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Beam-Beam Tune Distributions with Differing Beam Sizes

M.J. Syphers

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Abstract

The beam-beam interaction generates a spread in the tune distribution of the affected particles. Below we take an analytical approach to look at the distribution of tunes generated when the two interacting beams are of differing transverse size. To the extent that larger tune spread makes a beam of particles more susceptible to resonances, and hence potentially shorter particle and luminosity lifetimes, a quick look at the expected tune distributions due to the interaction of two round beams of differing size can provide insight. The case of recent Tevatron experience is discussed.

1 Introduction

For many years the Tevatron proton-antiproton collider had operated in a "weak-strong" regime, where the much higher intensity proton bunches affected the behavior of the lower intensity antiproton bunches, with little consequence in the reverse. However, as the intensity of the antiproton bunches in the Tevatron continue to rise, this situation is clearly no longer the case. The introduction of the Recycler storage ring into the complex, wherein more than 4×10^{12} antiprotons are often made available for a single Tevatron store, has increased the per-bunch antiproton intensity to approximately one third the proton bunch intensity. Additionally, the Recycler provides much smaller transverse emittances than can be attained in the proton bunches – again, approximately a factor of three smaller. Since beam-beam effects are characterized by the head-on tune shift parameter, ξ, which is determined by the ratio of bunch intensity to transverse emittance (assuming round beams) for the on-coming beam, then ξ and $\bar{\xi}$ (from the proton and antiproton beams, respectively) are now essentially of equal magnitude.

In recent operations it has often been found that while keeping the proton beam parameters constant, as the antiproton beam intensity is increased the proton beam lifetime is often significantly shortened when beam-beam interactions are present. This is presumably due to the beam spanning across various resonance lines in tune space, the spread in tunes being created by the beam-beam interactions. Purposefully increasing the antiproton transverse emittance to reduce $\bar{\xi}$ and improve the proton lifetime (and hence luminosity lifetime) must be balanced against the reduction of the instantaneous luminosity due to the same emittance increase.

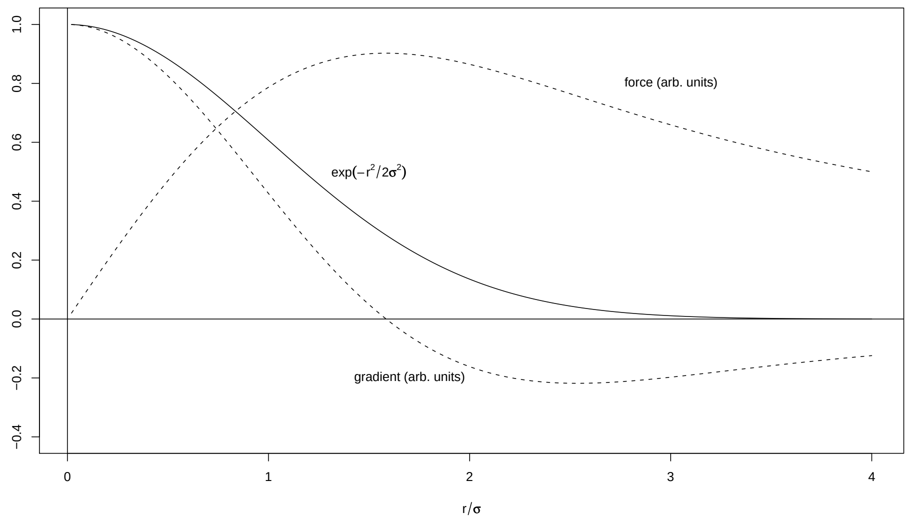

Figure 1: Characteristics of the beam-beam force.

