Tevatron Integrated Luminosity

A tutorial primer

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300x Design

- With ~2 years left in the program, the Tevatron is operating at its best performance, over 300 times its design luminosity
- People ask: Under our present conditions, how far from optimal are we running?
- Let's look at...
  - factors influencing luminosity
  - factors influencing its integration
  - optimization

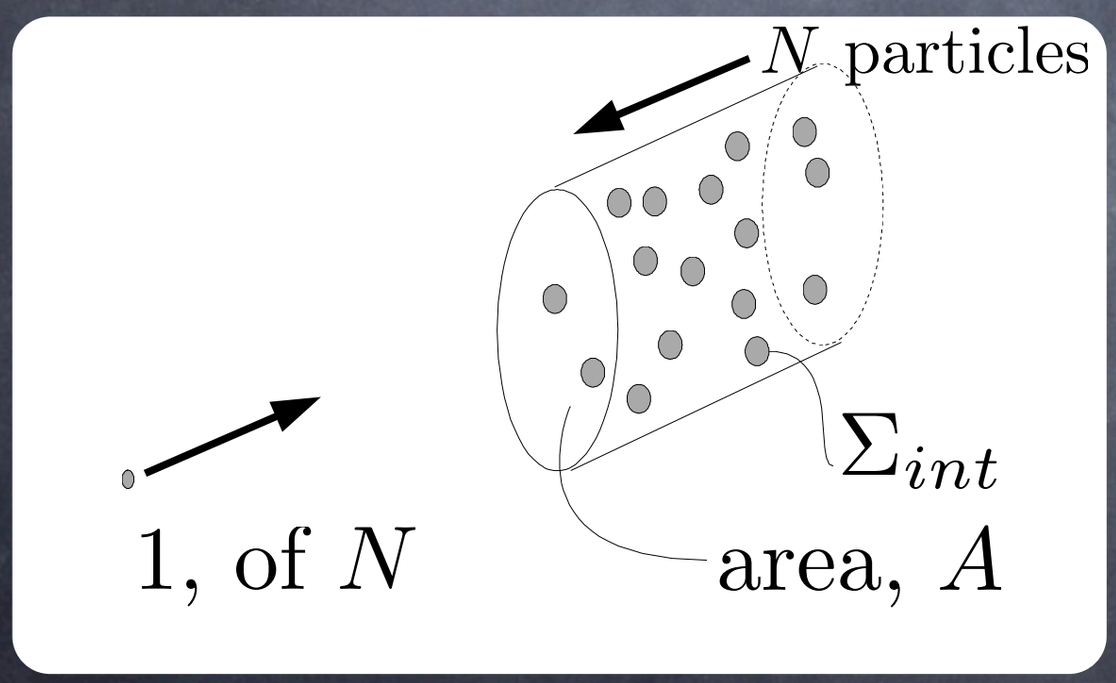
Luminosity

- Will take a rather "analytical" approach, with simplifying assumptions*
  - illustrate the basic principles, generate a reasonable analytical model
- First, look at equal beams under "ideal" conditions; then, unequal beam populations; then, add more realism...

* M. J. Syphers, FERMILAB-FN-0802-AD

Round, Uniform Beams

- Imagine two bunches passing through each other, each with N pariticles and of transverse cross sectional area A; the ``interaction cross section" for a collision is Σ_{int}. If they pass through each other with frequency f, then the rate at which particles collide will be:



$$\mathcal{R} = \left(\frac{\Sigma_{int}}{A} \cdot N\right) \cdot N \cdot f$$

$$= \boxed{\frac{f\ N^2}{A}}\ \Sigma_{int}$$

$$\equiv \mathcal{L} \cdot \Sigma_{int}$$

Luminosity

Round Gaussian Beams

- If B bunches of each beam are made to collide, and the revolution frequency is f_0, then $f = Bf_0$
- If the transverse extent of the beams are round and Gaussian, with variance σ^2, then the effective cross sectional area will give...

$$\mathcal{L} = \frac{f_0 B\ N^2}{4\pi\sigma^2} \cdot \mathcal{H}$$

- Here, $\mathcal{H}$ is a form factor, which decreases the provided luminosity due to the longitudinal extent of the bunches

Luminosity Evolution

- In our so-far perfect collider, particles will be ``lost" due to the collisions (which is what we want!). Suppose there are n detectors through which the beams pass and collide.
- Then, $$B\,\frac{dN}{dt} = -\mathcal{L}\,\Sigma_{int}\,n \quad \propto \quad N^2$$
- From which...

$$\mathcal{L}(t) = \frac{\mathcal{L}_0}{\left[1 + \left(\frac{n\mathcal{L}_0\Sigma}{BN_0}\right)t\right]^2}$$

Integrated Luminosity

- If we count the events in all detectors that occur over time, then we see that

$$N_{events} = n \int \mathcal{R} dt = n \int \mathcal{L}(t) dt \cdot \Sigma_{int}$$

- If all of the beams are ``used up" in collisions, then the **maximum** integrated luminosity we could expect from the store would be:

$$I_0 \equiv \int_0^\infty \mathcal{L}(t) dt = \frac{B N_0}{n \, \Sigma_{int}}$$

- Here, BN_0 is the initial total intensity of each beam

Integrated Luminosity

- Integrating our previous result,

$$I(T) \equiv \int_0^T \mathcal{L}(t)dt = \frac{\mathcal{L}_0 T}{1 + \mathcal{L}_0 T(n\Sigma/BN_0)} = I_0 \cdot \frac{\mathcal{L}_0 T/I_0}{1 + \mathcal{L}_0 T/I_0}$$

- Integrated luminosity begins to max out @ $T >> I_0/\mathcal{L}_0$
- $I(T) = I_0/2$ when $T = I_0/\mathcal{L}_0$
- The time to reach a fraction f of I_0 will be $T_f = \frac{I_0}{\mathcal{L}_0}\frac{f}{1-f}$

Time for some numbers...

- Assume 36 bunches in each beam, BN_0 = 250x10^{10} particles in each beam (typical of antiprotons in today's Tevatron operation), a spot size of $\sigma = 25\ \mu\mathrm{m}$, and an hour glass factor H = 0.6; take an inelastic cross section of 60 mb....

$$
\begin{aligned}
\mathcal{L}_0 &\approx 64 \times 10^{30}/\mathrm{cm}^2/\mathrm{sec} = 64\ \mu\mathrm{b}^{-1}/\mathrm{sec} = 0.23\ \mathrm{pb}^{-1}/\mathrm{hr}, \\
I_0 &\approx 21\ \mathrm{pb}^{-1}/\mathrm{store}, \\
I_{0.85} &\approx 18\ \mathrm{pb}^{-1}/\mathrm{store}, \\
T_{0.85} &\approx 3\ \mathrm{weeks}
\end{aligned}
$$

Unequal Bunch Intensities

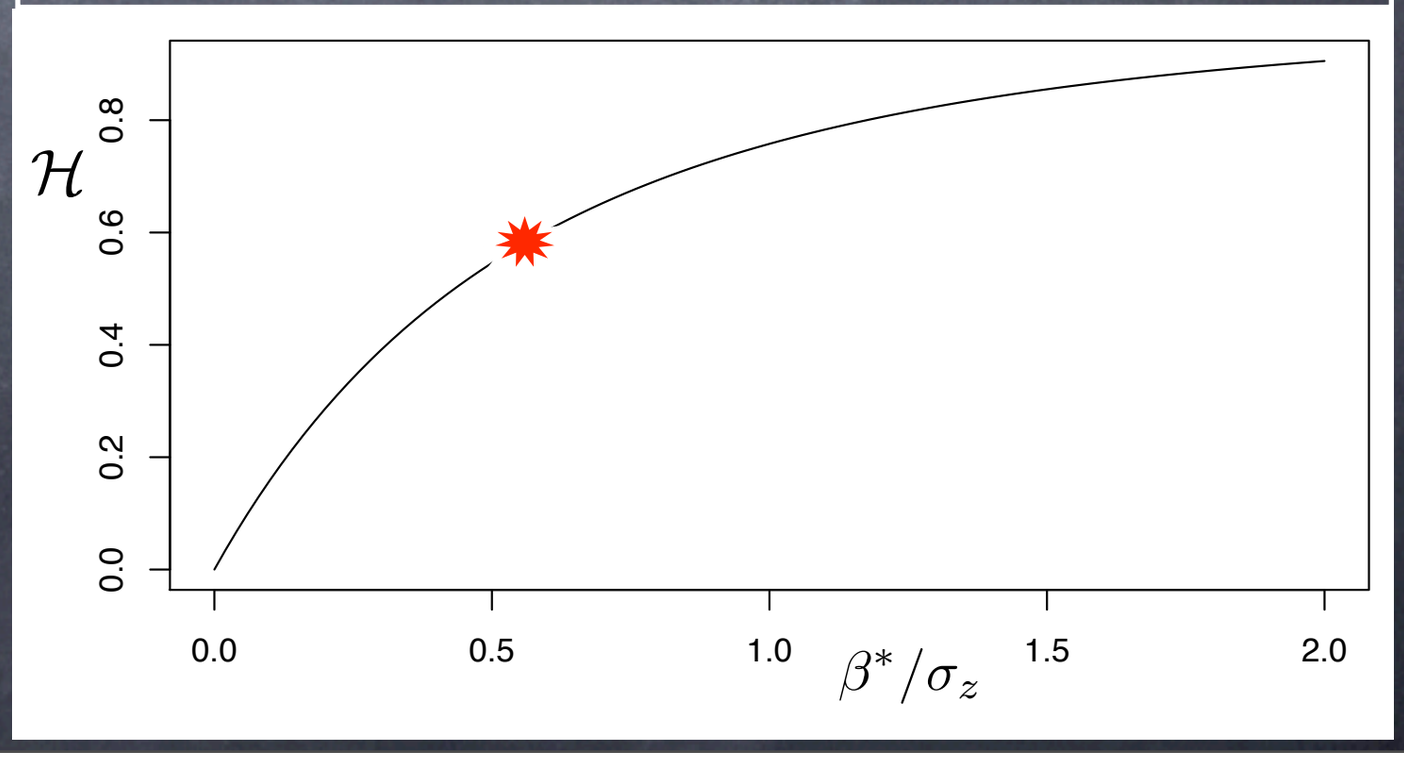
- For unequal beam emittances, bunch intensities ...

$$\mathcal{L} = \frac{f_0 B N_1 N_2}{2\pi({\sigma_1^*}^2 + {\sigma_2^*}^2)} \cdot \mathcal{H}$$

$$= \frac{3 f_0 \gamma B N_1 N_2}{\beta^*(\epsilon_1 + \epsilon_2)} \cdot \mathcal{H}$$

$$\sigma^* = \sqrt{\frac{\epsilon \beta^*}{6\pi\gamma}}$$

$$\mathcal{H} = \sqrt{\pi}\left(\frac{\beta^*}{\sigma_z}\right) e^{(\beta^*/\sigma_z)^2}\left[1 - \mathrm{erf}(\beta^*/\sigma_z)\right]$$



- Typically,
  - β^* ~ 30 cm
  - σ_z ~ 50 cm
  - ϵ ~ 5-20 μm

Luminosity Evolution ...

- In the Tevatron we have more protons than antiprotons, which increases the luminosity shown previously. So, assume whenever a proton is lost, so is an antiproton.
- Let $N_1(t) = N(t) + \Delta N$, and $N_2(t) = N(t)$; then,

$$\mathcal{L} = \frac{f_0 B N_1 N_2}{4\pi\sigma^{*2}} \cdot \mathcal{H} = \frac{f_0 B N(N + \Delta N)}{4\pi\sigma^{*2}} \cdot \mathcal{H}$$

$$\boxed{B\dot{N} = -\mathcal{L}\,\Sigma_{int}\,n} \qquad k \equiv n\mathcal{L}_0\Sigma/BN_1^0N_2^0 = nf_0\mathcal{H}\Sigma/4\pi\sigma^{*2}$$

- From which:

$$\mathcal{L}(t) = \mathcal{L}_0\,\frac{\Delta N^2 e^{\Delta N k t}}{(N_1^0 e^{\Delta N k t} - N_2^0)^2}$$

... with Unequal Bunch Intensities

- Integrating, we get

$$I \equiv \int_0^T \mathcal{L}(t)dt = \frac{BN_2^0}{n\Sigma} \cdot \left(\frac{e^{\Delta NkT} - 1}{e^{\Delta NkT} - \frac{N_2^0}{N_1^0}} \right) \quad \Longrightarrow \quad I_0,$$

- where now... $\boxed{I_0 \equiv \frac{BN_2^0}{n\Sigma}}$ (given by beam of lesser intensity)
- The time to integrate to a fraction f of I_0:

$$T_f = \frac{I_0/\mathcal{L}_0}{(1 - N_2^0/N_1^0)} \ln\left(\frac{1 - fN_2^0/N_1^0}{1 - f} \right)$$

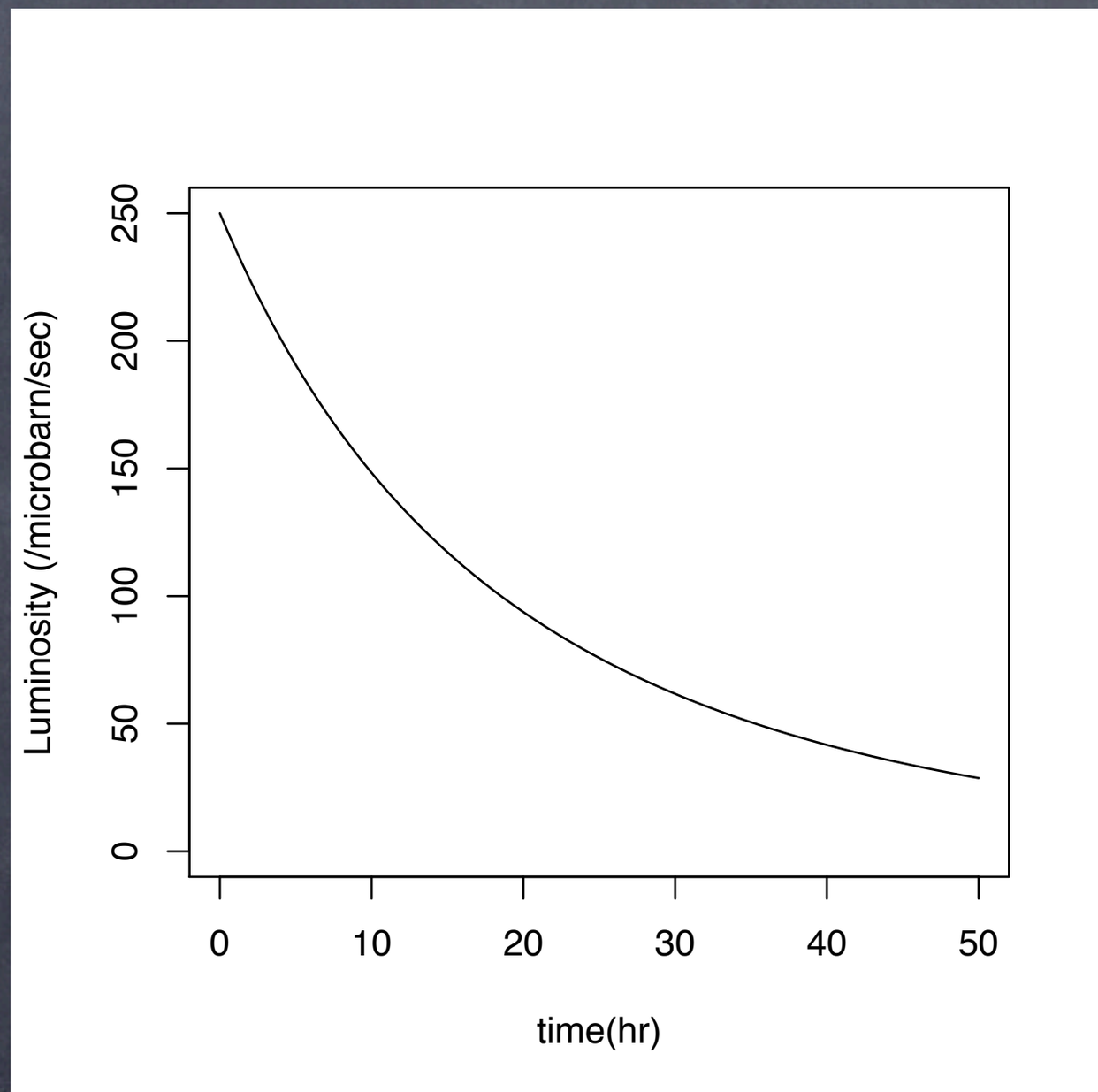
- for which:

