Physics Processes in RF cavities filled with H_2

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3-24-2009

FNAL

This work is intended to provide some of the tools necessary for understanding the experiments that will soon be carried out in MTA and supported by an SBIR Grant with MuonsInc. This is a summary of the results from the following paper that will be shortly available:

Handbook for gas filled rf cavity aficionados'
A.V. Tollestrup, Moses Chung, Katsuya Yonehara
Version 1.0 2-19-2009

Many of the calculations in the above report have been done in Mathematica and I can make them available on request.

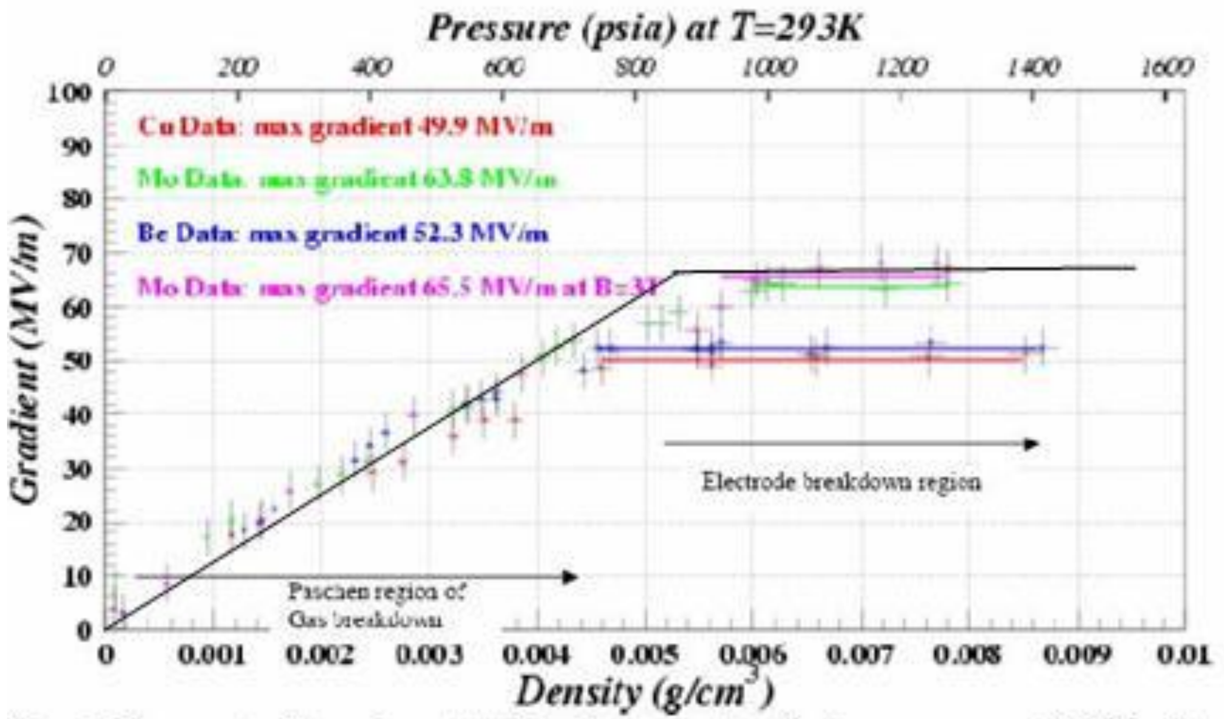


Figure 3: Measurements of the maximum stable TC gradient as a function of hydrogen gas pressure at 800 MHz with no magnetic field for three different electrode materials, copper (red), molybdenum (green), and beryllium (blue). As the pressure increases, the mean free path for ion collisions shortens so that the maximum gradient increases linearly with pressure. At sufficiently high pressure, the maximum gradient is determined by electrode breakdown and has little if any dependence on pressure. Unlike predictions for evacuated cavities, the Cu and Be electrodes behave almost identically while the Mo electrodes allow a maximum stable gradient that is 28% higher. The cavity was also operated in a 3 T solenoidal magnetic field with Mo electrodes (magenta); these data show no dependence on the external magnetic field, achieving the same maximum stable gradient as with no magnetic field.

P. Hanlet et al., “STUDIES OF RF BREAKDOWN OF METALS IN DENSE GASES” , PAC06, Knoxville, Tennessee. MuonsInc paper

Two Regions:

1. **Paschen Region**
2. **Electrode Region**

Paschen Region

$V_{breakdown} = f(E/p)$

Units: E/p in Volts/cm / P in mmHg or better

1 Td = 1 volt/cm /10^{17}molecules/cm^3.

Basic Physics: Double pressure and double E and the physics is the same. This is the linear region in 1st slide. As we cross the breakdown line, a Towsend Avalanche is generated: $n(x) = n(0)\,\mathrm{Exp}(ax)$

A free electron gains enough energy to ionize the hydrogen and the avalanche grows exponentially.

α/p vs E/p E=60 MV/m, rho = .005→ 56 atm = 42560 mmHg
Gives E/p = 14.1. This is the same as the DC value! Note on the curve below that α = 0 foe E/p< 14 indicating that no multiplication takes place.

If we α/p = 0.001 and p = 42560, then α = 42.5, or exponential length is

1/42.5 cm

Prob of ionization vs E_e The threshold for $e + H_2 \to H_2^+ + e$

15.3 eV

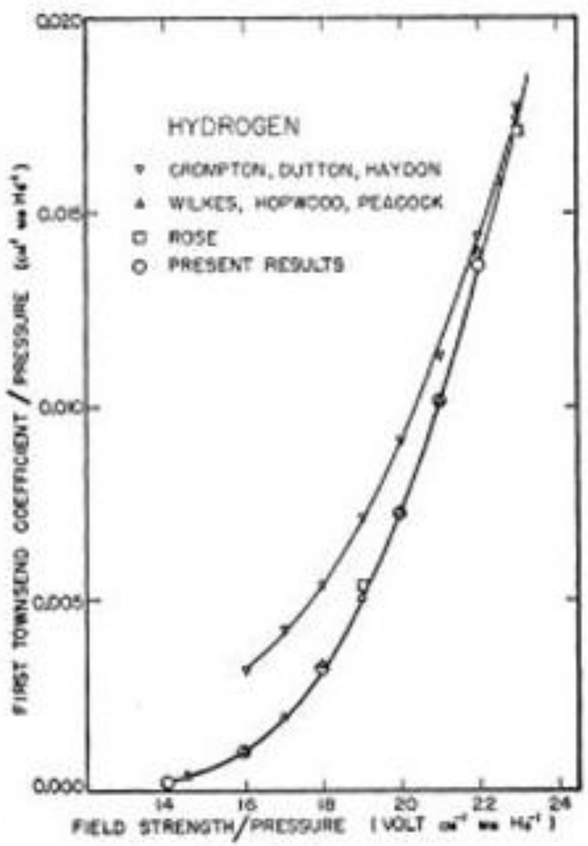
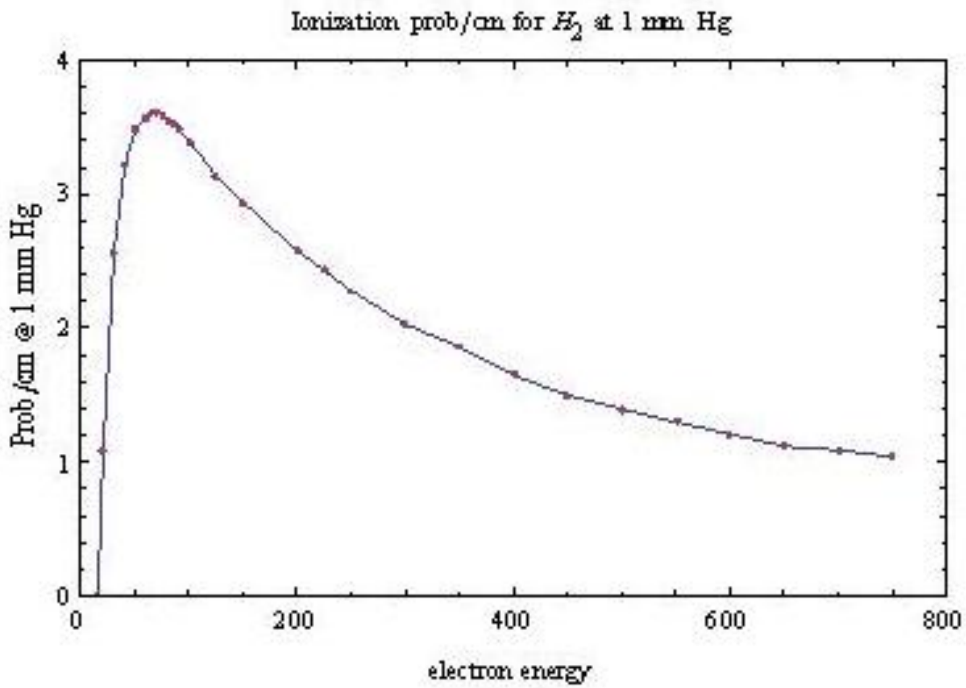


FIG. 4. α/p vs E/p for hydrogen.



2nd Towsend Coefficient, γ

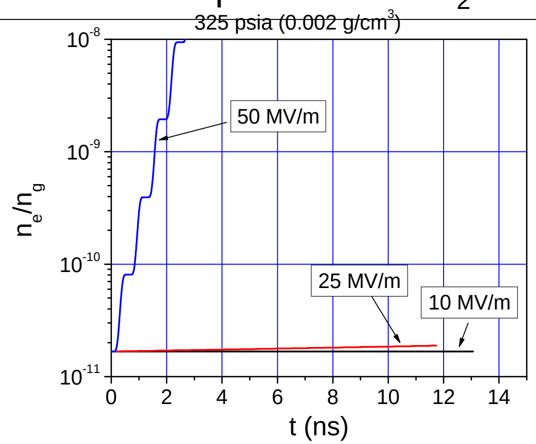
In addition to exponential growth, there is the possibility of feedback by photons or ions striking the cathode. This leads to the following equation for the growth of ions:

$$n(x) = n(0)\,\mathrm{Exp}(ax) / [1 - \gamma\,(\,\mathrm{Exp}(\alpha x)-1]$$

Note that since the exponential can be large, a very small γ can cause the denominator to go to zero and initiate breakdown.

