**FERMILAB-TM-2374-AD**

# Formulas for comparing the results of drag force and cooling rates measurements with a non-magnetized cooling model

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January 17, 2007

Abstract

A 0.1-0.5 A, 4.3 MeV DC electron beam provides cooling of 8 GeV antiprotons in Fermilab's Recycler storage ring [1]. Efforts are being made to improve understanding of the cooling process and eventually decrease the cooling time. In part, measurements of two types are being performed: recording of the cooling rates at operational parameters and of drag rates with a low number of antiprotons. To compare results of these measurements with the non-magnetized model, known formulas for the cooling force were integrated over Gaussian velocity distributions of both electron and antiproton beams. The paper presents the final expressions and their derivation.

## 1. The model

The electron cooler installed in the Fermilab's Recycler ring employs a weak 105 G longitudinal magnetic field to focus the electron beam in the cooling section. The simplest model to estimate the cooling rates of the antiproton beam is as follows:

1. Influence of the magnetic field on the cooling dynamics is neglected (so-called non-magnetized cooling).
2. The electron beam properties are assumed to be the same along the cooling section and across the area occupied by the antiproton beam.
3. The electron angle and energy spread are described by Gaussian distributions.
4. The antiproton angle and energy spread are described by Gaussian distributions as well.
5. The changes of the Coulomb logarithm over the range of relative velocities are ignored, and the logarithm is taken out of the integral.
6. In the cooling section, the electron velocity spread in $\boldsymbol{x}$ direction is equal to the $\boldsymbol{y}$ spread; the same is assumed for the antiproton velocity distribution. (The longitudinal component is $\boldsymbol{z}$).
7. Effects of the longitudinal dynamics of the antiproton beam are neglected, i.e. the beam is assumed to be either coasting or kept in a perfect rectangular potential well (idealized representation of barrier buckets).
8. Antiproton motion in all three directions is uncoupled.

## 2. Cooling force

Under these assumptions, the cooling force in the beam frame is expressed as [2]

$$\vec{F}_b(\vec{V}_p) = -4\pi \cdot m_e r_e^2 c^4 n_{eb} \eta \cdot L_c \cdot \int \frac{\vec{V}_p - \vec{v}_e}{|\vec{V}_p - \vec{v}_e|^3} f(\vec{v}_e) d^3 v_e \equiv A \cdot \int \frac{\vec{V}_p - \vec{v}_e}{|\vec{V}_p - \vec{v}_e|^3} f(\vec{v}_e) d^3 \vec{v}_e \tag{1}$$

where the following notation is used:

$\vec{V}_p$ - antiproton velocity in the beam frame,

$\vec{v}_e$ - electron velocity in the beam frame,

$m_e$ - electron mass,

$r_e$ - electron classical radius,

$c$ - speed of light,

$n_{eb}$ - electron density in the beam frame,

$\eta$ -portion of the ring occupied by the cooling section,

$L_c$ - Coulomb logarithm.

The electron velocity distribution is Gaussian in each direction,

$$f(\vec{v}_e) = \frac{1}{(2\pi)^{3/2} \sigma_{ex} \sigma_{ey} \sigma_{ez}} \exp\left( -\frac{v_{ex}^{\ 2}}{2\sigma_{ex}^2} - \frac{v_{ey}^{\ 2}}{2\sigma_{ey}^2} - \frac{v_{ez}^{\ 2}}{2\sigma_{ez}^2} \right). \tag{2}$$

