Fourier Analysis of Booster TBT data

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Beam position at the j-th BPM after a *single* kick:

$$z_n^j = \sqrt{\beta_z^j} \mathrm{e}^{i\Phi_z^j} A_z \mathrm{e}^{iQ_z(\theta_j + 2\pi n)} + c.c. \qquad (z = x, y)$$

$$n \equiv \text{ turn number} \quad A_z = |A_z| \mathrm{e}^{i\delta_z} \equiv \text{ constant of motion}$$

$$\Phi_z \equiv \int_0^\theta d\theta\, ' \frac{R}{\beta_z} - Q_z\theta \qquad \text{(periodic phase function)}$$

Twiss functions:

$$\beta_z^j = |Z_j(Q_z)|^2 / A_z^2 \qquad \mu_z^j = \arg\,(Z_j) - \delta_z \quad Z_j(Q_z) \equiv \text{ Fourier component of } z_j$$

Amplitude fit:

$$|A_z|^2 = \frac{\sum_j 1/\beta_{0z}}{\sum_j 1/|Z_j(Q_z))|^2}$$

In the presence of *coupling*, the excitation of one mode will excite the other too at the *same* frequency. Following for instance a horizontal kick (first order approximation):

$$y_n^j = \Big[\sqrt{\beta_y^j}\Big(\mathrm{e}^{-i\Phi_y^j} w_+^j - \mathrm{e}^{i\Phi_y^j} w_-^j\Big) - \sqrt{\beta_x^j}\mathrm{e}^{i\Phi_x^j}\sin\chi_j\Big] A_x \mathrm{e}^{iQ_x(\theta_j+2\pi n)} + c.c.$$

$$\chi_j \equiv \text{tilt of } j\text{-th BPM}$$

Coupling functions:

$$w_\pm(\theta) = -\int_0^{2\pi} d\theta' \frac{C_\pm(\theta')}{4\sin\pi Q_\pm} \mathrm{e}^{-iQ_\pm[\theta-\theta'-\pi\mathrm{sign}(\theta-\theta')]}$$

$$Q_\pm \equiv Q_x \pm Q_y$$

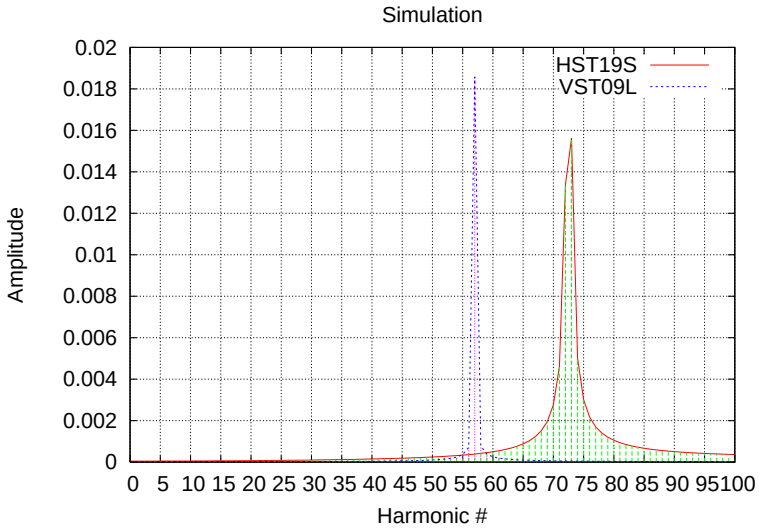
$$C_\pm(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\Big\{\Big(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\Big) + B_\theta\Big[\Big(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\Big) - i\Big(\frac{1}{\beta_x} \mp \frac{1}{\beta_y}\Big)\Big]\Big\}\mathrm{e}^{i(\Phi_x \pm \Phi_y)}$$