Note: At 50 MV/m there is an increase of about a factor of 5 / ½ cycle of RF. This would lead to very fast break down. At 25 MV/m, at the edge of breakdown, there is slow growth with breakdown requiring many cycles.

0D simulations of RF breakdown are in agreement with experiments in H_2 at 0.002 g/cm^3:



Seed plasma population has a density of 10^{10} cm^{-3}, a very small fraction of the initial neutral gas density (6×10^{20} cm^{-3}).

The 25 MV/m simulation shows a very slow growth in electron density (red curve) and the 50 MV simulation (blue curve) shows an extremely rapid breakdown of the gas.

At 25 MV/m, breakdown is initially slow but finite (borderline Paschen level).

50 MV/m is well above Paschen level.

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See D. V. Ross. “Low Emittance Workshop, FNAL, April 21-25 2008”

Electron drift under an electric field

Consider an ensemble of electrons with a distribution of random directions and velocities, V_r. If we apply a field E, there will be a superposed drift velocity **v= μE.** This equation defines the mobility its derivation is as follows (crudely!)

$$v = \tfrac{1}{2}\, a\, t \;=\; \tfrac{1}{2}\, e/m\, E\, t \;=\; \tfrac{1}{2}\, e/m\, E \,\langle \lambda / V_r \rangle$$

$$\lambda = 1 / N \sigma$$

N is the density of molecules that the electrons are moving thru.

$$\mathbf{v} = \tfrac{1}{2}\, e/m \;\langle 1 / (N \sigma V_r) \rangle\; \mathbf{E} \;=\; \mu\, \mathbf{E}$$

.

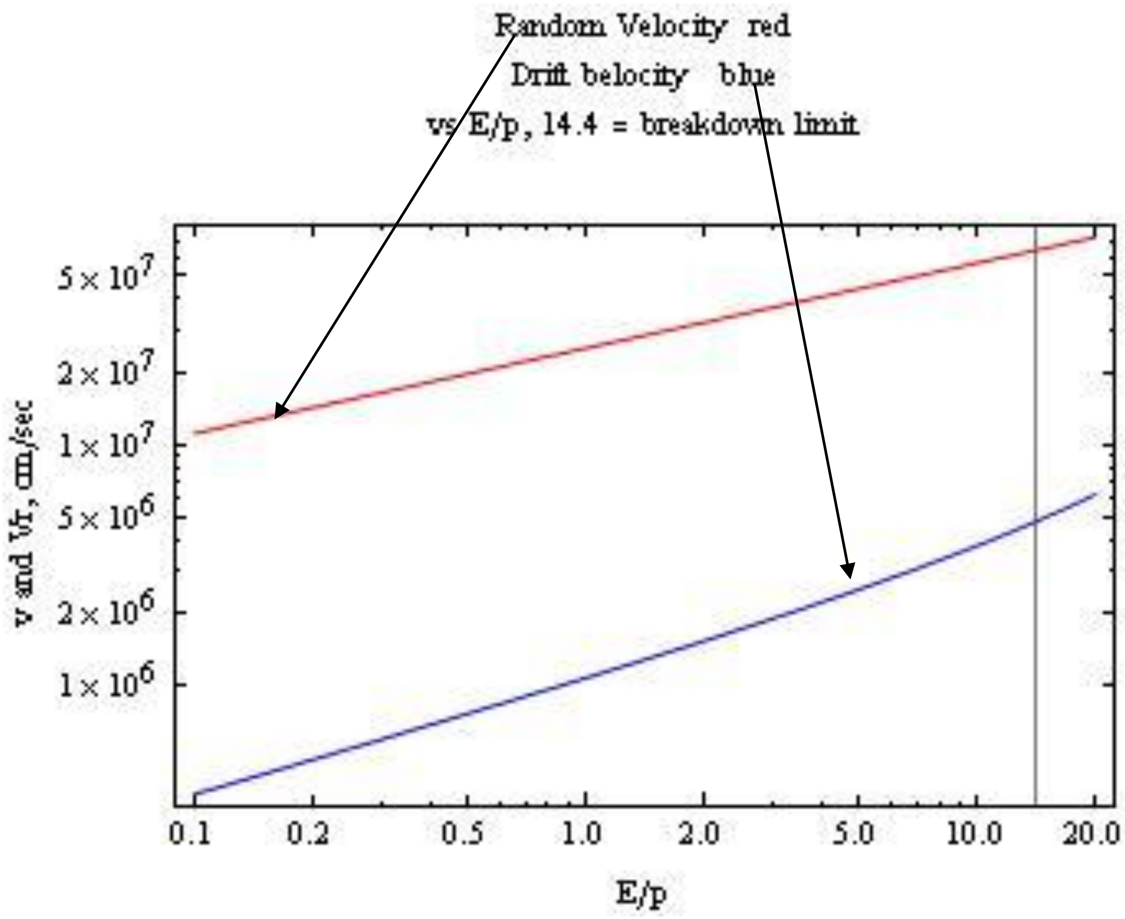
V_r. is determined by the temperature of the swarm. This temperature is set by the electrons gaining energy from the field and loosing energy be inelastic collisions with the gas molecules.

(a) $\epsilon_m = 0.357\, (E/P)^{0.71}$

(b) $\mu[E / P] = 1.72\; 10^{-2}\, [\, 1 - 2.4\;\; 10^{-2}\, (E/P)^{0.71}\,]^{-1.75}\, (E/P)^{-.53}$

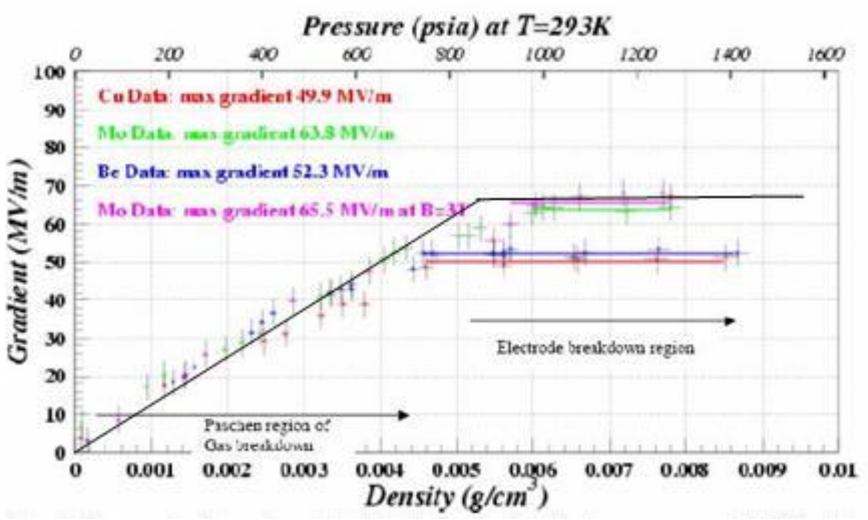
(c) $v[E/P] = \mu[E / P]\;\; E/P\; 5.93\; 10^{7}$ cm/sec where E/P is in V/cm/torr

A.E.D. Heylen "Calculated electron mobility in hydrogen" Proc. Roy Soc. 76, 779 (1960)



At E/p =14.1 the rms swarm energy is 2.33 eV

Plateau Region



Basic Facts:

1. Breakdown V independent of p
2. Breakdown V depends on metal
3. The break down gradient is similar to that observed in vacuum cavities. We will assume that field emission is taking place in the gas filled cavities in the same manner as has been observed in vacuum cavities
   See "J. Norem, et al. PRST 6, 072001-1, (2003).",

Three possible causes

Not all!

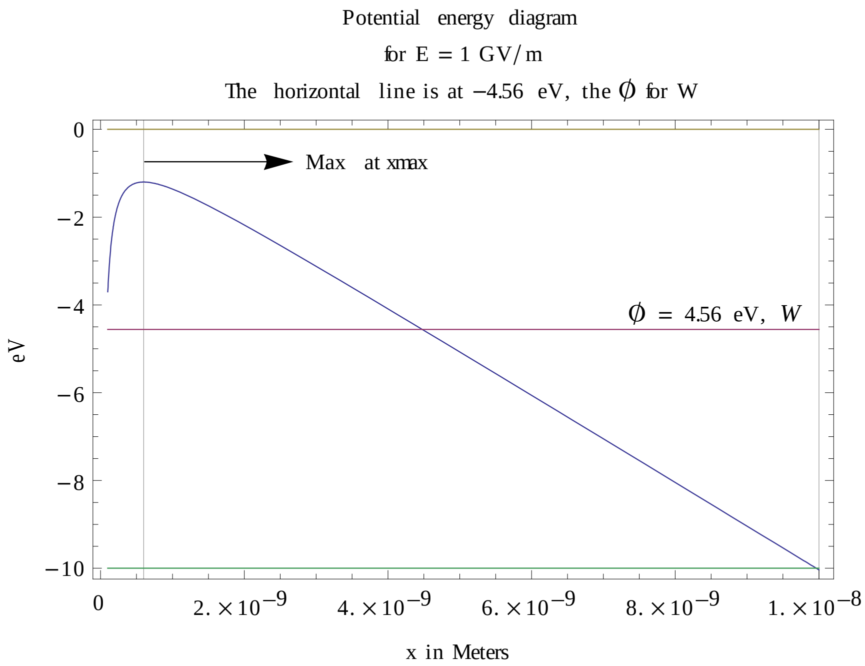
1. Field emission
2. RF heating
3. Surface failure du e high forces.
4. Many other possibilities have been proposed for vacuum cavities.