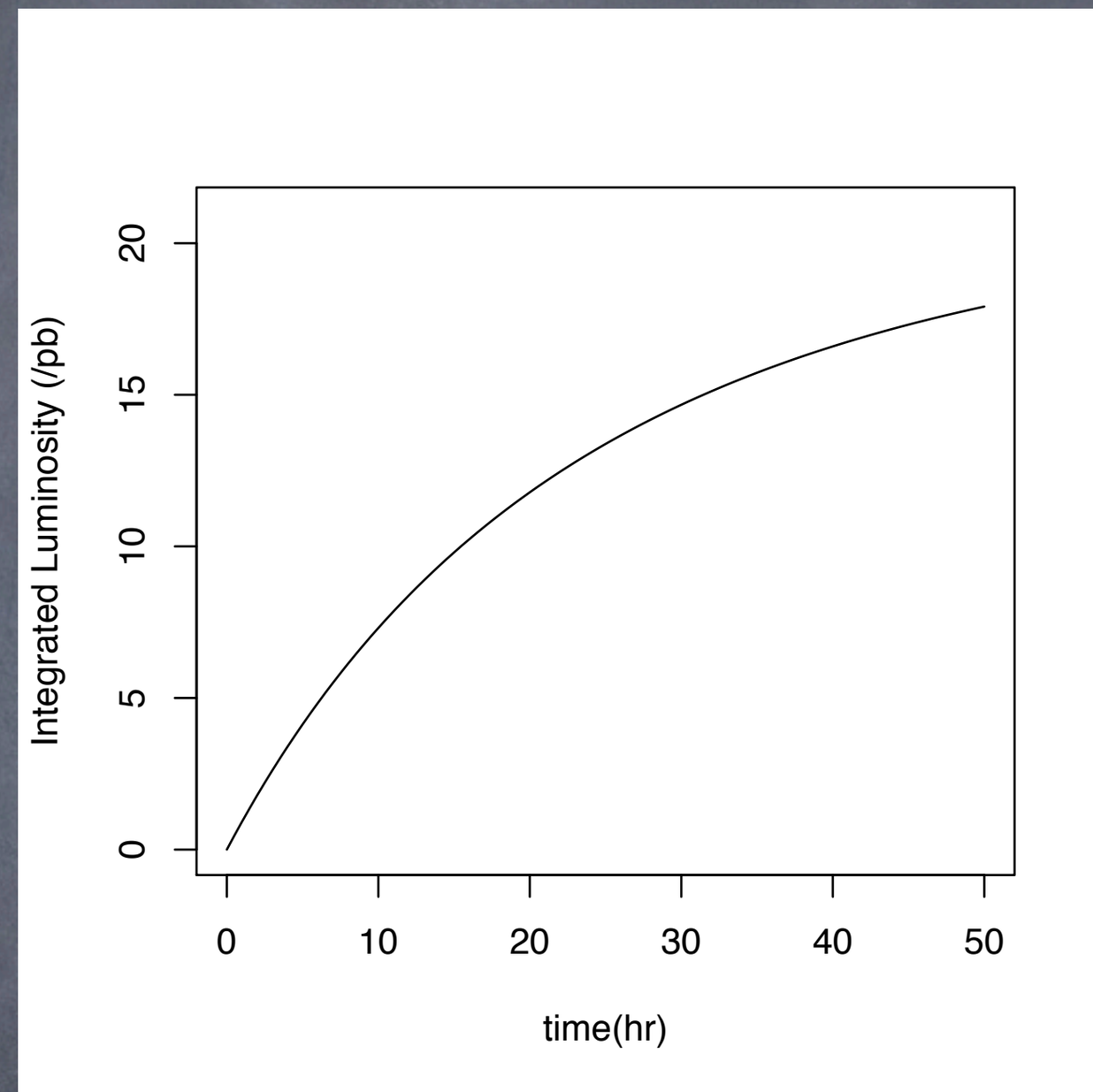
$$\mathcal{L}(T_f)/\mathcal{L}_0 = (1 - f)(1 - fN_2^0/N_1^0)$$

Numbers again...

- Take 250x10^9 for N_1^0 (protons) and 70x10^9 for N_2^0 (pbars) and keep other parameters as before...
- Suppose store stays in until 85% of I_0 is reached, (for which luminosity reaches 10% of original value):

$$
\begin{aligned}
\mathcal{L}_0 &\approx 250 \times 10^{30}/\mathrm{cm}^2/\mathrm{sec} = 250\ \mu\mathrm{b}^{-1}/\mathrm{sec} = 0.9\ \mathrm{pb}^{-1}/\mathrm{hr}, \\
I_0 &\approx 21\ \mathrm{pb}^{-1}/\mathrm{store}, \\
I_{0.85} &\approx 18\ \mathrm{pb}^{-1}/\mathrm{store}, \\
T_{0.85} &\approx 2\ \mathrm{days}
\end{aligned}
$$





- Instantaneous (left) and integrated (right) luminosity vs. time through a ``perfect'' store, using parameters above. Here, the number of particles in one beam is ~30% that of the other beam.

Add a bit more realism...

- Tevatron stores do not integrate to 18 pb^{-1} ...
- In the above, particles are lost only due to collisions
- Not the only source of particle loss, however
- Need to include effect of emittance growth and corresponding particle lifetime due to other causes, such as:
  - beam-gas scattering, RF noise, PS ripple, ...

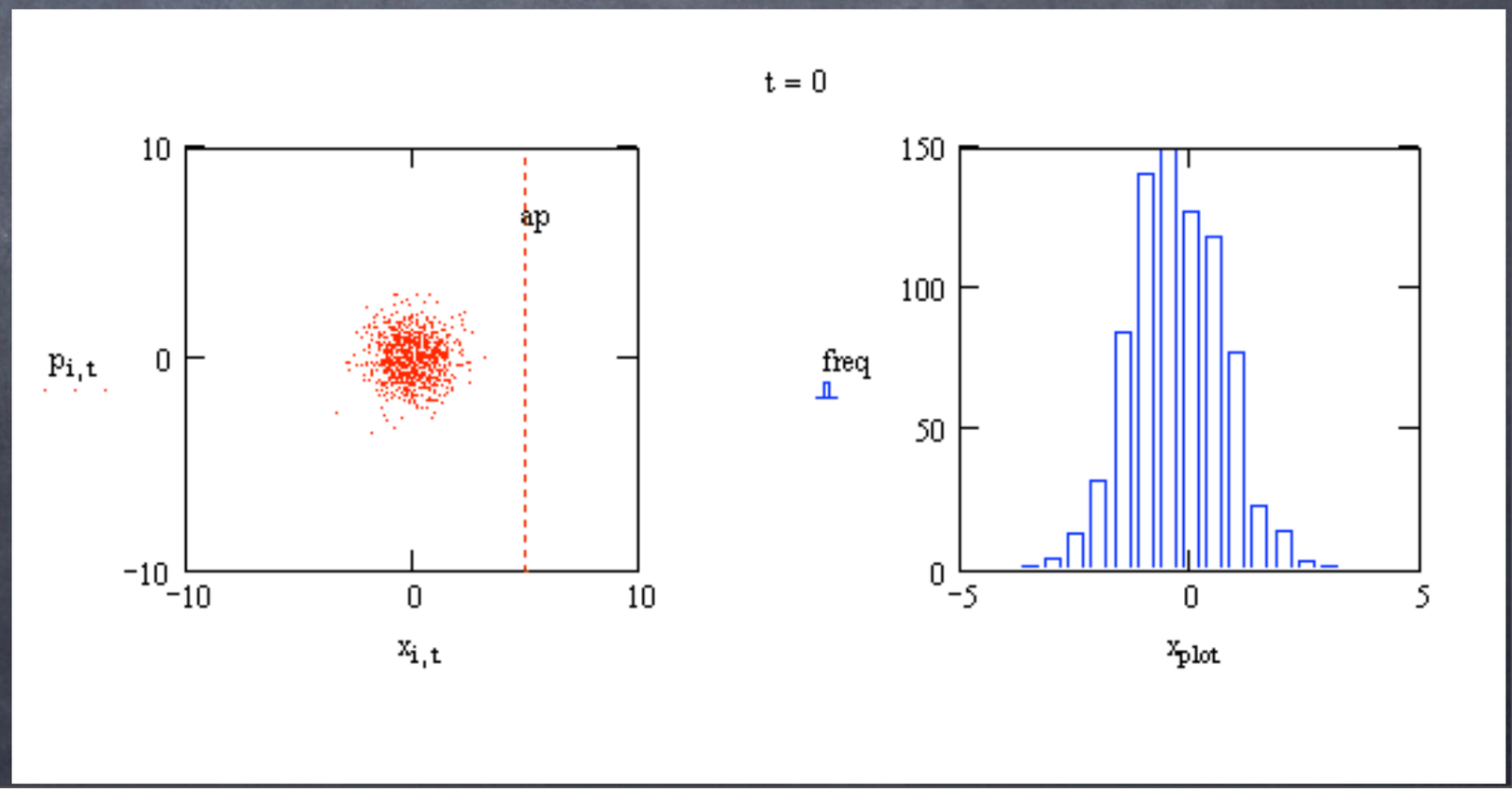
Emittance Growth

- Just before we ``initiate collisions,'' the beam is scraped and the aperture is defined by the collimators
- From then on, assume that single-particle emittance growth mechanisms drive particles transversely into the aperture
- Assume an ``effective emittance'' $\hat{\epsilon}$ and an effective emittance growth rate $\dot{\epsilon}$

Transverse Diffusion

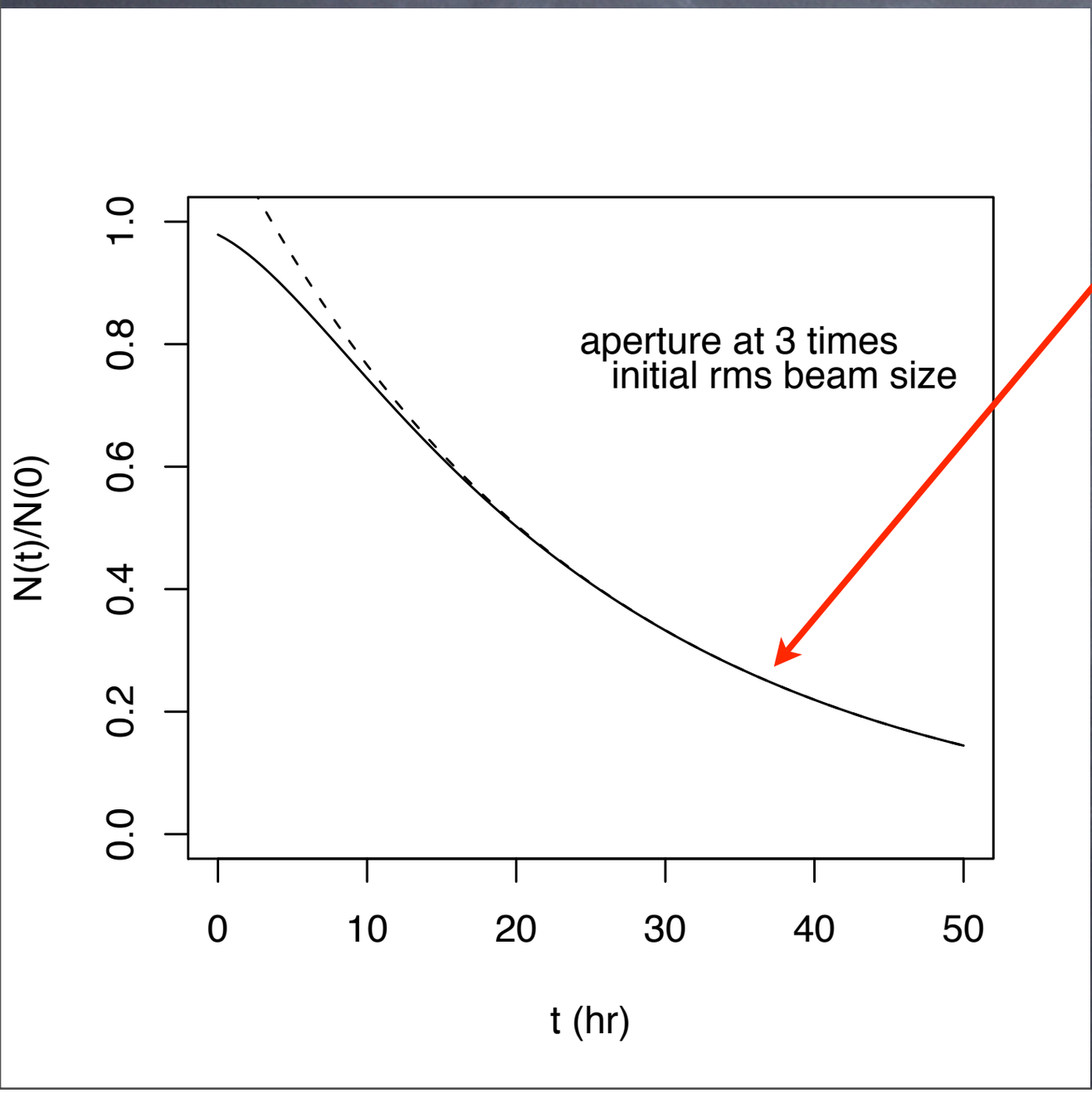
- Particle emittances grow due to diffusion processes, until they reach an aperture
- An equilibrium distribution will form, with an equilibrium lifetime
- Define Emittance: $\epsilon \equiv 6\pi\gamma\langle x^2\rangle/\beta$ *(lattice function)* $\hat{\epsilon} \approx 0.92\pi\gamma a^2/\beta$
- Growth rate, in absence of an aperture:

$$\dot{\epsilon} = (6\pi\gamma/\beta)\cdot\frac{d\langle x^2\rangle}{dt}$$



Diffusion Loss Rates

- Once equilibrium distribution is reached, an equilibrium lifetime will develop



$$\tau = \frac{2a^2}{\lambda_1^2 d\langle x^2\rangle/dt} \approx \frac{2\hat{\epsilon}}{\dot{\epsilon}}$$

$\lambda_1 = 2.405$

- Use this as our model for particle loss
- Assume loss mechanisms are same/similar for both beams, with equal equilibrium lifetimes in absence of collisions

Differential Equations for bunch intensities

- With this model in mind...

$$\dot{N}_1 = -\mathcal{L}\cdot\Sigma\cdot n/B - \frac{1}{\tau}N_1 = -kN_1N_2 - \frac{1}{\tau}N_1$$

$$\dot{N}_2 = -\mathcal{L}\cdot\Sigma\cdot n/B - \frac{1}{\tau}N_2 = -kN_1N_2 - \frac{1}{\tau}N_2$$

  - where, again, $k = \mathcal{L}_0\Sigma n/(BN_1^0N_2^0) = \mathcal{L}_0/(I_0N_1^0)$

- Subtracting: $N_1(t) - N_2(t) = (N_1^0 - N_2^0)e^{-t/\tau}$

- If let $N_2(t) = N(t)$:

$$\boxed{\dot{N} + \left(\frac{1}{\tau} + k\Delta N e^{-t/\tau}\right)N + kN^2 = 0}$$

Luminosity, w/ Diffusion

- Above DiffEq can be solved analytically, which gives $N_2(t) = N(t)$. Then we also know

$$N_1(t) = N_2(t) + (N_1^0 - N_2^0)e^{-t/\tau} \equiv N_2(t) + \Delta N\, e^{-t/\tau}$$

- from which we get the luminosity:

$$\mathcal{L}(t) = \mathcal{L}_0 \frac{\Delta N^2 e^{-2t/\tau} e^{-(1-e^{-t/\tau})\Delta N k \tau}}{\left(N_1^0 - N_2^0 e^{-(1-e^{-t/\tau})\Delta N k \tau}\right)^2}$$

- Reduces to our previous result, when $\tau \to \infty$.