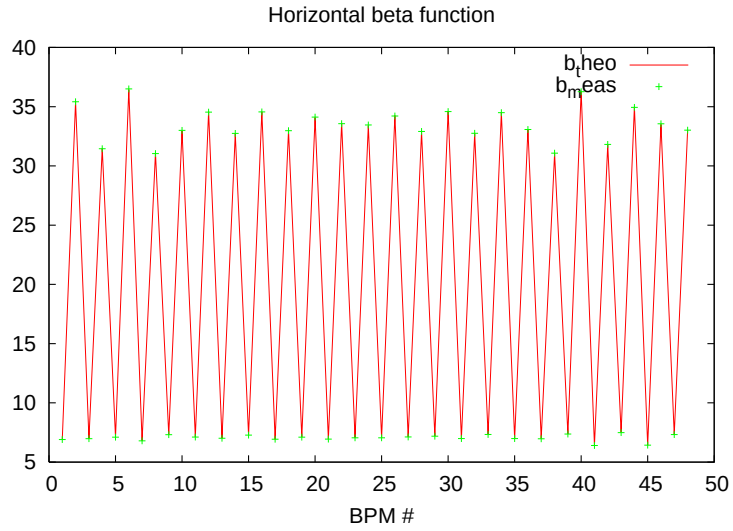
Minimum tune split:

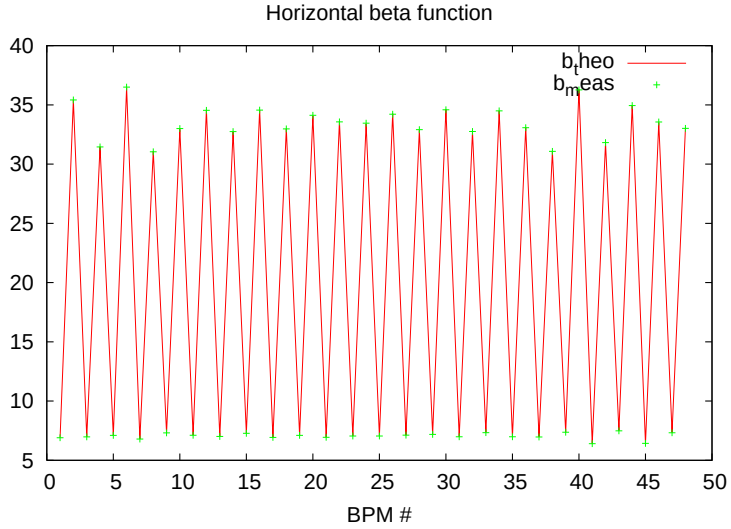
$$\Delta \equiv |\bar{C}_-| \quad \bar{C}_\pm = \frac{n_\pm - Q_\pm}{\pi}\int_0^{2\pi} d\theta\; w_\pm \mathrm{e}^{in_\pm\theta} \quad n_\pm \equiv \mathrm{Round}(Q_x \pm Q_y)$$

This analysis is *routinely* used for correcting the coupling at Tevatron during shot set up.

