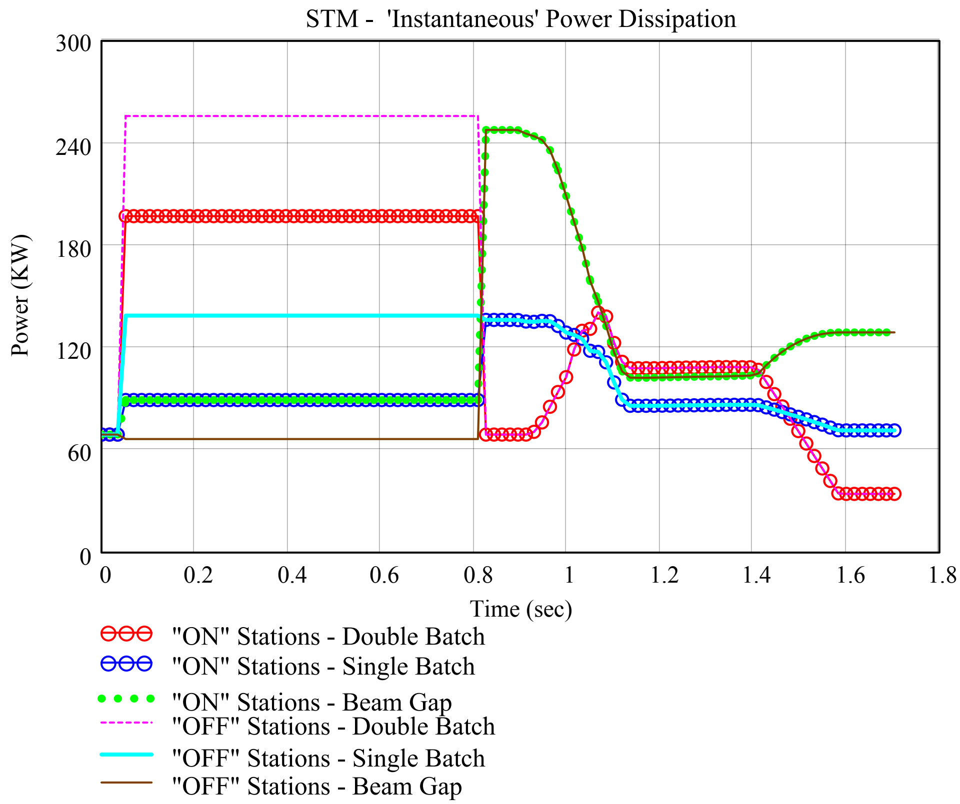

PA 'Instantaneous' DC Anode Current
Current (Amps)
15
12
9
6
3
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Time (sec)
"ON" Stations - Double Batch
"ON" Stations - Single Batch
"ON" Stations - Beam Gap
"OFF" Stations - Double Batch
"OFF" Stations - Single Batch
"OFF" Stations - Beam Gap

