A Model for Lorentz Detuning in a Superconducting RF Cavity Controlled with a Vector Phase Modulator

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Introduction

This note examines the behavior of a HINS Superconducting Single Spoke Resonator cavity in the presence of Lorentz detuning. This note is based on the model described in Beams Document 2910 "A Simple Model for a Superconducting RF cavity with a Vector Phase Modulator".

Envelope Equations

The cavity voltage and the total cavity current can also be separated into fast varying and slow varying parts:

$$v(t) = Re\left\{V(t)e^{j\omega t}\right\} \tag{1}$$

$$i_s(t) = Re\left\{I_s(t)e^{j\omega t}\right\} \tag{2}$$

where V(t) and I_S(t) are slowly varying complex phasors. The cavity voltage and current phasors can be separated into real and imaginary parts:

$$V = V_r + jV_i \tag{3}$$

and:

$$I_s = I_{s_r} + jI_{s_i} \tag{4}$$

The response of the cavity voltage to the source current is described by two coupled equations:

$$\frac{2Q}{\omega_o}\frac{dV_r}{dt} + V_r + tan(\varphi_D)V_i = RI_{s_r} \tag{5}$$

and:

$$\frac{2Q}{\omega_o}\frac{dV_i}{dt} + V_i - tan(\varphi_D)V_r = RI_{s_i} \tag{6}$$

where:

$$tan(\varphi_D) = Q\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right) \tag{7}$$

where ω_o is the resonant frequency of the cavity. If the excitation frequency ω is close to the resonant frequency ω_o:

$$\omega = \omega_o + \Delta\omega \tag{8}$$

then Eqn. 7 becomes:

$$tan(\varphi_D) = -\frac{2Q}{\omega_o}\Delta\omega \tag{9}$$

Lorentz Detuning Approximation

For intense electromagnetic fields inside the cavity, the radiation pressure of the fields will push against the walls of the cavity and the resonant frequency of the cavity will shift to a lower value. As an approximation, assume that the shift in resonant frequency is linearly proportional to the magnitude of the cavity voltage:

$$\frac{\Delta\omega}{\omega_o} = -\frac{2\pi}{\omega_o}\mathrm{K_L}|\mathrm{V}|^2 \tag{10}$$

where K_L is a constant describing the Lorentz detuning. The total detuning angle of the cavity can be broken into a static and dynamic part:

$$tan(\varphi_D) = tan(\varphi_{DS}) + 2\pi\tau_f\mathrm{K_L}|\mathrm{V}|^2 \tag{11}$$

where ϕ_{DS} is the static shift in phase angle due to the initial geometry of the cavity at zero voltage and τ_f is the loaded filling time of the cavity. From Equation 60 in Beams Document 2910, the generator phase required during the beam pulse is:

$$tan(\varphi_g) = \frac{I_p tan(\varphi_b) - I_{p_0} tan(\varphi_D)}{I_{p_0} + I_p} \tag{12}$$

Where I_p is the RF beam current and I_{po} is the RF beam current for which the coupling was tuned for optimum power transfer. For optimum power transfer to the cavity fg is zero. The static detuning of cavity would need to be:

$$tan(\varphi_{DS}) = \frac{I_p}{I_{p_0}} tan(\varphi_b) - 2\pi\tau_f\mathrm{K_L}|\mathrm{V}|^2 \tag{13}$$

Examples

Equations 5 and 6 are non-linear but can be solved numerically. In the following examples, the cavity R/Q of 262 Ohms was used. This value is typical of a superconducting single spoke resonator used in HINS. The cavity voltage and synchronous phase angle were chosen from the design of the first cryomodule in HINS. The cavity voltage of 1472kV with a synchronous angle of 30 degrees was used. It was assumed that the cavity coupling would be optimized for optimum power transfer to the beam at a beam pulse current of 15mA. The loaded Q of the cavity is about $2.2\text{x}10^5$ and the fill time of the loaded cavity is about 212uS. Beam injection time should be about 147uS after the generator current turns on. The beam pulse length is 1mS. This example used a detuning coefficient of 0.46 kHz/MV^2 which would detune the cavity by about 1kHz at the operating voltage. If there was no Lorentz detuning, then the cavity would have to be detuned by +30 degrees to keep the generator current in phase with the cavity voltage for a synchronous phase angle of 30 degrees. However, with a detuning coefficient of 0.46 kHz/MV^2, the cavity must be statically detuned to an angle of -36.8 degrees. The parameters of this example are listed in Table 1. Cavity voltages, phase angles, and reflection coefficient are shown in Figures 1-4.

If the cavity is not statically detuned, then the generator current will have to be -20.5 degrees out of phase with the cavity voltage during the beam pulse. This phase shift requires 14% more klystron power in order for the vector modulator to still keep 10% overhead margin.

Parameter	Value	Units
Cavity Voltage	1472	kV
Klystron Power	23.61	kW
R/Q	262	Ohms
Reference Beam Current	15	mA
Actual Beam Current	15	mA
Synchronous Phase Angle	30	degrees
Detuning Angle	-36.835	degrees
Generator phase during filling	-0.7	degrees
Generator phase during beam	0	degrees
Vector modulator gain during filling	0.9	
Vector modulator gain during beam	0.9	
Beam injection time	0.147	mS
Beam Pulse Length	1	mS
Lorentz Detuning	0.46	kHz/MV^2

Table 1. Parameters for Example 1.

