

# More Rebunching Options for the $\mu 2e$ Conversion Experiments

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## Abstract

Some scenarios for bunching beam in the Fermilab Accumulator and Debuncher for future  $\mu$ -e experiments are discussed. In these scenarios 8 GeV proton batches from the the Booster are debunched, stacked and compressed to a single long bunch in the Debuncher ring, where protons are slow extracted to feed the  $\pi/\mu$  production target for a  $\mu$ -e conversion experiment[1]. Updated variations from scenarios developed in a previous note[2] are presented, as well as somewhat different examples. A new scenario in which the beam is adiabatically bunched in the Accumulator followed by a fast phase-energy rotation in the Debuncher and capture by higher harmonic rf is also presented.

## Introduction

In previous notes,[1, 2] we discussed plans to use 8-GeV proton beam from the Fermilab Booster, stacking it for multiple turns in the Accumulator, where the beam is debunched, transferring the beam into the Debuncher, and then rebunching it to a single bunch, which will then be slow-extracted toward target which will produce secondary  $\pi$ 's, that decay to  $\mu$ 's and are stopped by a target where  $\mu$ -e conversion events can be detected, similar to the proposed MECO experiment.[3] The scenario is designed to be compatible with future Fermilab proton plans, which are being developed for the time when the Tevatron Collider is no longer operated and the Recycler, the Accumulator and the Debuncher are no longer used for antiprotons and can be adapted to assist the proton beam program. Since that note the proton plan has changed and the potential  $\mu$ -to-e program is being developed.

The initial scenario is discussed in more detail in ref. [1], and a more recent variant is presented in ref. [4] In the initial scenario, 1 to 4 booster batches are transferred into the Accumulator and debunched, the beam is extracted into the Debuncher. In the initial scenario Debuncher the beam is rebunched to a single bunch with full width less than  $\sim 200$ ns. The rebunched beam is slowly extracted over the remainder of the  $\sim 1.5$ s cycle time, with the extracted proton beam sent to the external target that produces  $\pi$ 's for the  $\mu$ -e conversion experiment.

The proton beam in the Accumulator is debunched to a relatively small energy spread, and fills the circumference (474m or 1590ns). After accumulation of 4 Booster batches the full emittance is estimated to  $84 \times 0.38 = 32$  eV-s, which implies an energy spread of 20MeV (full width) in the fully debunched beam with length of  $\sim 1600$ ns. An extraction gap of  $>45$ ns is introduced into the circumference-length bunch (by a barrier bucket or harmonic rf) and a fast kicker extracts the long bunch into the Debuncher. The Debuncher has an  $\sim 30$ m greater circumference, which increases the initial gap in the circulating beam to  $>145$ ns.

This scenario was designed around a Fermilab proton plan in which Booster batches are stacked in the Accumulator and then stacked in the Recycler for transfer into the Main Injector, providing a maximum number of 120 GeV protons for neutrino experiments (NUMI and NOVA). The  $\mu$ -to-e program would transfer a small fraction of Accumulator proton stacks into the Debuncher for rebunching and slow extraction.

However, in the present Fermilab proton plan, the use of the Accumulator to feed the Recycler is deferred, and the Recycler alone will be used for stacking Booster batches. In that case the Accumulator and Debuncher can still be used in a dedicated mode for the  $\mu$ 2e source. C. Ankenbrandt et al. [4] have presented a new scenario in which the accumulator is used to accumulate and stack 1 to 6 booster batches for transfer to the Debuncher per 1.5s Fermilab proton cycle time. The timeline for this cycle is displayed in Figure 1.

The  $\mu$ -e experiment requires that the beam be bunched to a small fraction of the ring circumference. The bunching must conserve phase space. The expected total longitudinal emittance of a batch of 80 booster bunches at 0.08eV-s per bunch is 6.4 eV-s. (These are full emittances; the rms emittance is  $\sim 6\pi$  times smaller.) Allowing an  $\sim 25\%$  dilution, a booster batch has a longitudinal emittance of 8 eV-s, and we have used that as a reference batch emittance. If we stack 4 of these without further dilution, we find a total emittance of 32 eV-s, and 6 of these would obtain a total emittance of 48 eV-s. This can be compare with a reference emittance of 10 eV-s for a BNL AGS bunch. Thus, if we stack more than one Booster batch, we obtain a larger emittance, which implies a longer bunch and/or a larger momentum spread than in the AGS.