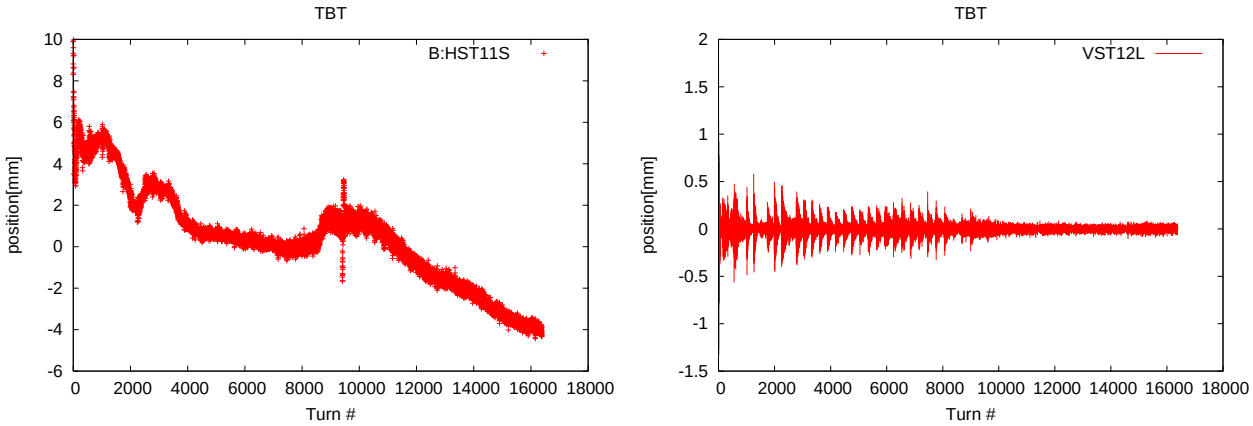
Code testing on simulated Booster data

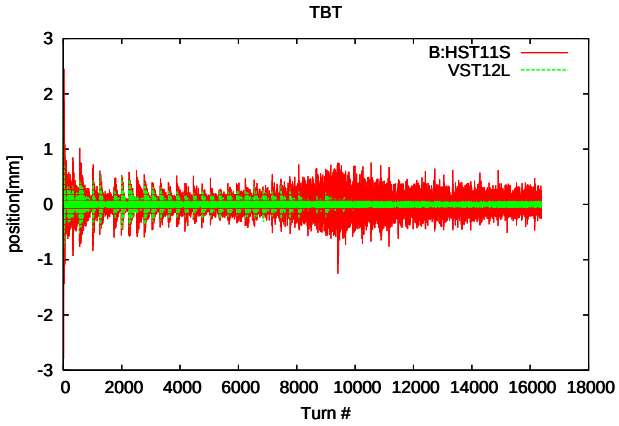






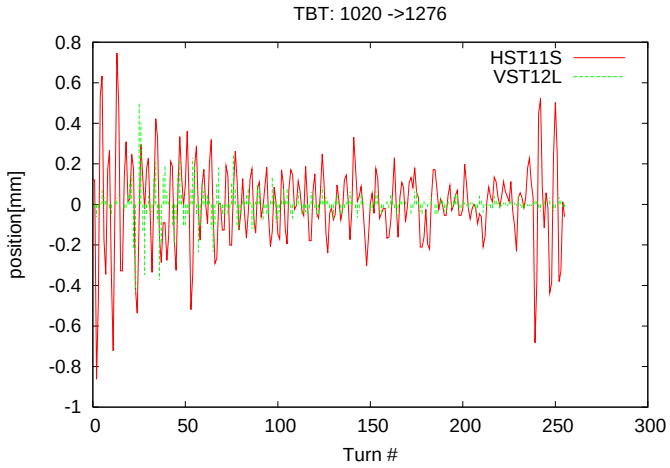
Actual Booster data - AC mode (August 31)



