Y. Alexahin 2009

I try here to answer two questions:

- how strong is the effect of Coulomb scattering in a D-T collider, is it forbidding, manageable or negligible?
- is it possible compensate it with ionization cooling?

1. Coulomb scattering starting from first principles:

Consider a particle with momentum p=mv moving between scattering centers of density n_2. (Transformation from the lab frame where particles move with velocities $v_{1,2}$: $v= v_1- v_2$, $m= m_1m_2/(m_1+ m_2) = 6/5\ m_p$ in d+t case).

The change in the transverse momentum $p_\perp$ ($p_\perp<<p$ by assumption) after passing one scattering center is:

$$\Delta p_\perp = \frac{e^2 r}{v}\int_{-\infty}^{\infty}\frac{ds}{(s^2+r^2)^{3/2}} = \frac{2e^2}{vr} \tag{1}$$

Let us take for certainty x-component of the momentum ($p_x = p_\perp \cos\varphi$). After passing a number of centers

$$\Delta p_x = \frac{2e^2}{v}\sum_i \frac{\cos\varphi_i}{r_i} \tag{2}$$

For randomly distributed scattering centers the average over realizations is

$$\langle(\Delta p_x)^2\rangle = \left(\frac{2e^2}{v}\right)^2 \frac{1}{2}\sum_i \frac{1}{r_i^2} \tag{3}$$

For a continuous distribution with density $n_2(r)$ the summation should be replaced with integration:

$$\sum_i \rightarrow 2\pi\int n_2(r) r\,dr\,dz \tag{4}$$

The lower limit of integration is ~ the nucleus radius, the upper limit in the absence of the Debye screening is ~ the beam radius *a*.

Introducing (small) scattering angle $\theta = p_\perp/p$ we have in the case of constant density:

$$\frac{d\theta^2}{dz} = \frac{d}{dz}\langle(\Delta p_x)^2 + (\Delta p_y)^2\rangle/p^2 = 2\pi n_2\left(\frac{2e^2}{pv}\right)^2 ln\frac{a}{r_{min}} \tag{5}$$

Probability of fusion increases at the same time as

$$\frac{dP_f}{dz} = \sigma_f(v) n_2 \tag{6}$$

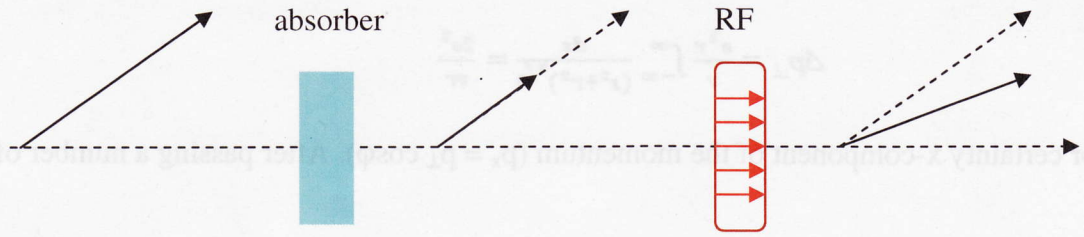
While it reaches 1 the scattering angle increases to

$$\theta_{fin}{}^2 = \frac{2\pi}{\sigma_f(v)}\left(\frac{2e^2}{pv}\right)^2 ln\frac{a}{r_{min}} \tag{7}$$

Let us make a numerical estimate: v=0.01c, $\sigma_f = 5\cdot 10^{-28}$ m^2, ln=30, then $\theta_{fin}{}^2 \approx 200$. This number can be made somewhat smaller by increasing v, but still it will remain large enough to make cooling necessary.
The scattering angle in the lab frame for both species is exactly the same as (7) in the case of equal momenta.

2. Can ionization cooling help?

The principle of ionization cooling (or any other based on losses) is illustrated in this drawing:



Let δ<<1 be the fraction of total momentum lost in the absorber:

$$\frac{\Delta p}{p} = \frac{\Delta p_z}{p_z} = \frac{\Delta p_\perp}{p_\perp} = -\delta \tag{8}$$

RF restores only the longitudinal component to its initial value. The amount of energy taken from the cavity therefore is:

$$\Delta T = \frac{p_z \Delta p_z}{m} \approx 2T\delta \tag{9}$$

The decrease in the trajectory slope (cooling) can be linked to this energy as

$$\Delta\theta^2 = 2\theta\Delta\theta \approx -2\theta^2\delta = -\theta^2\,\Delta T/T \tag{10}$$