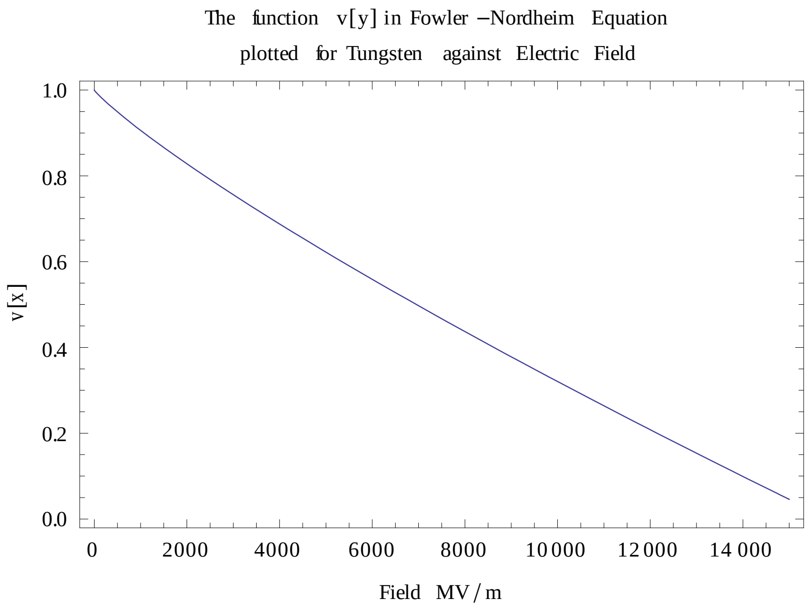
Field emission

$$j = A\frac{E^{2}}{t(y)\phi}e^{B\phi^{1.5}v[y]/E}$$

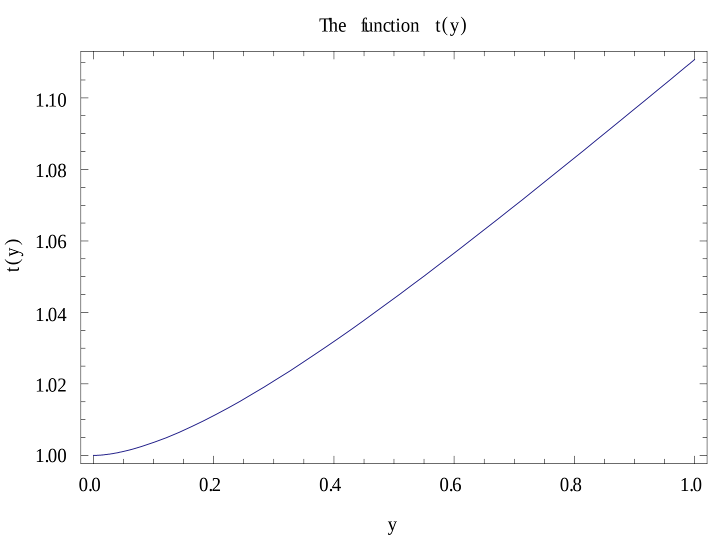
y = .0362 $E^{1/2}$ / f
j = current density in A/cm^2
A= 154   B= 6830
E is the DC field in MV/m, ϕ is the work function in eV
The functions t(y) and v(y) are shown below.

The functions t[y] and v[y] were not calculated correctly until 1953! See
H. E, Burgess and H. Kroemer, Physical Review, 90, 515, (1953)



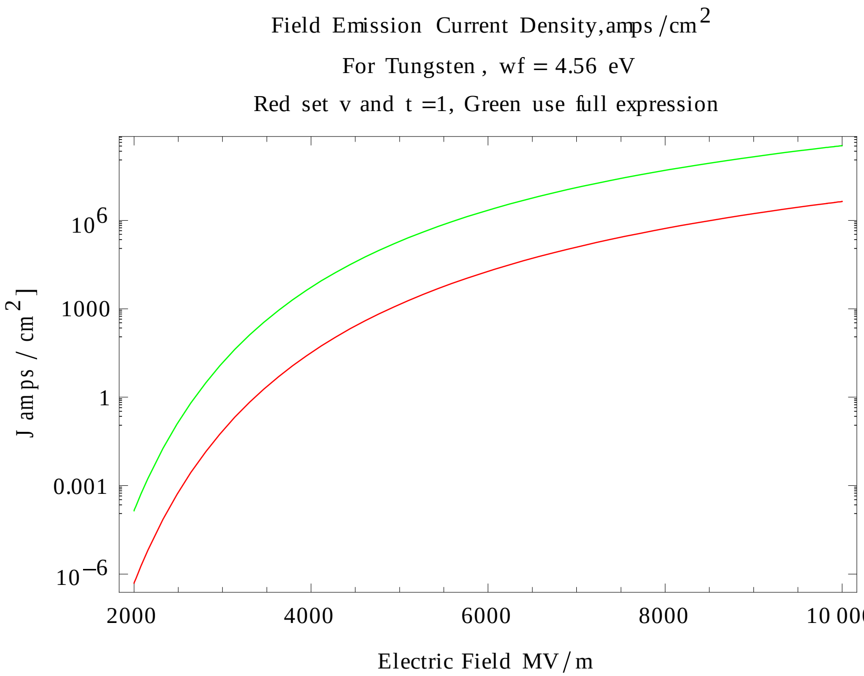


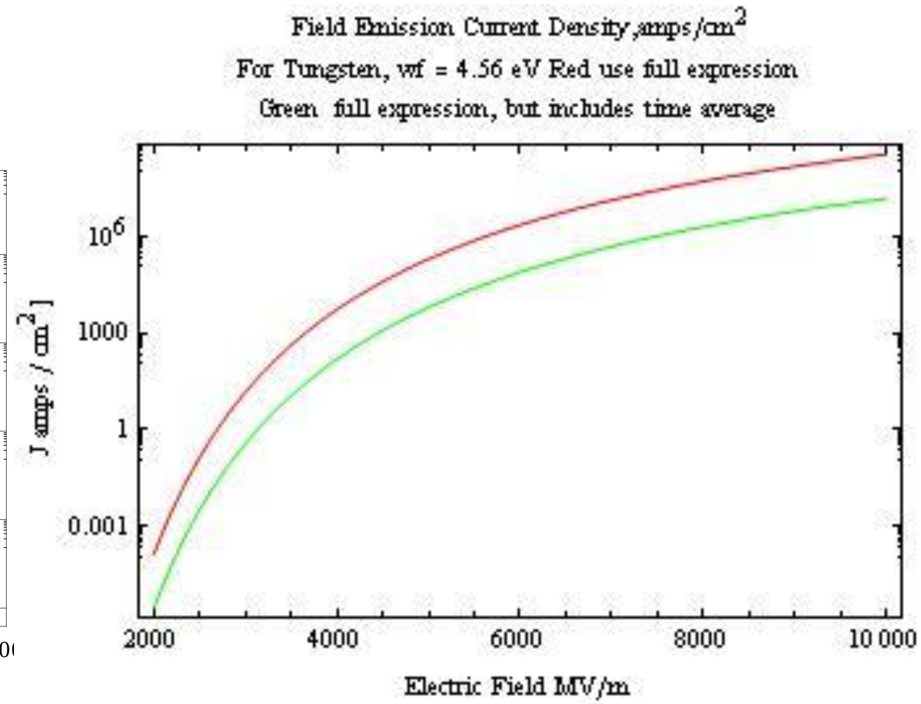
V[y] appears in exponent and has a large effect.



The function v[y] shown above for copper is important although many times is set to 1.0!. The function t[y] is also frequently ignored, but it's variation is rather small.

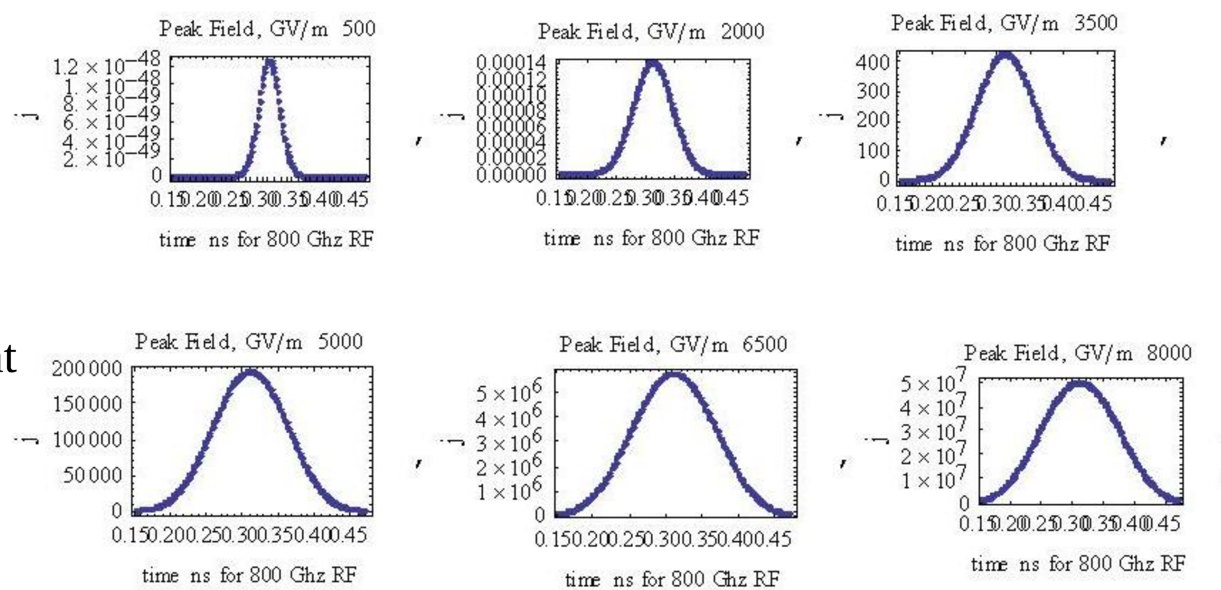
Note: Increasing E lowers the barrier and makes it also narrower.