2 Characterization of the Beam-Beam Interaction

The beam-beam interaction is an inherently nonlinear interaction. While estimating the effects of beam-beam collisions on single particle behavior and particle lifetime is tricky at best, some insights into the problem can be seen with a simple calculation involving two round beams interacting where one beam can be larger in transverse size than the other. Imagine a particle circulating a synchrotron suddenly encounters an on-coming distribution of particles which has a round bi-Gaussian transverse profile with rms size σ. This "test particle" will experience a transverse force from passing through the on-coming bunch that is of the form

$$\text{Force} \propto \frac{1 - e^{-x^2/2\sigma^2}}{x}$$

where x is the transverse distance from the center of the distribution. If the amplitude of the test particle is small, the force is approximately linear with x, which leads to the small amplitude tune shift $\Delta\nu = \xi$, given by

$$\xi = \frac{3r_oN}{2\epsilon}$$

where N is the intensity of the on-coming bunch, ϵ the 95%, normalized transverse emittance of the on-coming bunch, and r_o is the classical radius of the particles of the on-coming bunch. However, as can be deduced by Figure 1, the nonlinear x dependence of the force will lead to a variation in tune which depends upon the amplitude of an individual particle's motion with respect to the center of the on-coming bunch.

By examining the average effect over many encounters with the on-coming beam, a particle's tune as a function of the amplitude of its motion can be calculated. Figure 2 shows the change in

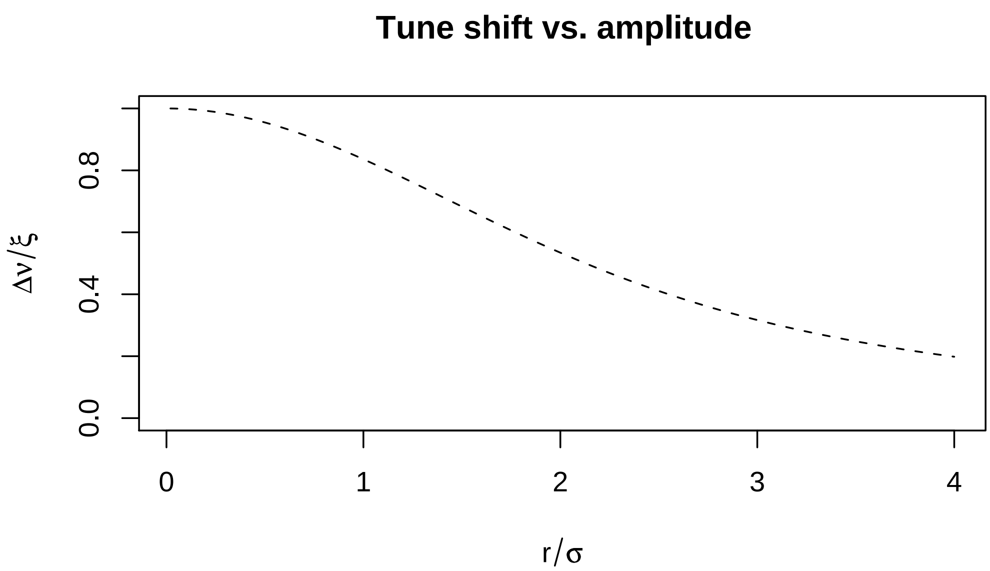

Figure 2: Beam-beam force.

tune, $\Delta\nu$, of a particle with maximum transverse amplitude r due to an on-coming round beam with rms extent σ. The functional form of the tune shift is approximately

$$\Delta\nu(r) = \xi \cdot \frac{1 - e^{-(r/2\sigma)^2} I_0[(r/2\sigma)^2]}{(r/2\sigma)^2} \tag{1}$$

where $I_0(z)$ is the modified Bessel function of order zero. We can use this result to arrive at a distribution of tunes for a beam of "test particles." We continue to use Gaussian distributions, and note that particles with amplitude r will have their tune shifted from the unperturbed value by an amount given by Eq. 1. For a Gaussian distribution of particles within our test bunch, having the same rms beam size σ as the on-coming bunch, the number of particles with an amplitude between values r and $r + dr$ is given by

$$dN = \frac{1}{\sigma^2}\, e^{-r^2/2\sigma^2}\; r\; dr.$$

The distribution is schematically shown in Figure 3, along with a plot of the number of particles per dr at radius r. By associating this number with the value of tune shift at that radius, we can plot the tune distribution shown in Figure 4.