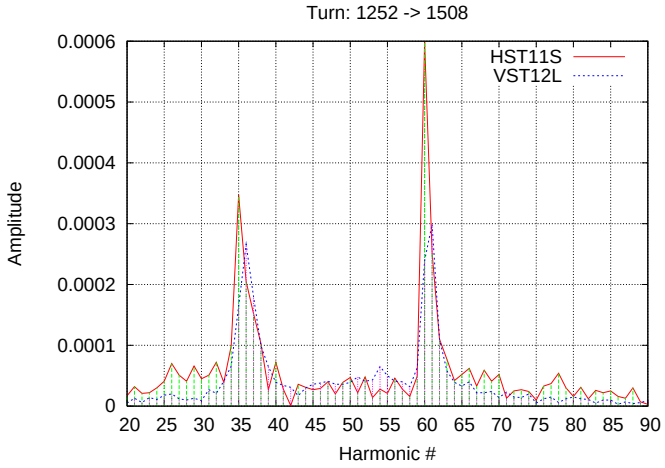


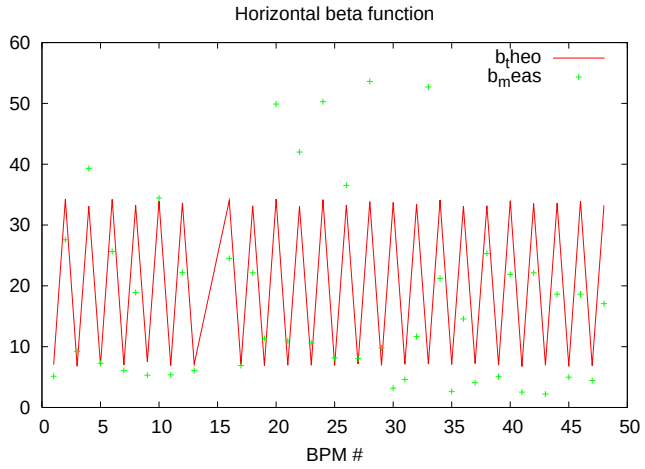
After subtracting the closed orbit (piecewise average)

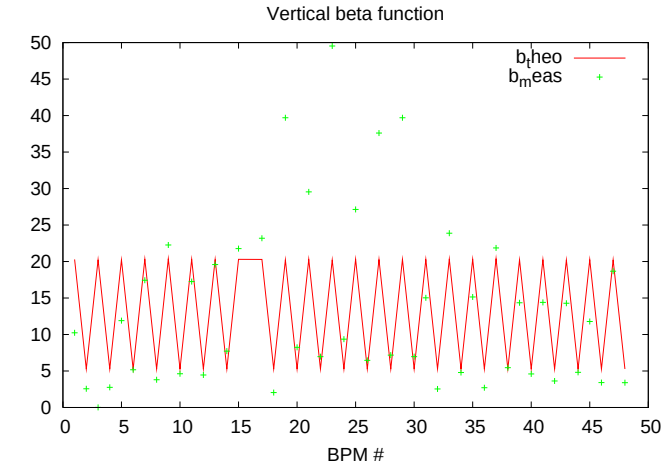
Observation: BPMs are not "synchronised"!

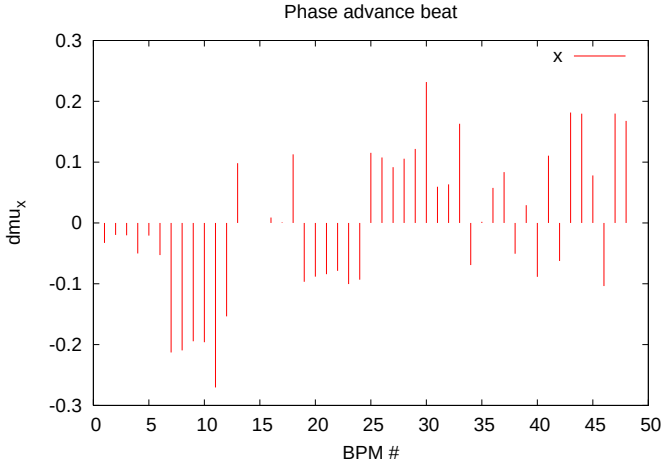


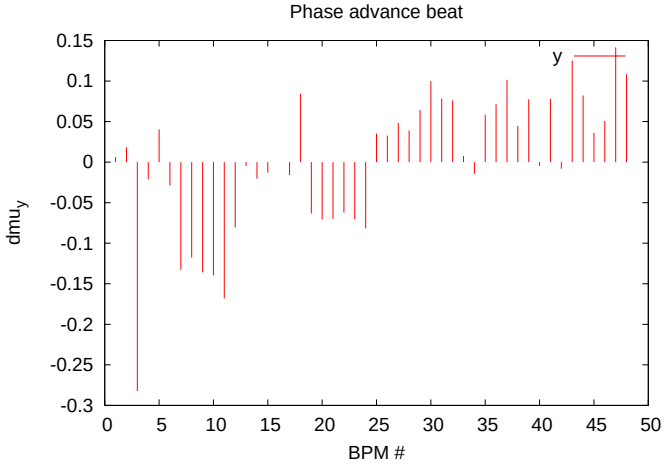
Trick used for a "crude"(?) ordering of the turns.