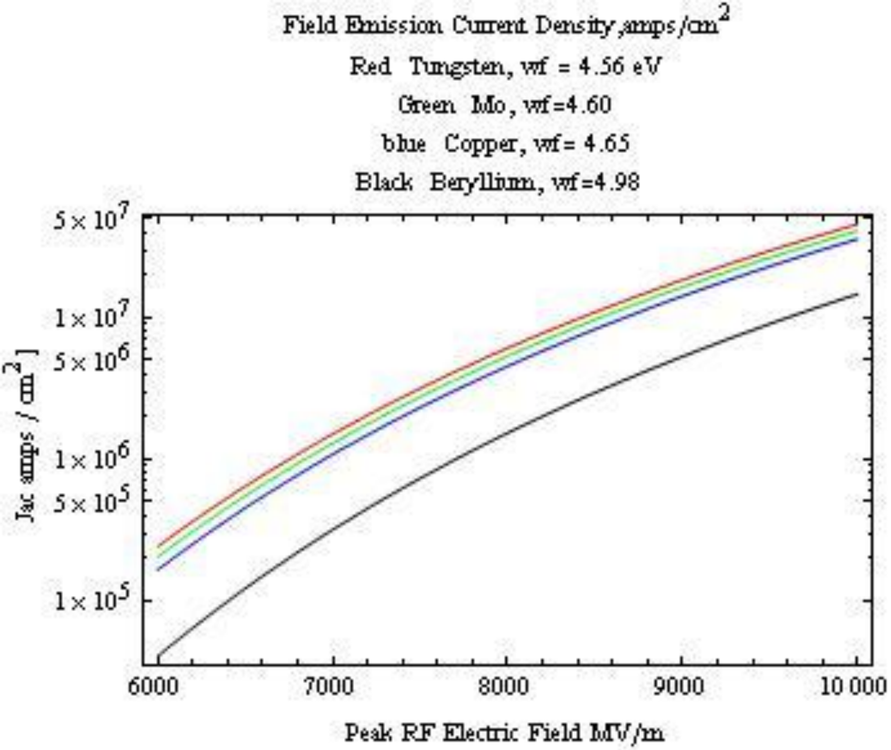


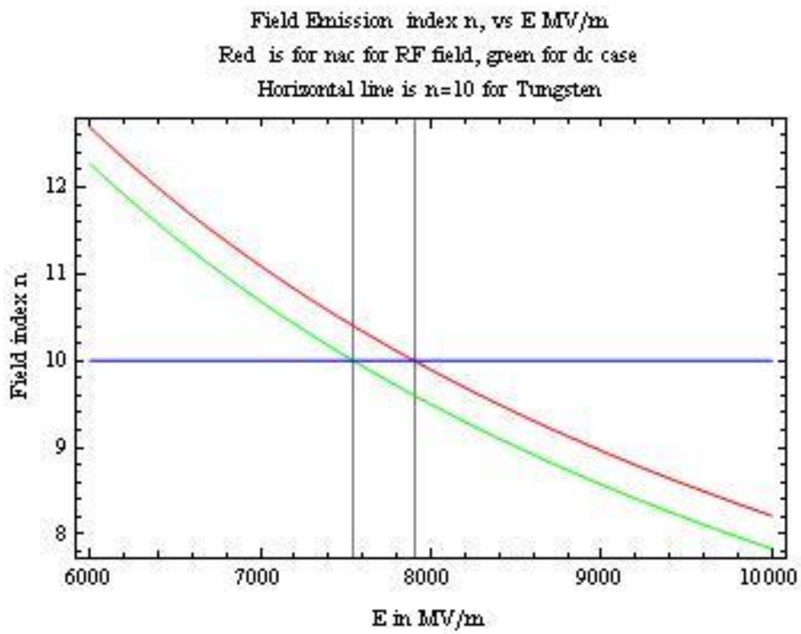


Above: Green is full expression
Red sets v,t =1

With RF, there is only emission at the very peak. The curve top, right shows comparison of DC and AC average current. Curve to right shows j[t] for 800 Mhz RF.





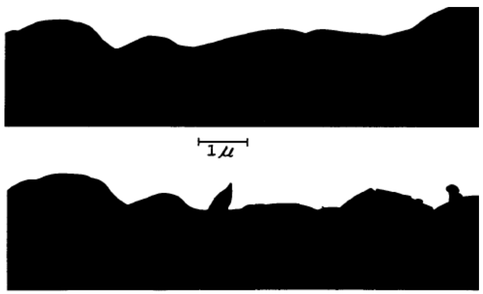


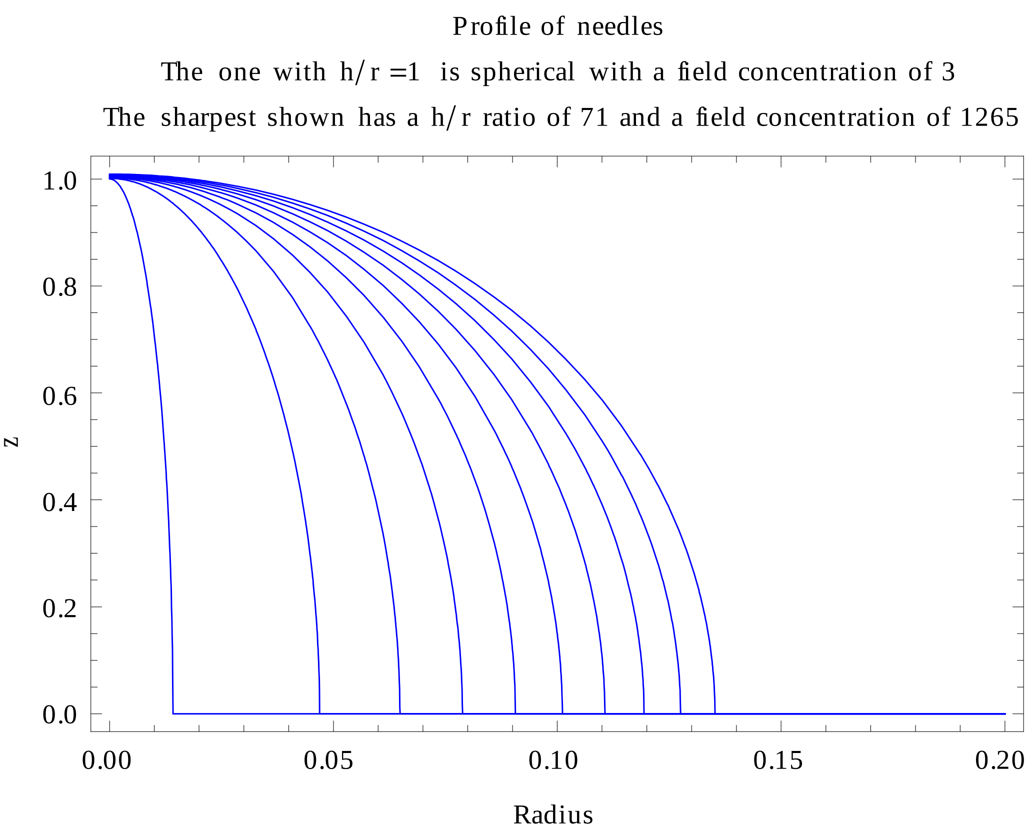
Field emission is a powerful tool

Left: jac vs field for W, Mo, Cu, and Be. Right: Log Derivative of jac for W.

1. J is hard to measure, but in vacuum cavity, the x-rays from dark current are easy to measure. Thus one can get the value of n at break down and since n varies with E, one has the local field at the emitter. It is many times bigger than the ambient field and allow us to get information about the emitter! E~ 8000 MV/m
2. Given E, one can calculate j.
3. This provides us a measure of the surface asperities for different metals and also a monitor during training. See previous reference to Norem.

Model for emitters





Prolate spheroidal coordinates give a solution of Laplace eq and give us a model to play with. One would guess that the local field is increased by sucking in lines of force from a circle or radius = emitter height. So the field would be enhanced by $(h/r)^2$. For the prolate spheroids, the factor is $0.5(h/r)^2$ for h/r >6.

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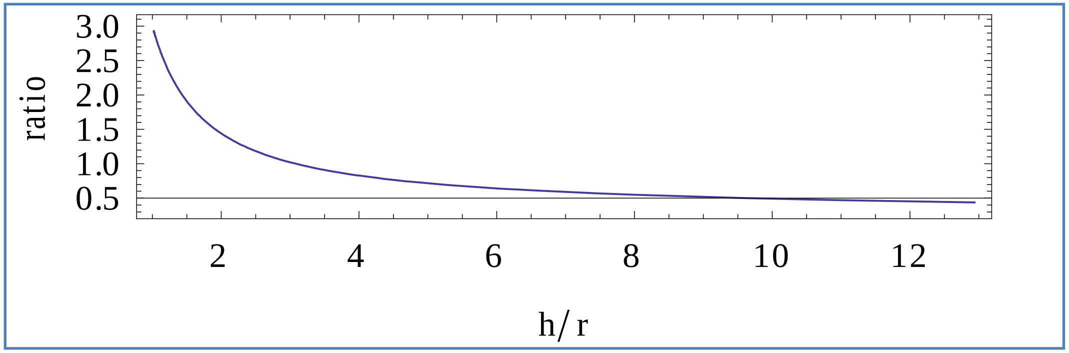
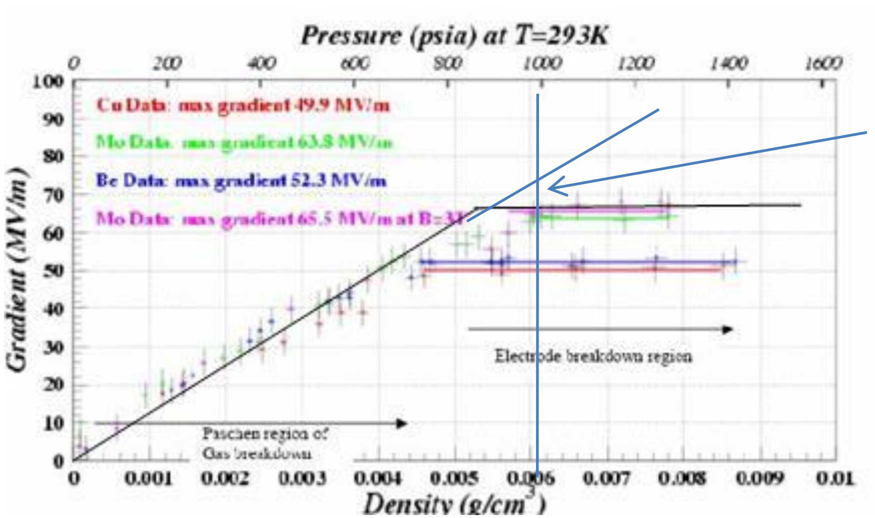
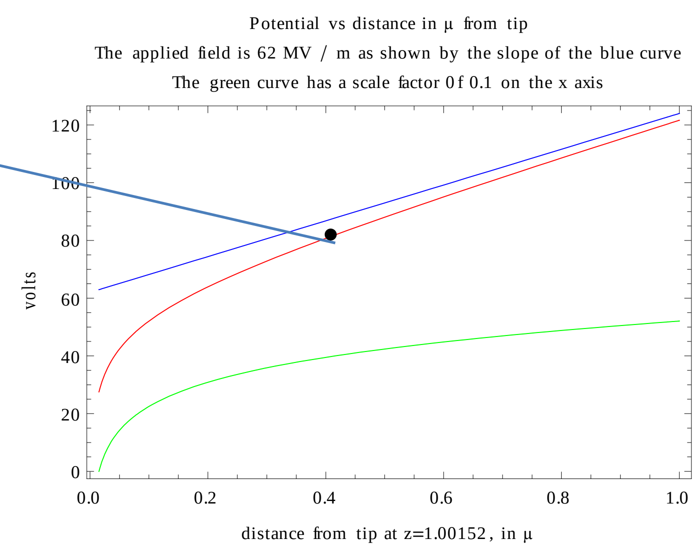


Example

1000 psia; 62M/m ; Tungsten Electrodes n=10, → E= 7900; → jac= 5.4 10^6 A/cm^2 . 7900/62 = 127 → h/r = 15.9 If we have a 1 micron high emitter, the radius is .063 microns. If we assume 10% of the emitter area is in the tip, we get a current of 0.2 mA and if it is emitted in one rf cycle, we get about 1,500,000 electrons injected into the hydrogen. Reduce the gradient by a factor 2.5 (25MV/m) and you get 1 electron/cycle. Maybe shouldn't expect light E<25MV/m.