The Debuncher momentum aperture  $\delta p/p$  is, approximately,  $\pm 2\%$  and in a typical example we have allotted a full energy spread of  $\pm 150$  MeV, corresponding to an rms energy spread of  $\sim 60$  MeV. With this rms energy spread, a 48 eV-s beam would have an rms bunch length of 42.5ns or a full width of  $\pm 106$ ns. This is about 4 times larger than the expected bunch length for the BNL-MECO scenario, however.

### Rf Bunching

Scenarios for rf bunching in the debuncher have been explored, initially by considering only the longitudinal motion of the beam. The equations of motion are:

$$\frac{d\phi}{dn} \cong \frac{2\pi}{\beta^2 \gamma} \left( \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right) \frac{\Delta E}{mc^2} = \frac{2\pi}{\beta^2 \gamma} \alpha_p \frac{\Delta E}{mc^2}$$

$$\frac{d\Delta E}{dn} = eV_{RF}(\phi)$$

where  $n$  is the number of turns,  $\phi$  is the particle phase (longitudinal position) and  $\Delta E$  is the particle energy offset from the reference energy.  $V_{rf}(\phi)$  is the rf voltage,  $m$  is the proton mass,  $\beta=v/c$  and  $\gamma = (1-\beta^2)^{-1/2}$  are the usual kinematic factors, and  $\alpha_p = 1/\gamma^2 - 1/\gamma_t^2$  is the momentum compaction factor. In the Debuncher,  $\gamma=9.52$  and  $\gamma_t=7.6$ , so  $\alpha_p = -0.006$ . The small value of  $\alpha_p$  means the motion is relatively isochronous, which implies that the longitudinal motion is relatively slow. That also means that only a relatively small amount of  $V_{rf}$  may be needed in the rebunching.

### Barrier Bucket rf

A simple model for bunching would be to follow the bunching procedure used in the Recycler, which uses barrier bucket rf waveforms to bunch and debunch antiproton beams. In an initial example we can grow a square wave potential at two ends of the bunch and then move the phases of that square wave together until the two ends of the potential meet. In the previous paper we considered square waves of  $+$  and  $- 20$  kV amplitude, with  $40^\circ$  phase widths ( $\sim 180$ ns). After this bunch formation, bunching in a

single harmonic  $h=4$  (2.38MHz) rf system is imposed at 25kV, and the amplitude increased to 75kV to form the beam into a bunch with less than 200ns full width. This case corresponds to a 4-batch bunch ( $\epsilon_L < 32\text{eV}\cdot\text{s}$ ). An  $h=4$  rf system is used since it is similar in voltage and frequency to an rf system that is currently used in the Main Injector for Tevatron bunch coalescing. (That rf will not be needed in the Main Injector after 2010.)

For a 6-batch bunch ( $\epsilon_L \cong 48\text{eV}\cdot\text{s}$ ), the same procedure can be followed but more rf is required. In figure 3, we present results of a simulation of barrier bucket bunching and  $h=4$  bunch compression. In that scenario, the barrier bucket amplitude is 30 kV, and the bunch is initially compressed to short length over 0.135s. Then an  $h=4$  rf system of 50kV is imposed and ramped to 150kV over  $\sim 0.05\text{s}$ , compressing the beam to an rms bunch length of  $\sim 40\text{ns}$ , with the entire beam held within a  $\sim 200\text{ns}$  full width. The energy full width of the beam is increased to  $\sim \pm 150\text{MeV}$ .

### Multi-harmonic rf buncher

The barrier bucket approach requires use of an inefficient low-Q rf system, that may be somewhat expensive, and the rf waveforms will be less ideal than our initial model. We may also consider a bunching system consisting of a combination of low-harmonic rf systems, and these could be obtained from high-Q fixed-frequency cavities. For a  $\sim 6$ -batch bunch, we also consider a combination of  $h=1, 2, 3, 4$  rf cavities with maximum voltages of 38, 19, 12.7 and 9.5kV respectively, and combined in order to obtain an approximately linear rf wave form. In an initial simulation these cavities are ramped from 0 to full strength over  $\sim 0.033\text{s}$ . Then the 4<sup>th</sup>-harmonic rf is ramped from 50 kV up to 125kV over 0.05s and then held fixed at 125kV hold the beam for the remainder of the 1.33s operational cycle. Simulation results are displayed in figure 3. (These rf voltages are somewhat larger than that needed for a 6-batch bunch.)