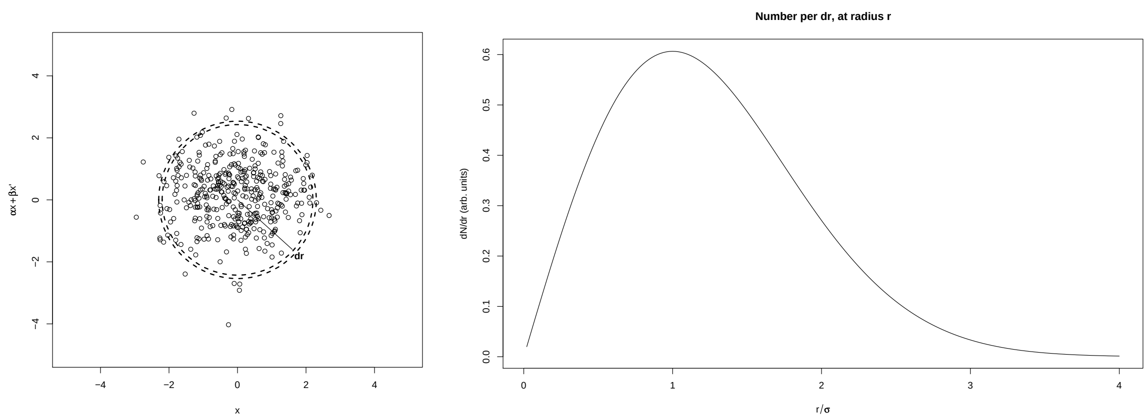

Figure 3: Schematic of particle distribution in phase space, and a plot of the distribution function of particles per shell of thickness dr at radius r.

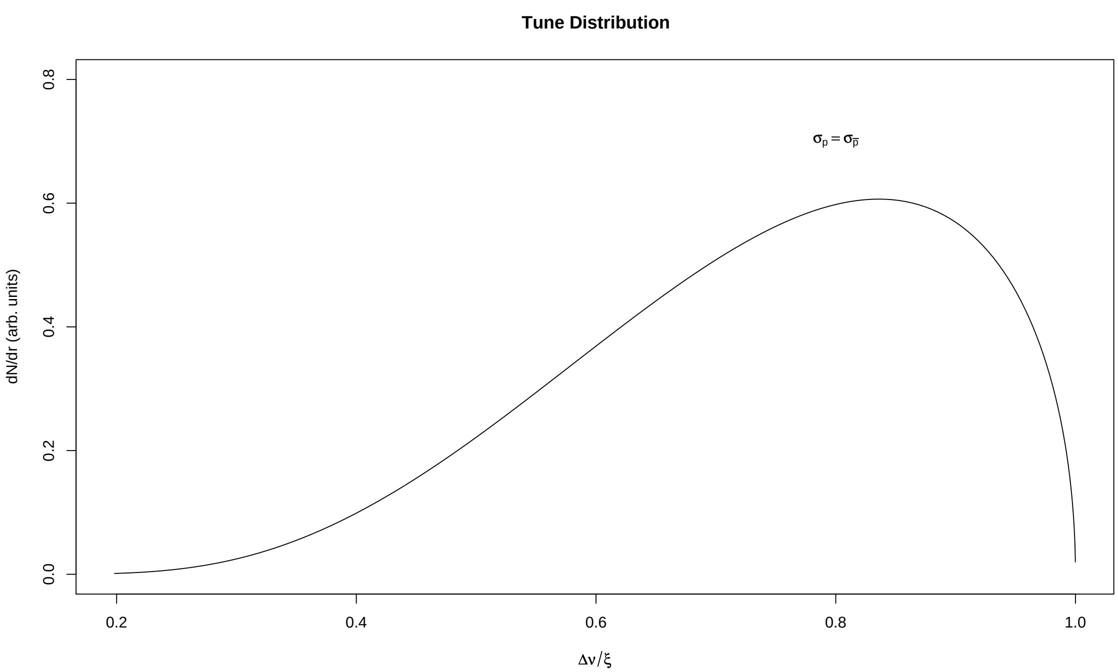

Figure 4: Tune distribution for equal sized beams colliding head-on.