The following integration of (1) is done [3] with a mathematical trick used in [4]. First, following to the analogy with electrostatics, an effective potential is calculated:

$$U(V_{px}, V_{pz}, V_{pz}) \equiv \int \frac{1}{|\vec{V}_p - \vec{v}_e|} f(\vec{v}_e) d^3 \vec{v}_e =$$

$$\frac{1}{(2\pi)^{3/2} \sigma_{ex} \sigma_{ey} \sigma_{ez}} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dv_{ex}\, dv_{ey}\, dv_{ez} \frac{\exp\left( -\frac{v_{ex}^{\ 2}}{2\sigma_{ex}^2} - \frac{v_{ey}^{\ 2}}{2\sigma_{ey}^2} - \frac{v_{ez}^{\ 2}}{2\sigma_{ez}^2} \right)}{\sqrt{(V_{px} - v_{ex})^2 + (V_{py} - v_{ey})^2 + (V_{pz} - v_{ez})^2}} \tag{3}$$

The expression in the integrand denominator is replaced by an integral

$$\frac{1}{\sqrt{(V_{px} - v_{ex})^2 + (V_{py} - v_{ey})^2 + (V_{pz} - v_{ez})^2}} = \frac{2}{\sqrt{\pi}} \int_0^{\infty} dt \exp\left[ -t^2 \left( (V_{px} - v_{ex})^2 + (V_{py} - v_{ey})^2 + (V_{pz} - v_{ez})^2 \right) \right]. \tag{4}$$

This substitution allows taking integrals over electron velocities:

$$U(V_{px},V_{pz},V_{pz}) = \frac{2}{\sqrt{\pi}} \cdot \int_0^\infty dt \frac{\exp\left(-\frac{t^2 V_{px}{}^2}{1+2\cdot\sigma_{ex}^2 t^2} - \frac{t^2 V_{py}{}^2}{1+2\cdot\sigma_{ey}^2 t^2} - \frac{t^2 V_{pz}{}^2}{1+2\cdot\sigma_{ez}^2 t^2}\right)}{\sqrt{\left(1+2\cdot\sigma_{ex}^2 t^2\right)\left(1+2\cdot\sigma_{ey}^2 t^2\right)\left(1+2\cdot\sigma_{ez}^2 t^2\right)}} \quad (5)$$

Now each component of the cooling force is given by derivation of Eq. (5) over the correspondent component of the antiproton velocity (*xi= x ,y, or z*):

$$F_{b_xi}(\vec{V}_p) = -A \cdot \frac{\partial}{\partial V_{xi}} U(V_{px},V_{py},V_{pz}) \quad (6)$$

For example, for the longitudinal component,

$$F_{bz}(V_{px},V_{pz},V_{pz}) = \frac{4A}{\sqrt{\pi}} \cdot \int_0^\infty dt \frac{\exp\left(-\frac{t^2 V_{px}{}^2}{1+2\cdot\sigma_{ex}^2 t^2} - \frac{t^2 V_{py}{}^2}{1+2\cdot\sigma_{ey}^2 t^2} - \frac{t^2 V_{pz}{}^2}{1+2\cdot\sigma_{ez}^2 t^2}\right)}{\sqrt{\left(1+2\cdot\sigma_{ex}^2 t^2\right)\left(1+2\cdot\sigma_{ey}^2 t^2\right)\left(1+2\cdot\sigma_{ez}^2 t^2\right)}} \cdot \frac{t^2}{1+2\cdot\sigma_{ez}^2 t^2} \quad (7)$$

The cooling force expressed in the form of Eq. (7) is close to what is called “Binney's formula” in publications of the BNL/JINR group (see, for example, [5]).

## 3. Cooling rate calculation

A dedicated set of cooling measurements was made with the beam initially kept under stochastic cooling only. In this case, the equilibrium distributions are Gaussian, and the initial cooling rates immediately after applying electron cooling can be compared with those calculated under assumptions of this model. Discussion of diffusion rates is outside of the framework of this paper, and the diffusion is neglected.