The final beam state is similar to the above example, with an rms bunch length of  $\sim 40$  ns and an energy full width  $< \sim \pm 150\text{MeV}$ . The finite number of harmonics resulted in a slightly more dilute phase space than a more idealized linear or barrier bucket rf would obtain; however, a more complete optimization and a more accurate evaluation of barrier bucket nonlinearity may remove that difference. The bunching did occur in a significantly shorter time than the above barrier bucket example, however.

### Discussion

Further study of instability intensity limitations is needed. The compressed beam has an enhanced space charge tune shift. A formula for that tune shift is:

$$\delta\nu \cong \frac{r_p N}{4\pi\beta\gamma^2 B_F \epsilon_{N,rms}},$$

where  $B_F$  is the bunching factor. With  $B_F = 0.06$ , and  $N = 1.5 \times 10^{13}$ ,  $\epsilon_{\text{Fermi}} = 6\pi\epsilon_{N,rms} = 20\pi\text{mm}\cdot\text{mr}$ , we obtain  $\delta\nu \cong 0.1$ , which should be acceptable. However 6 batches of  $5 \times 10^{12}$  protons would give us  $N = 3 \times 10^{13}$ , which would then obtain  $\delta\nu \approx 0.2$ , a somewhat more difficult value.

While the simulations show bunch compression, a complete assessment would show some small amount of beam leaking into the interbunch gap, and the desired extinction factor of  $10^{-9}$  cannot be obtained by the bunching rf alone. The bunching rf can provide an extinction of at best  $\sim 10^{-3}$ . Additional extinction procedures will be needed, including additional gap-clearing rf and kickers, adjustment of the slow extraction dynamics and kickers to improve extinction, and extracted beam transport clearing kickers. Improved event discrimination in the  $\mu$ -e decay detector will help.

## Hybrid Scenario

In the previous scenarios we used only the Debuncher for bunching and storage of the beam for slow extraction, although our simulations did begin with a gap in the continuous beam, which was assumed to be formed in the Accumulator to obtain an extraction gap for transfer into the Debuncher.

With the more recent Fermilab proton plan[4], the Accumulator is not used for NUMI and its entire cycle time is available for bunching the proton beam before transfer into the Debuncher. In the cycle of Fig. 1, this independent beam preparation time could be up to 0.8s, after which the beam could be transferred for slow extraction use in the Debuncher for the full 1.33s cycle time. The Accumulator is less isochronous than the Debuncher, with  $\gamma_t = 5.5$  in the Tevatron I lattice and  $\gamma_t = 6.5$  in the Tevatron II lattice, and this increases the speed of bunching. This feature is particularly useful for initial adiabatic bunching. For final compression the more isochronous Debuncher can provide bunching to minimal length with less rf voltage and more gradual compression. Therefore we propose to use a hybrid system in which the initial adiabatic bunching is performed in the Accumulator and the final compressions are performed in the Debuncher.

In the hybrid scenario we propose to use the less isochronous Accumulator for initial adiabatic bunching. An  $h=1$  rf system is ramped from 0 to 6 kV over a relatively long period ( $\sim 0.3$ s in the case presented in Fig. 5). This adiabatically bunches the beam to a single bunch of  $\sim$ half the Accumulator circumference. The beam is then transferred to the Debuncher, where a larger  $h=1$  rf system (40kV) phase-energy rotates the beam by  $90^\circ$  to a short bunch and an  $h=4$  250kV system captures and compresses the beam to a short bunch (38ns rms length) and large energy spread ( $< \pm 200$  MeV).

The hybrid system eliminates the separate step of adiabatically forming a gap in the Accumulator beam for extraction. The bunched Accumulator beam has a beam free gap of  $> 500$  ns, which enables a relatively easy kicker system for the Accumulator. Also there is time for additional beam halo clearing in the Accumulator without reducing the slow extraction time in the Debuncher.

The system could be made more linear by adding third-harmonic or other rf variations and that may improve the bunching significantly. The present scenario is optimized for a 6-batch bunching and phase-energy rotation. With fewer batches, some scenario

variation could be needed to avoid phase-space dilution. The smaller momentum spread would mean that more time may be needed to develop the adiabatic bunching.

The system would need readaptation if the previous scenario in which the Accumulator is also needed for NUMI beam stacking is implemented. A shorter, less adiabatic, bunching time in the Accumulator may be used, and more harmonics in the Debuncher may be needed to reduce the nonlinear longitudinal mismatch.