To counteract scattering which would otherwise increase θ up to the value θ_{fin} given by eq.(7) the total energy gain in RF should be

$$\Delta T/T = \theta_{fin}{}^2/\theta_{eq}{}^2 \tag{11}$$

Assuming $\theta_{eq}{}^2 \approx 0.1$ (which is already too large) and taking from the previous section $\theta_{fin}{}^2 \approx 200$ we see that amount of RF energy which should be imparted into a d+t pair in the case of equal d and t momenta is

$$\Delta T \sim \frac{\theta_{fin}{}^2}{\theta_{eq}{}^2}(T_d + T_t) \sim 2\cdot 10^3 \times 60keV = 120\ MeV \tag{12}$$

Taking into account that only ~20MeV is released in d+t fusion and that scattering in the absorber itself will significantly contribute to $\theta_{fin}{}^2$ the whole idea seems hopeless.

M. Albrow

$d - t \to \alpha + n$ fusion with colliding d-t beams.

α (3.5 MeV), n 14.1 MeV

$n + Li^6$ (tank) $\to t + \alpha + 5.8$ MeV

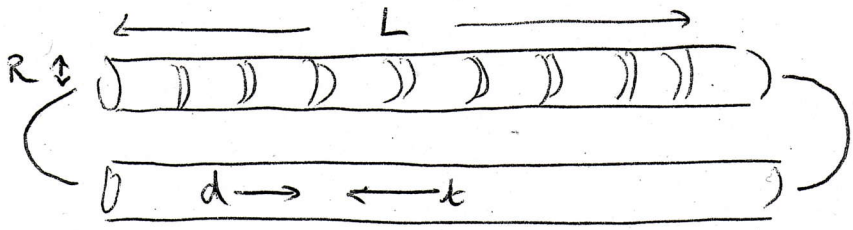
↳ recovered

$\sigma(dt \to n\alpha) > 3b$ from $p_d = p_t \sim 10$ MeV/c $\to 16$ MeV/c
max $\sim 5b$ at $p_d = p_t \sim 12$ MeV/c
} assume equal momentum d & t.

However probably want equal velocity d & t, to keep in phase.

Concept:

LSS = long straight sections



pair of linear "reactors" long solenoids. (20m? 50m?)

How strong a field can we get? H.T. Superconductors. (minimize losses)

Cavities to maintain constant cm energy. speeds.

Ionisation cooling (gas or foils)?

Electron cooling? probably not, KE(e) for same speed ~6eV!

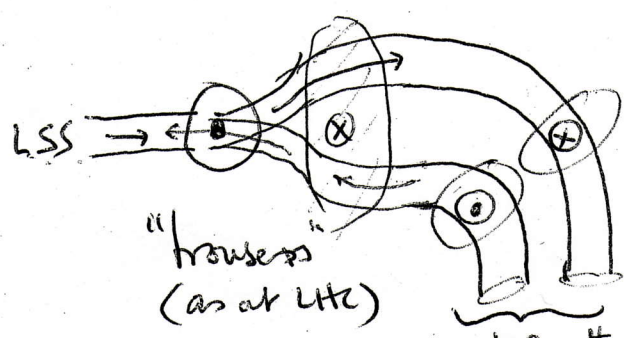
$v = \beta c$ small, $\beta \ll 1$, non-relativist approx $KE = \frac{1}{2}mv^2$

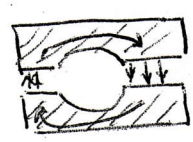
Want equal speed, $\beta_= \sim 0.005 = 1.5$ m/μsec

Then $p_d = 9.4$ MeV/c & $p_t = 14.05$ MeV/c

Arcs: Permanent magnets. Say R = 2m, bending dipole ≈ 156 gauss (d), 233 gauss (t)

Scheme:

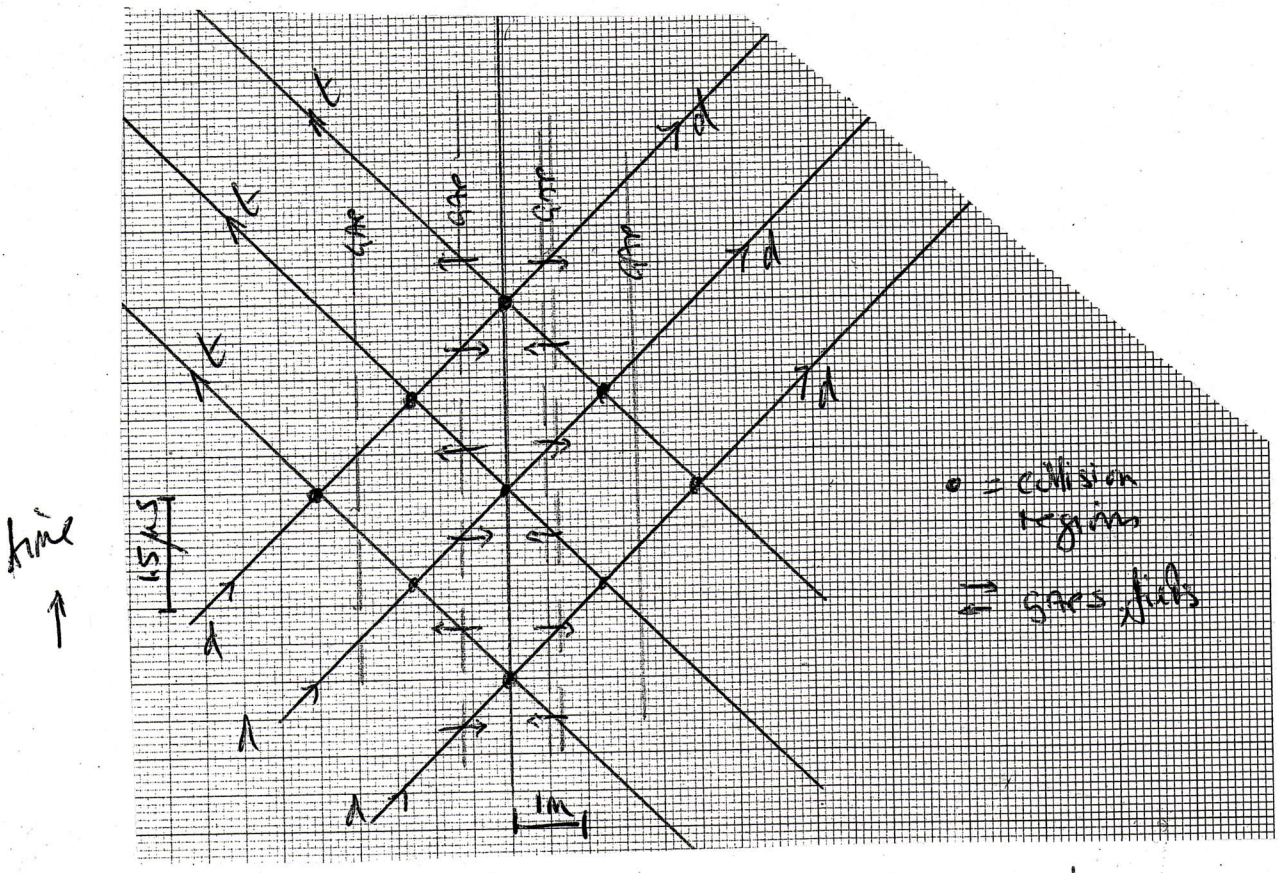




(like Foster VLHC magnets, but different gaps → diff. H.

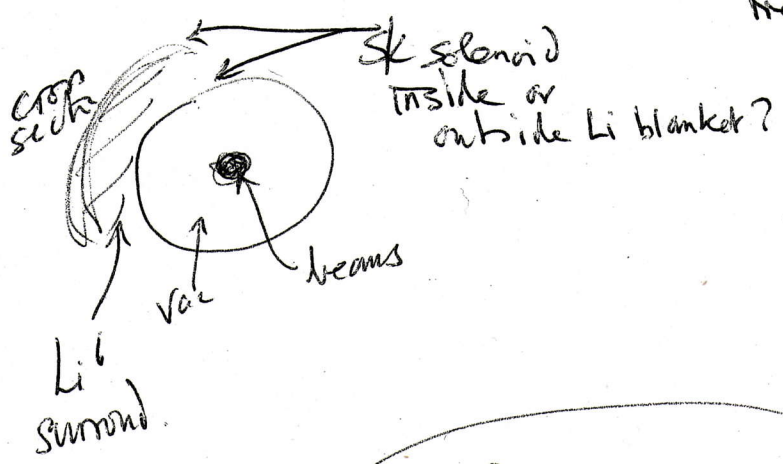
But can cross over X to compensate

Simultaneous "acceleration" of opposing +ve beams!



$\overrightarrow{z}$ for $\beta = v = 1.5\,m/\mu sec = \frac{dz}{dt}$

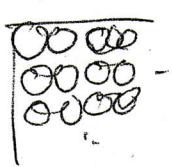
Want ~ lossless RF – power oscillates between cavities.
Frequency ~ 1 MHz, z-unit ~ 1 m.