3 Unequal Beam Sizes

The next step is to examine the resulting tune distributions when the two beams have unequal transverse extent. When one beam is much smaller than the other, then this beam will see more of the "linear" region of the on-coming beam and thus much of its distribution will be shifted toward the maximum value, ξ. The other, larger beam will be spread out over the nonlinear region of the smaller beam, and so will have a much larger number of particles with smaller tune shifts. Figure 5 shows the results of having beams with ratios of rms transverse size that vary by up to a factor of two. The peaking of the tune distribution of the smaller beam, and the spreading of the larger beam's distribution are readily evident.

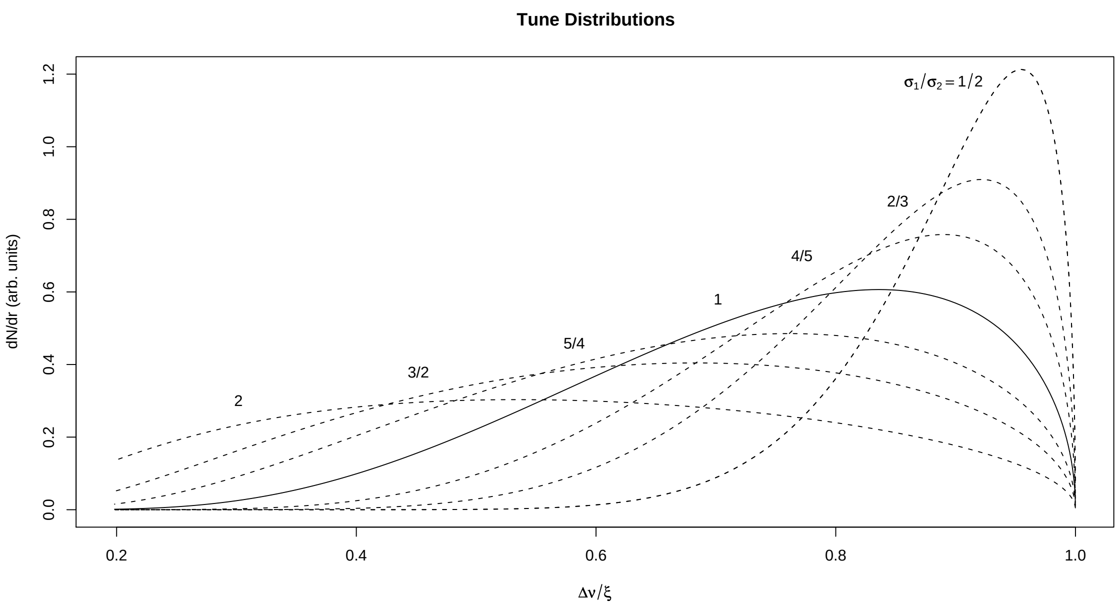

Figure 5: Tune distributions for a variety of ratios of beam size.