Below avalanche



Evidence that field emission is involved in breakdown

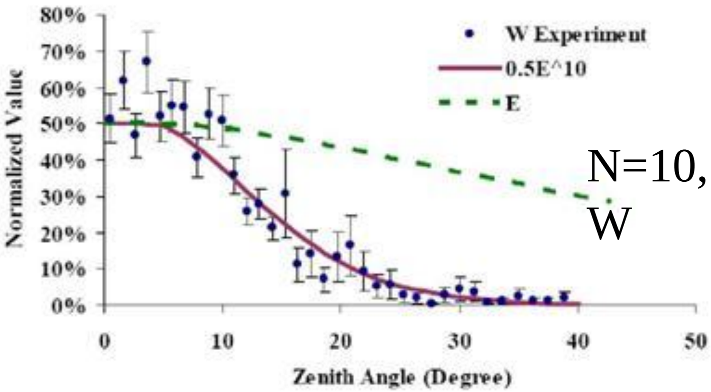


Figure 12: W breakdown area fraction vs. zenith angle.

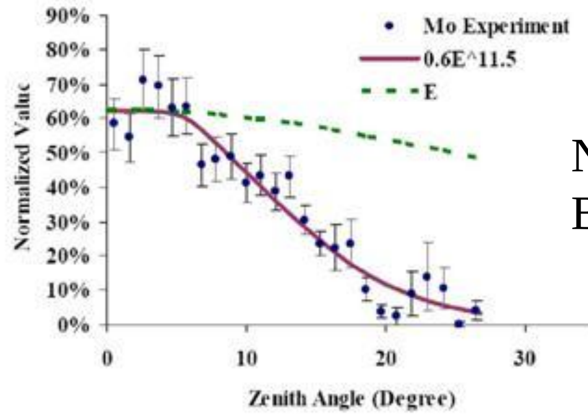


Figure 11: Mo breakdown area fraction vs. zenith angle.

N=11.5
Be

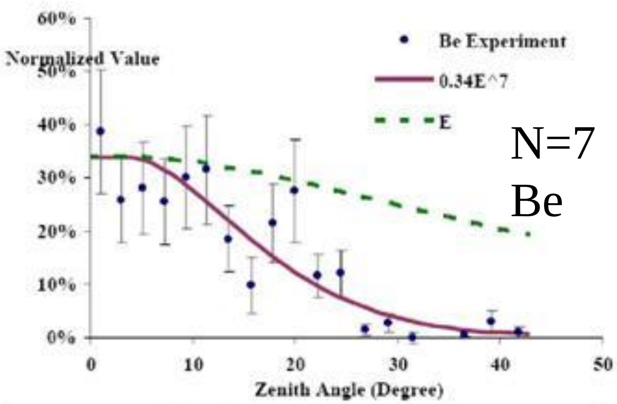


Figure 10: Be breakdown area fraction vs. zenith angle.

Measurement of the density of pits in the cavity electrode vs polar angle on the hemispherical electrode for three different electrode materials. The green line shows the variation of field and the solid line raises this field to a power n which is chosen to fit the data.

M. BastaniNejad et al, PAC07 Albuquerque, paper WEPMS071 (MuonsInc)

What causes break down?

1. Field emission injecting a large charge into the gas and triggering a gas streamer can't be the cause. The limiting voltage is independent of gas pressure and any phenomenon involving the gas would depend on pressure.
2. Maybe the field emission current heats the tip and the current increases by thermionic emission.

Melting electrode metal

dW = $j(t)^2$ / s(T) dt dV – Cooling = C(T) Rho dV dT

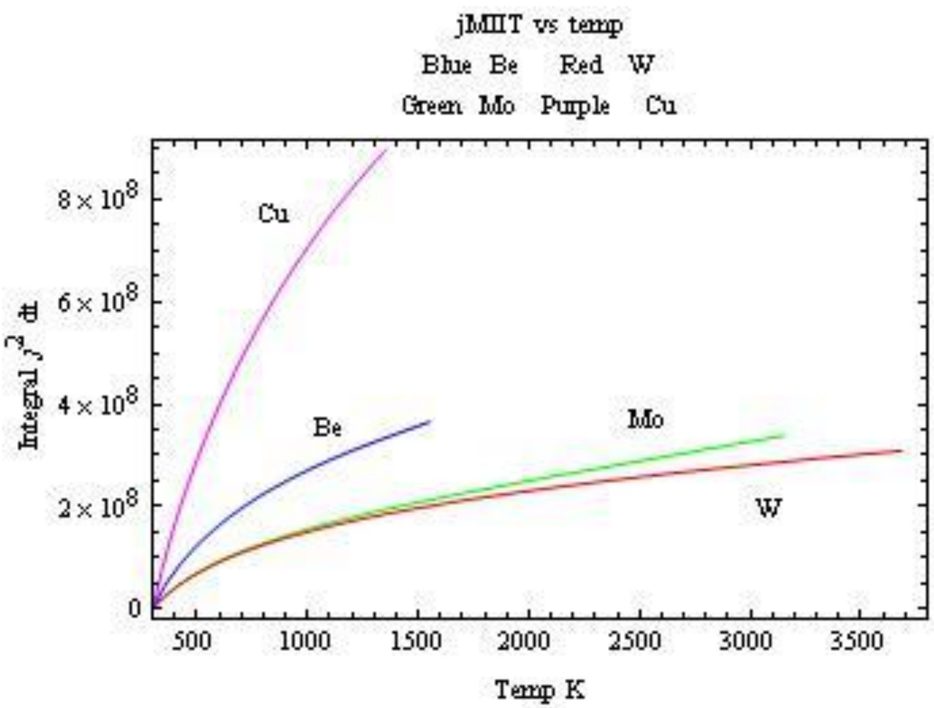
Rearrange and assume no cooling. Gives a limit.

$$\int_0^t j(t)^2\,dt = \int_{273}^{T} \frac{C(T)Rho}{\sigma(T)}\,dT$$

The heat capacity and conductivity are known functions of temperature. We integrate the above equation for Be, Cu, Mo and W up to the melting point. Since we have neglected cooling, we can set a lower limit on what the left hand side must be to achieve a given temperature.

How much

$$\int_0^t j(t)^2\,dt = \int_{273}^{T} \frac{C(T)Rho}{\sigma(T)}\,dT$$



Integral over one cycle

Gradient, MV/m	$\int jac^2 dt$
2000	1.17642×10^{-18}
2500	1.33457×10^{-12}
3000	1.59554×10^{-8}
3500	0.0000139495
4000	0.00235773
4500	0.132472
5000	3.42923
5500	50.3779
6000	482.857
6500	3326.64
7000	17658.8
7500	76011.
8000	275709.
8500	867963.
9000	2.42675×10^{6}

	Density	M.P.	Joules to M.P.	Heat of Fusion	Sum	$Int[j^2 dt]$
W	19.3	3683	11602.8	3705.6	15308.4	3.08907×10^{8}
Mo	10.2	3156	11064.	2927.4	13991.4	3.38976×10^{8}
Cu	8.96	1356	4295.26	1818.88	6114.14	8.96933×10^{8}
Be	1.84	1551	6944.55	2493.2	9437.75	3.65227×10^{8}

What causes break down? continued

1. Field emission injecting a large charge into the gas and triggering a gas streamer can’t be the cause. The limiting voltage is independent of gas pressure and any phenomenon involving the gas only would depend on pressure.
2. Maybe the field emission current heats the tip and the current increases by thermionic emission or positive ion bombardment causes run away heating.
3. Maybe RF surface currents?

   1. This is a good try. CLIC actually sees surface disruption at grain boundaries. Their frequency is higher and the surface current density is much higher than ours. It seems to be more of a fatigue plus heating effect. This was shown at the ANL High Gradient Conference.
   2. The surface currents go to zero just where our breakdowns are occuring.

What causes break down? continued

1. Field emission injecting a large charge into the gas and triggering a gas streamer can't be the cause. The limiting voltage is independent of gas pressure and any phenomenon involving the gas would depend on pressure.
2. Maybe the field emission current heats the tip and the current increases by thermionic emission or positive ion bombardment causes run away heating.
3. Maybe RF surface currents.
4. Disruption of the surface from the large electrostatic forces. This is being pursued by Norem for the vacuum case. I suspect that it is the solution for the gas filled case.

QUESTIONS:

1. Are the forces large enough to pull out the atoms?

Metal	F^+	F^{++}
W	1190	530
Re	980	460
Ta	1100	450
Mo	795	388
Nb	800	342
Ir	955	541
Pt	761	488
Zr	700	345
Au	570	505
Fe	490	352
Ni	445	385
Cu	410	475
Zn	383	388

Field Strength for this table is in MV/cm

These fields are about 100 times greater than the fields we have been considering.

The table below shows the tensile strength of some materials and the equivalent electrostatic field strength to achieve this stress.