If linear reactor parallel (compact) Could have both in same Li blanket.

- Could even have both in same solenoid? (Is $\vec{H}$ important

Can have arrays of these twin-pipes



say 30 × 30 = 1000

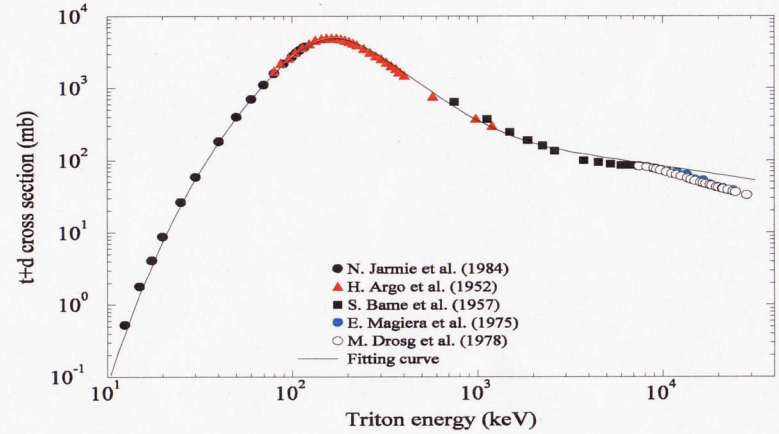
MW → GW.

S. Striganov

Coulomb scatterinig x-section in CMS

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\, m_d\, m_t}{m_d + m_t}\right)^2 \frac{1}{(1-\cos\vartheta)^2 p^4}$$

Maximum of fusion cross section at deuterium energy 110 keV or tritium energy 165 keV in lab system → momentum P =12.181 MeV/c in CMS

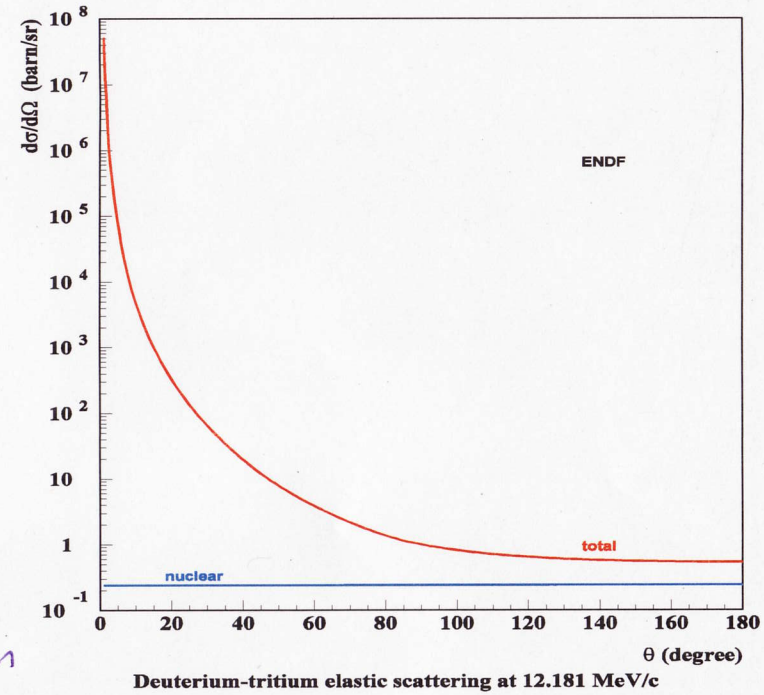


Near maximum Coulomb x-section

$$\frac{d\sigma}{d\Omega} = 1.192\left(\frac{12.181\,\text{MeV/c}}{p}\right)^4 \frac{1}{(1-\cos\vartheta)^2}\,\frac{\text{barn}}{\text{sr}}$$

X-section for angle ≥ Θ

$$\sigma = 3.75\,\frac{1+\cos\theta}{1-\cos\theta}\left(\frac{12.181\,\text{MeV/c}}{p}\right)^4 \text{barn} \approx \frac{15\ \text{barn}}{\theta^2}\left(\frac{12.181\,\text{MeV/c}}{p}\right)^4$$



$E_{CM} = 400$ keV

$\theta^* = 200$ mrad

$\sigma_f = 0.85$ barn

$\sigma_c(\theta > \theta^*) = 77.5$ barn

Particle-beam scattering

- 10^{14} per cm^3 in each of d or t beam
- Particle from one beam goes ***L*** cm through another beam. Probability of scattering to angle > Θ is

$$\frac{1.5\ 10^{-9}}{\theta^2}\left(\frac{12.181\quad \mathrm{MeV/c}}{p}\right)^4 L$$