## **Discussion**

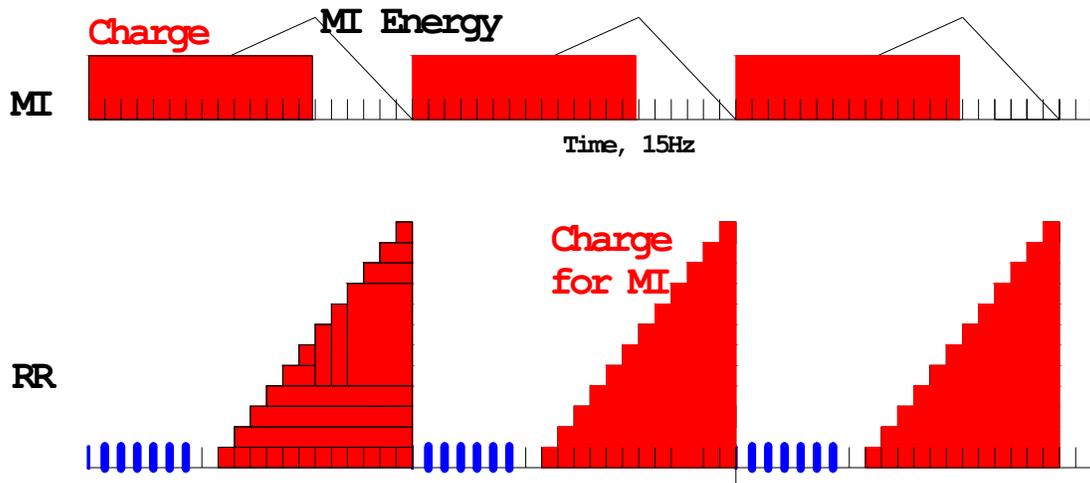
Potential bunching plans for developing a short bunch in the Debuncher compatible with the present Fermilab proton plan have been presented. A favored scenario is a hybrid scenario in which the beam is first adiabatically bunched in the Accumulator and then transferred into the Debuncher for  $\phi$ -E rotation and  $h=4$  bunch compression, and then held in a short bunch for slow extraction.

## **References**

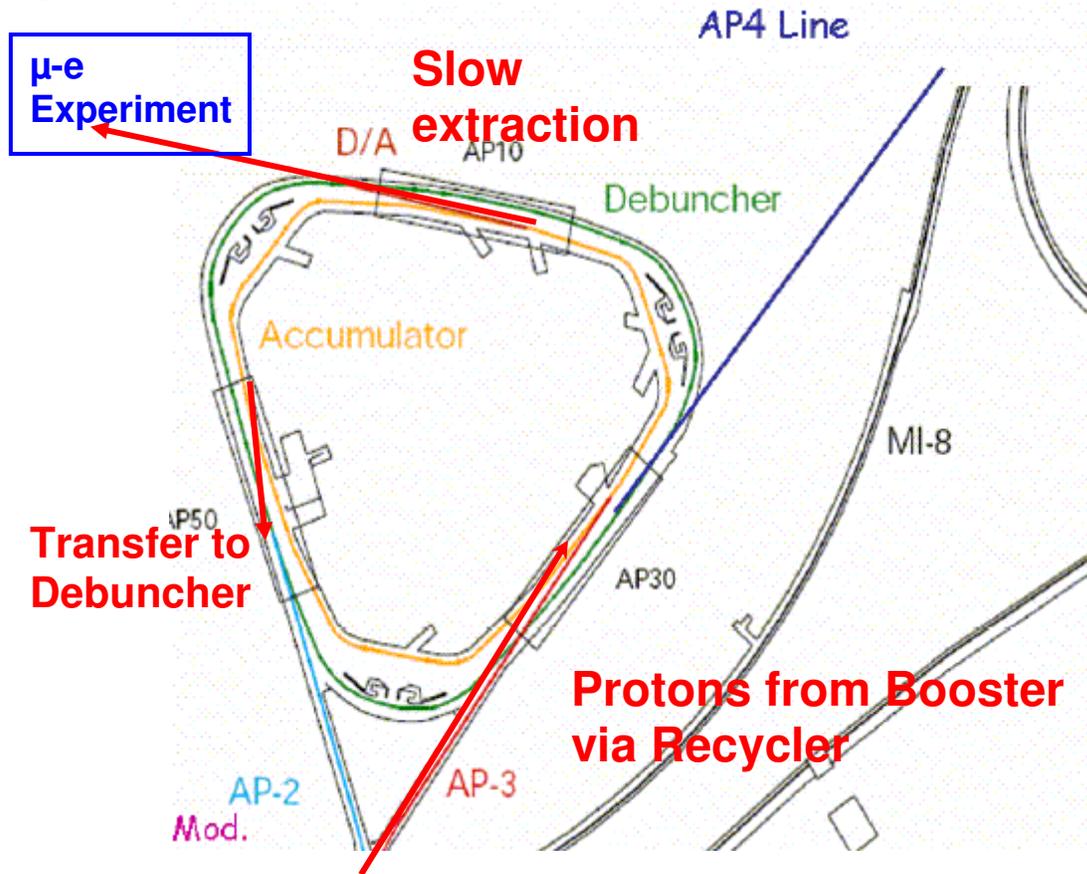
- [1] C. Ankenbrandt et al., "Using the Fermilab Proton source for a  $\mu$ -e Conversion Experiment", FERMILAB-TM-2368-AD-E, 6 November 2006
- [2] D. Neuffer, "Rebunching in the Debuncher for mu-e Conversion Experiments, Beams-Doc-2505, October 2006.
- [3] W. Molzon, Spokesperson, BNL P940 proposal "A search from  $\mu^+N \rightarrow e^+N$  with Sensitivity Below  $10^{-16}$ " (Muon-Electron CONversion), September 1997.
- [4] C. Ankenbrandt, D. Harding, D. Johnson, D. McGinnis, M. Popovic, "Delivering Protons to the Antiproton Source after the Tevatron Collider Era", Beams-Doc-2678, March 2007.