The longitudinal cooling rate in the lab frame can be expressed as a time derivative of the r.m.s. antiproton momentum width $\delta p = \gamma \sigma_{pz} M_p$, where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ is relativistic factor, $\sigma_{pz}$ is the r.m.s. spread of the longitudinal velocities of antiprotons in the beam frame, and $M_p$ is the proton mass. For a coasting beam,

$$M_p \dot{\sigma}_{pz} \cdot \sigma_{pz} = \int F_{bz} V_{pz} f_p(\vec{V}_p)\, d\vec{V}_p, \quad (8)$$

where the time derivative is taken in the beam frame. Assuming a Gaussian distribution of antiproton velocities

$$f_p(\vec{V}_p) = \frac{1}{(2\pi)^{3/2}\sigma_{px}\sigma_{py}\sigma_{pz}} \exp\left( -\frac{V_{px}^2}{2\sigma_{px}^2} - \frac{V_{py}^2}{2\sigma_{py}^2} - \frac{V_{pz}^2}{2\sigma_{pz}^2} \right), \tag{9}$$

one can rewrite Eq. (8) using (5)- (7):

$$M_p \dot{\sigma}_{pz} \cdot \sigma_{pz} = \frac{4}{\sqrt{\pi}} \frac{A}{(2\pi)^{3/2}\sigma_{px}\sigma_{py}\sigma_{pz}} \cdot \int_0^\infty dt \int_{-\infty}^{\infty} dV_{px} \frac{\exp\left( -\frac{V_{px}^2}{2\sigma_{px}^2} - \frac{V_{px}^2 t^2}{1+2\sigma_{ex}^2 t^2} \right)}{\sqrt{1+2\sigma_{ex}^2 t^2}} \cdot \tag{10}$$

$$\int_{-\infty}^{\infty} dV_{py} \frac{\exp\left( -\frac{V_{py}^2}{2\sigma_{py}^2} - \frac{V_{py}^2 t^2}{1+2\sigma_{ey}^2 t^2} \right)}{\sqrt{1+2\sigma_{ey}^2 t^2}} \int_{-\infty}^{\infty} dV_{pz} \frac{\exp\left( -\frac{V_{pz}^2}{2\sigma_{pz}^2} - \frac{V_{pz}^2 t^2}{1+2\sigma_{ez}^2 t^2} \right)}{\left(1+2\sigma_{ez}^2 t^2\right)^{3/2}} \cdot V_{pz}^2$$

Taking the last three integrals yields

$$\dot{\sigma}_{pz} \cdot \sigma_{pz} = \frac{4A \cdot \sigma_{pz}^2}{M_p \sqrt{\pi}} \cdot \int_0^\infty dt \frac{t^2}{\sqrt{1+2(\sigma_{ex}^2+\sigma_{px}^2)t^2}\sqrt{1+2(\sigma_{ey}^2+\sigma_{py}^2)t^2}\left(1+2(\sigma_{ez}^2+\sigma_{pz}^2)t^2\right)^{3/2}} \tag{11}$$

For the case of axial symmetry in the electron beam and equality of antiproton velocity spreads in the cooling section

$$\sigma_{ex} = \sigma_{ey}, \sigma_{px} = \sigma_{py}, \tag{12}$$

the integral can be rewritten as follows:

$$\dot{\sigma}_{pz} \cdot \sigma_{pz} = \frac{4A \cdot \sigma_{pz}^2}{M_p \sqrt{\pi}\left[2(\sigma_{ex}^2+\sigma_{px}^2)\right]^{3/2}} \int_0^\infty du \frac{u^2}{(1+u^2)(1+\alpha u^2)^{3/2}}, \tag{13}$$

$$u = t\sqrt{2(\sigma_{ex}^2+\sigma_{px}^2)}, \quad \alpha = \frac{\sigma_{ez}^2+\sigma_{pz}^2}{\sigma_{ex}^2+\sigma_{px}^2}$$

For the case of $\alpha < 1$, the integral can be taken by the substitution $u = \frac{\cos v}{\sqrt{\sin^2 v - \alpha}}$, and the final formula is

$$\dot{\sigma}_{pz} = \sqrt{\frac{2}{\pi}} \frac{A \cdot \sigma_{pz}}{M_p \sqrt{\sigma_{ez}^2+\sigma_{pz}^2}(\sigma_{ex}^2+\sigma_{px}^2)} \cdot f_{long}(\alpha) \tag{14}$$

$$f_{long}(\alpha) = \frac{1 - \sqrt{\frac{\alpha}{\alpha-1}} \cdot \arccos(\sqrt{\alpha})}{1-\alpha}$$