The case of recent Tevatron operation is represented by the largest ratio shown, namely a 2-to-1 ratio of proton to antiproton beam size. Consider the conditions where the proton bunches have $N = 250 \times 10^9$ particles each, with typical transverse emittances of $\epsilon = 14\pi$ mm-mrad. And consider antiproton bunches with intensities of $\bar{N} = 70 \times 10^9$ with emittances of $\bar{\epsilon} = 4\pi$ mm-mrad resulting from the Recycler ring. The corresponding beam-beam tune shift parameters are $\bar{\xi} = 3r_o\bar{N}/2\bar{\epsilon} = \xi = 3r_oN/2\epsilon = 0.0125$. However, the beam sizes at the collision point will have a ratio of $\bar{\sigma}/\sigma = \sqrt{4/14} \approx 1/2$. The situation is exemplified in Figure 6. Note that the proton tunes are spread nearly uniformly over a region of extent $\Delta\nu \approx 0.025$ (for two collision points!), whereas the antiproton beam has a spread of extent $2 \cdot \Delta\nu_{rms} \approx 0.004$. One might not be surprised if the proton beam lifetime were much worse than the antiproton beam lifetime, as was indeed observed under similar conditions.

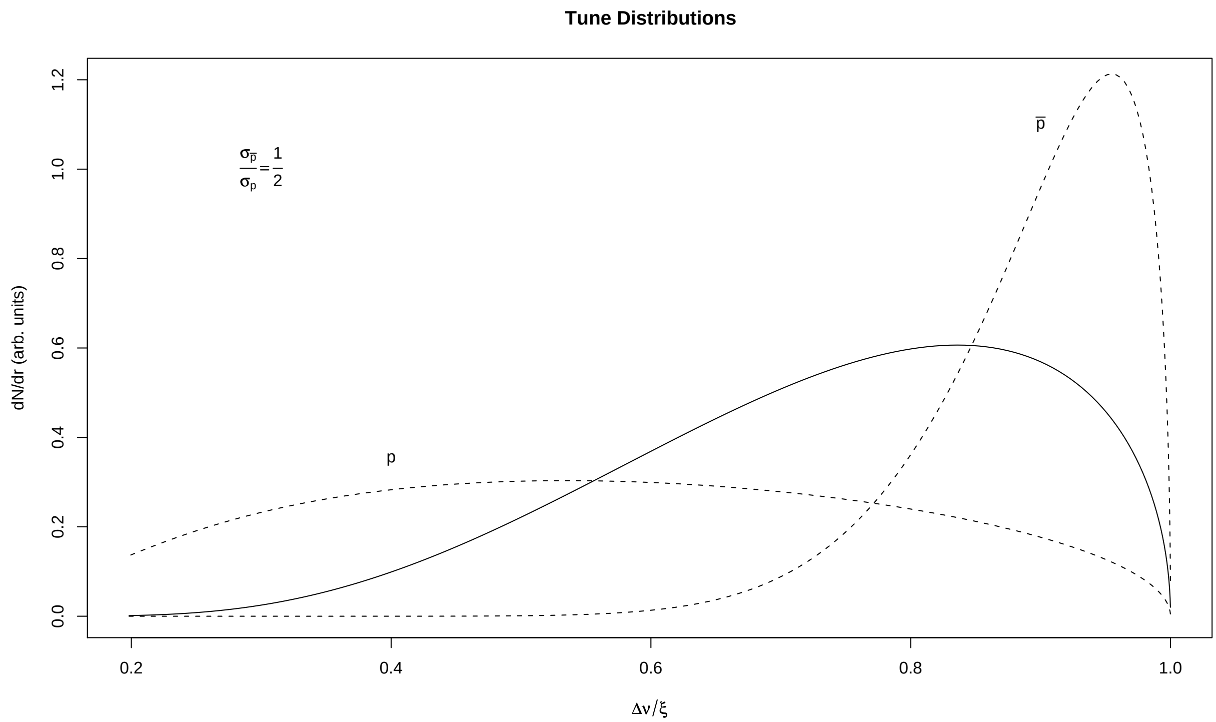

Figure 6: Tune distributions when the two beams have equal beam-beam "strengths," but one beam is twice the transverse size of the other beam. This corresponds to recent Tevatron operation, where the proton bunches (p) have been twice a large as the antiproton bunches ($\bar{p}$), though the beam-beam parameters are comparable. Most of the antiprotons pass through the central, linear region of the proton bunch giving them similar tune shifts, whereas the proton distribution spans the strongly nonlinear region of the antiproton bunch, resulting in a large, nearly flat tune distribution over the allowed values. The solid curve is the same as shown in Figure 5 for comparison. For the Tevatron, ξ is typically 0.025 due to two interaction points.