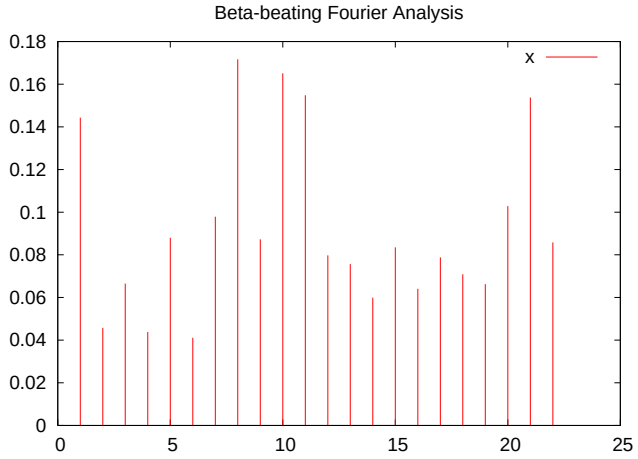


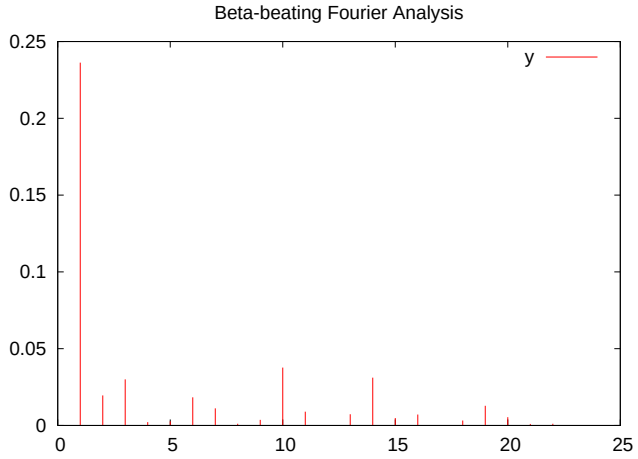

Horizontal beta function
b_theo
b_meas
BPM #

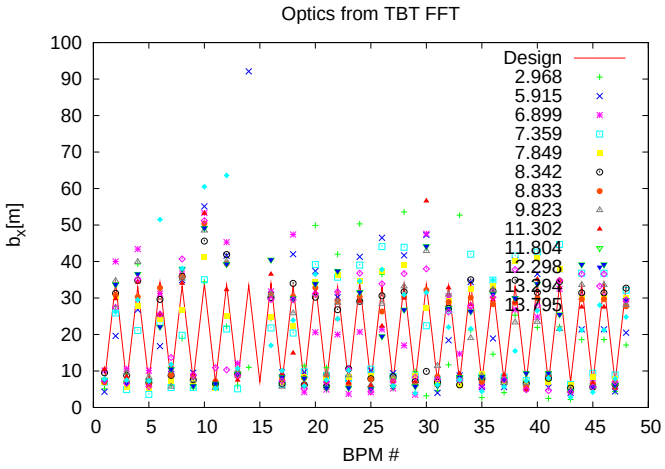

Vertical beta function
b_theo
b_meas
BPM #

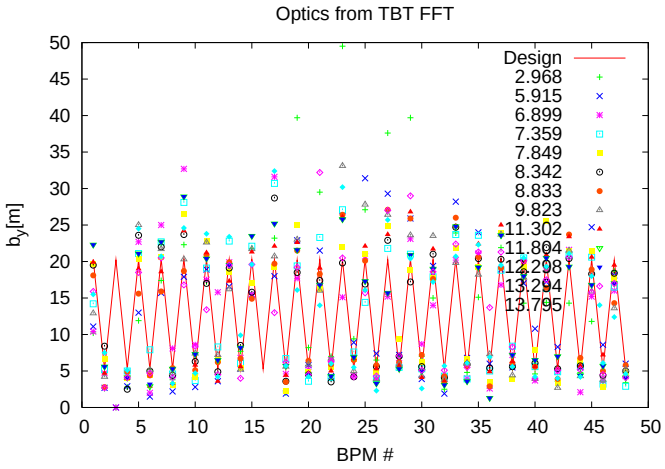

Phase advance beat
x
dmu_x
BPM #

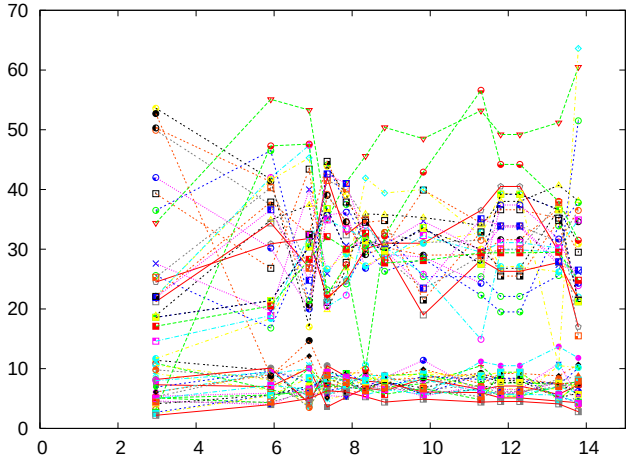