In the opposite case of $\alpha > 1$, the integral is taken with the substitution $u = \frac{\sinh v}{\sqrt{\alpha - \cosh^2 v}}$ , and the function is

$$f_{long}(\alpha) = \frac{\sqrt{\frac{\alpha}{\alpha - 1}} \cdot \cosh^{-1}(\sqrt{\alpha}) - 1}{\alpha - 1}. \tag{14a}$$

At the boundary case of $\alpha = 1$, the integral is equal to 1/3.

In the Recycler Electron Cooler (REC), the transverse electron velocities are always significantly larger than other components, and the condition $\alpha < 1$ is fulfilled. For simplicity, only this case will be considered further in this paper.

Transition to the lab frame is made by substituting Eq. (14) into

$$\frac{d}{dt_l}\delta p = \dot{\sigma}_{pz} M_p \cdot \tag{14b}$$

Note that the transverse antiproton velocities enter Eq. (14) only in a sum with electron transverse velocities, both squared. For typical REC parameters, this model dictates that longitudinal cooling rate doesn't depend on the antiproton emittance. Moreover, in operation the antiproton momentum spread changes by only about a factor of 2, and always $\sigma_{ez}^2 << \sigma_{pz}^2 << \sigma_{ex}^2$. For such parameters, the longitudinal cooling rate predicted by Eq. (14) varies by less than ±7%.

The transverse cooling rate over a single pass through the cooling section is identical to Eq. (8) with replacement of indexes *z* by *x*. Taking into account averaging over betatron phases decreases the rate by factor of two. Once more assuming symmetry (12), one can write the rate as the integral

$$\dot{\sigma}_{px} \cdot \sigma_{px} = \frac{2A \cdot \sigma_{px}^2}{M_p \sqrt{\pi}\left[2(\sigma_{ex}^2 + \sigma_{px}^2)\right]^{3/2}} \int_0^\infty du \frac{u^2}{\left(1+u^2\right)^2 (1+\alpha u^2)^{1/2}} \quad . \tag{15}$$

$$u = t\sqrt{2(\sigma_{ex}^2 + \sigma_{px}^2)}, \quad \alpha = \frac{\sigma_{ez}^2 + \sigma_{pz}^2}{\sigma_{ex}^2 + \sigma_{px}^2}$$

The integral is taken using the same substitution that gives the following equation for the transverse cooling rate:

$$\frac{d}{dt_b}\left(\sigma_{px}^2\right) = \sqrt{\frac{2}{\pi}} \frac{A \cdot \sigma_{px}^2}{M_p (\sigma_{ex}^2 + \sigma_{px}^2)^{3/2}} \cdot f_{tr}(\alpha) \tag{16}$$

$$f_{tr}(\alpha) = \frac{\arccos\sqrt{\alpha} - \sqrt{\alpha(1-\alpha)}}{2(1-\alpha)^{3/2}}$$

(case of $\alpha < 1$). Transition to the transverse antiproton emittance $\varepsilon$ and lab frame gives

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dt_l} = \sqrt{\frac{2}{\pi}}\frac{A}{\gamma M_p(\sigma_{ex}{}^2 + \sigma_{px}{}^2)^{3/2}} \cdot f_{tr}(\alpha) \cdot \tag{16a}$$

Under the assumptions of this model, the logarithmic derivative of the emittance varies by less than ±15% in the typical range of REC parameters.

Functions appearing the formulas for the cooling rates are shown in Fig. 1. At small $\alpha << 1$, $f_{tr}(\alpha) \approx \pi/4 - \sqrt{\alpha}/2$, $f_{long}(\alpha) \approx 1 - \pi/2 \cdot \sqrt{\alpha}$ .