Material	Yield MPa	Equivalent E, GV/m
Be	350 415	{8.9, 9.7}
Al	15 20	{1.8, 2.1}
Cu	33	2.7
SS	520	10.8
Mo	550 1150	{11.1, 16.1}
W	550 620	{11.1, 11.8}

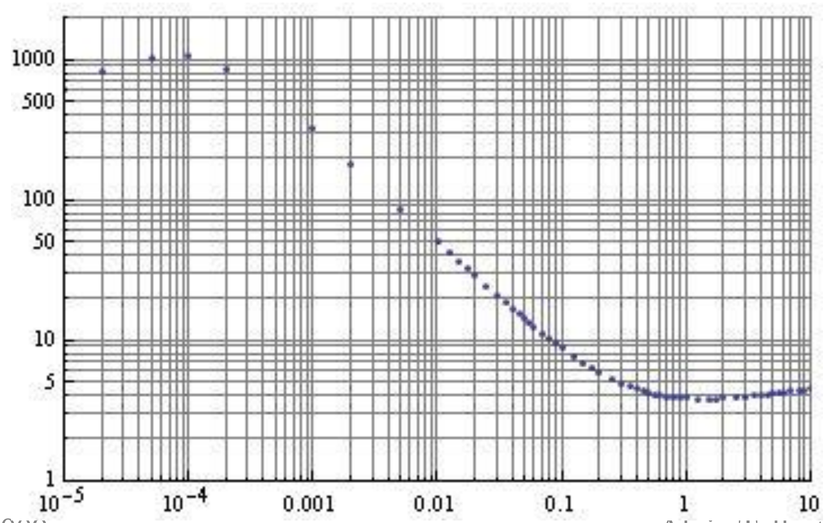
A simple argument

Consider the previous example with a 1 micron high emitter with a radius of .063 microns. All of the field lines that wind up on the emitter come from a circle far away with an area of 0.5 Pi h^2 with a total force of ε_0 E^2 (Pi 0.5 h^2). The base of the emitter has an area of Pi r^2 so the stress is given by Stress = ε_0 E^2 (0.5 $(h/r)^2$) . So we just use the effective field given by the field emission calculation to get the stress in the emitter.

The physics of what happens with a broken piece of emitter in a gas filled cavity has yet to be written!

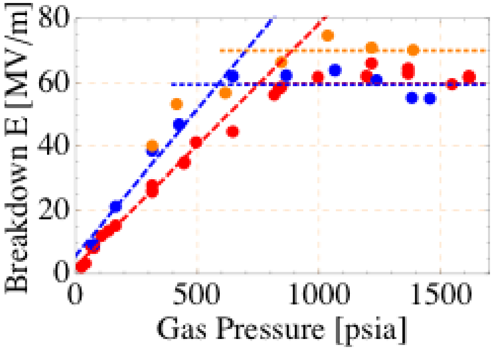
What causes break down? continued

1. Field emission injecting a large charge into the gas and triggering a gas streamer can't be the cause. The limiting voltage is independent of gas pressure and any phenomenon involving the gas would depend on pressure. No
2. Maybe the field emission current heats the tip and the current increases by thermionic emission. No.
3. Maybe RF surface currents. Doesn't seem likely at our gradients and frequencies.
4. Disruption of the surface from the large electrostatic forces.   Good idea!
5. Run away electrons. The emitter injects a bunch of electrons into the gas and the force from the electric field is greater than the dE/dx force from collisions in the gas. Doesn't work.   Electrons must be injected with energies greater than 3 KeV before the dE/dx is smaller than the breakdown E along the Paschen line (E/p = 14.1).



dE/dx for electrons in H_2 with density normalized to 1 grm/cm^3. The Horizontal scale is in MV and the vertical scale is in MV/cm.

Cu electrode

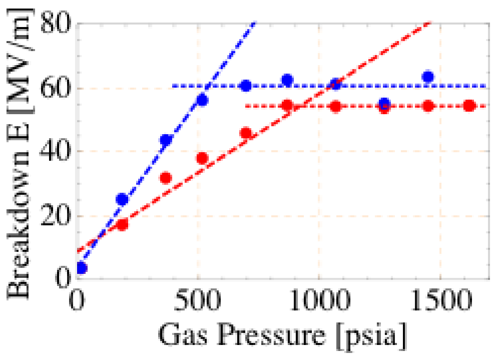


H2+SF6 (0.2 %) (E_{max}=70.0 MV/m)

H2 (E/P = 0.0762 MV/m/psia, E_{max}=60.0 MV/m)

H2+SF6 (0.01 %) (E/P = 0.0919 MV/m/psia, E_{max}=59.4 MV/m)

Al electrode



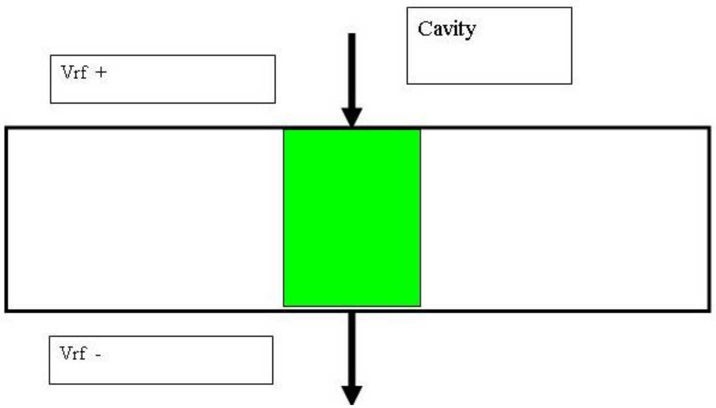
H2+SF6 (0.01 %) (E/P = 0.105 MV/m/psia, E_{max}=60.7 MV/m)

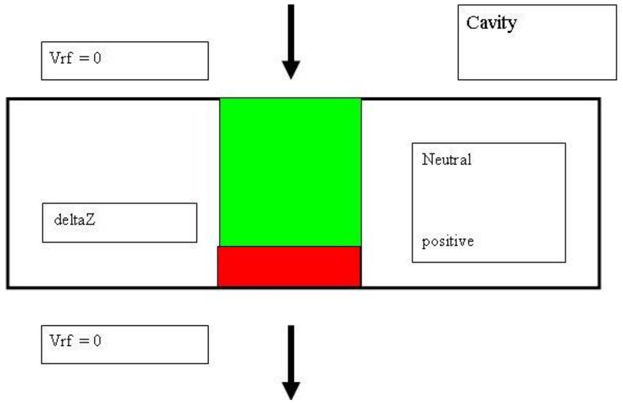
H2 (E/P = 0.0494 MV/m/psia, E_{max}=54.3 MV/m)

From Yonehara, Last run MTA

Material	Be	Cu	Mo	W
n	7.	12	11.5	10
wf	4.98	4.65	4.6	4.56
Max Grad	52.3	49.9	63.8	72

Let there be beam





The left figure shows the cavity just after a delta function beam has passed thru at peak field. The green represents ionized hydrogen left behind. The left figure shows ¼ cycle later. The electrons have drifted up leaving behind a layer of positive ions.

Some cavity numbers using an example takne from the LEMC

P= 200.` Bo=10.` betaPerp= 0.133

emit= 0.000422 minEmit= 0.000285

Nbeam= $1. \times 10^{11}$ rhoGas= 0.016`

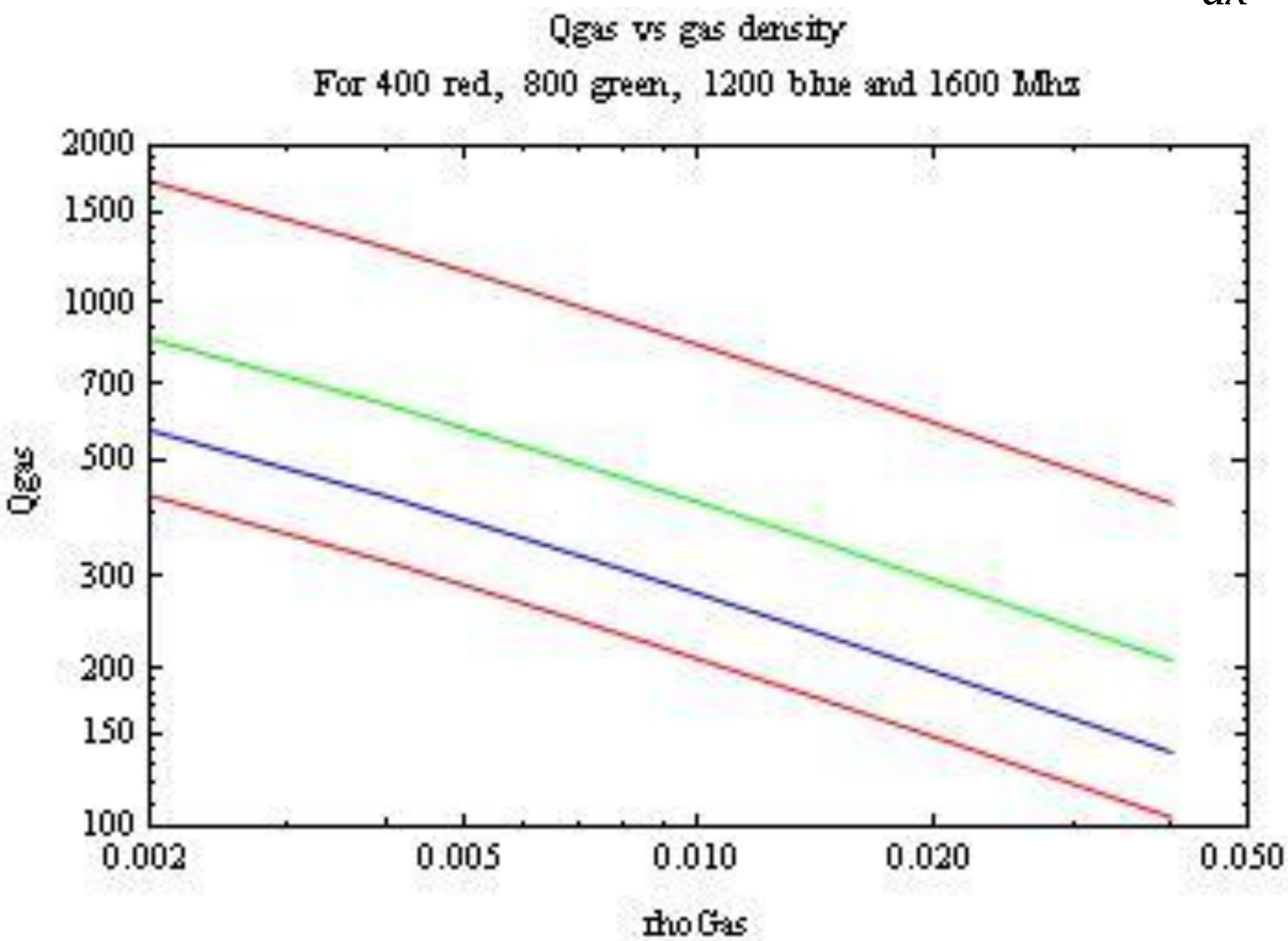
RF Gradient V/m =$16. \times 10^{6}$,Frf Hz= $800. \times 10^{6}$ h cm =$\frac{5}{2}$,