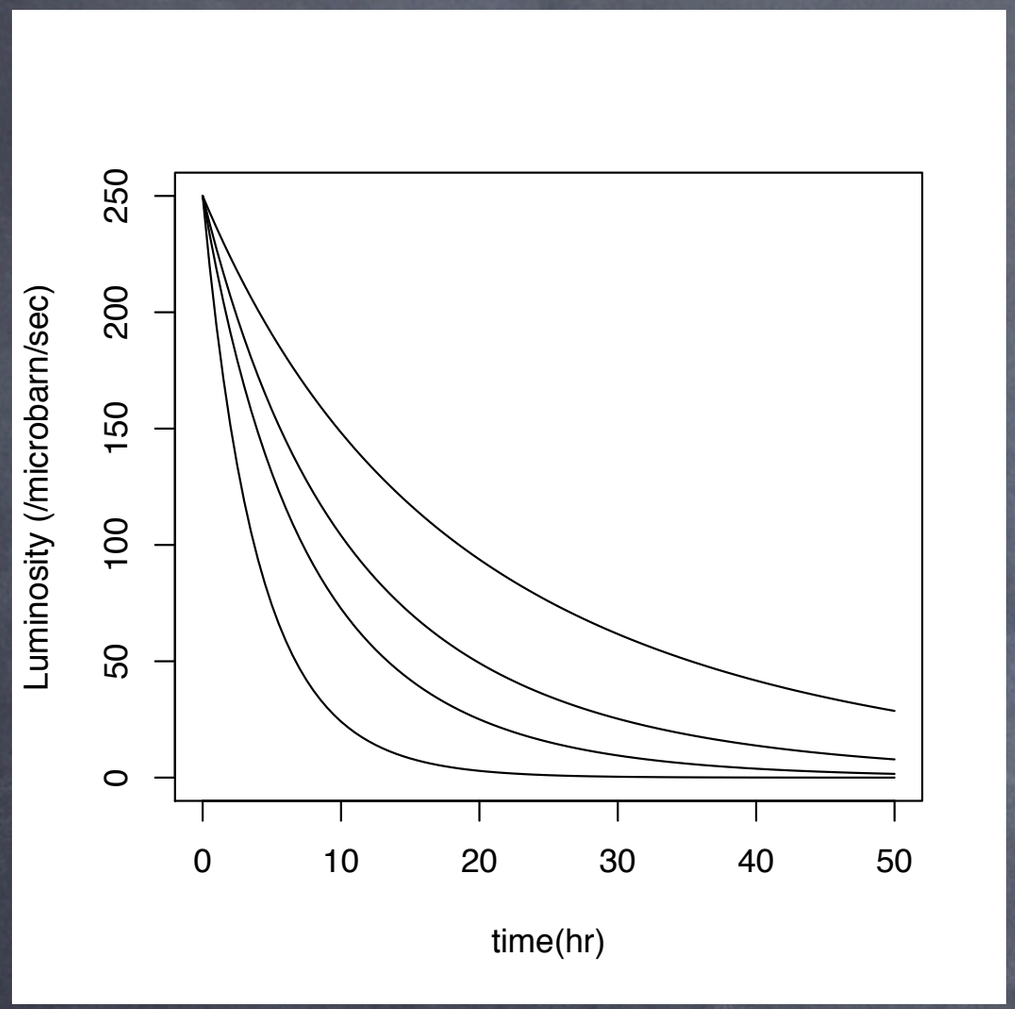
Integrated Luminosity, w/ Diffusion

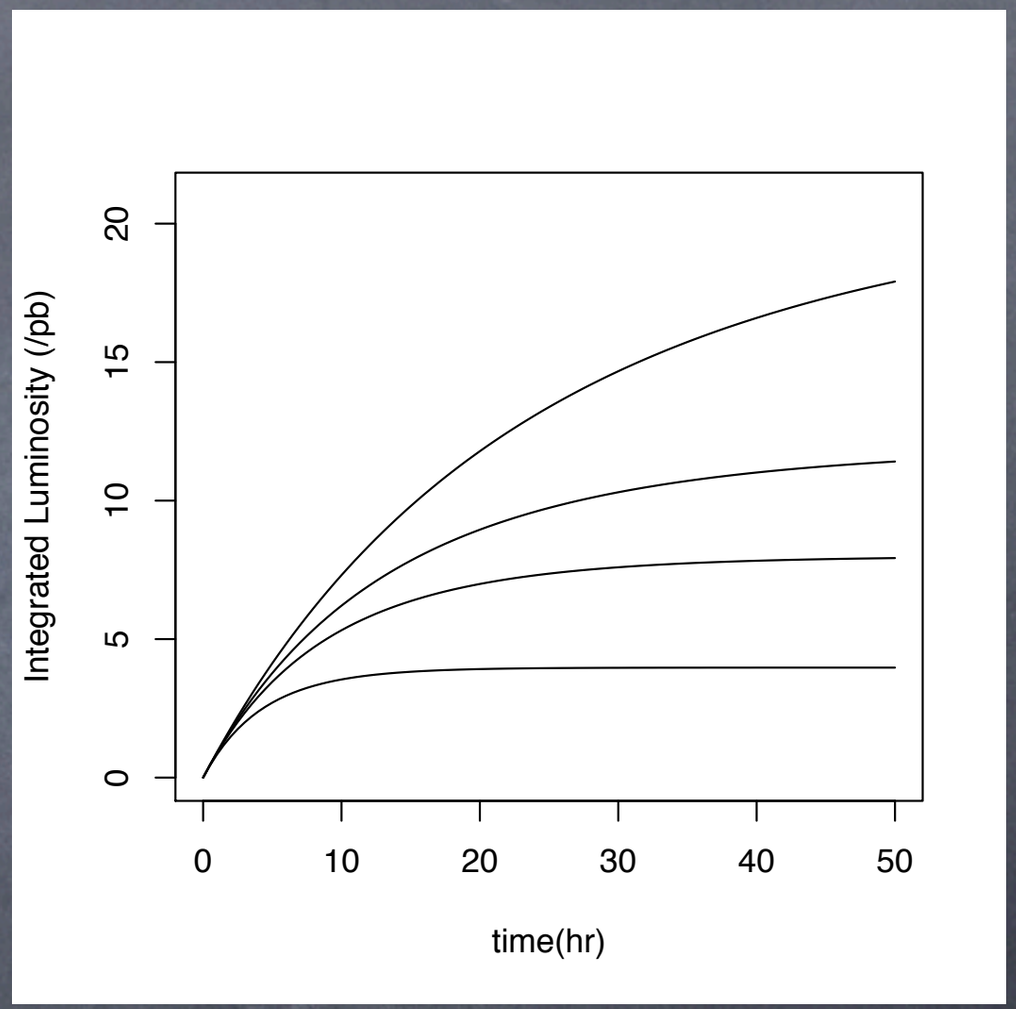
- Integrating the previous result:

$$I(t) = I_0 \left[1 - \frac{(N_1^0 - N_2^0)e^{-t/\tau}}{N_1^0 e^{(1-e^{-t/\tau})\Delta N k\tau} - N_2^0} - \frac{I_0}{\mathcal{L}_0 \tau} \frac{N_1^0}{N_2^0} \ln\left(\frac{N_1^0 - N_2^0 e^{-(1-e^{-t/\tau})\Delta N k\tau}}{N_1^0 - N_2^0} \right) \right]$$

- Asymptotic limit: $I(t \to \infty) \to I_0 \left[1 - \frac{I_0}{\mathcal{L}_0 \tau} \frac{N_1^0}{N_2^0} \ln\left(\frac{N_1^0 - N_2^0 e^{-\Delta N k\tau}}{N_1^0 - N_2^0} \right) \right]$
- Note: for $t > 0$, $I(t)$ is always less than I_0; can never get there (losing particles all the time due to mechanisms other than collisions)

Curves, with diffusion





- Curves with τ = 10 hr, 25 hr, 50 hr, and infinity, using same parameter values as before. Note that for this set, indicative of recent Tevatron performance, I_0/L_0 ~22 hr.

Diffusion in the Tevatron

- We know that in the absence of collisions, the beam emittance growth rate in the Tevatron is on the scale of $\dot{\epsilon} \approx 1\ \pi\ \mathrm{mm\text{-}mr/hr}$.
- In our model, put in typical values for initial bunch intensities, effective emittance, etc., and adjust the only remaining free parameter -- $\dot{\epsilon}$ -- to arrive at a typical integrated luminosity for a store

Numbers once again...

- Take 250x10^9 for N_1^0 (protons) and 70x10^9 for N_2^0 (pbars) and keep other parameters as before.
- Use $\dot{\epsilon} \approx 1\ \pi\ \mathrm{mm\text{-}mr/hr}$ as a numerical estimate
- Suppose store stays in until 85% of the predicted asymptotic value is reached:

$$
\begin{aligned}
\mathcal{L}_0 &\approx 250 \times 10^{30}/\mathrm{cm}^2/\mathrm{sec} = 250\ \mu\mathrm{b}^{-1}/\mathrm{sec} = 0.9\ \mathrm{pb}^{-1}/\mathrm{hr}, \\
I_0 &\approx 21\ \mathrm{pb}^{-1}/\mathrm{store}, \\
I_{asym} &\approx 8\ \mathrm{pb}^{-1}/\mathrm{store}, \\
I_{0.85} &\approx 7\ \mathrm{pb}^{-1}/\mathrm{store}, \\
T_{0.85} &\approx 20\ \mathrm{hours}
\end{aligned}
$$

Sources of emittance growth

- Beam-gas scattering (accounts for ~>0.5 π mm-mr/hr)*
- Intra-beam and beam-beam effects
- RF noise
- Power supply noise
- Orbital motion
- ...

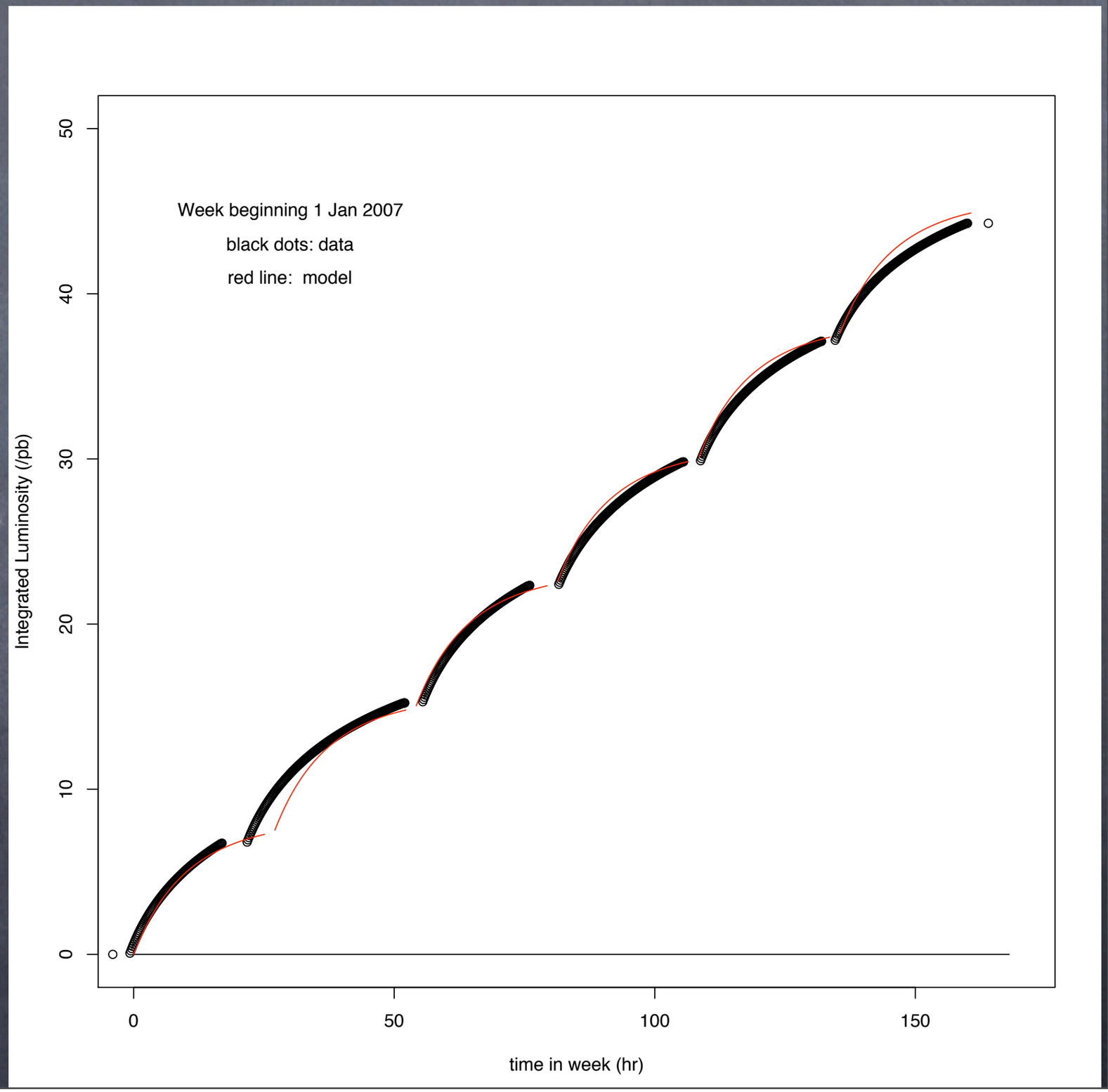
*V. A. Lebedev, L. Y. Nicolas† , A. V. Tollestrup , *Residual Gas, Emittance Growth and Beam Lifetime in Tevatron at 150 GeV*, Beams-doc-1155 (2004).

A good Tevatron week

- In the first week of 2007, the Tevatron ran 6 consecutive stores, all ending intentionally, and integrated a total of 45/pb.
- Using our model, and taking average initial parameters for these six stores, then adjusting the average effective emittance growth rate for that week to a value of ~0.8 π mm-mr/hr, yielded the following result:

January 2007

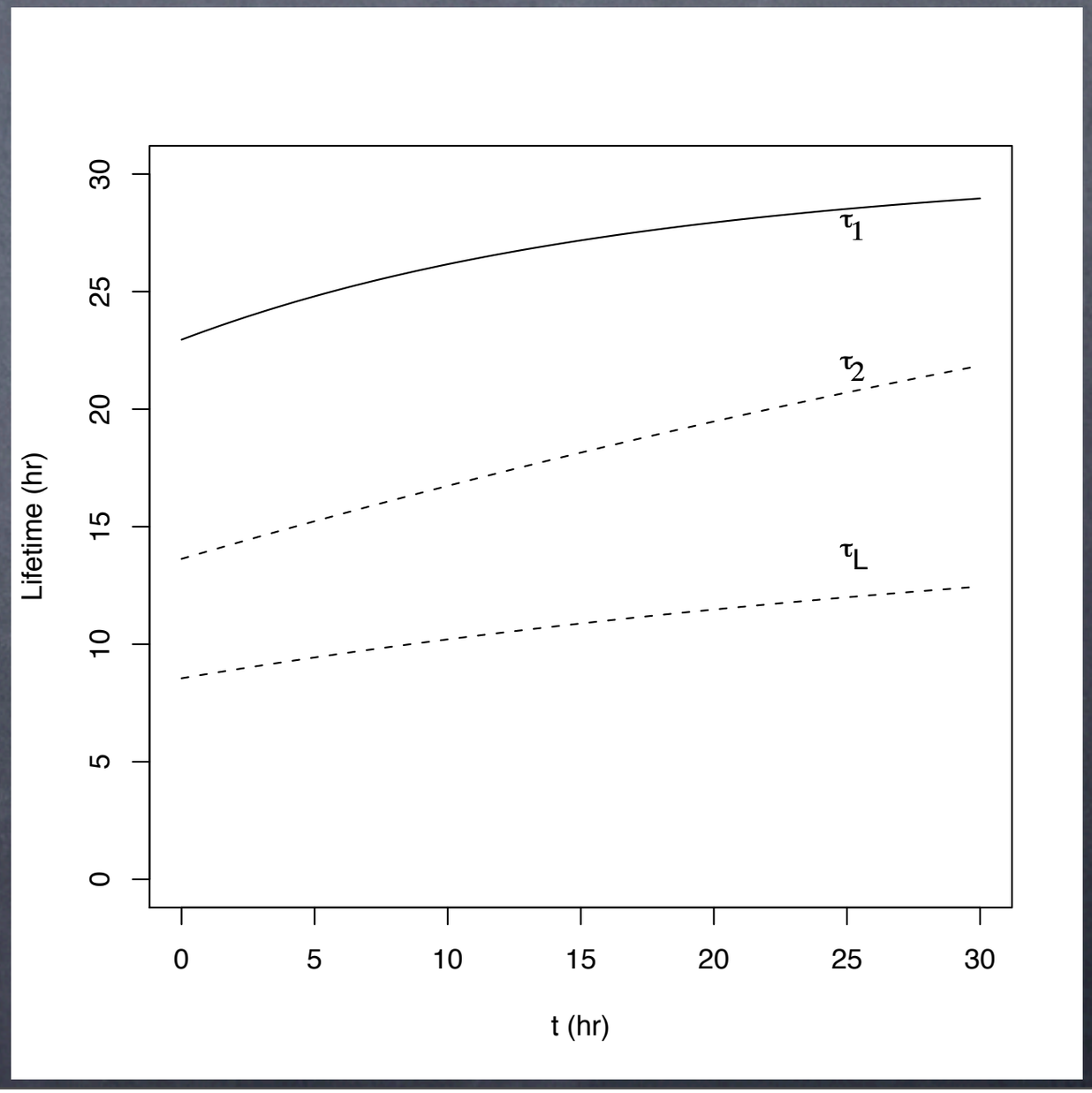
- black: data
- red: model
- assumes 6 equal stores, equally spaced, each using parameters that have been averaged over all 6 stores



Lifetimes

$$\tau_1 \equiv \frac{N_1(t)}{dN_1(t)/dt}, \quad \tau_2 \equiv \frac{N_2(t)}{dN_2(t)/dt} \quad \tau_L \equiv \frac{\mathcal{L}(t)}{d\mathcal{L}(t)/dt}$$

- Differentiate curves to find lifetimes...
- Results are similar to numbers reported by SDA during that week...