Phase advance beat
y
dmu_y
BPM #

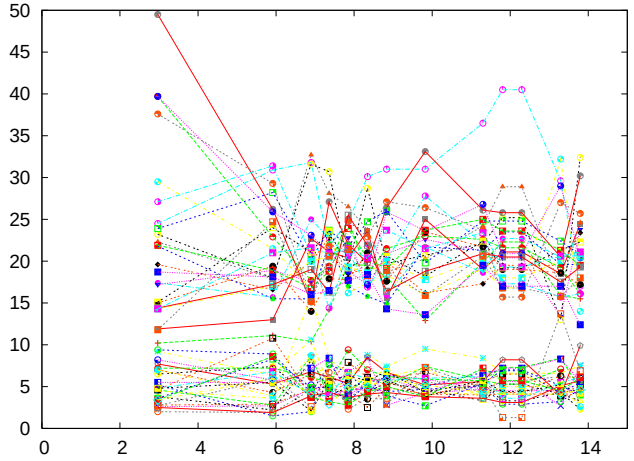

Beta-beating Fourier Analysis
x
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25

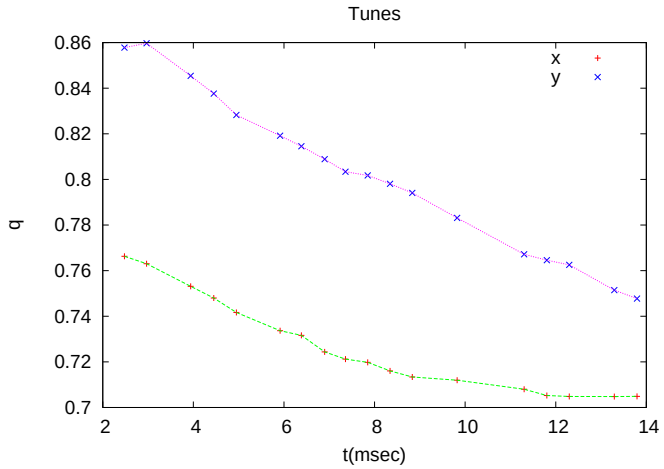

Beta-beating Fourier Analysis
y
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25