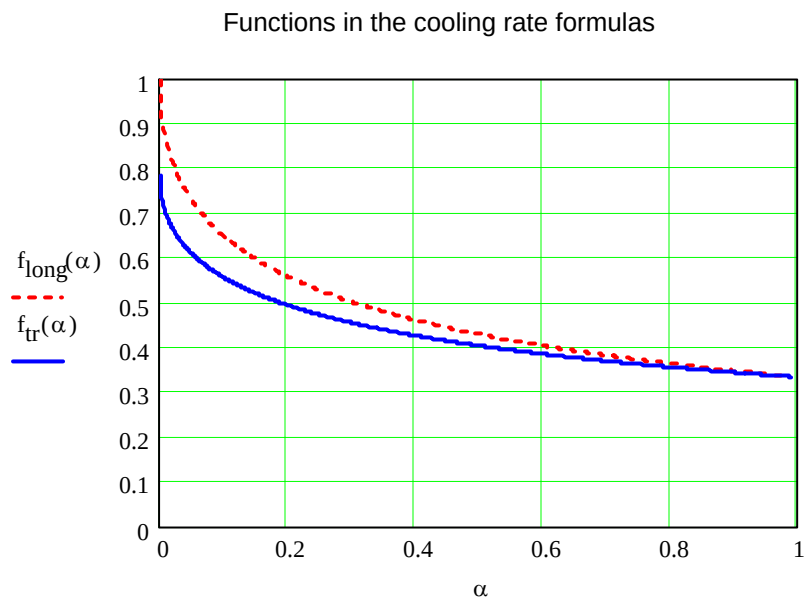


Figure 1. Functions in Eq. (14) (red dashed line) and (16) (blue solid line).

## 4. Formula for fitting the drag rate measurements

The most detailed description of the longitudinal cooling force at the Recycler cooler is given by the voltage jump measurements [6]. In these measurements, the antiproton beam with a small number of particles is deeply cooled, and the electron energy is changed by a known value. The antiprotons are dragged toward the new equilibrium with electrons, and the drag rate of the antiproton beam average momentum and the momentum width evolution contain the information about the cooling force. This chapter describes formulas used to fit the measured data.

The time derivative of the average longitudinal velocity in the electron beam rest frame is determined by the longitudinal cooling force averaged over the antiproton velocity distribution:

$$\dot{\bar{V}}_{pz} = \frac{1}{M_p}\int F_{bz}(\vec{V}_p)\, f_p(\vec{V}_p)\, d\vec{V}_p \,. \tag{17}$$

Assuming the Gaussian distribution over the transverse antiproton velocities and equality (12), the average force can be expressed similar to Eq. (10)-(11):

$$\overline{F}_{bz}\left(V_{pz}\right)=\frac{4A\cdot V_{pz}}{\sqrt{\pi}}\cdot\int_0^\infty dt\exp\left(-\frac{V_{pz}^2t^2}{1+2\sigma_{ez}{}^2t^2}\right)\frac{t^2}{\left[1+2(\sigma_{ex}{}^2+\sigma_{px}{}^2)t^2\right]\left(1+2\sigma_{ez}{}^2t^2\right)^{3/2}}. \quad (18)$$

To speed up calculations and make fitting more stable, it is found beneficial to transform the integral in Eq. (18) to the form

$$\overline{F}_{bz}\left(V_{pz}\right)=\frac{2A}{\sqrt{\pi}\,\sigma_t{}^2}\cdot\int_0^{\frac{V_{pz}}{\sqrt{2}\sigma_{ez}}} du\,\frac{e^{-u^2}u^2}{u^2+\frac{V_{pz}{}^2}{2\sigma_t{}^2}}, \quad (19)$$

$$\sigma_t{}^2=\sigma_{ex}{}^2+\sigma_{px}{}^2-\sigma_{ez}{}^2$$

As it was for the cooling rate, the drag force is insensitive to the transverse antiproton velocity at typical REC parameters.