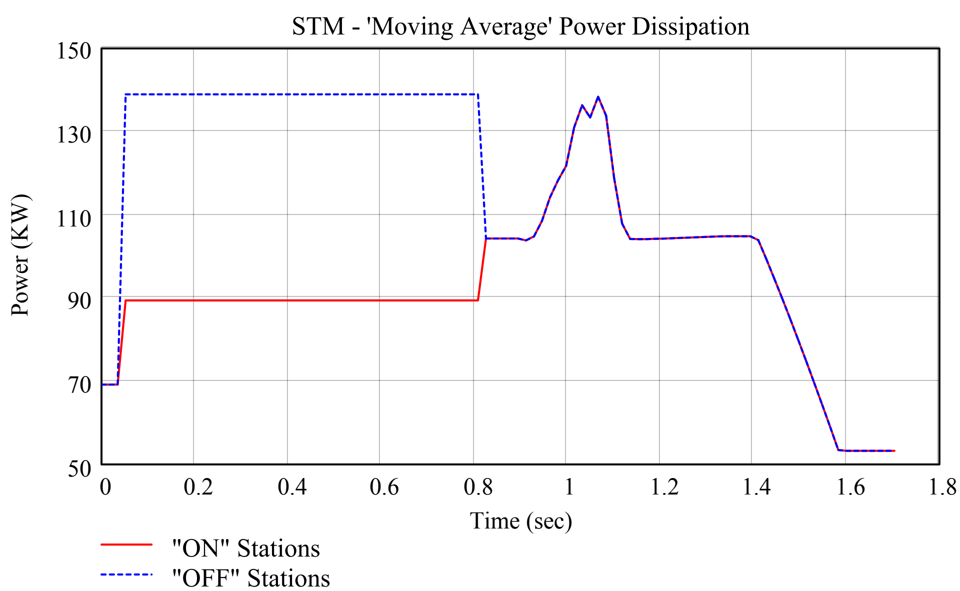

PA 'Moving Average' DC Current
Current (Amps)
15
12
9
6
3
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Time (sec)
"ON" Stations
"OFF" Stations

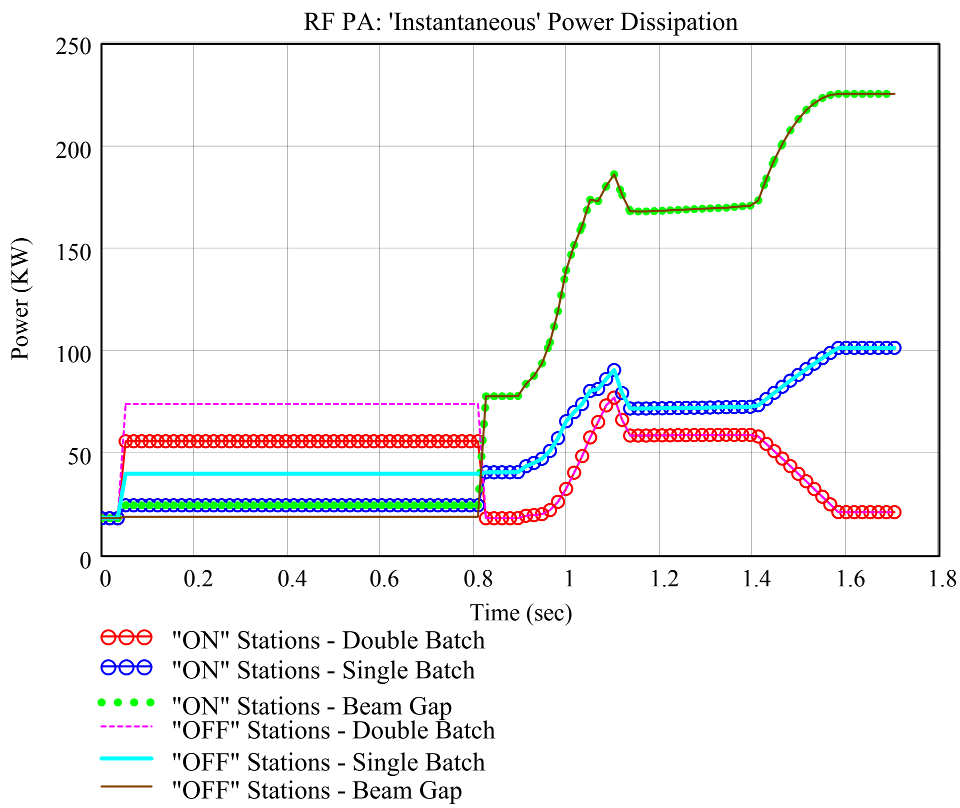

RF PA: Peak Anode Current
Amps
60
48
36
24
12
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Time (sec)
"ON" Stations - Double Batch
"ON" Stations - Single Batch
"ON" Stations - Beam Gap
"OFF" Stations - Double Batch
"OFF" Stations - Single Batch
"OFF" Stations - Beam Gap