15% of particles undergo scattering on Θ > 1 mrad after 100 cm

3/6/09 **ECO fusion.** V. Shiltsev

$\beta_e = 0.005$

$U_e = 6\,eV$

E. cooling vs ionization cooling:

Energy exchange / loss rates vary by factor

cooling enhanced

$$\frac{\text{e-cool}}{\text{ionz. cool}} = \frac{n_e \cdot l \cdot \frac{1}{(\Delta v/v)^2}}{n_{atom} \cdot l}$$

3 m

$$n_e \sim \frac{J_e \cdot \frac{l}{\beta c}}{l \cdot \pi R^2} = \frac{10\,(\cancel{A})\,\cancel{10^{-}}}{1.6 \cdot 10^{-19}} \cdot \frac{\left(\frac{3\,m}{0.4 \cdot c}\right)^{10 \cdot 10^{-8}\,s}}{3 \cdot 10^{2}\,cm \cdot 3 \cdot 0.2\,cm^2} = \frac{0.6 \cdot 10^{20} \cdot 10^{-7}}{4 \cdot 0.2} \quad 10^2 =$$

0.4×0.4 $\quad$ 1

$$= 0.8 \cdot 10^{11} = 8 \cdot 10^{10}\ 1/cm^3$$

in Lithium

$$n_e = \frac{6 \cdot 10^{22}\,atoms \times 4(e)}{12\,cm^3} = 2 \cdot 10^{22}\ 1/cm^3$$

$6g = 0.5\ g/cm^2$

$\boxed{V = 12\,cm^3}$

if

$$\frac{\text{e cool}}{\text{e-ionz cool}} = \frac{8 \cdot 10^{10}}{2 \cdot 10^{22}} \cdot \left(\frac{300}{0.3}\right) \cdot \frac{1}{(\Delta v/v)^2} = 4 \cdot 10^{-11} \cdot 10^{5} = 4 \cdot 10^{-6}\ !$$

10^3 $\quad$ 10^{-2}

10 kA D $v = 10^{-2}$

$U_k = 30\,MV$ we need to control e-beam to 1 eV

$\eta = 10^{-4} \cdot 10^{-6}$

$\theta = \frac{\Delta v}{v} \sim 0.1$

aperture of 200 mrad

fusion cross section @ 400 eV

$\sigma_f \sim 1$ barn

to scatter in 200 mrad $\sigma_{sc} = 80$ barns

10 T → 400 keV D-T has $r \sim 0.3$ mm

Y. S.
12/22/08

On ratio of ioniz. losses to d-t energy release

$$R = \frac{(dE/dx)_{ioniz}}{\Delta E_{nucl.}/cm} = \frac{(dE/dx)_{ion} \cdot \rho_D}{\sigma_{nucl} \cdot n_D \cdot \Delta E_{nucl.}}$$

density

1)

$$\left(\frac{dE}{dx}\right)_{ioniz} \approx \left(\frac{dE}{dx}\right)_{min} \times \left(\frac{m_t c^2}{E_{kin\,t}}\right) = 2\,\frac{MeV}{g/cm^2} \cdot \frac{3 \cdot 10^9}{2 \cdot 10^5} = 3 \cdot 10^4\,\frac{MeV}{g/cm^2}$$

3 GeV

200 keV

THIS $\frac{1}{\beta^2}$ factor in Bethe-Bloch

So total ioniz. loss for tritium:

@ 1 atm $(\rho = 0.17 \cdot 10^{-3})$ $\frac{dE}{dx} = 5 \cdot 10^3\ MeV/cm \cdot 10^{-3} = 5\ \frac{MeV}{cm}$

@ 10 atm $(\rho = 1.7 \cdot 10^{-3}\ \frac{g}{cm^3})$ $\frac{dE}{dx} = 5 \cdot 10^4\ MeV/cm \cdot 10^{-3} = 50\ \frac{MeV}{cm}$

2)

$$\Delta E_{nucl}/cm = \sigma \cdot n_D \cdot \Delta E_{released} = 5\ barn \left(5 \cdot 10^{20}\ \frac{d}{cm^3}\right) \cdot \underbrace{20 \cdot 10^6\ eV}_{20\ MeV} =$$

$$= 5 \cdot 10^{-24} \cdot 5 \cdot 10^{20} \cdot 20\ MeV = 0.05\ MeV/cm$$

So, the ratio:

$$\boxed{R = 10^2 \div 10^3}$$