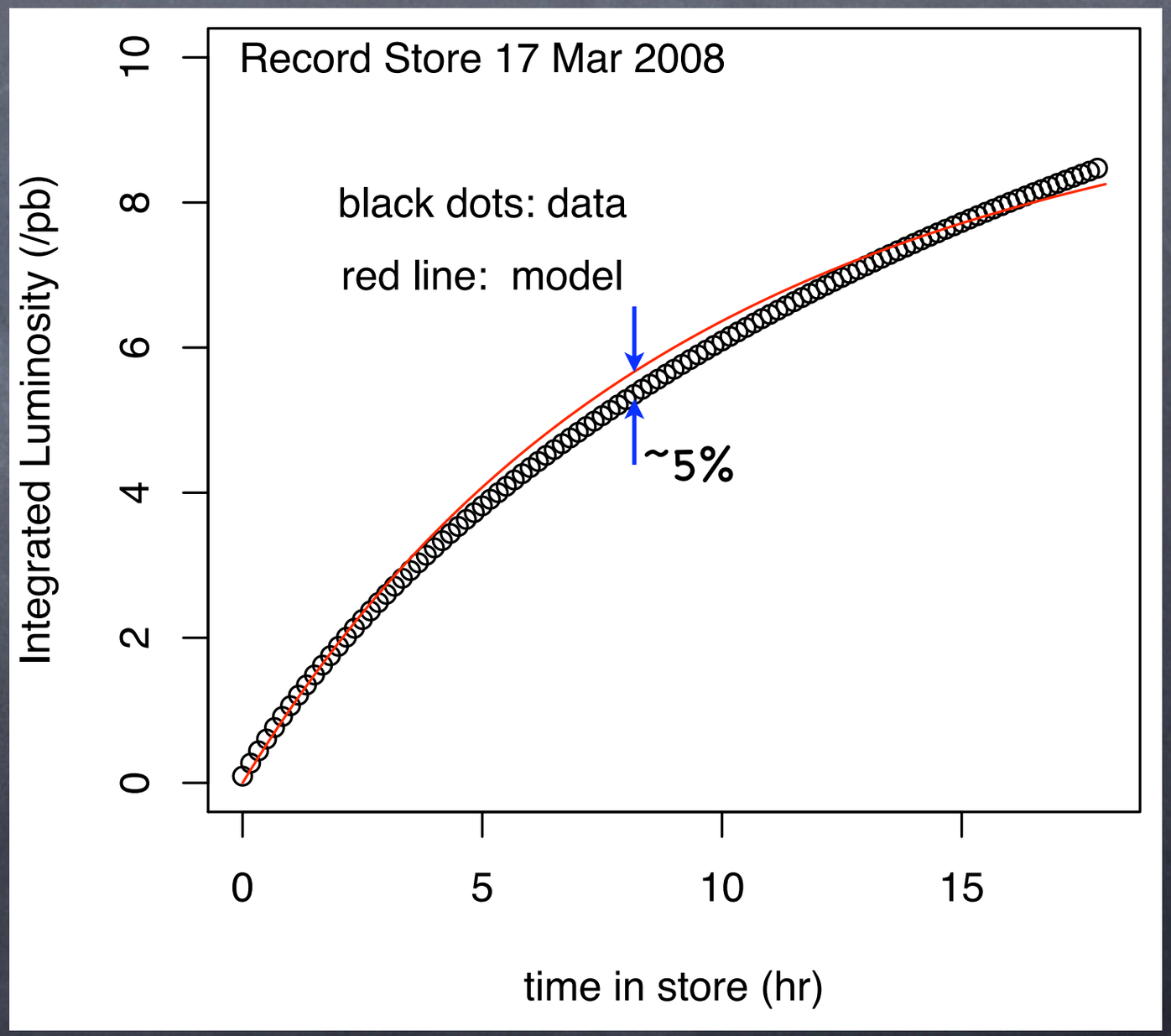


Record Store

- How well does this model the Store 5989 (315/μb/sec)?
- Put in store initial parameters, from SDA
- Varied $\dot{\epsilon}$ ...
  - used:

$$\dot{\epsilon} = 0.95\pi \ \mu\mathrm{m/hr}$$

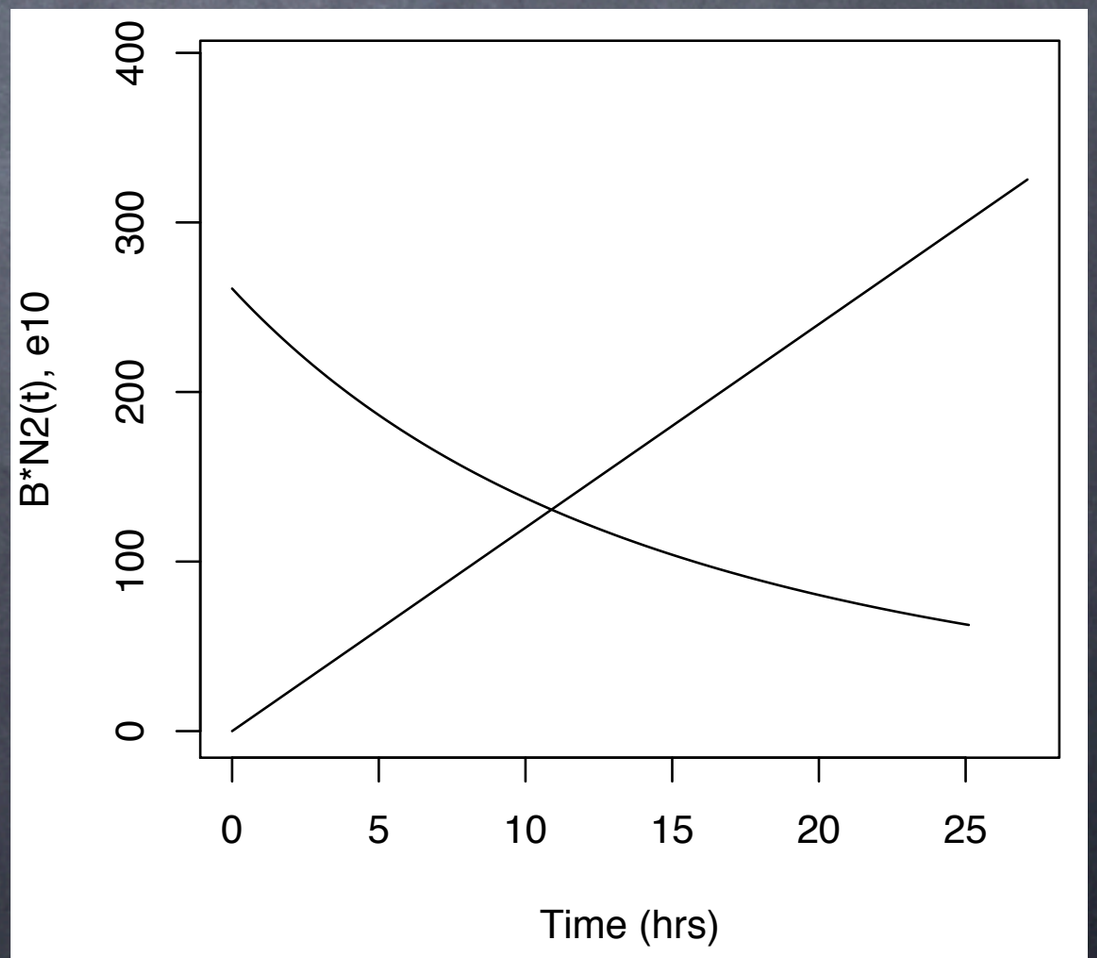
  - Result:



Store Length Optimization

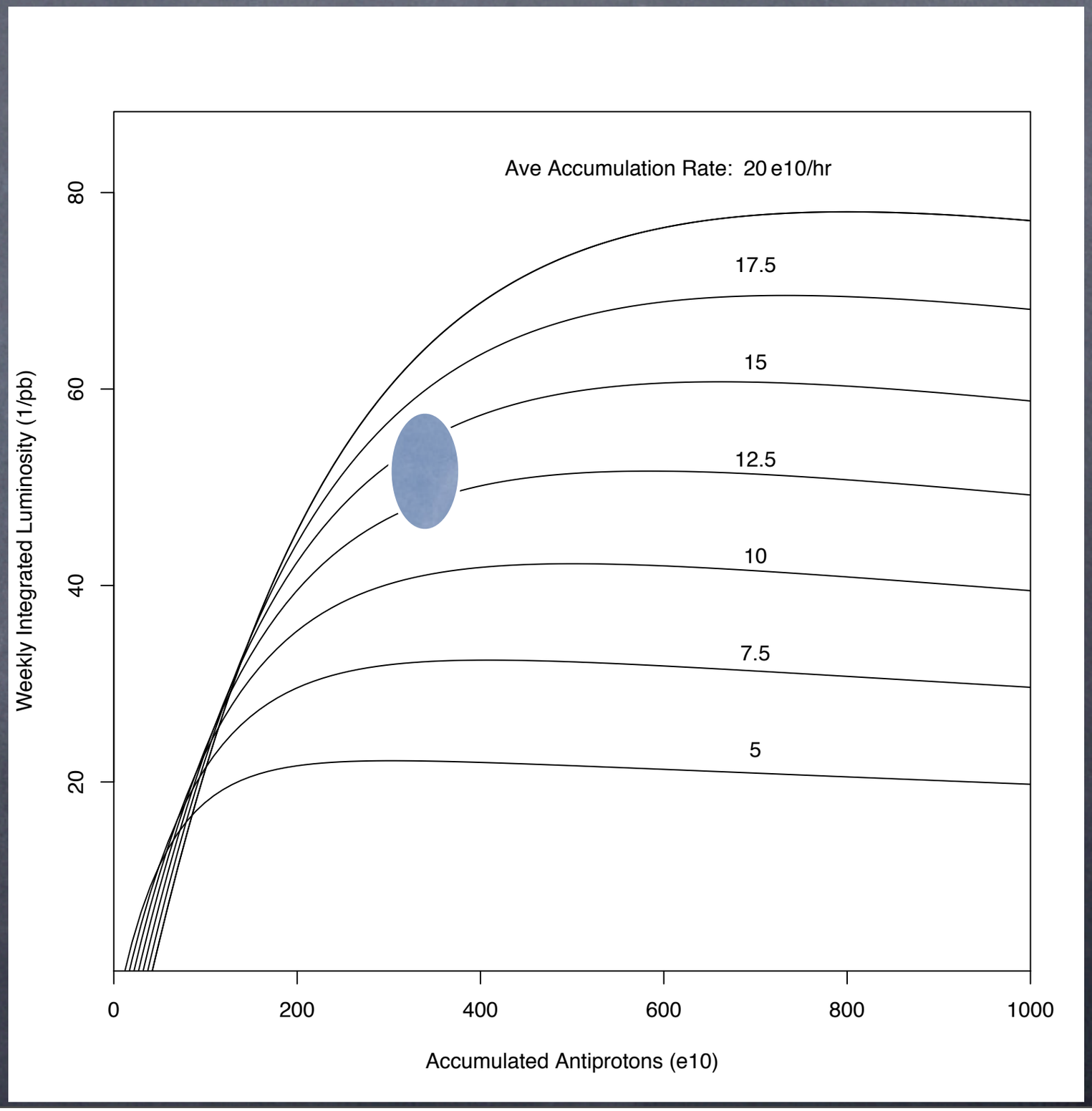
- Stack/Stash antiprotons during a store, prepare for the next one; store lasts a time T
- Suppose we ``reproduce" the same store each time
- Assume a 2 hr set-up time and assume a fraction $\mathcal{F}$ of the stashed antiprotons make it to collision in the Tevatron; $\mathcal{R}$ is the average accumulation rate:

$$\mathcal{F} \cdot (\mathcal{R} \cdot T) = B \cdot N_2^0$$



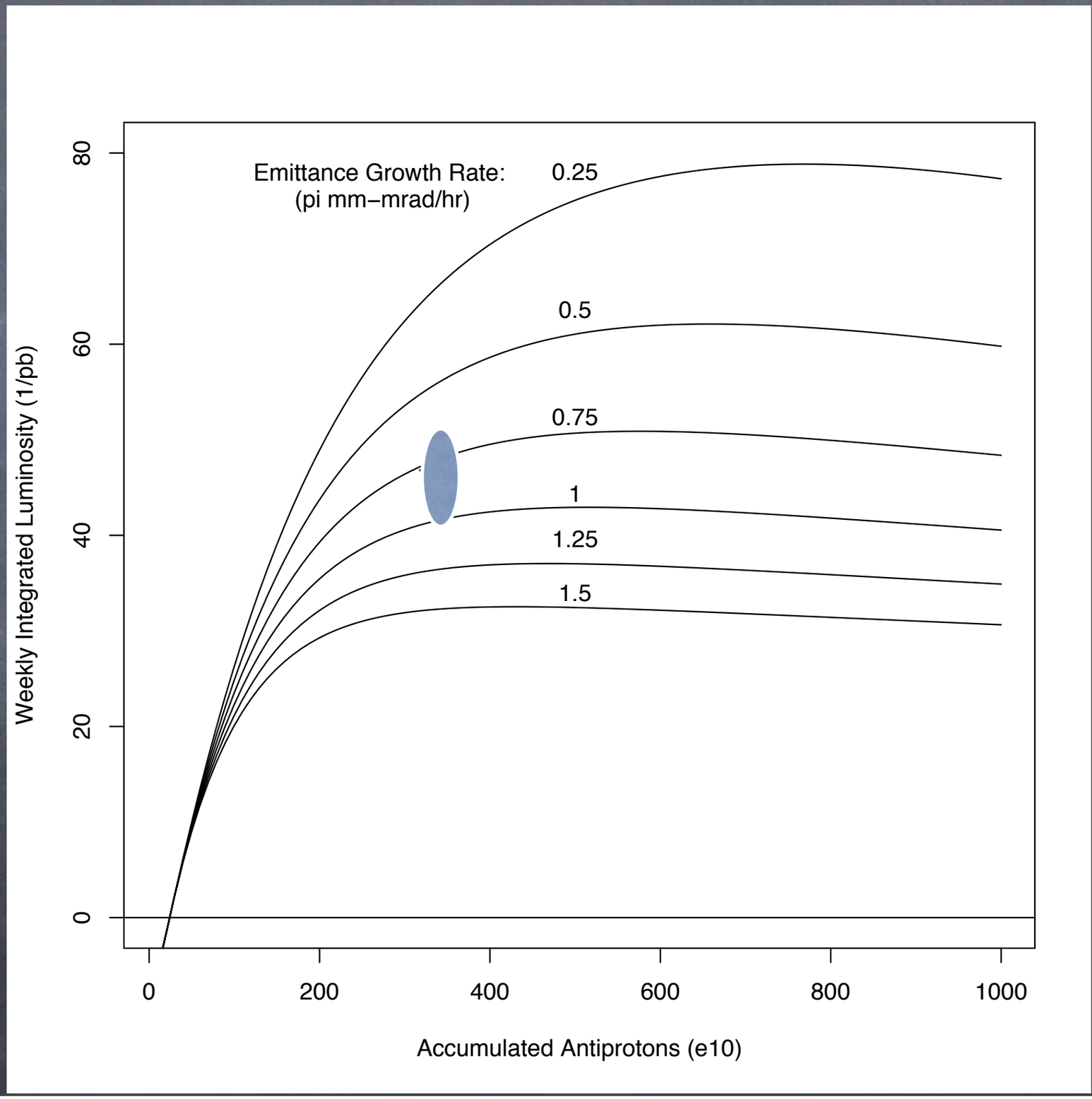
Optimizing Luminosity

- Put in equal stores from the same number of accumulated pbars
- Optimal initial luminosity given by the obtainable accumulation rate (all else being constant, of course)



Optimizing Luminosity

- If could improve upon effective emittance growth rate, would of course also improve integrated luminosity...
- (In figure, have assumed a value of R = 12e10/hr)



Optimization Process

- Keeping stores in a long time, to build up pbars and get a larger initial luminosity, doesn't necessarily help; eventually, not integrating much during this time
- If could have ``pbars on demand," would fill frequently; shot set-up time becomes the dominant factor
- For a given average pbar accumulation rate, there will be an optimum initial number of pbars which, if stores reproducible, would optimize the weekly integrated luminosity
- We're pretty close

Some Conclusions

- Essential features of Tevatron store described well by:
  - conditions setting the initial luminosity
  - a particle lifetime, for example as generated by an emittance growth rate due to diffusion processes.
- Several details of actual operation have been left out
  - hourglass form factor actually develops with time
  - influence due to beam-beam evolves during store, etc
  - But, when well tuned, mostly **COLLISIONS** dominate
- Interesting to note that simple analytical model can describe overall features of stores and integrated luminosity per week; helps to sort out important parametric choices to be made during operations

Speaking of beam-beam...