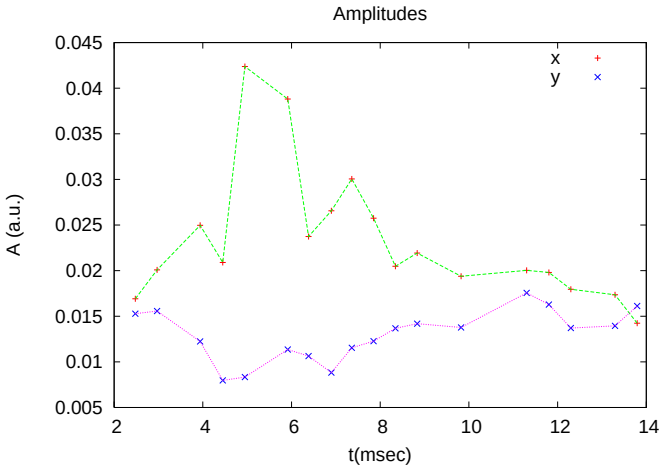

Optics from TBT FFT
Design
2.968
5.915
6.899
7.359
7.849
8.342
8.833
9.823
11.302
11.804
12.298
13.294
13.795
b_x[m]
BPM #

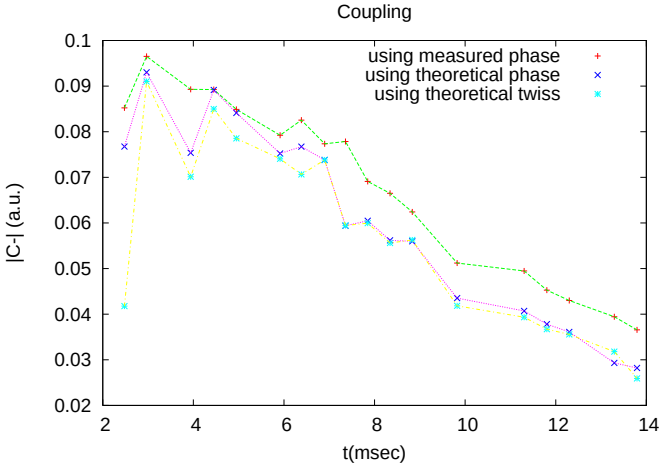

Optics from TBT FFT
Design
2.968
5.915
6.899
7.359
7.849
8.342
8.833
9.823
11.302
11.804
12.298
13.294
13.795
b_y[m]
BPM #

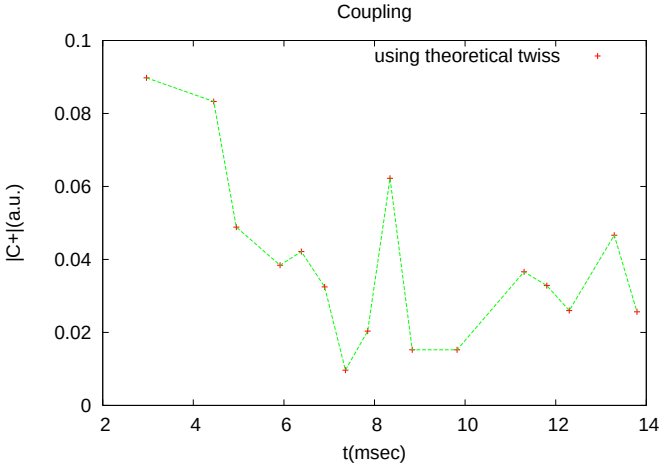





Tunes
q
t(msec)
x
y
0.86
0.84
0.82
0.8
0.78
0.76
0.74
0.72
0.7
2
4
6
8
10
12
14


Amplitudes
A (a.u.)
t(msec)
x
y
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
2
4
6
8
10
12
14


Coupling
|C-| (a.u.)
t(msec)
using measured phase
using theoretical phase
using theoretical twiss
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
2
4
6
8
10
12
14


Coupling
|C+|(a.u.)
t(msec)
using theoretical twiss
0.1
0.08
0.06
0.04
0.02
0
2
4
6
8
10
12
14

Conclusions

- B:HST16L and B:VST16S are suspect.
- The data are not suitable for a fine optics measurement through FFT analysis.
- The tunes could be clearly tracked.
- The coupling computation looks reliable and indicates a large coupling.