For a narrow antiproton distribution, the force can be expanded around the average velocity:

$$\overline{F}_{bz}\left(V_{pz}\right)\approx\overline{F}_{bz}\left(\overline{V}_{pz}\right)+\overline{F}_{bz}\left(\overline{V}_{pz}\right)\left(V_{pz}-\overline{V}_{pz}\right)+\overline{F}_{bz}''\left(\overline{V}_{pz}\right)\frac{\left(V_{pz}-\overline{V}_{pz}\right)^2}{2}. \quad (20)$$

Substitution of (20) into (17) gives the relationship between the drag rate and the longitudinal cooling force:

$$\dot{\overline{V}}_{pz}\approx\frac{1}{M_p}\left[\overline{F}_{bz}\left(\overline{V}_{pz}\right)+\overline{F}_{bz}''\left(\overline{V}_{pz}\right)\frac{\sigma_{pz}{}^2}{2}\right], \quad (21)$$

where

$$\sigma_{pz}{}^2\equiv\int_{-\infty}^{\infty}\left(V_{pz}-\overline{V}_{pz}\right)^2 f_{pz}\left(V_{pz}\right)dV_{pz} \quad (22)$$

is the r.m.s. velocity spread of the antiproton distribution.

In the lab frame, Eq. (21) is transformed into

$$\dot{\overline{p}}=F_{lz}\left(\overline{p}\right)+F_{lz}''\left(\overline{p}\right)\cdot\frac{\delta p^2}{2}\ , \quad (23)$$

where the momentum deviation $p \equiv P - P_0 = \gamma M_p V_{pz}$ is the difference between the antiproton momentum $P$ and its equilibrium value $P_0 = p_e \frac{M_p}{m_e}$, and $p_e$ is the average electron momentum.

In most of measurements, the drag rate was recorded at small velocity spreads, when the term with the second derivative of the cooling force in Eq. (20) can be neglected. In the lab frame, it means that the longitudinal cooling force is equal to the time derivative of the average antiproton momentum deviation:

$$F_{lz}(\bar{p}) \approx \dot{\bar{p}} \ . \tag{24}$$

The measured values of $\dot{\bar{p}}$ are fitted to

$$F_{lz}(p) = -F_0 \cdot \int_0^{p/p1} du \frac{e^{-u^2} u^2}{u^2 + \left(\frac{p}{p2}\right)^2} \quad , \tag{25}$$

and the fitting parameters are expressed in terms of electron beam parameters as follows:

$$\begin{aligned} p1 &= \delta W_e \cdot \sqrt{2} \frac{M_p}{\beta m_e c} \\ p2 &= \vartheta_t \cdot \sqrt{2}\, \gamma^2 \beta c M_p \\ F_0 &= \frac{n_{el}}{\vartheta_t^2} \cdot \frac{2}{\sqrt{\pi}} \cdot \frac{4\pi \cdot m_e r_e^2 c^2 \eta \cdot L_c}{\gamma^3 \beta^2} \end{aligned} \quad , \tag{26}$$

where

$\vartheta_t = \frac{1}{\gamma\beta c}\sqrt{\sigma_{ex}^{\ 2} + \sigma_{px}^{\ 2} - \sigma_{ez}^{\ 2}}$ - an effective angle,

$\vartheta_e = \frac{\sigma_{ex}}{\gamma\beta c}$ - 1D electron r.m.s. angle in the cooling section,

$\delta W_e = p_e \sigma_{ez}$ - effective r.m.s. ripple of the electron energy,

$n_{el} = \gamma n_{eb} = \frac{j_e}{\beta c}$ - electron density in the lab frame.

Typically, in the Recycler measurements $\sigma_t^{\ 2} \approx \sigma_{ex}^{\ 2} >> \sigma_{px}^{\ 2}, \sigma_{ez}^{\ 2}$, and $\vartheta_t \approx \vartheta_e$. An example of fitting is shown in Fig.2. The results are very close to what has been reported in Ref. [7], where an approximate expression for the integral Eq. (1) was used with an assumption of zero transverse antiproton velocities.