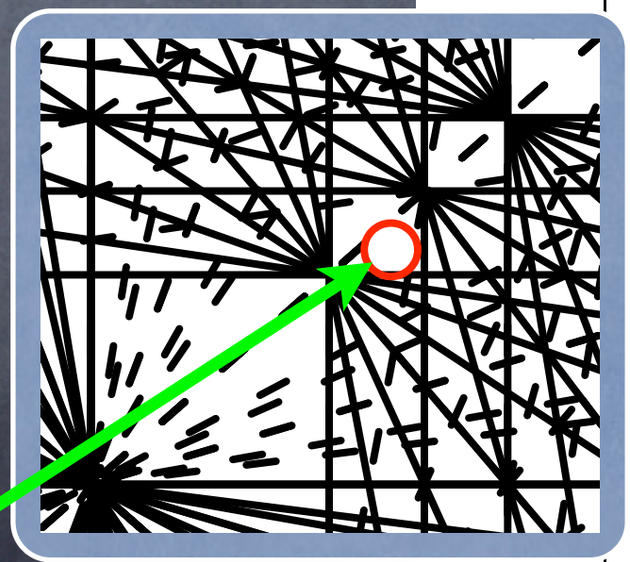
- Take one example of optimizing beam conditions
- Lifetime, loss rates often associated with betatron tune(s) (transverse oscillation frequencies) of the individual particles in the ring

(tune = osc. freq / rev. freq)

- When values of tunes correspond to resonant conditions, particle amplitudes can grow rapidly, leading to particle loss
- Beam-beam interaction causes a spread of oscillation tunes; even when center of beam distribution is far enough from resonance, some particles may not be

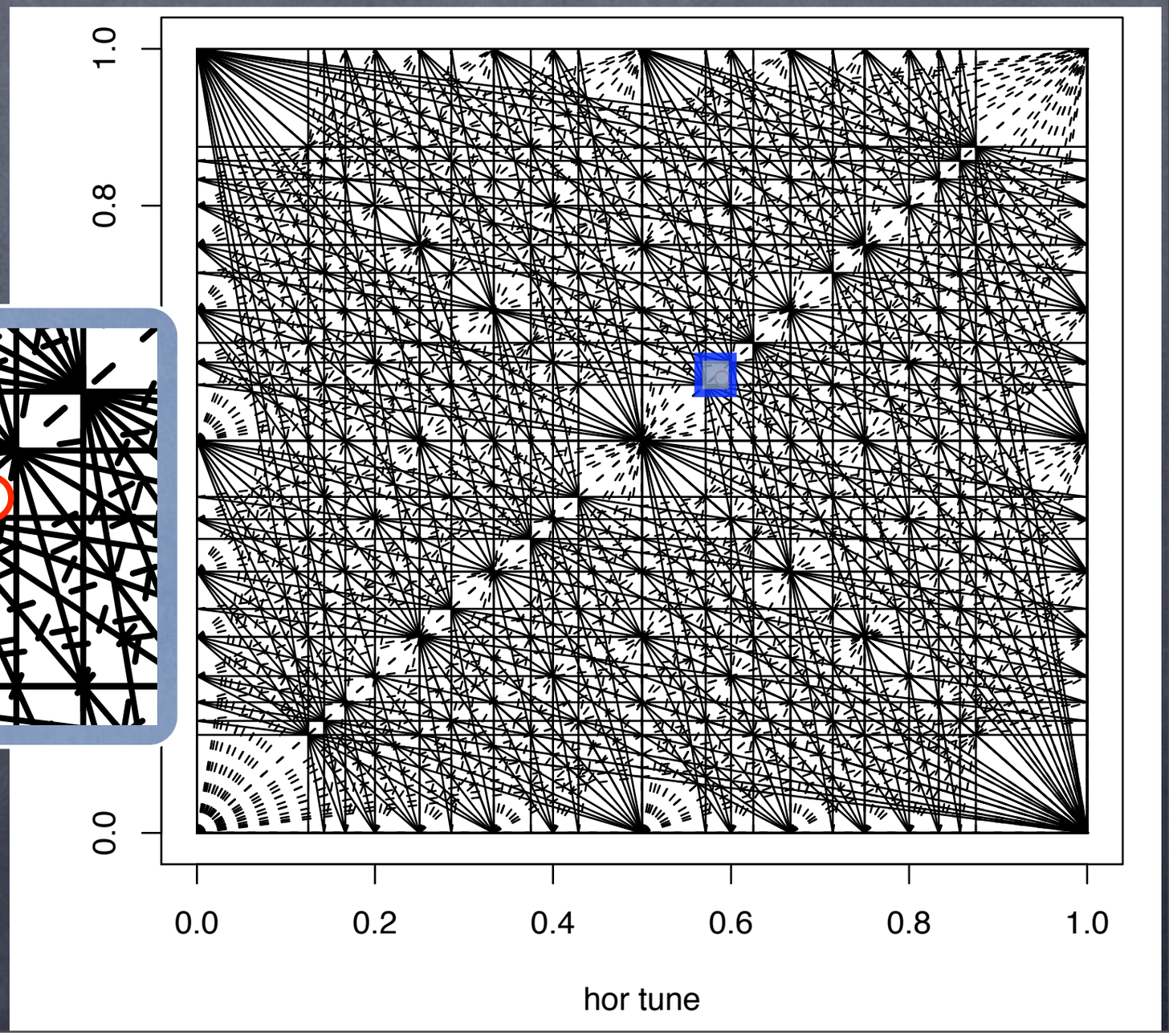
Tune Diagram

- Resonance Lines in tune space indicate potential problem spots for operation
- Tev operation:
  - ~ 20.59, 20.58



width ~ 0.025

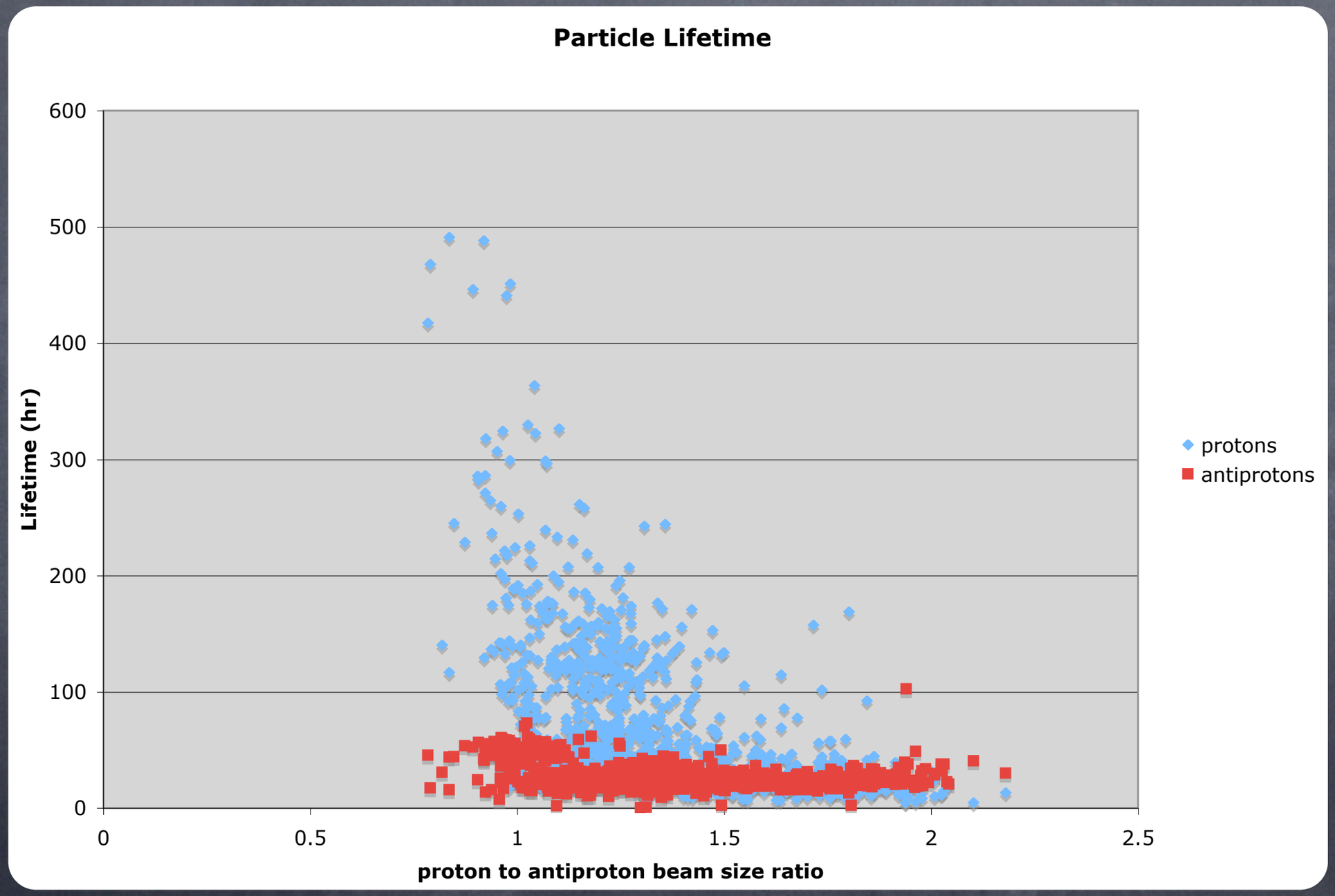
(through 8th order shown)



Present Conditions

- Have traditionally considered the Tevatron as operating in a regime of ``weak" antiproton bunches in the presence of ``strong" proton bunches
- The operation of the Recycler and of Electron Cooling have produced very intense, small pbar bunches
- Acting as a lens, the strength of the beam-beam interaction of pbar on p is now essentially the same as for p on pbar
- However, the Tevatron beams do have different sizes, and the effects are nonlinear; this influences the tune spread of the two beams differently

