Growth Rates of coupled Bunch Modes due to the Fundamental RF Impedance

7/17/2006

Introduction

The growth rates for the coupled bunch modes due to the fundamental rf Impedance in MI are calculated as a function of the detuning angle for an intensity of 5.5E13 ppp. The growth rates are compared with the synchrotron Frequency and the Landau damping rate.

Detuning Angle and coupled bunch driving terms.

Here we assume a full Ring and a total beam Intensity of 5.5 E13 particles. For this Intensity the detuning angle for optimum detuning is:

$$\tan\phi z = \frac{I_B}{I_0} = \frac{2\times I_{DC}}{V_{rf}/R_T} = \frac{2\times 0.8A}{1MV/(18\times 353.6K\Omega)} = 10, \quad (1)$$

$$OR\phi_Z = 84.2^0$$

The coupled bunch dipole mode (m=1) with coupled bunch mode number n has spectral lines at the frequencies:

$$f_{n,p} = (pM + n) f_0 + f_s \quad (2)$$
$$-\infty \prec p \prec \infty$$

Where f is the revolution frequency, M is the harmonic number and the synchrotron frequency.
There are a total of M couple bunch modes which for convenience are numbered for positive numbers from 0 to M/2-1 and with negative numbers from M/2 to M-1(i.e. the mode M-1 is numbered as -1).

As far as interaction with the fundamental RF resonance is concerned, only two of the spectral lines in (0) contributes for each mode n, namely p=1,-1. One is an upper sideband to a revolution harmonic $f_+ = f_{rf} + nf_0 + f_s$, and the other is a lower sideband, $f_- = f_{rf} - nf_0 - f_s$. The growth rate then is proportional to the difference in the real part of the impedance evaluated at the two mode sidebands located symmetrically on either side of the rf frequency:

$$1/\tau_n = \frac{I_0 \eta f_{rf}}{2Q_s \beta^2 (E/e)} \mathrm{Re}\{Z_l(f_+) - Z_l(f_-)\} \quad (3)$$

The growth of damping rates are caused by the asymmetry in the Impedance due to the Cavity detuning.

The magnitude and the real part of the total rf cavity Impedance as well as the driving terms for a detuning angle of 84.2 degrees are plotted in Fig. 1