<b>Table: Accumulator and Debuncher Ring Parameters</b>			
Parameter	Symbol	Accumulator	Debuncher
Circumference	$C=2\pi R$	474m	504m
Beam Momentum	P	8.89	8.89 GeV/c
Transition	$\gamma_t$	5.4	7.52
Betatron functions (maxima)	$\beta_x, \beta_y, \eta$	61, 38, 9.1m	19.8, 17, 2.2m
Tunes	$\nu_x, \nu_y$	6.7, 8.67	9.66, 9.76
Period	$C/\beta c$	1590	1690ns

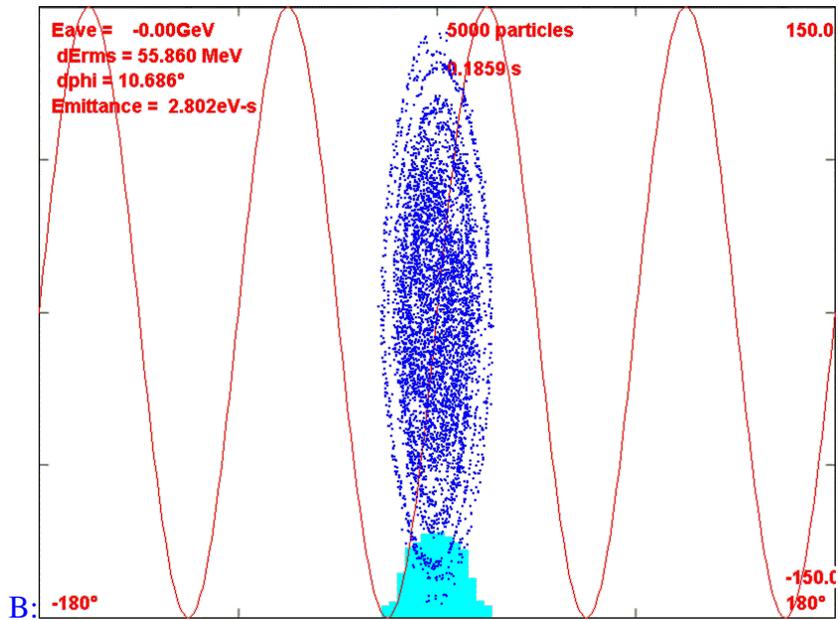
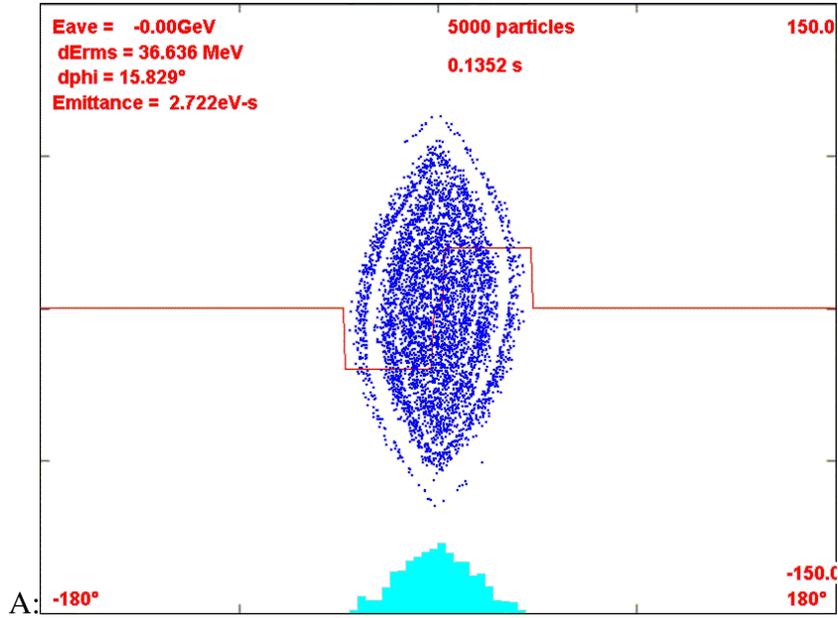
**Figure 1:** Timeline for new Proton plan with  $\mu e$  available pulses. (from ref. 4) The baseline scenario has a period of 20 booster cycles (15Hz) or a period of 1.33. In each scenario cycle 