1	beamRadius, cm	0.750729
2	H molecule density	4.8176×10^{21}
3	av. molecule Spacing in microns	0.000592191
4	muons/cm^{2} =	5.64785×10^{10}
5	Averge μ spacing microns	0.0420783
6	Radius 2 eV electron, Bo T field, microns	0.476562
7	spacing between ions along track, microns	5.46875
8	positive ion density/cm^{3}	1.03275×10^{14}
9	plasma Frequency	5.7291×10^{11}
10	area bohr orbit	8.79146×10^{-17}
11	EoverP, V/cm/torr =	1.17481
12	Mobility=	0.0165644
13	electron velocity cm/sec=	1.15416×10^{6}
14	deltaZ, cm	0.000275821
15	Charge on end of cavity	1.54159×10^{13}
16	Plasma Charge/cm x deltaZ/2	5.04358×10^{10}
17	plasma Charge/beam charge	0.504358
18	plasma + beam charge/cavity charge	0.00975851
19	Gas loss/cycle/cm cavity length	0.00375996
20	Qgas, 1/Q = 1/Qcav + 1/Qgas	329.742
21	Total Plasma Charge	4.57143×10^{14}
22	RF power to plasma,MW	0.0585143
23	Peak energy stored in cavity	0.493307

Table produced by Mathematica

Cavity Q reduced by loss to electrons moving in the gas.

$$deltaW = \oint i[t]E[t]dxdydt = \oint (v[E_{rf}Sin[\omega t]/P]N_{beam}\rho_{gas}\frac{dE}{dx}e/wip)(.01E_{rf}Sin[\omega t])dt$$



10^{11} Muons
400 MHz pill box
5 cm long cavity
Above dimensions scaled with frequency. See Table slide 25.

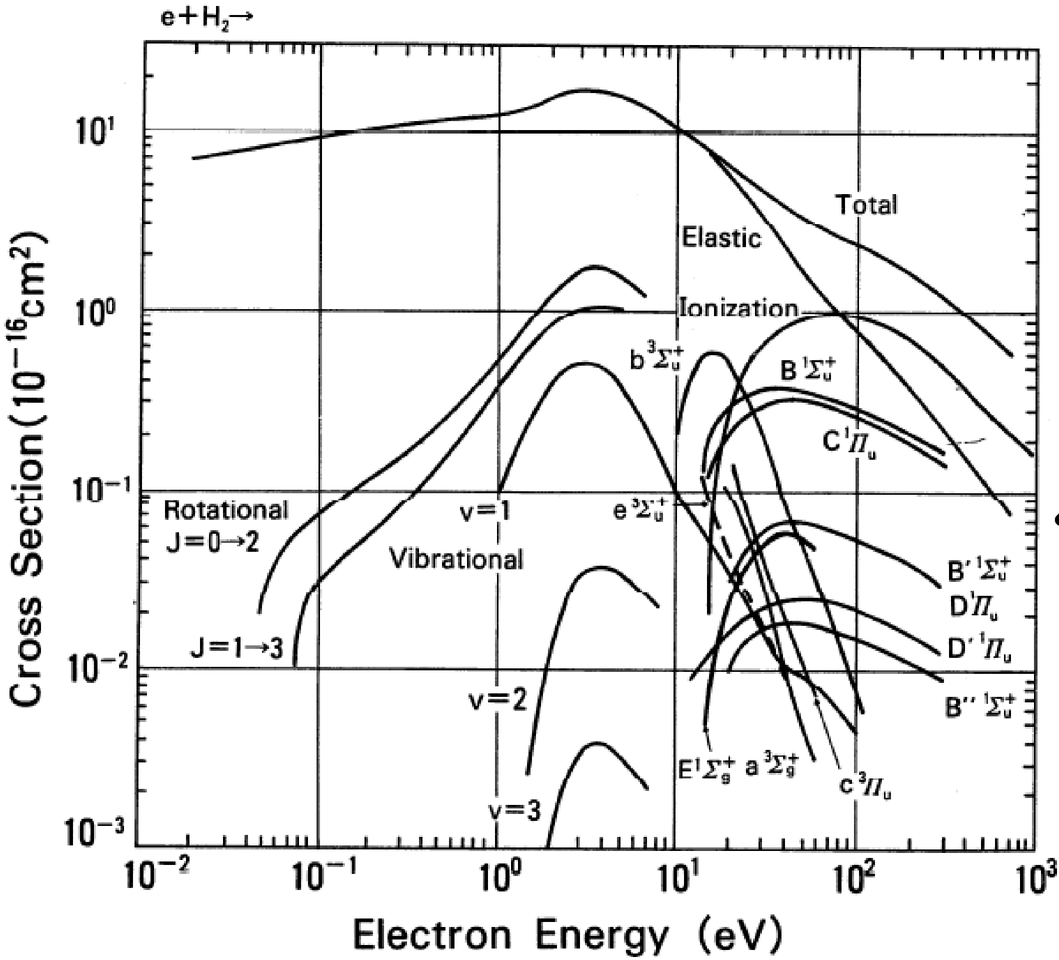
Details of cavities on last slide

f	400	800	1600
Q	12886.3	9112.	6443.16
Power	770016.	272242.	96252.
mJ/cycle	1.92504	0.340302	0.0601575
Stored Energy	3.94811	0.493513	0.0616892
Qgas 1 bunch	10991.4	5495.7	2747.85
Qgas 16 bunches	659.484	329.742	164.871
TotalPower@10^{11}	1.577×10^{7}	7.79224×10^{6}	3.85625×10^{6}

Plasma Problems

1. The free electrons transfer energy from the field to the gas and lower the Q. If the electron can be eaten by a heavy molecule, it doesn't absorb energy
2. Something needs to remove the ions before the next cycle. There is a huge amount of stored charge. 10^{11} muons going thru 5 cm of .016 grms/cm^3 H_2 generates 170 micro C. of + and – charge that must either neutralize or be swept out by some mechanism.

Hydrogen ion chemistry



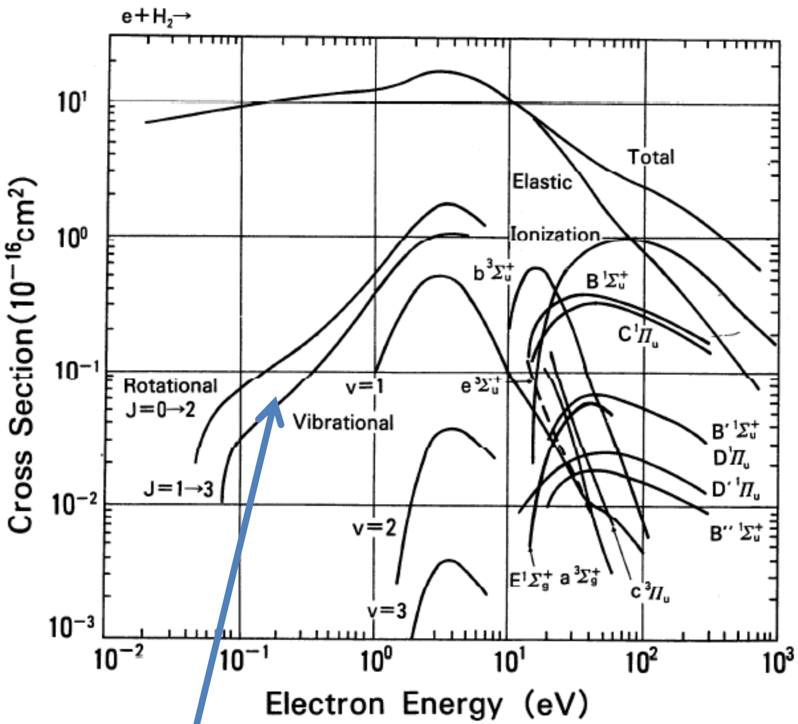
$$\frac{dN_1}{dt} = RN_1N_2$$

$$R = <\sigma V_1>$$

$$\varepsilon_e = 0.357\left(\frac{E}{p}\right)^{0.71} = 1/2m_eV^2$$

Question 1: We used the electron velocity to calculate the RF cavity losses. Is this correct? Does the velocity follow the RF voltage?

What is the relaxation time of the electron and does it get eaten? It has an average energy of 0.2



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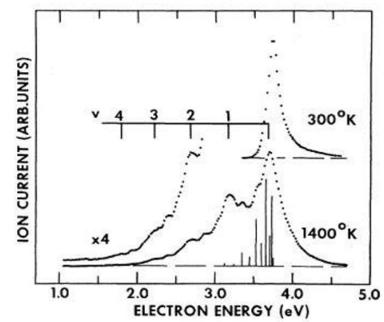

FIG. 1. Threshold region of H$^-$/H$_2$ cross-section curves at 300 and 1400°K. Note that in the 1400°K curve, peak intensities from excited molecules are much larger than expected from the vibrational population (e.g., 1.4% for $v=1$) and rotational population (vertical bars), reflecting a drastic increase of cross section with vibrational and rotational quanta. The vertical lines ($v=0-4$) indicate expected peak positions for rotational-vibrational profiles at 1400°K.