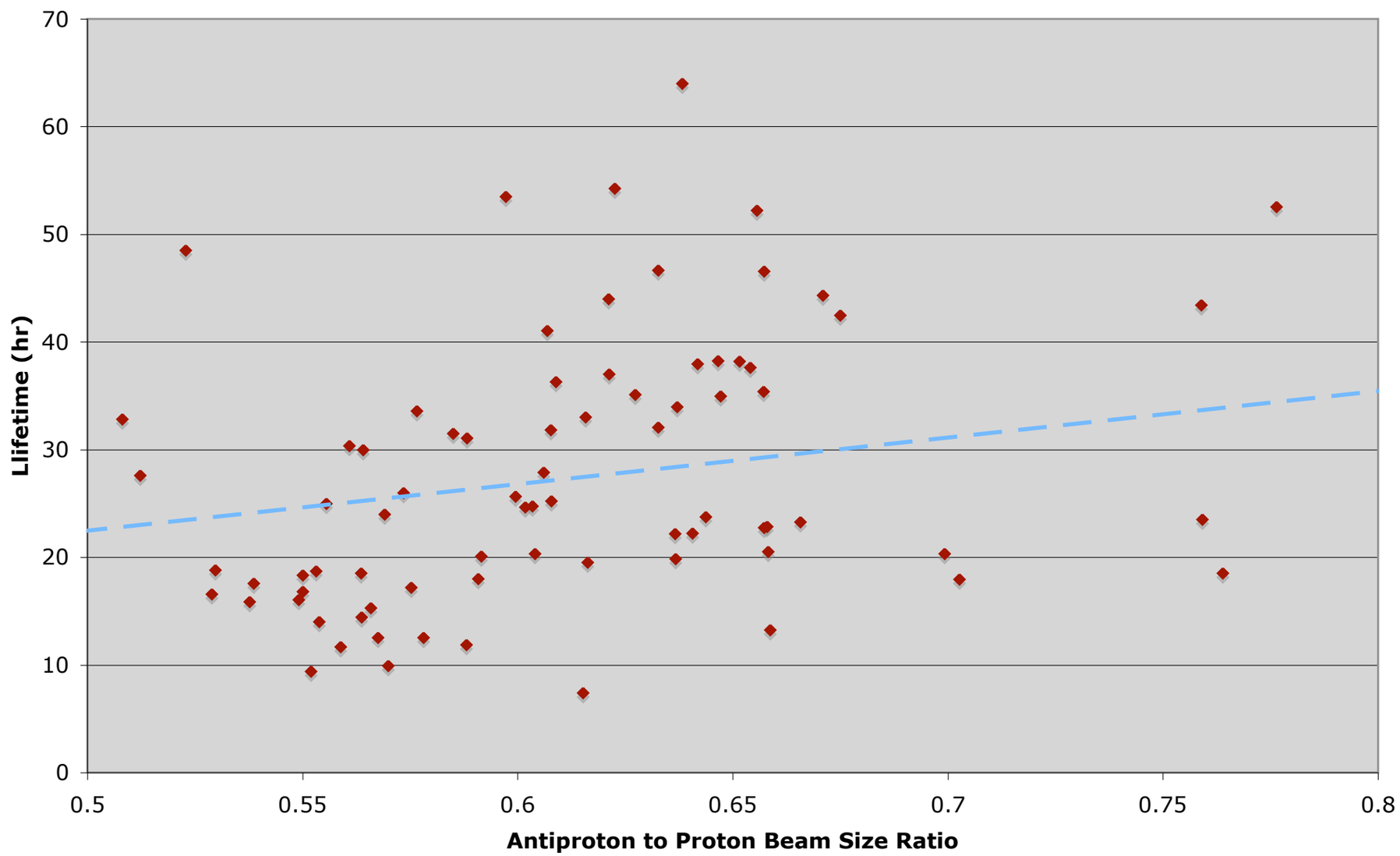
Lifetimes during 1st two hours



Data available thru SDA;
thanks -- J. Annala

~1000 stores

Non-luminous Lifetimes



~100 stores since last long shutdown

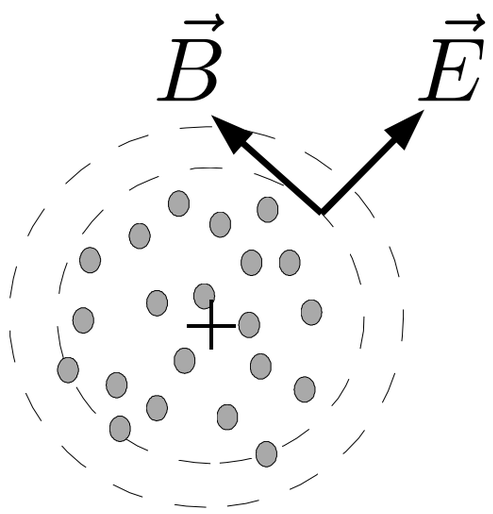
Beam-Beam Interactions

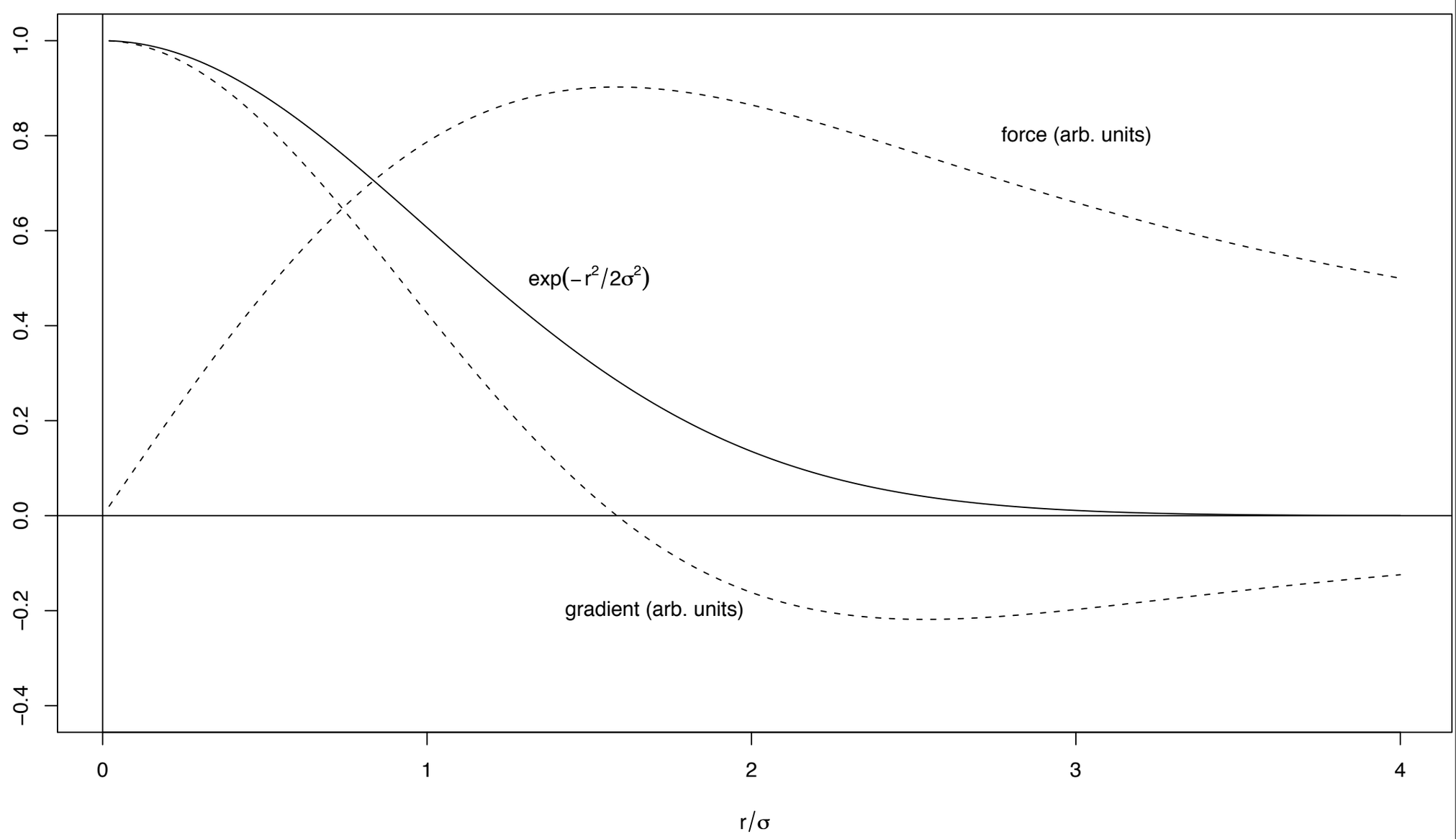
- A rich subject, with many papers (books?) written on it*
- Head-on vs. Long-range interactions
  - will concentrate on the first
- Will assume round, Gaussian beams, but with potentially different transverse size, at the interaction points
- Ultimately, wish to optimize collider operation in this regime

* Here's another one: M. J. Syphers, Beams-doc-3031

The Beam-Beam Force

- Force, and hence Gradient, vary with position ...





Assumes round, Gaussian beam

The Beam-Beam Tune "shift"

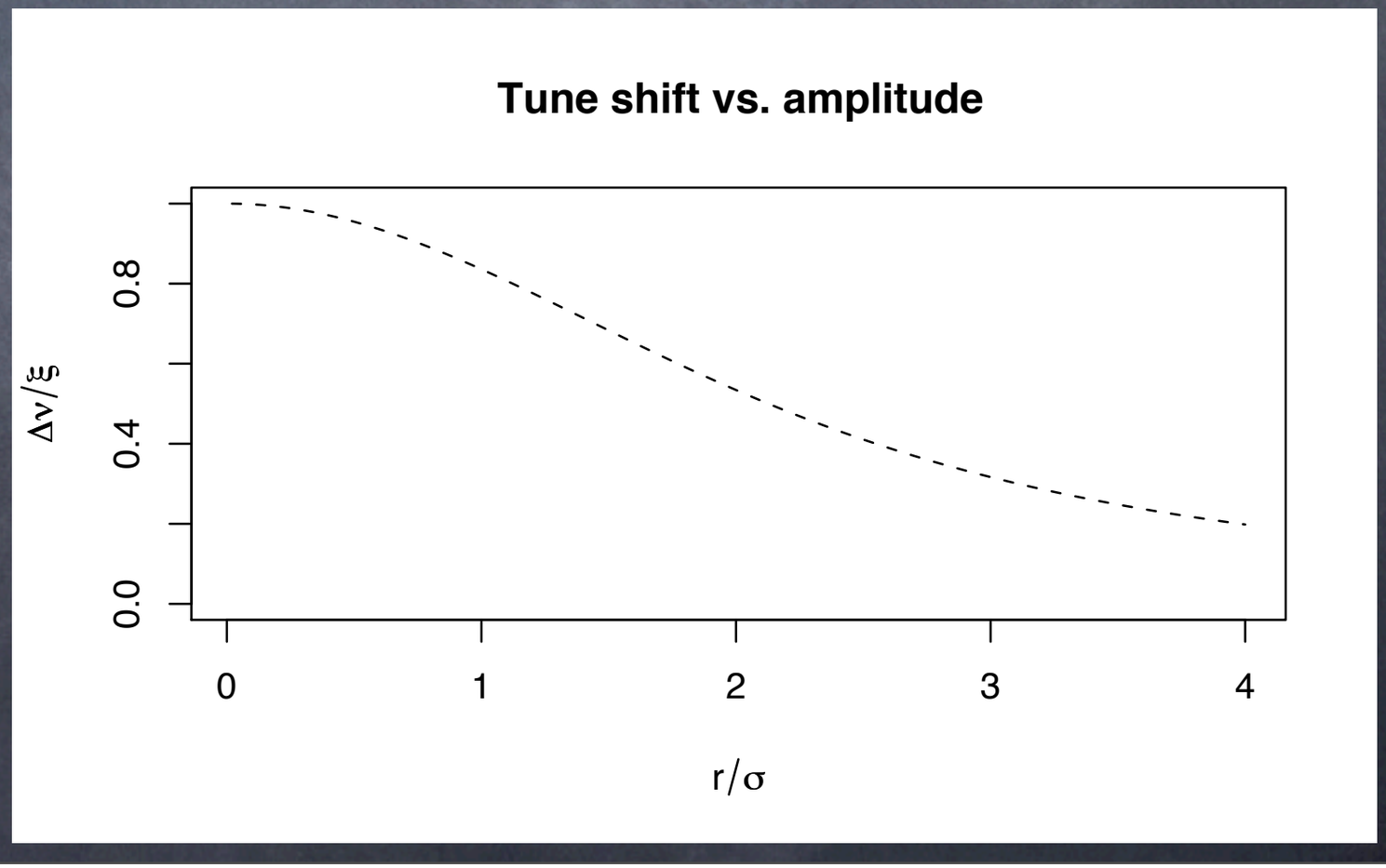
- ``The tune" only has meaning in an average sense, when nonlinear fields involved

$$\Delta\nu(r) \approx \xi \cdot \frac{1 - e^{-(r/2\sigma)^2} I_0[(r/2\sigma)^2]}{(r/2\sigma)^2}$$

- Tune shift parameter:

$$\xi = \frac{3 r_o N}{2\epsilon}$$

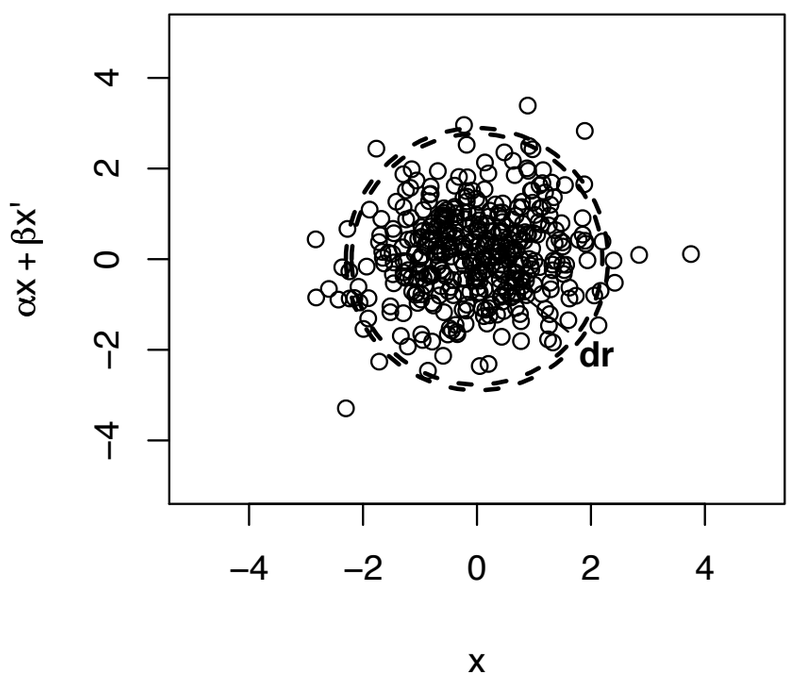
N = no./bunch (on-coming bunch)
r_o = classical radius
ϵ = 95%, norm. emittance

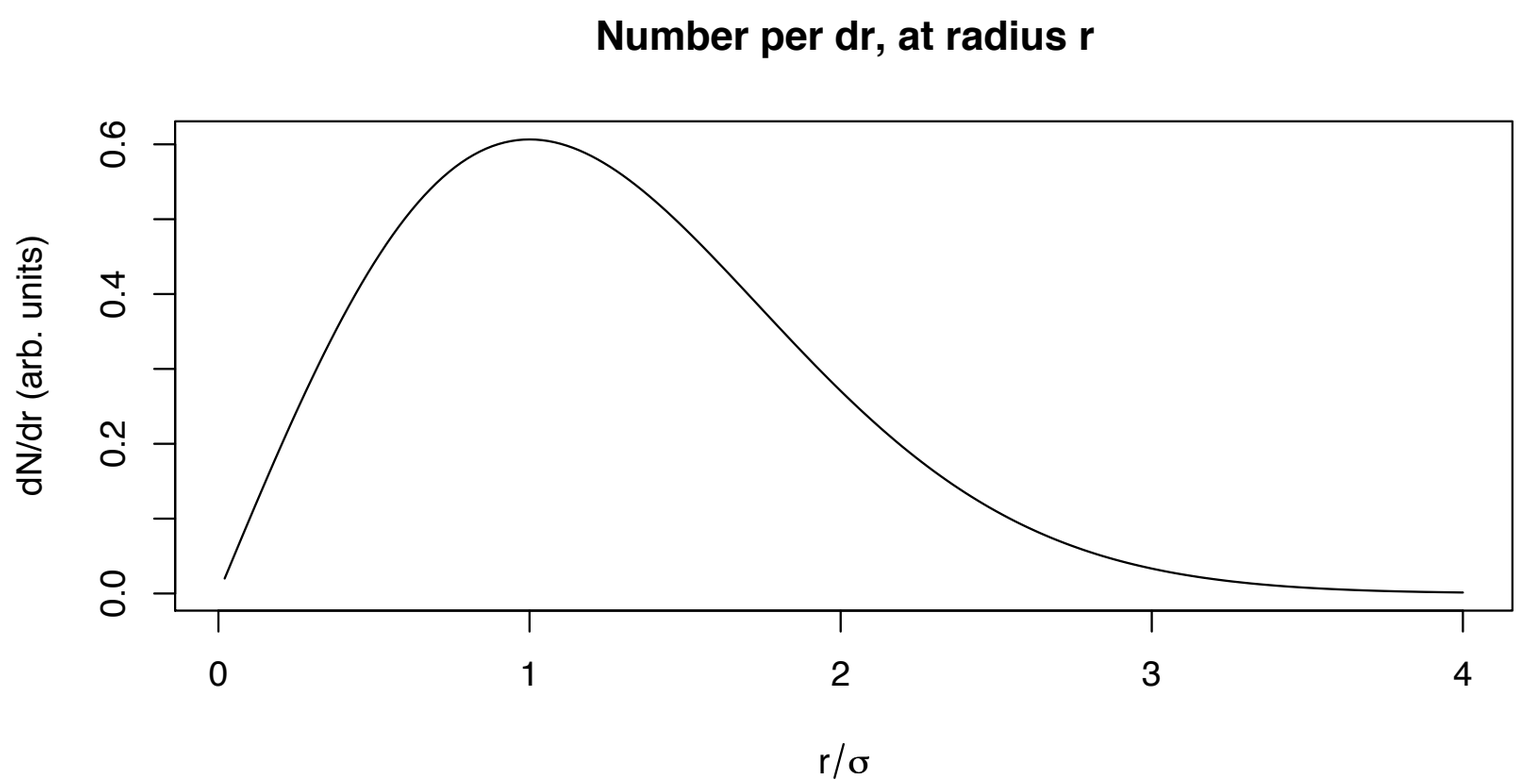


Estimating the Tune Spread due to Head-On Collisions

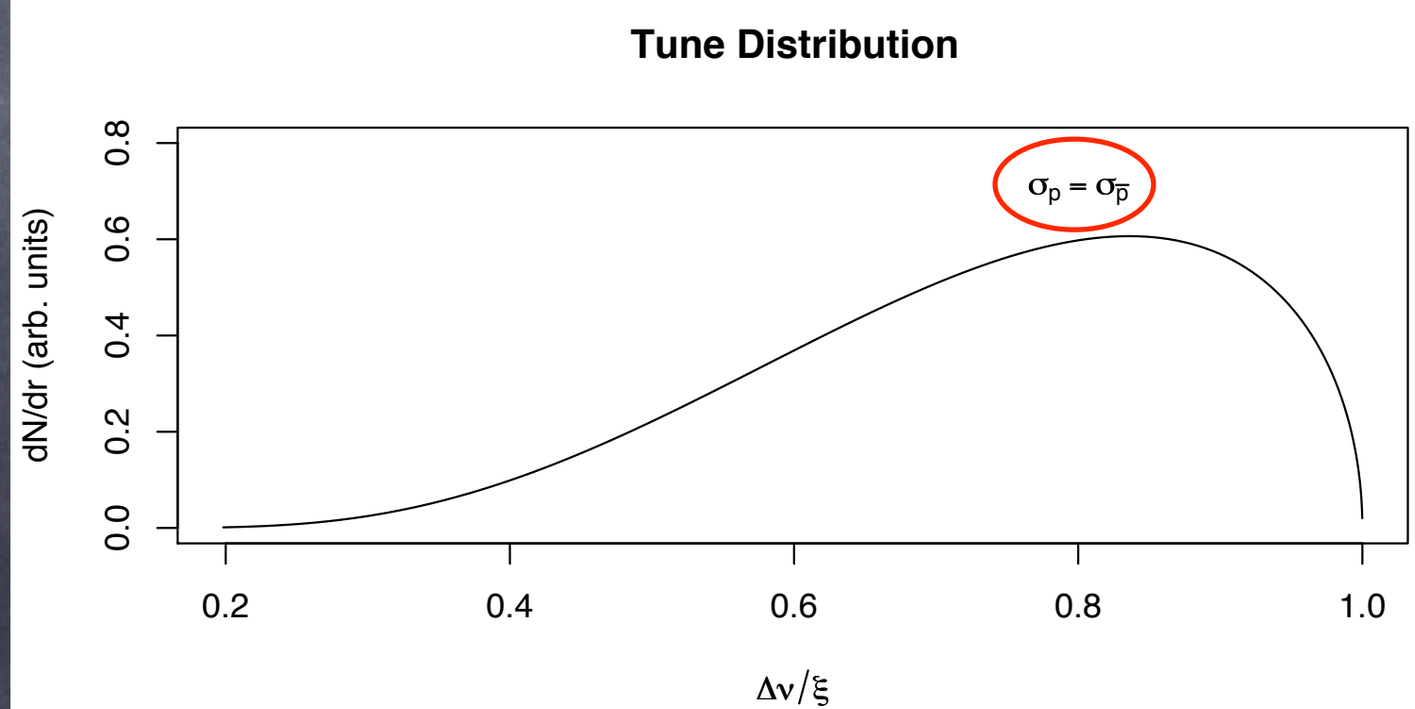
- Assume tune varies only with phase space amplitude, as given on previous slide
- Since each amplitude has a corresponding "tune," look at how many particles exist at each amplitude and plot no. particles vs. tune