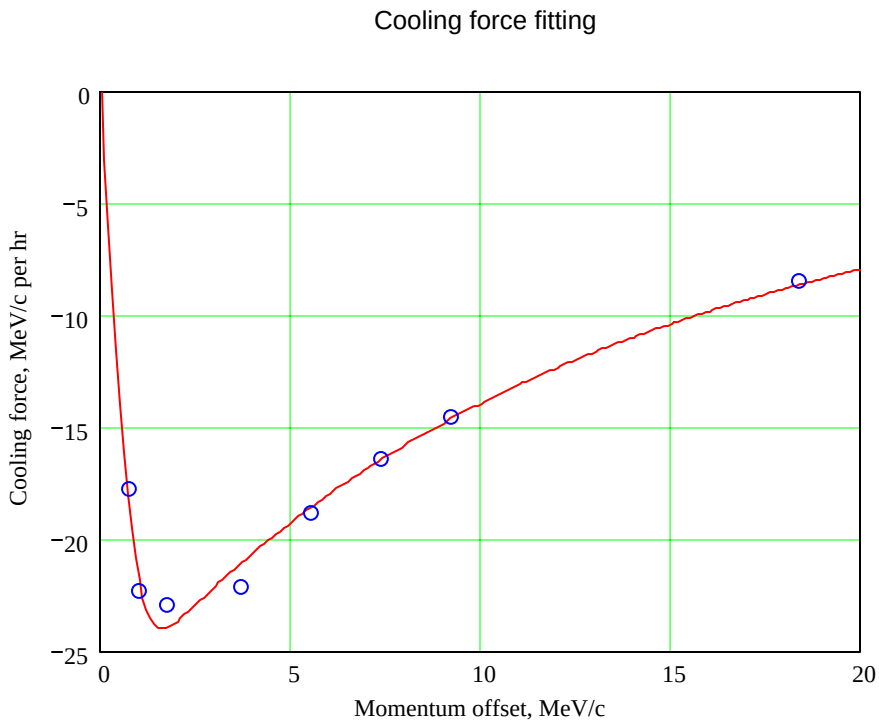


Figure 2. Example of fitting Eq. (25) to the most complete data set of the voltage jump measurements (blue circles). The data were taken on February 6, 2006 with 0.1 A electron beam. The coasting antiproton beam contained $3.5 \cdot 10^{10}$ particles with the transverse emittance of $\varepsilon$ =1.5 $\pi$ mm·mrad (95%, normalized) and initial momentum spread of $\delta p$ = 0.2 MeV/c. The red solid line represents Eq. (25) at $F_0$ = 31 MeV/c per hr, *p1*= 0.95 MeV/c, *p2*=24 MeV/c. Corresponding electron beam parameters are as follows: $\delta W_e$ = 370 eV, $\vartheta_e$ = 0.2 mrad, $j_e$= 1.3 A/cm$^2$. The Coulomb logarithm is 10.

For measurements of the force at small momentum offsets or at a large momentum spread, the relative contribution of the second derivative term in Eq. (23) may be important. Fig. 3 shows its value for the case of $\delta p$ = 1 MeV/c and the cooling force with parameters of Fig.2. If this term is comparable with the value of the force, expansion Eq. (23) is not converging any more. Therefore, the simple interpretation of the voltage jump measurements with Eq. (23) is valid only when