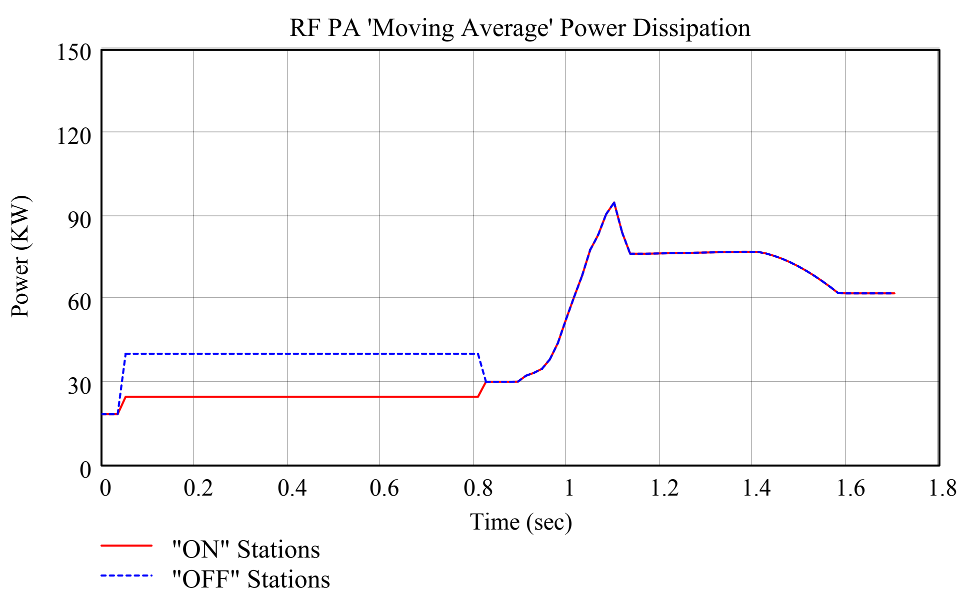
Assumed Modulator Input Voltage:

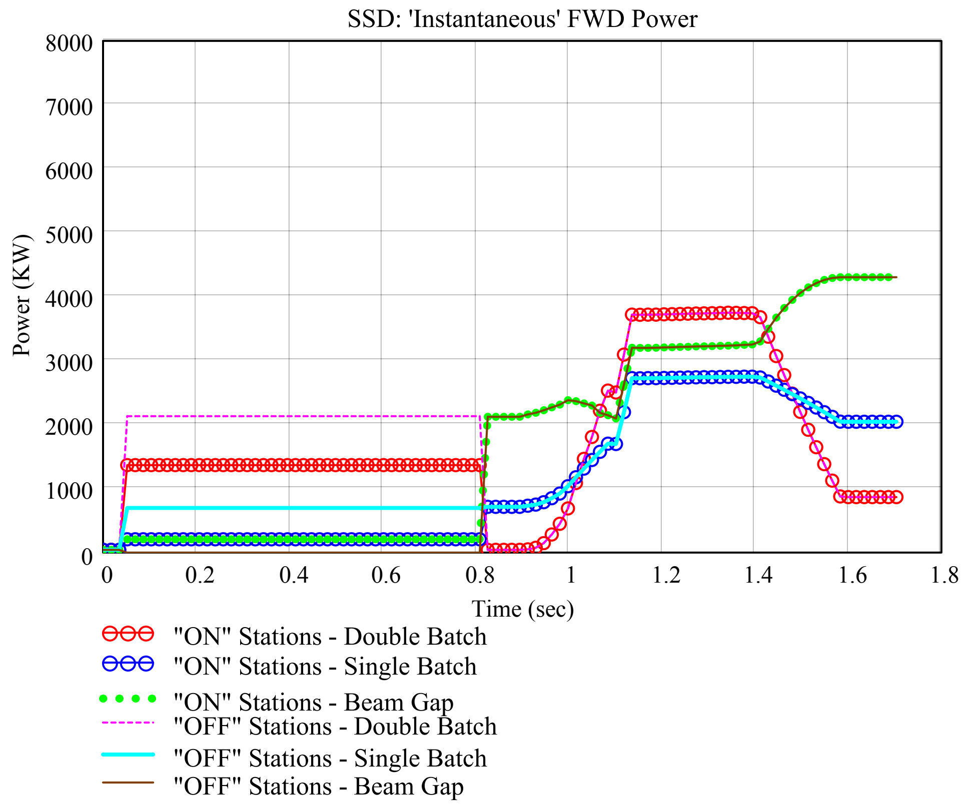
$V_{APS} = 28\,kV$

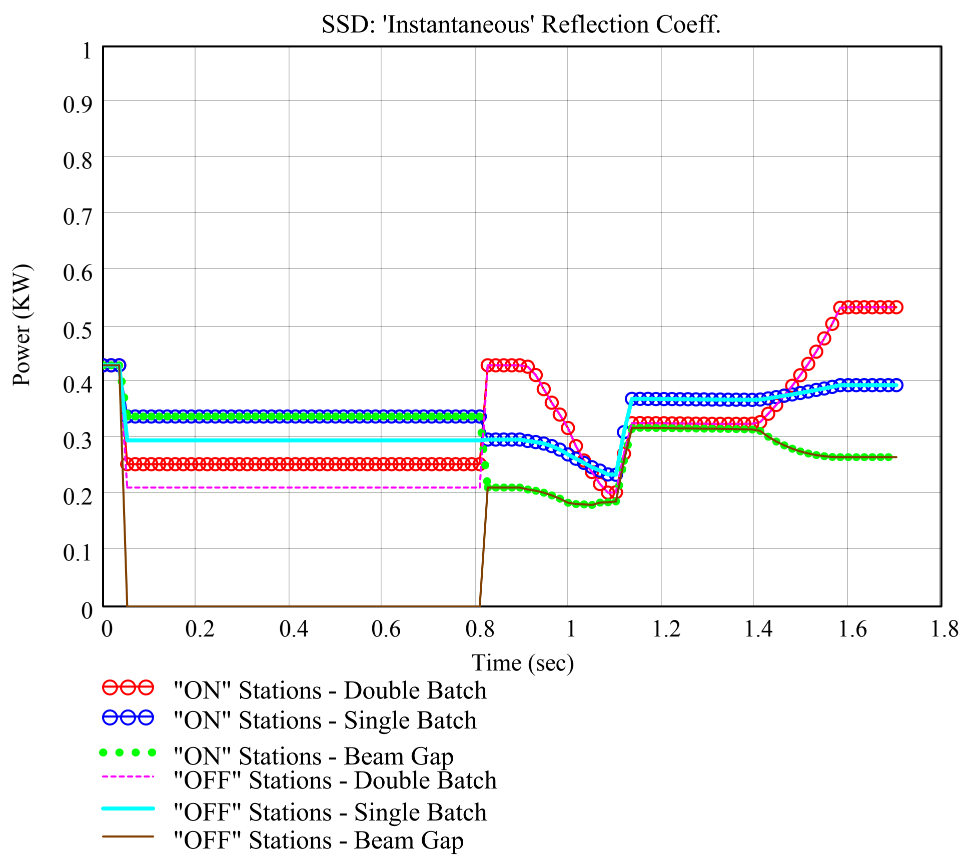


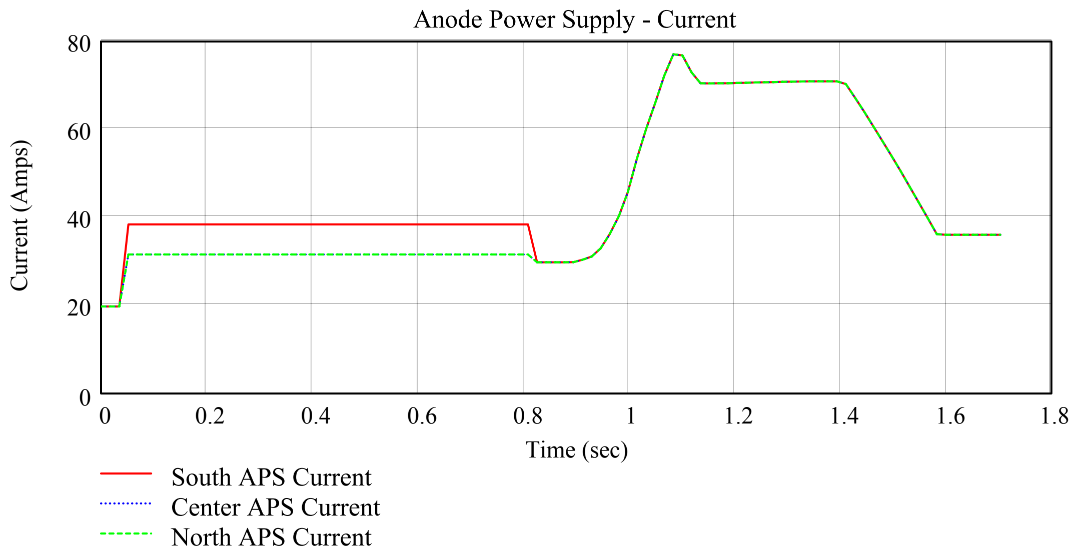



RF PA: 'Instantaneous' Power Dissipation
Power (KW)
250
200
150
100
50
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Time (sec)
"ON" Stations - Double Batch
"ON" Stations - Single Batch
"ON" Stations - Beam Gap
"OFF" Stations - Double Batch
"OFF" Stations - Single Batch
"OFF" Stations - Beam Gap


RF PA 'Moving Average' Power Dissipation
Power (KW)
150
120
90
60
30
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Time (sec)
"ON" Stations
"OFF" Stations


SSD: 'Instantaneous' FWD Power
Power (KW)
Time (sec)
0
1000
2000
3000
4000
5000
6000
7000
8000
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
"ON" Stations - Double Batch
"ON" Stations - Single Batch
"ON" Stations - Beam Gap
"OFF" Stations - Double Batch
"OFF" Stations - Single Batch
"OFF" Stations - Beam Gap


SSD: 'Instantaneous' Reflection Coeff.
Power (KW)
Time (sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
"ON" Stations - Double Batch
"ON" Stations - Single Batch
"ON" Stations - Beam Gap
"OFF" Stations - Double Batch
"OFF" Stations - Single Batch
"OFF" Stations - Beam Gap