4 Optimizing Tevatron Luminosity

To optimize the integrated luminosity of a particular store, one may wish to make the tune spread of the proton beam similar to the tune spread of the antiproton beam, and make each as small as practical. While the Recycler ring can produce very small emittances, suppose we control the final value to make $\bar{\xi}$ smaller than ξ. This will automatically reduce the tune spread of the proton bunch. And if we keep the antiproton bunch small enough in transverse size, its tune spread can still be smaller than in the case of identical beam sizes. Always running the proton beam at the beam-beam limit ($\xi = 0.0125$ at each interaction point), the luminosity of the collider can be expressed accordingly:

$$\mathcal{L} = \frac{2f_0\gamma\xi}{r_o\beta^*} \cdot \frac{B\bar{N}}{1+\bar{\epsilon}/\epsilon} \cdot f$$

where f_0 is the revolution frequency, $\gamma = E/mc^2$, β^* is the amplitude function at the interaction points, f is the "hour glass factor" (typically ~ 0.6), and $B\bar{N}$ is the total number of antiprotons distributed among B bunches in the synchrotron.

As an example, take the Tevatron parameters used earlier and increase the emittance of the antiprotons from 4π mm-mrad to 8π mm-mrad. The parameter $\bar{\xi}$ will be reduced by a factor of two, and the two beam sizes will have a ratio of $\sqrt{8/14} \approx 3/4$. The shapes of the tune distributions will closely resemble those found in Figure 5, but with the extent of the proton distribution reduced appropriately. The resulting distributions are shown in Figure 7. In this case, the tune spread in each beam is approximately $2 \times 0.1 \times 0.025 = 0.005$. The reduction of the initial luminosity from increasing the antiproton emittance by this factor of two is approximately 18%.

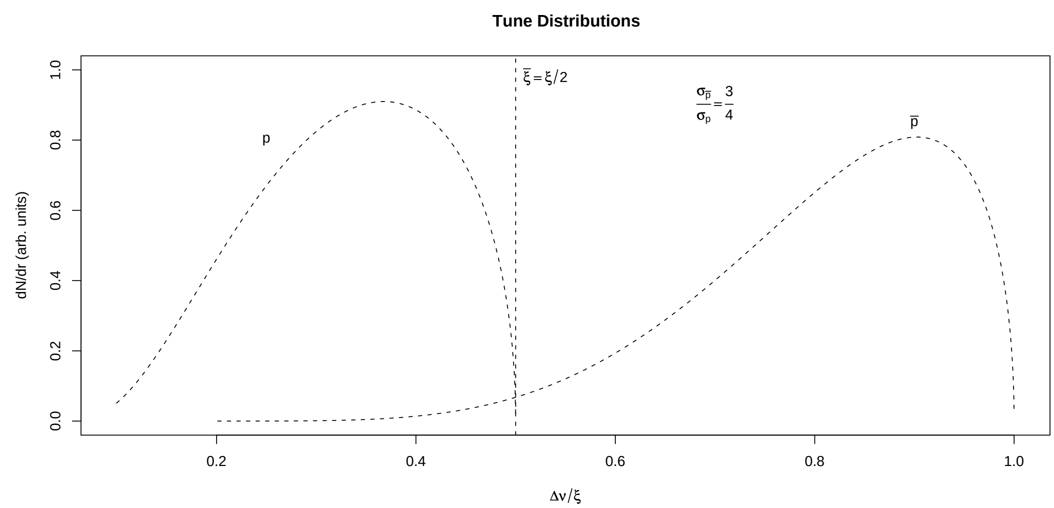

Figure 7: Result of increasing the emittance of the antiproton beam by a factor of two, thus reducing the maximum beam-beam tune shift of the protons by the same factor, but also increasing the beam size ratio from 1/2 to about 3/4. The resulting tune spreads of the two particle distributions are more closely matched.