$$\delta p^2 << \frac{2F_{lz}(\overline{p})}{F_{lz}''(\overline{p})}. \tag{27}$$

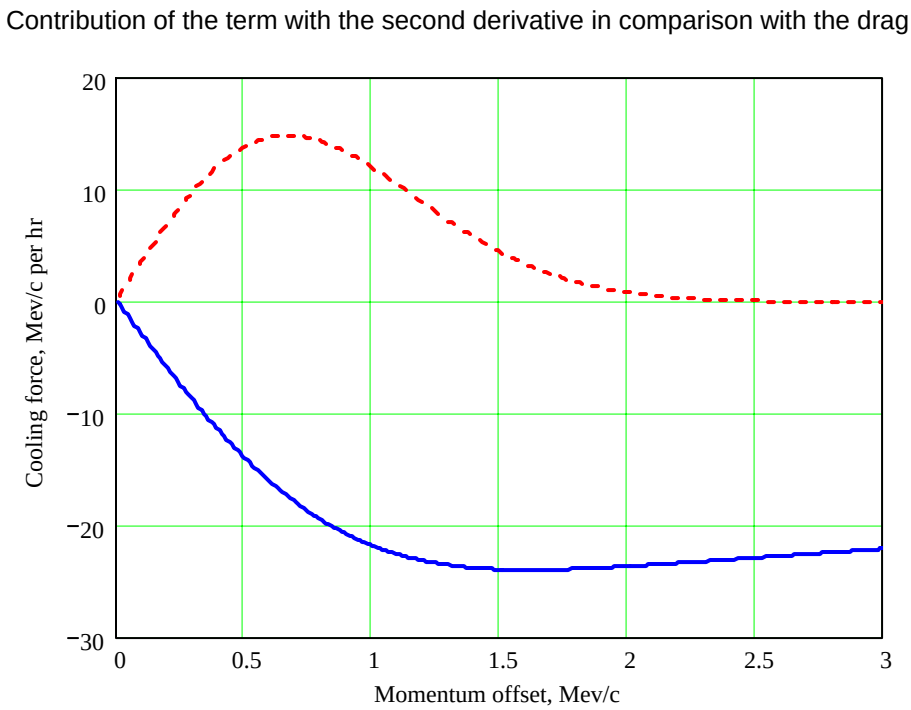


Figure 3. Comparison of the contribution of the second derivative term $F_{lz}''(\overline{p})\cdot\frac{\delta p^2}{2}$ (red dashed line) with the longitudinal cooling force $F_{lz}(\overline{p})$ (solid blue line) at the antiproton momentum spread of 1 MeV/c. The curves are calculated for the parameters of Fig.2.

## 5. Conclusion

1. In the framework of the considered model, formulas for electron cooling rates of an initially Gaussian antiproton beam have been derived. In the range of REC parameters, the longitudinal cooling rate and the logarithmic derivative of the transverse emittance depends weakly on the antiproton momentum spread and emittance.
2. Integration of the non-magnetized cooling force over a Gaussian distribution of transverse antiproton velocities gives a compact representation of the drag force. For typical REC parameters, the drag force is insensitive to transverse antiproton velocities.
3. The voltage jump measurement provides a simple estimation of the drag force only at a low enough antiproton momentum spread, determined by Eq. (27).

## Acknowledgement

I am thankful for many useful discussions to A. Burov, A. Fedotov, V. Lebedev, S. Nagaitsev, L. Prost, and A. Sidorin.

## References


1. A.Shemyakin et al., Proc. of EPAC'06, Edinburgh, UK, June 26-30, 2006, 1654-1656
2. Ya.S. Derbenev and A.N. Skrinsky, Particle Accelerators **8** (1977) 1
3. This calculation of the cooling force is a reproduction of formulas in a MathCad file provided to me by V. Lebedev
4. J.D.Bjorken and S.K.Mtingwa, Particle Accelerators, 13, 115 (1983)
5. I.N.Meshkov et al., Physics guide of BETACOOL code, v.1.1, BNL note C-A/AP#262, p. 17
6. H. Danared *et al.*, Phys. Rev. Lett. **72** (1994) 3775
7. L.Prost *et al.*, Electron cooling of 8 GeV antiprotons at Fermilab's Recycler: Results and operational implications, to be published in Proc. of HB2006, Tsukuba, Japan, May 29-June 2, 2006