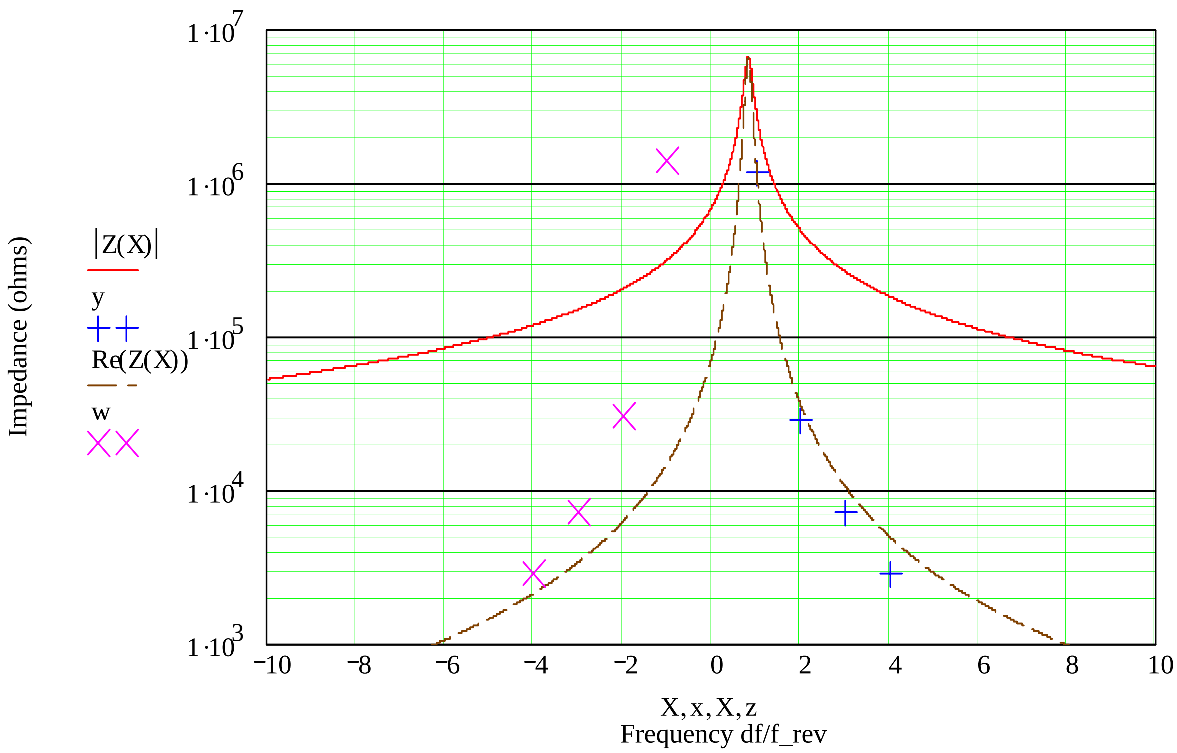

Fig. 1: Magnitude(Red) and Real part(Brown) of the total MI rf Impedance along with the driving terms of the modes 1to 4 (stable) and (-1)to(-4) (unstable).

Growth Rates

The growth rates for the first 4 unstable modes for 5.5E13 protons vs. detuning angle are plotted in Fig. 2

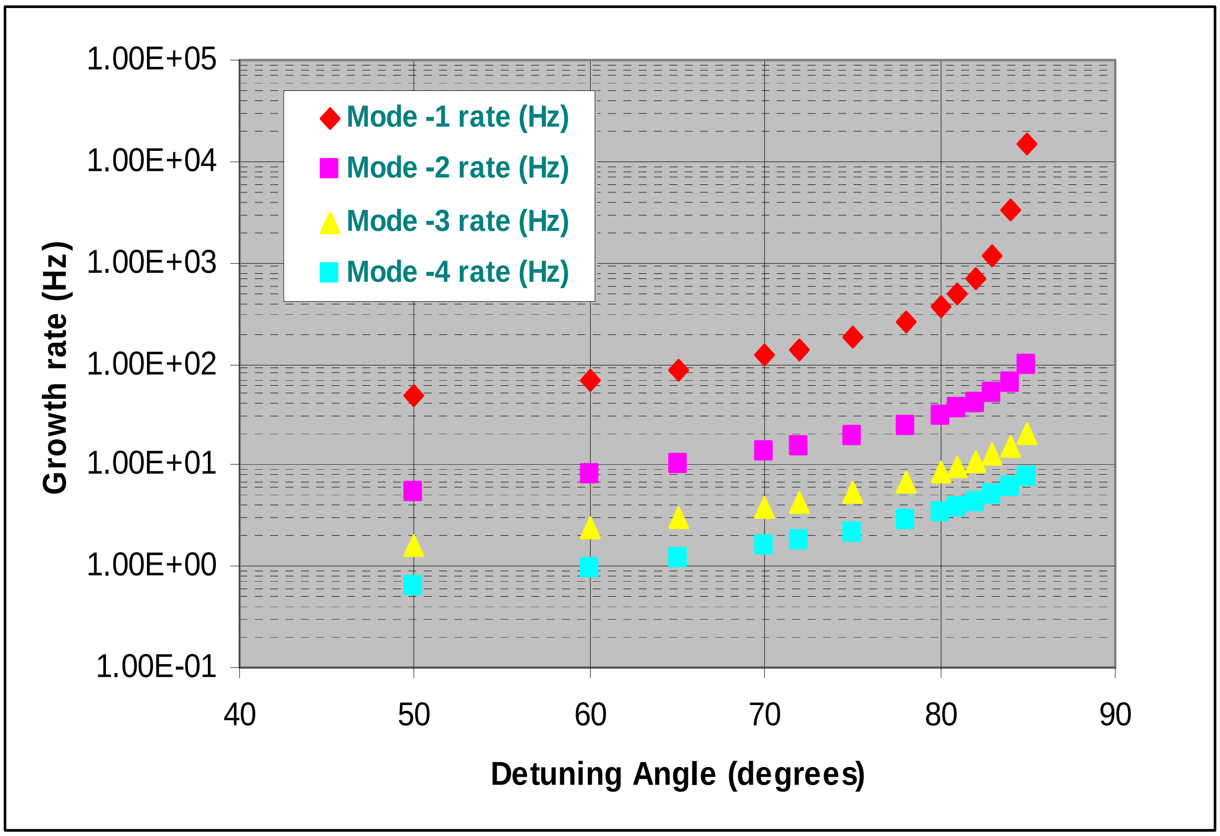

Fig. 2: Growth rates vs. Detuning angle for 5.5E13protons

The growth rates normalized to the synchrotron frequency are plotted in Fig. 3. All modes except mode -1 have growth rates much smaller than the synchrotron frequency making it possible to design a multi-batch feedback system for damping if necessary.