The Tevatron operation is limited (within reason) in the number of antiprotons that can be produced for a store. Although very small emittances can now be obtained with the Recycler, the effect on the proton beam lifetime can be quite dramatic. Additionally, while protons are plentiful the lower limit of their emittance is governed by the performance of the injector chain. To routinely arrive at the conditions of Figure 7, one could contemplate complying with a recipe, such as the following: Suppose $B\bar{N}$ antiprotons are available at collision energy. Pick a proton bunch intensity given by $N = 7\bar{N}/2$. If the beam-beam tune shift limit is not yet reached, continue; otherwise, dilute the proton emittance to a value of $\epsilon = 3r_oN/2\xi$. Next, dilute the antiproton emittance to a value of $\bar{\epsilon} = 4\epsilon/7$. This will generate the conditions $\bar{\xi} = \xi/2$ and $\bar{\sigma}/\sigma \approx 3/4$. The resulting tune spreads will be similar for the two beams and, presumably, so will their lifetimes with appropriate choices of base tunes. This scenario is shown schematically in Figure 8, where the initial luminosity is plotted *vs.* available number of antiprotons in the Recycler (and assuming 80% of those make it to collision).

If the luminosity lifetime can be improved by a large enough factor while taking a smaller hit in initial luminosity, then clearly the program wins. Recent operational experience seems to bear this out. Regularly executing a particular algorithm in daily operation with repeatable results is challenging, but is certainly within reach. Methods for systematically increasing the antiproton beam emittance at 980 GeV without affecting the protons are being developed and tested at this writing.

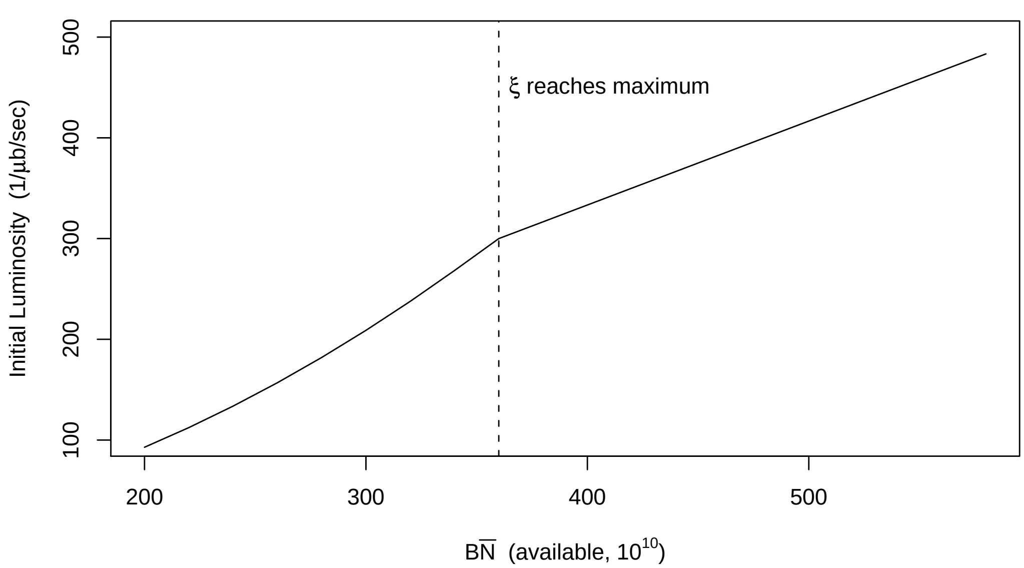

Figure 8: Initial luminosity *vs.* Recycler stash size, following the recipe in the text. The luminosity becomes “linear” when the tune shift parameter on the antiprotons reaches about 0.025 (total). The proton and antiproton emittances and the proton intensity are tailored to produce roughly the same tune spread for each beam in each case.