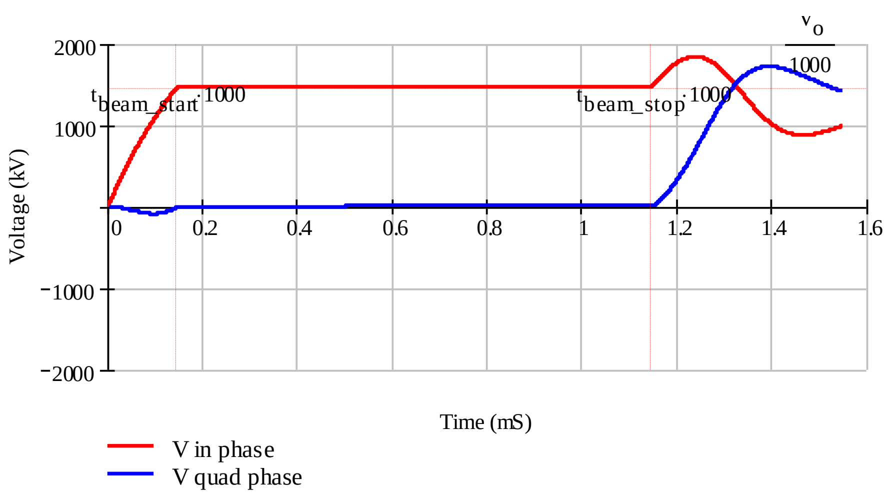

Figure 1. Cavity voltage for Example 1.

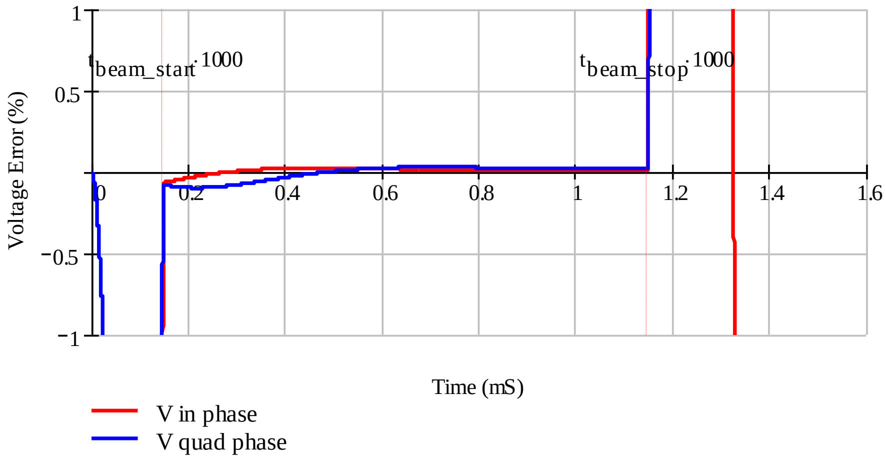

Figure 2. Cavity voltage error for Example 1.

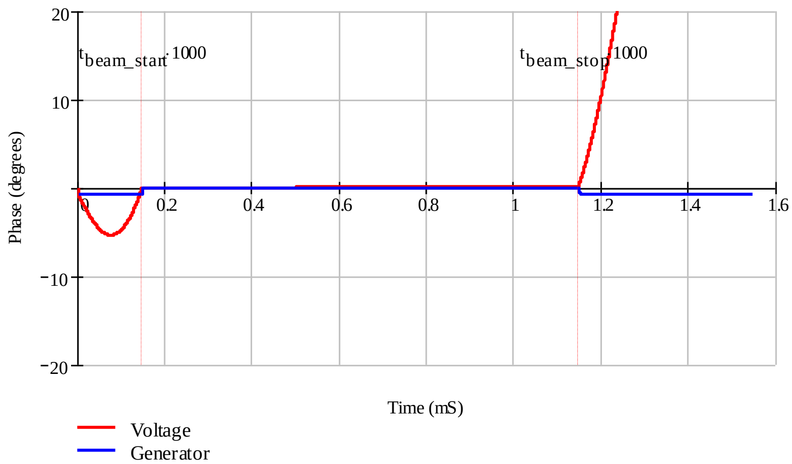

Figure 3. Voltage and Generator phase for Example 1.

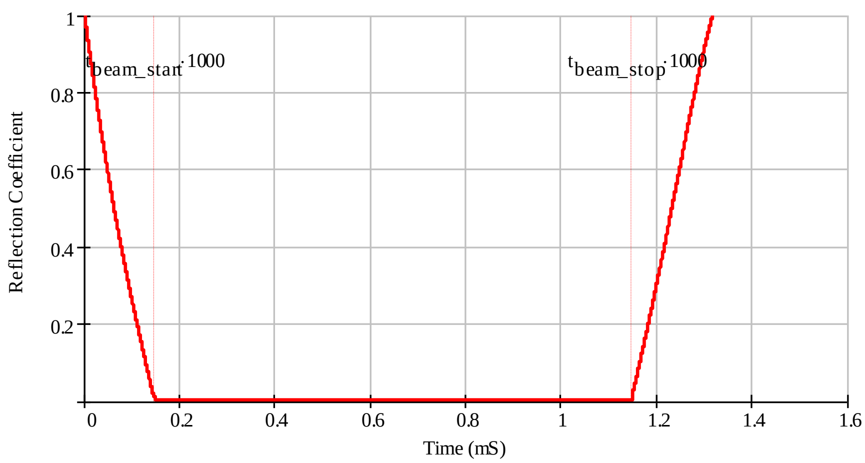

Figure 4. Reflection coefficient for Example 1.