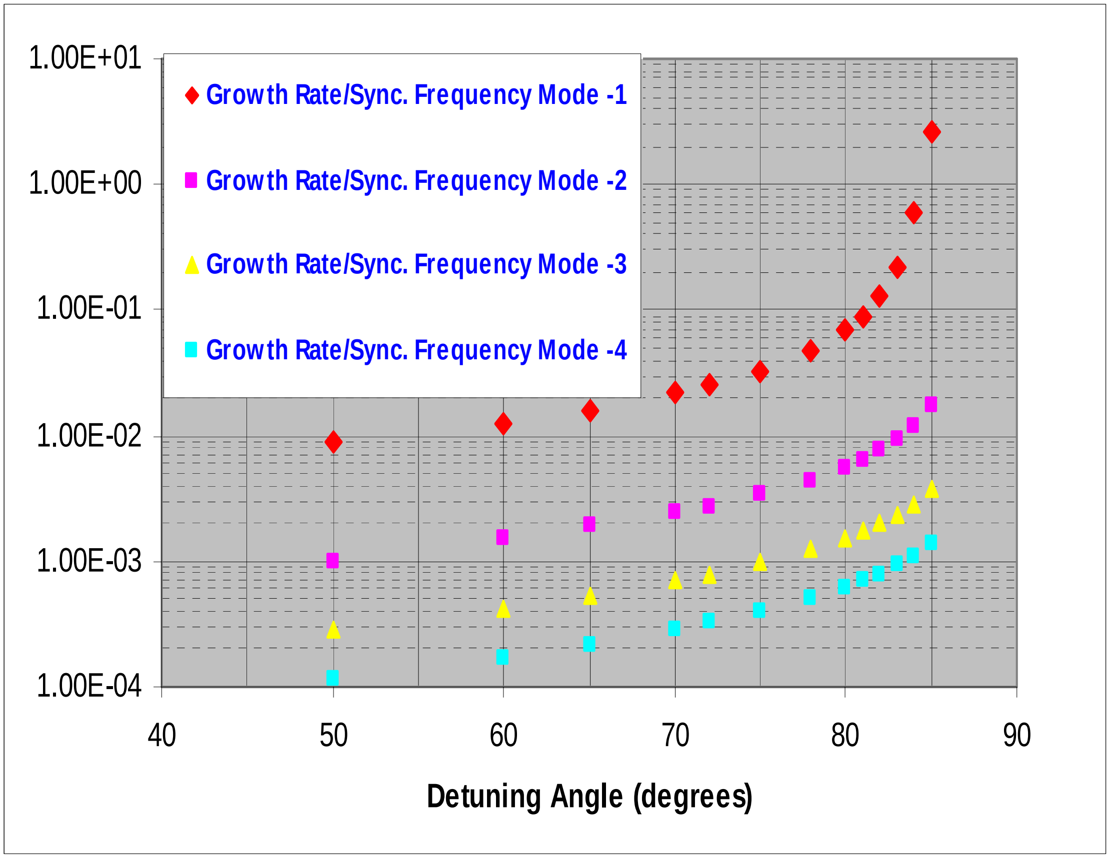

Fig.3: Growth rates normalized to synchrotron frequency for the first 4 unstable modes.

3. Landau damping

Landau damping from the spread of the synchrotron frequencies inside an rf bucket can stabilize the beam. The spread due to a nonlinear sinusoidal rf wave form can be written as [2]:

$$\frac{\Delta\omega_s}{\omega_s} = \left(\frac{\pi^2}{16}\right)\left(\frac{1+\sin^2\phi_s}{1-\sin^2\phi_s}\right)(h\tau_L f_0)^2 \quad (4)$$

Where τ_L is the total bunch length of the bunch in sec, h is the rf harmonic number, f_0 is the revolution frequency and ϕ_s is the synchronous phase angle. For MI at injection the typical bunch length is 9 nsec and $\omega_s = 5.5 \times 10^3 \sec^{-1}$ giving a spread $\Delta\omega_s = 850 Hz$. A particular couple bunch mode will be stable if [3]:

$$\frac{1}{\tau_n} = \Delta\omega_n \leq \frac{1}{4}\Delta\omega_s \quad (5)$$

The growth rates for the first 4 unstable modes normalized to Landau frequency spread are plotted as function of the detuning angle in Fig. 4. As we can see from the plot all modes except mode -1 will be stable. In order to be stable for mode -1 we need either to operate with detuning angles of less than 75 degrees or to reduce the rf impedance at the first revolution harmonic sidebands about a factor of 10 (the optimum detuning angle at 5.5E13 is 84 degrees).