Tune Distribution



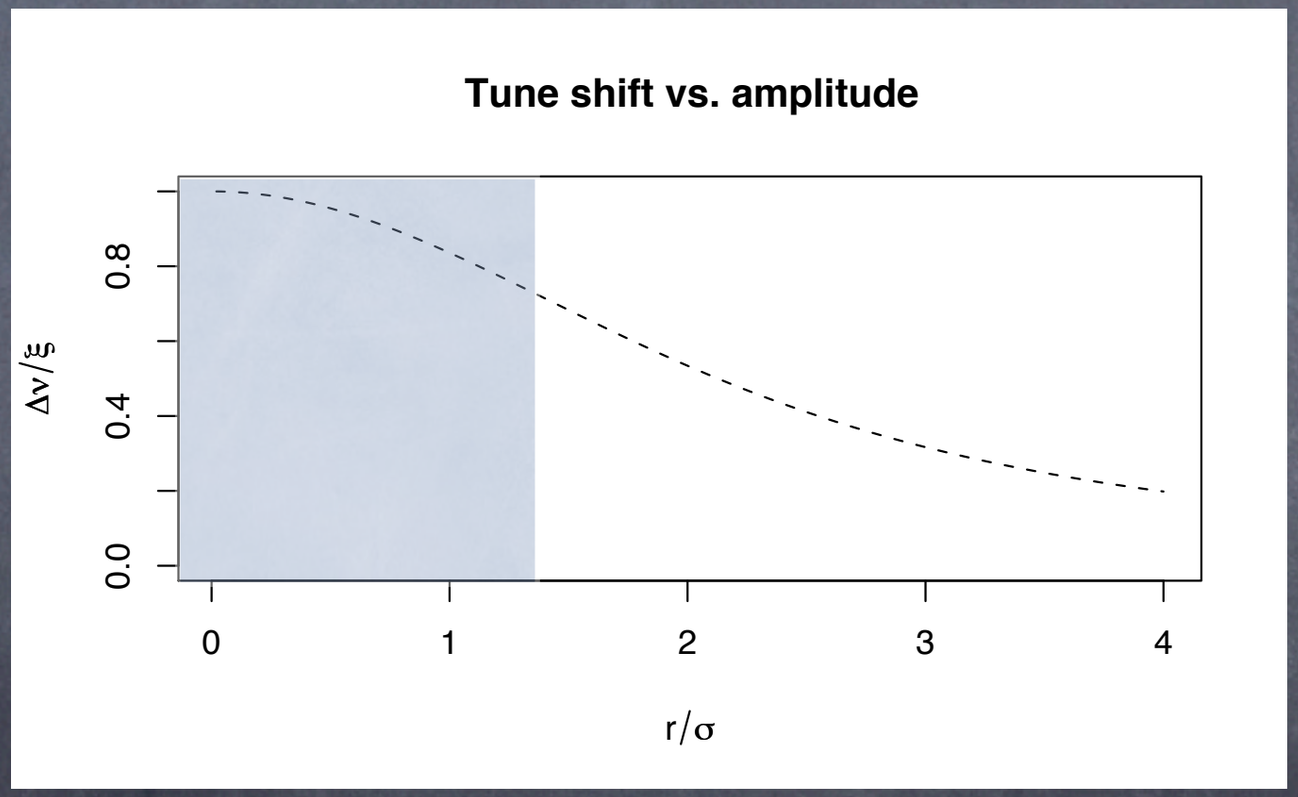


No. of particles per dr at radius r, and thus with tune ν:



Unequal Beam Sizes

- When unequal sizes, "smaller" beam sees more of the "linear" region of the other beam, and thus its tune distribution will peak toward the maximum tune shift



- On the other hand, particles of the "larger" beam experience a wider range of nonlinear force, and hence have a potentially larger tune spread

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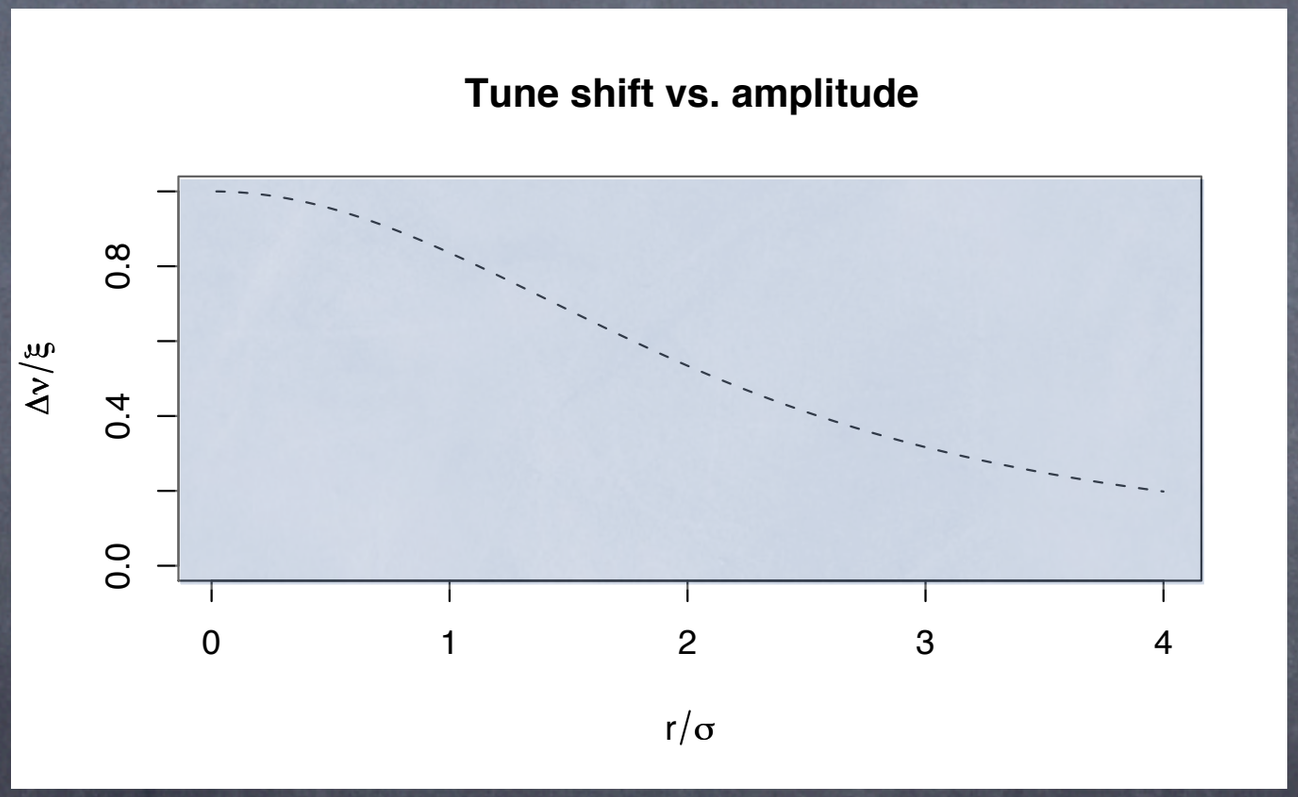

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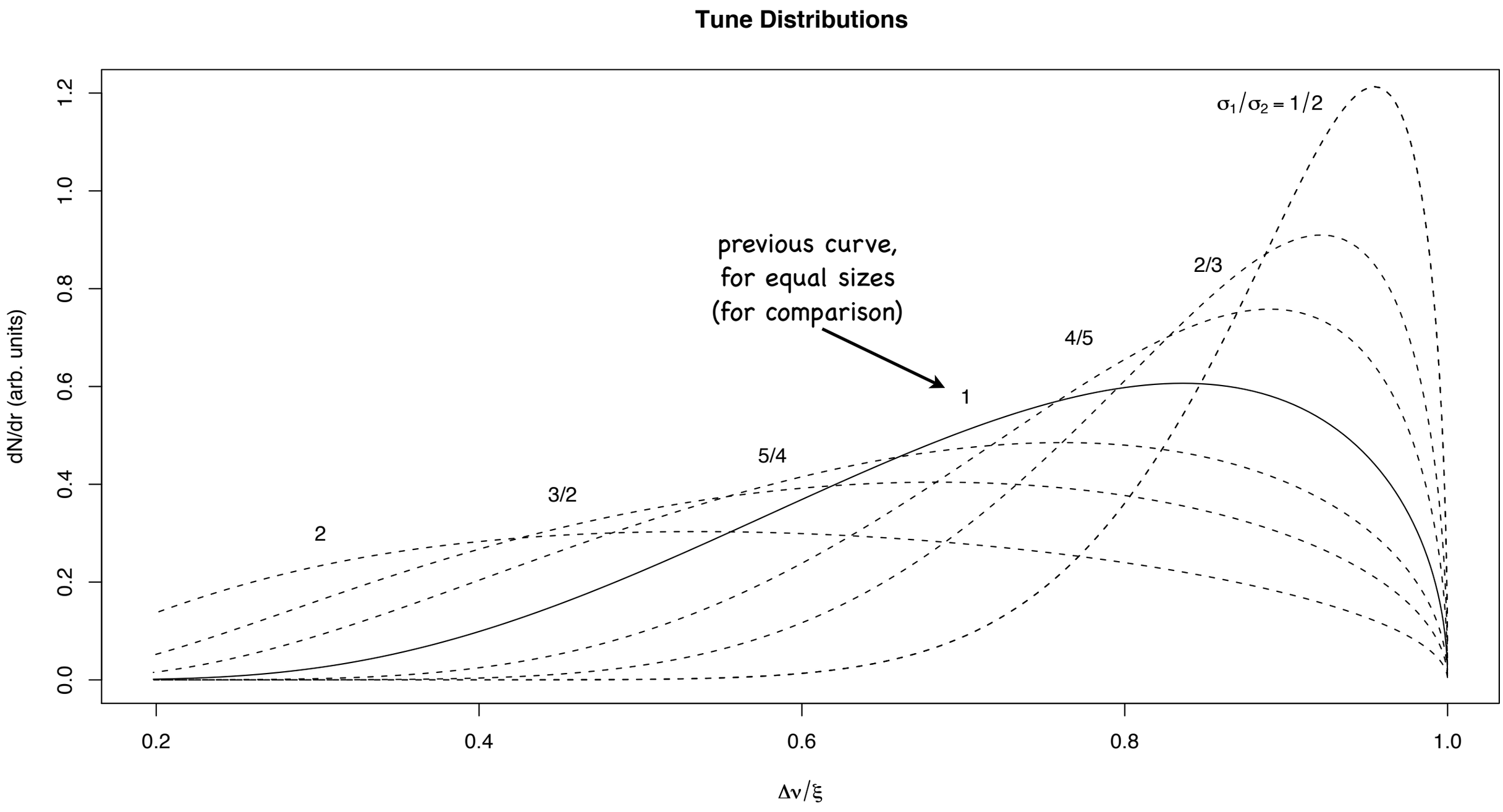
Tune Distribution

same tune shift param, unequal beam sizes



Typical Tevatron Params

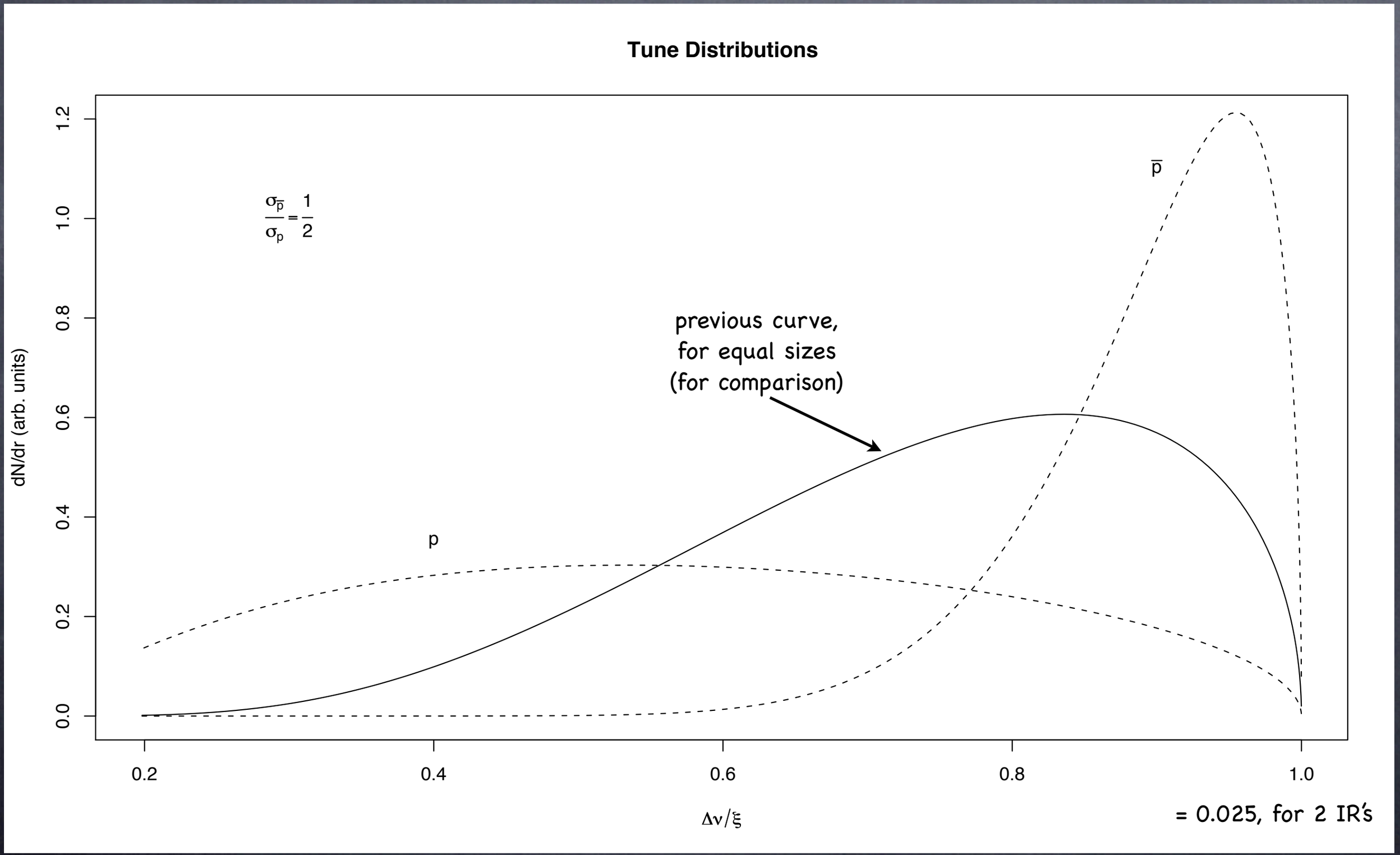
- Let's use: $\xi = \frac{3(1.5 \times 10^{-18})(250 \times 10^9)}{2 \cdot 14\pi\ 10^{-6}} = 0.0125$ (due to p)

  $\bar{\xi} = \frac{3(1.5 \times 10^{-18})(70 \times 10^9)}{2 \cdot 4\pi\ 10^{-6}} = 0.0125$ (due to pbar)

Note:
2 IR's make total of 0.025

- But: $\sigma_{\bar{p}}/\sigma_p = \sqrt{\frac{4}{14}} \approx \frac{1}{2}$

Cold Pbars...