12 consecutive booster batches are sent to the Recycler for the Main Injector-based neutrino oscillation studies (Mivos). (These twelve are shown in red.) 8 booster cycles are available for the Accumulator/Debuncher, and we consider using up to 6 of these (shown in blue). With up to  $5 \times 10^{12}$  protons/booster batch, or up to  $3 \times 10^{13}$  protons per cycle, we have  $\sim 2.2 \times 10^{20}$  protons per  $10^7$ s Snowmass-year of accelerator operation.



**Figure 2:** Overview of Booster-Accumulator-Debuncher system.

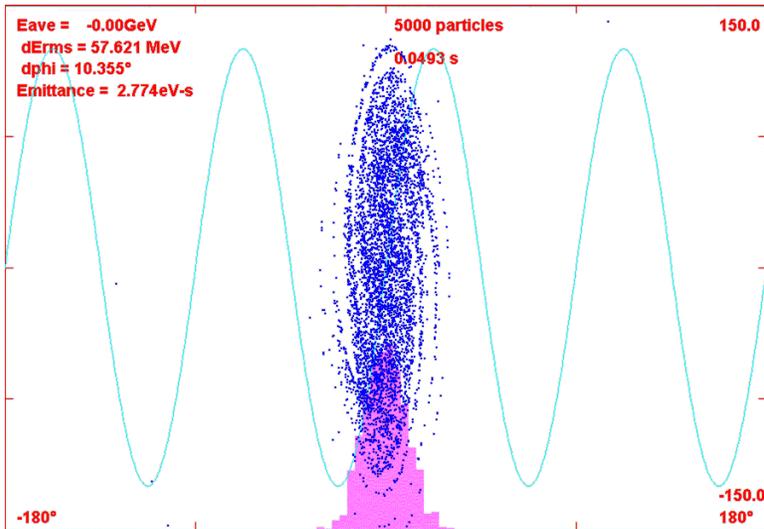
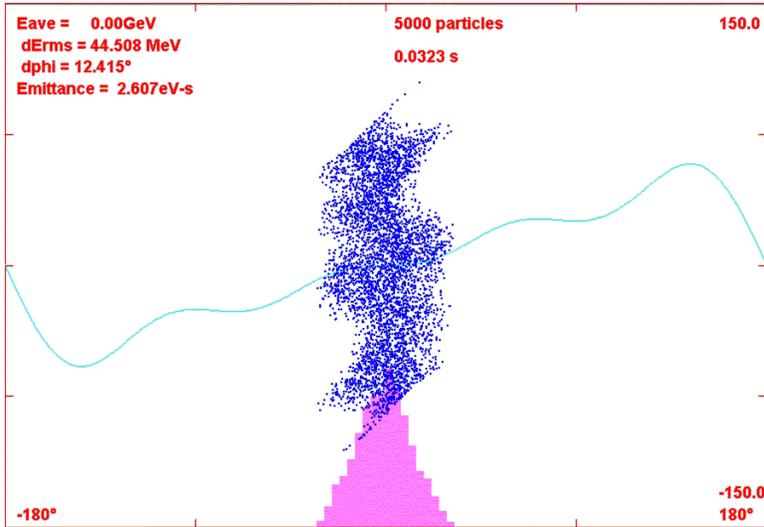