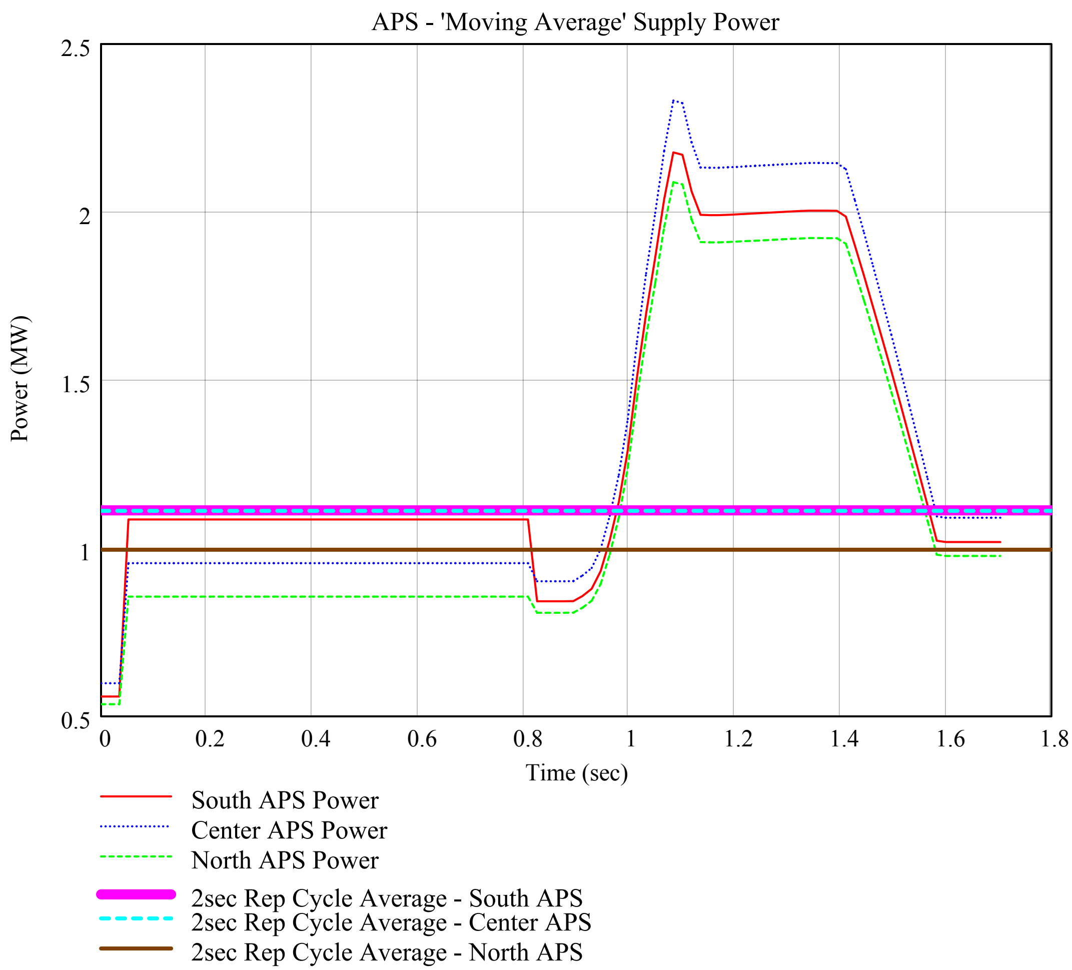


Actual APS Voltages:

$V_{SAPS} = 28.25$ kV

$V_{CAPS} = 30.25$ kV

$V_{NAPS} = 27.1$ kV



Summary:

The estimated Series Tube Modulator (STM) dissipation is highest for the Slip-Stacking 'OFF' stations during the slipping process. Even with the increase in the PA anode bias from 3.5 kV to 6.3 kV afforded by the combination of MRF and not using screen current regulation during slipping, the STM dissipation could be relaxed by further increasing the anode bias. The RF PA tube anode dissipation is well under the tube maximum rating of 150kW. Although the average Anode Power Supply (APS) power is well under the thermal ratings of the APS's, the South APS exceeds it's 1.8KVA instantaneous maximum specification and the Center APS has little headroom from its 2.4KVA instantaneous maximum specification.

References:

[1] D. McGinnis, "Main Injector RF Requirements for a 1.3 Megawatt 120 GeV Proton Source", Fermilab Beams-doc-#2253-v5, May 2006.
< https://beamdocs.fnal.gov/AD-private/DocDB/ShowDocument?docid=2253>

[2] F. Pedersen, "RF Cavity Feedback", CERN/PS 92-59 (RF).

[3] D. Boussard, "Beam Loading", Sec. 2.5.2 of "Handbook of Accelerator Physics and Engineering" by Chao and Tigner, pp.108-112.

[4] D.A. Edwards & M.J. Syphers, "An Introduction to the Physics of High Energy Accelerators", John Wiley & Sons, Inc., New York, 1993.