Tune Distributions
dN/dr (arb. units)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.2
0.4
0.6
0.8
1.0
Δν/ξ
$\frac{\sigma_{\bar{p}}}{\sigma_p} = \frac{1}{2}$
$\bar{p}$
p
previous curve,
for equal sizes
(for comparison)
= 0.025, for 2 IR's

Example of Possible Tevatron Beam Preparation

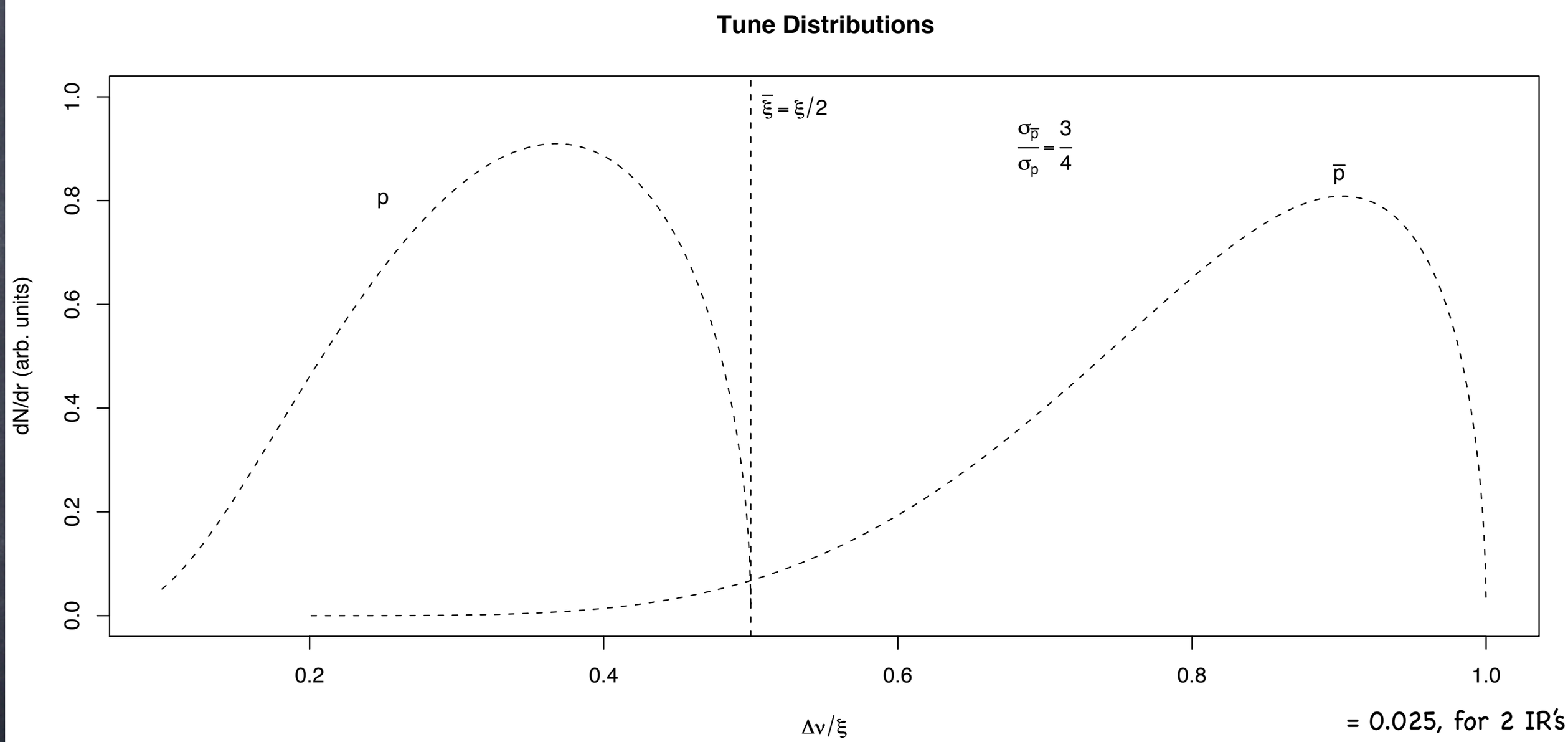
- Imagine taking previous parameters, and doubling the antiproton emittance to 8π mm-mrad
- Results:

$$\bar{\xi} = \frac{3(1.5 \times 10^{-18})(70 \times 10^{9})}{2 \cdot 8\pi\ 10^{-6}} = 0.0062 = \xi/2$$

$$\sigma_{\bar{p}}/\sigma_p = \sqrt{\frac{8}{14}} \approx \frac{3}{4}$$

- Look at new tune distributions...

Optimal?

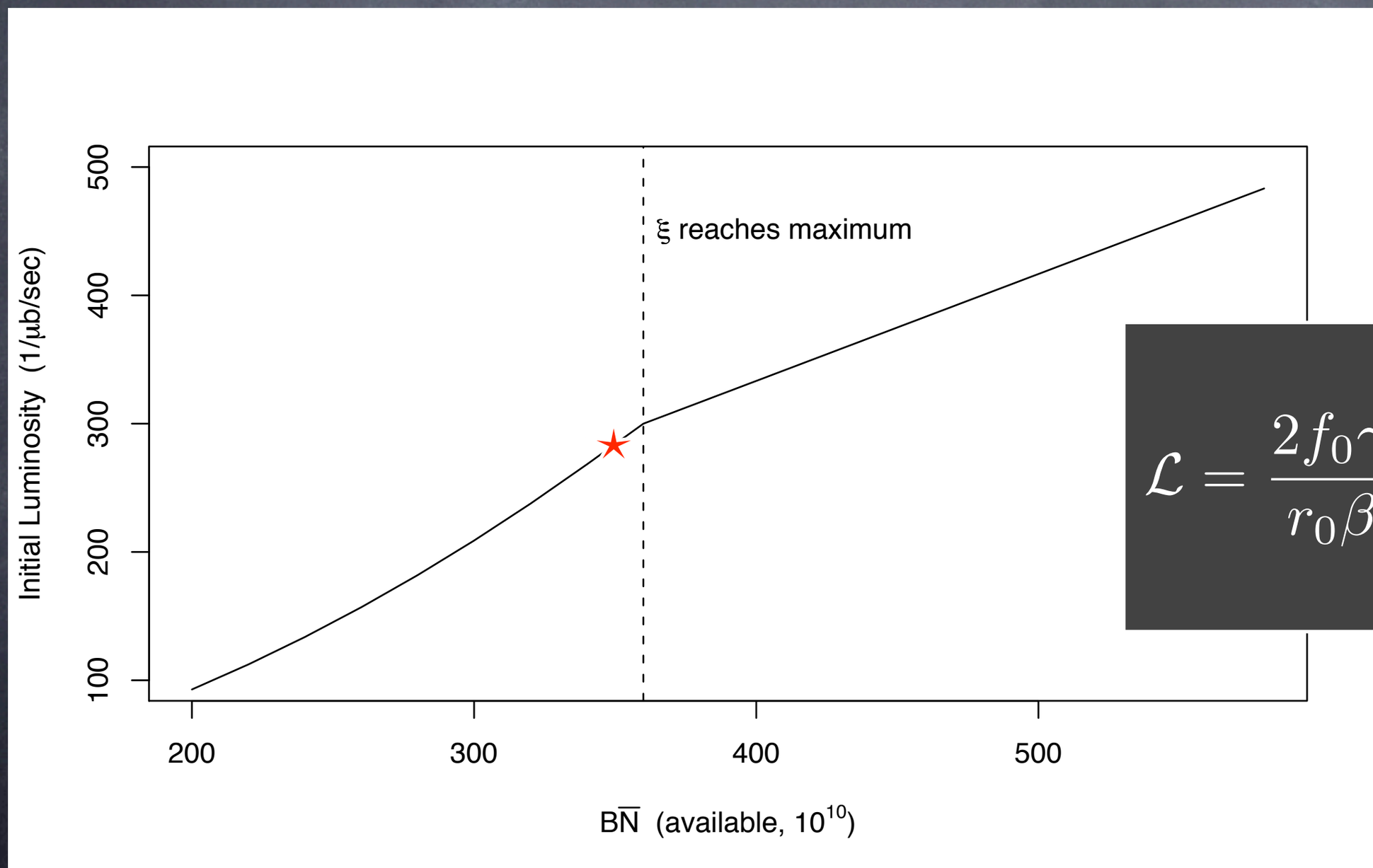


Roughly equal tune spreads in both beams
--> does this optimize lumi lifetime??

Example Recipe...

- Suppose we like the conditions of previous slide...
- Given no. of pbars available for a shot, determine no. of protons to use and their emittance to keep $\bar{\xi} = \xi/2$ and tailor pbars accordingly to keep $\sigma_{\bar{p}}/\sigma_p \approx 3/4$
  - Ex: $$N = \frac{7}{2}\bar{N}; \quad \epsilon = (3r_o/2\xi) \cdot N; \quad \bar{\epsilon} = \frac{4}{7}\epsilon$$
- Run proton beam at beam-beam limit; if its emittance is already too large, leave as is
  - i.e., make $\xi \leq 0.012$

Initial Luminosity vs. Stash Size

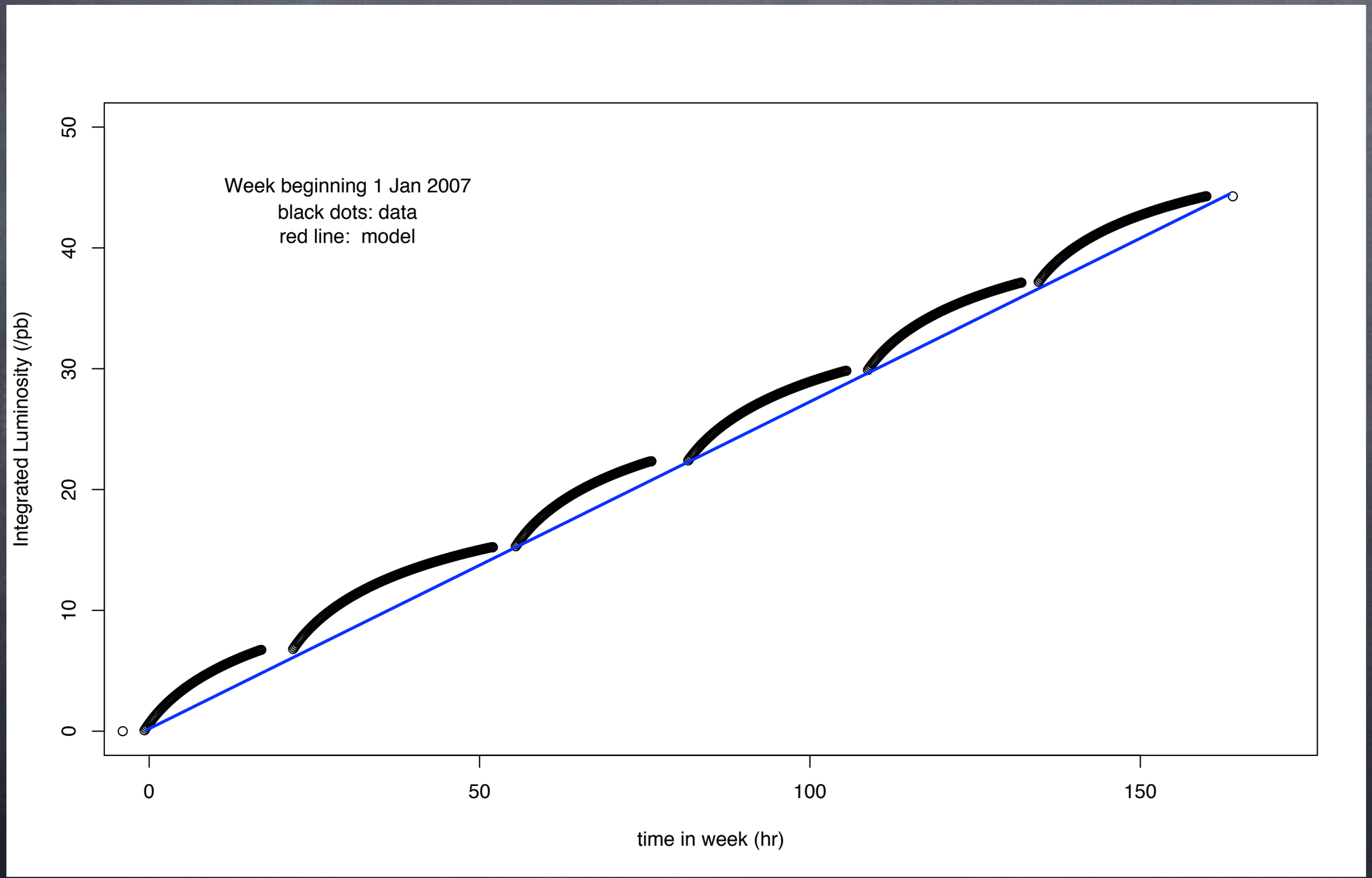


Example only

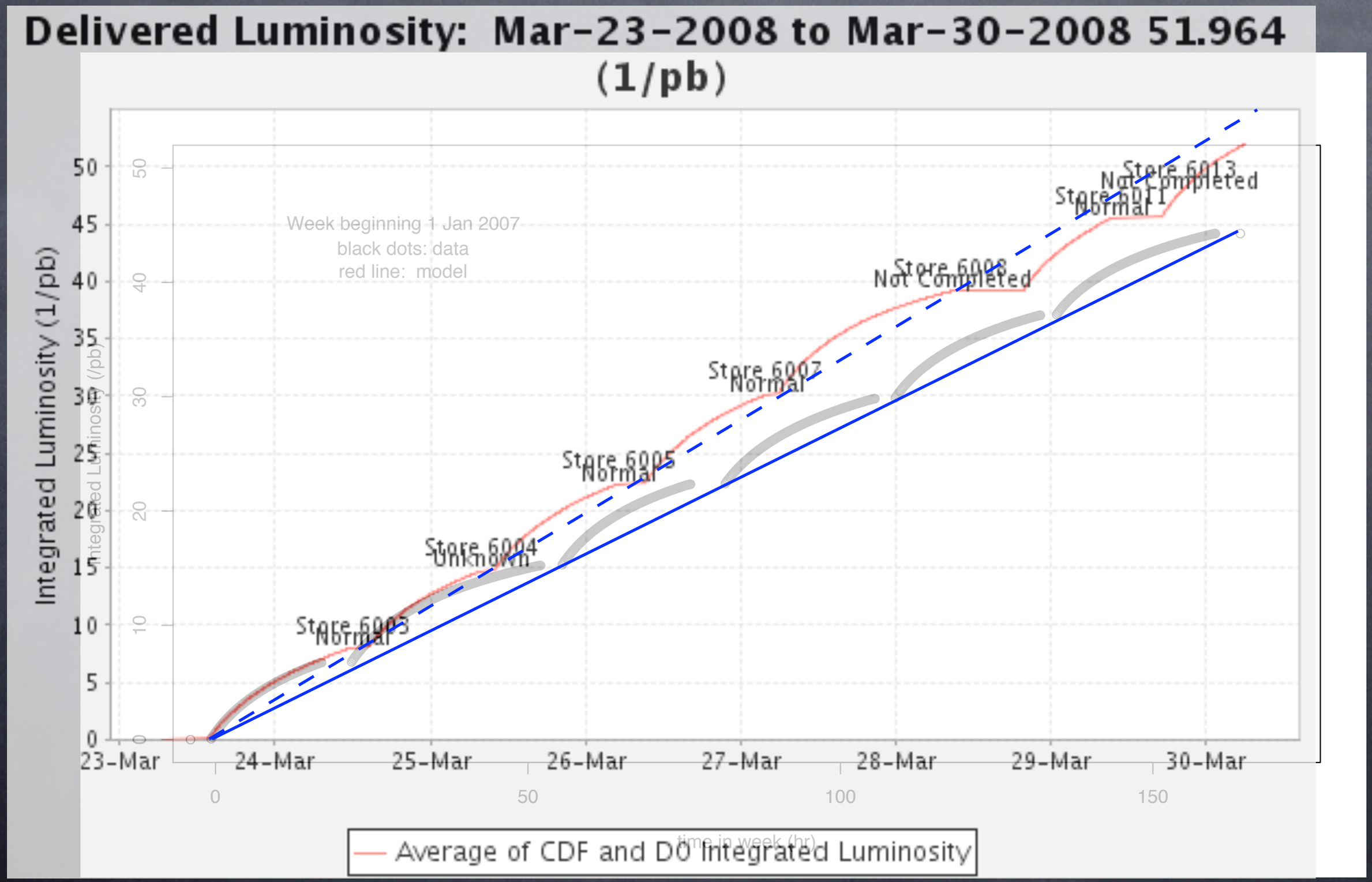
$$\mathcal{L} = \frac{2 f_0 \gamma \xi}{r_0 \beta^*} \cdot \frac{B\bar{N}}{1 + \bar{\epsilon}/\epsilon} \cdot \mathcal{H}$$

- Assumes 80% make it to collisions, and that the conditions above "optimize" the luminosity lifetime

Best 7-days To-Date



Best 7-days To-Date



General Remarks

- under steady collider conditions, accumulation rate determines optimum initial luminosity, store length
  - at or near this optimum today
- want to run steadily under ``adiabatically changing conditions"; repeatability is key
- p-pbar collider is VERY tricky, complicated; now essentially at optimum efficiency -- quite a feat and lots to be proud of
- 2002: struggling to get to 30; now, daily at ~300×10^{30}
- run run run run run run run run run run run run run run r

Thanks, and References

- Thanks to: Jerry Annala, Cons Gattuso, Salah Chaurize for data and discussions
- Further reading:
  - Beams-doc's: 3031, 2798, 2685, 1478, 1348, (MJS)
  - ... 1155, 2230, 2645, ... (many: search Luminosity)
  - PRST-AB 8:101001 (2005) Shiltsev, et al., ...
  - Acc Phys Books -- Edwards and Syphers, Lee, Wiedemann, Wilson, etc.