**Figure 3.** 1-D simulation of bunch compression of a “6-batch” bunch in the Debuncher .  
A: Longitudinal distribution after 0.135s of barrier bucket compression  
B: Longitudinal distribution after further h=4 bunching, beam can be compressed to  
(Emittance numbers on plots are rms emittances; full emittances are a factor of  $6\pi$  larger)

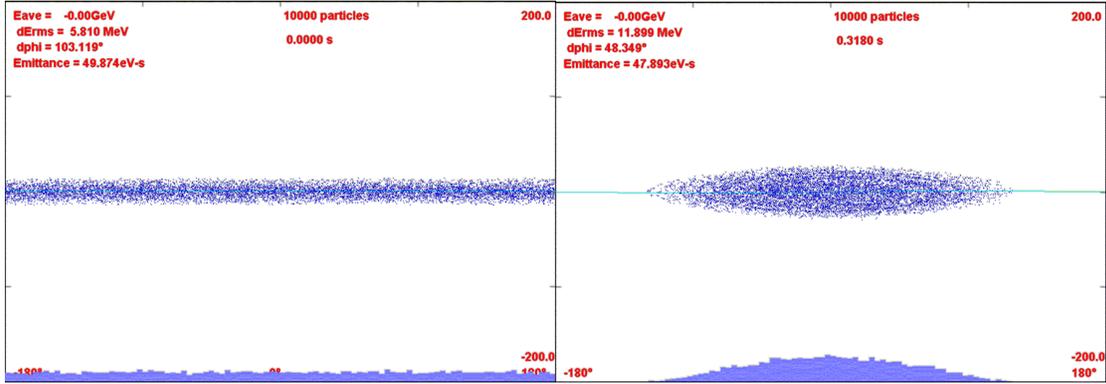


**Figure 4.**

Beam after simulation of h=1, 2, 3, 4 bunching, followed by h=4 bunch compression; beam properties are similar to the previous barrier-bucket example.

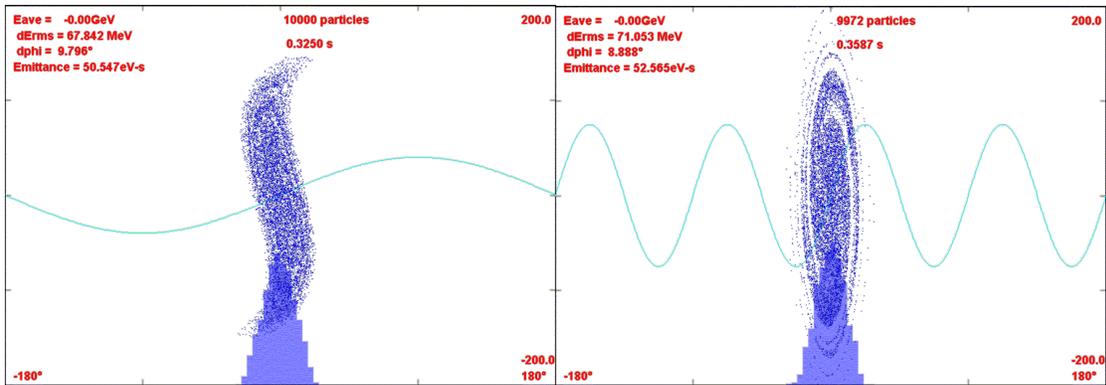


**Figure 5:** Accumulator + Debuncher bunching and phase-energy rotation. The beam is first adiabatically bunched in the Accumulator using an  $h=1$  rf system (0 to 6 kV), then transferred into the Debuncher where it is phase-energy rotated (40 kV) and then bunched at  $h=4$  (250 kV).



A: initial debunched beam.

B: After adiabatic bunching in Accumulator.



C: After  $\phi$ -E rotation in Debuncher

D: After  $h=4$  bunching in Debuncher.