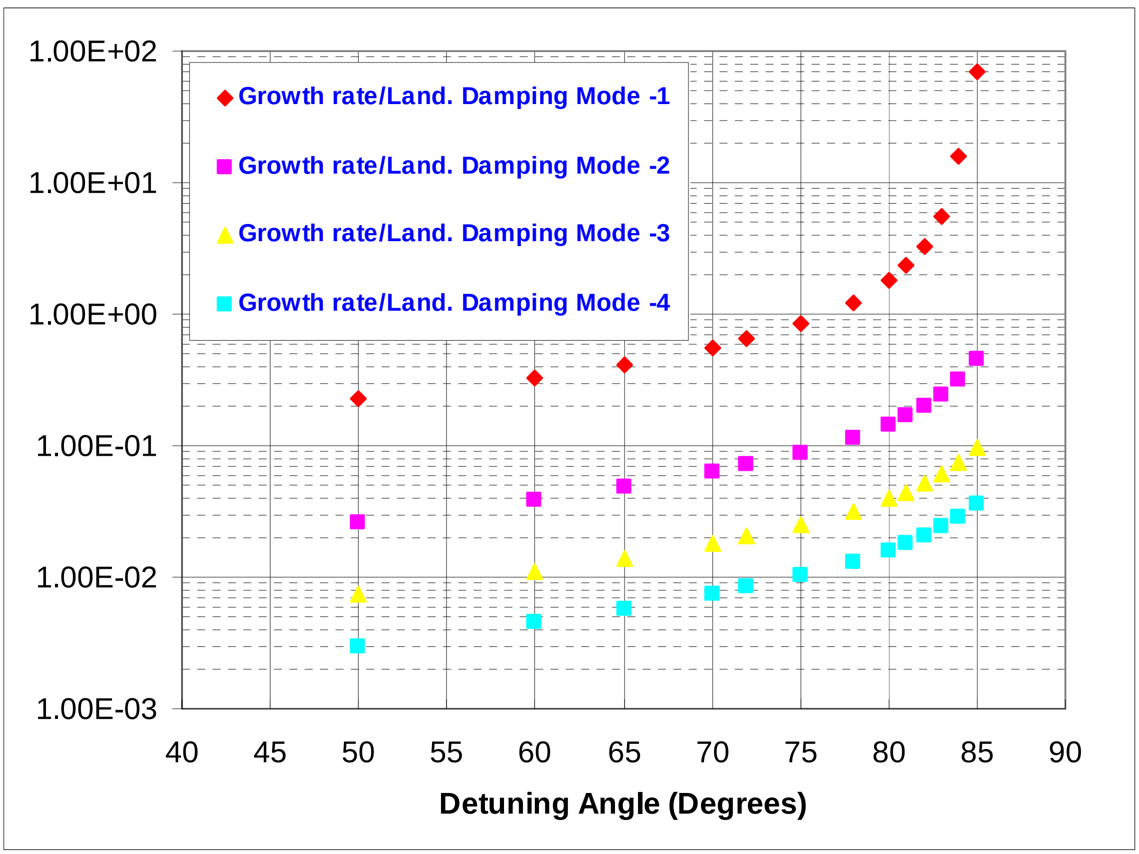

Fig. 4: Growth rates for the first 4 unstable modes normalized to Landau frequency spread as a function of detuning angle.

Conclusions

The growth rates for the coupled bunch dipole modes due to the rf Impedance were calculated for a peak intensity of 5.5E13 particles. We assumed a full ring thus overestimating the growth rates as much as a factor of 3[4]. It appears that if we manage to reduce the impedance on the first revolution harmonic sidebands by a factor of 10 we are going to be stable for all the dipole modes up to a detuning angle of 84 degrees.

References

[1] F. Pedersen, *RF Cavity Feedback*
[2] K.Y. Ng, *Physics Of Intensity Dependent Beam Instabilities,* Fermilab-FN-0713
[3] F.J. Sacherer, *A Longitudinal Stability Criterion for Bunched Beams,* IEEE Trans. Nuclear Sci. **NS 20,** 3, 825 (1973).
[4] X. Huang, *Calculation of Longitudinal Couple-Bunch Dipole Mode for Main Injector,* Beams-doc-2058-v1