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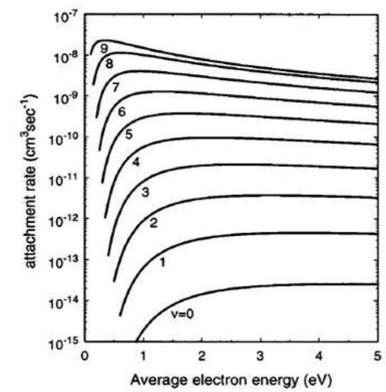
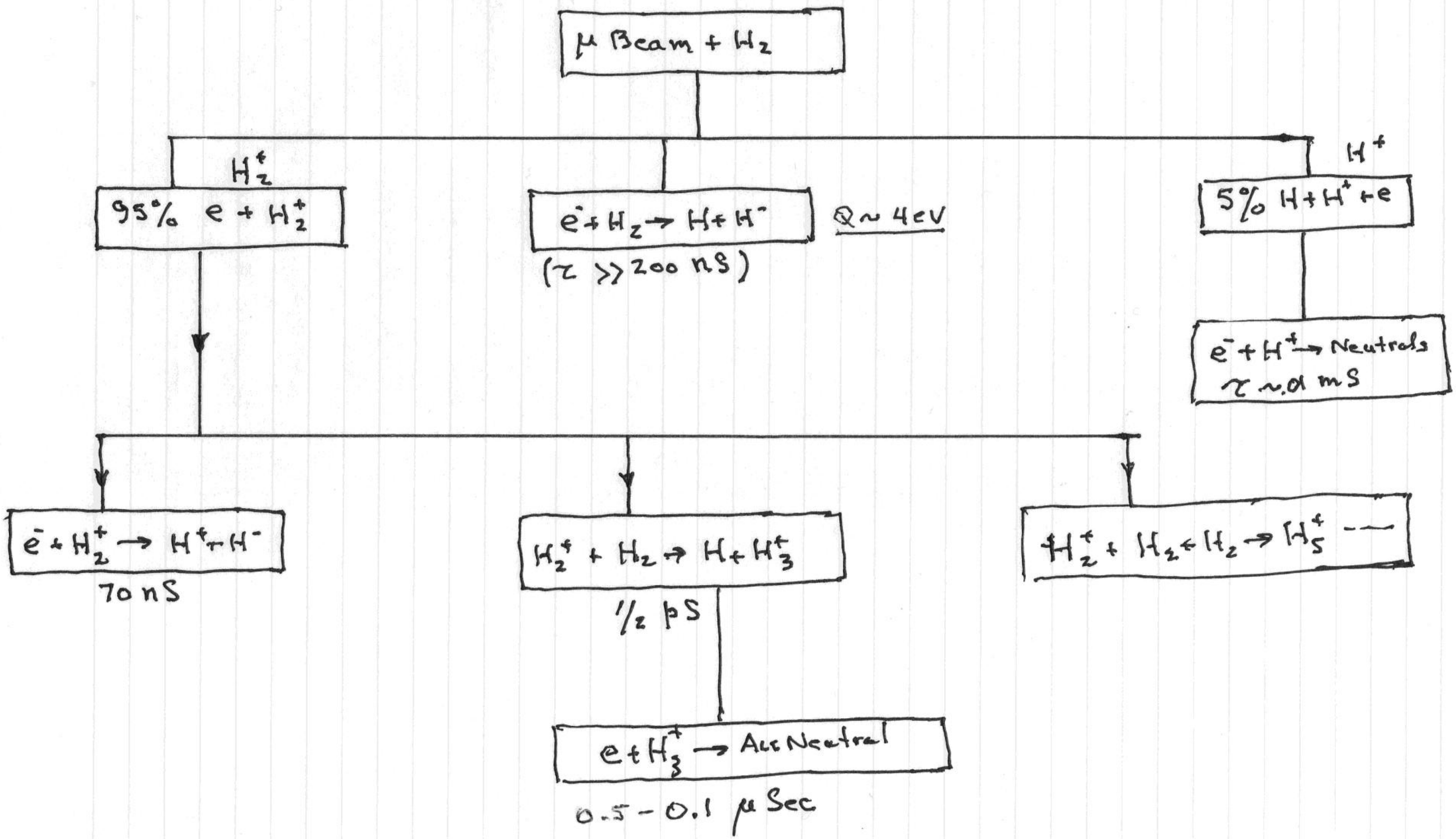


FIG. 2. Attachment rates for vibrational states of H$_2$ with $v\leq 9$ and $j=0$.

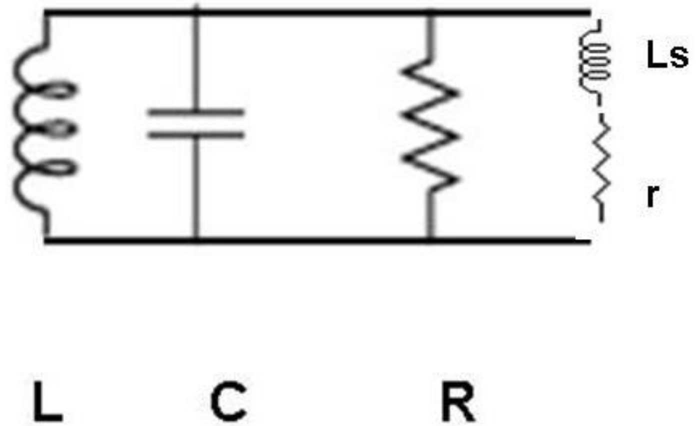
e+ H_2 → H^- + H eats electrons, but Q is about 4 eV and the cross section is very dependent on the hydrogen molecule temperature.

The inelastic rotational collisions damp the energy faster than delta E/E = 2 m_e /M_{hmol} . From the above cross sections, we calculate a relaxation time of about 1 ps which is much shorter than the RF period.

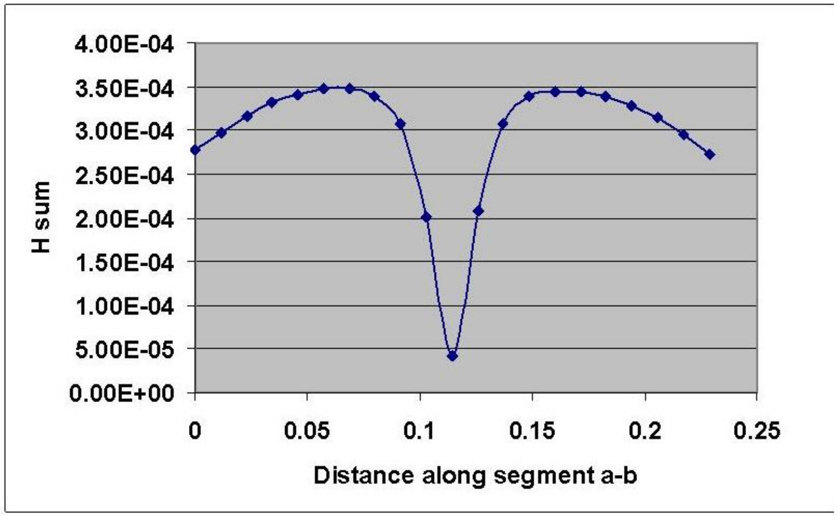
Hydrogen ion chemistry

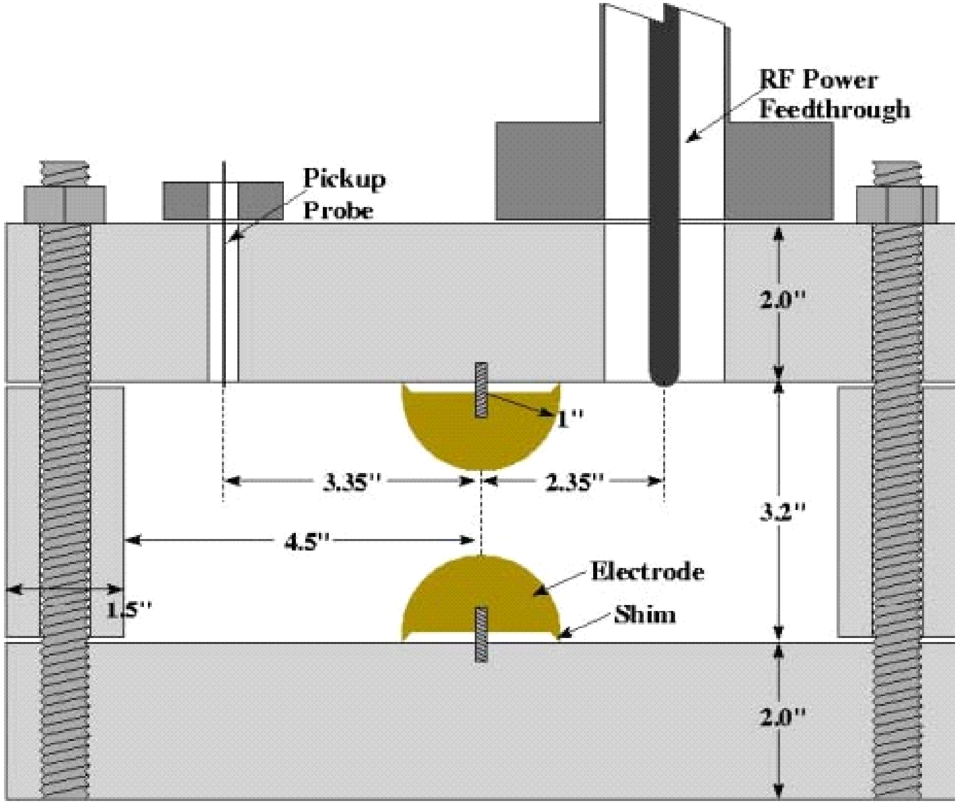


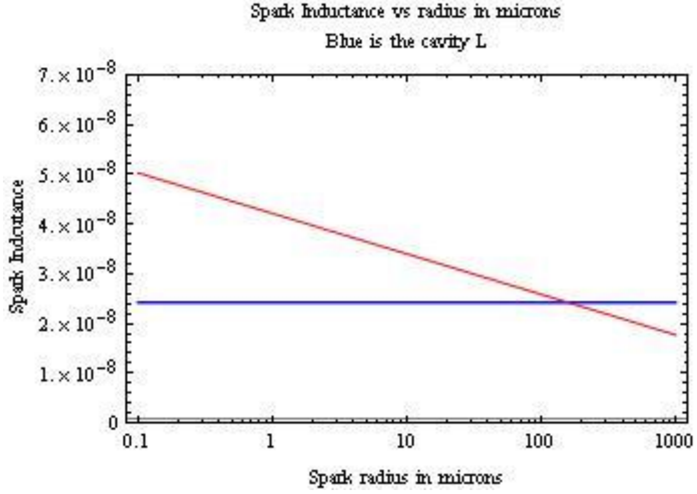
The discharge



$$Ls = \frac{\mu_o}{2\pi} h Log[r1/r2] + \frac{\mu_o}{8\pi} h$$



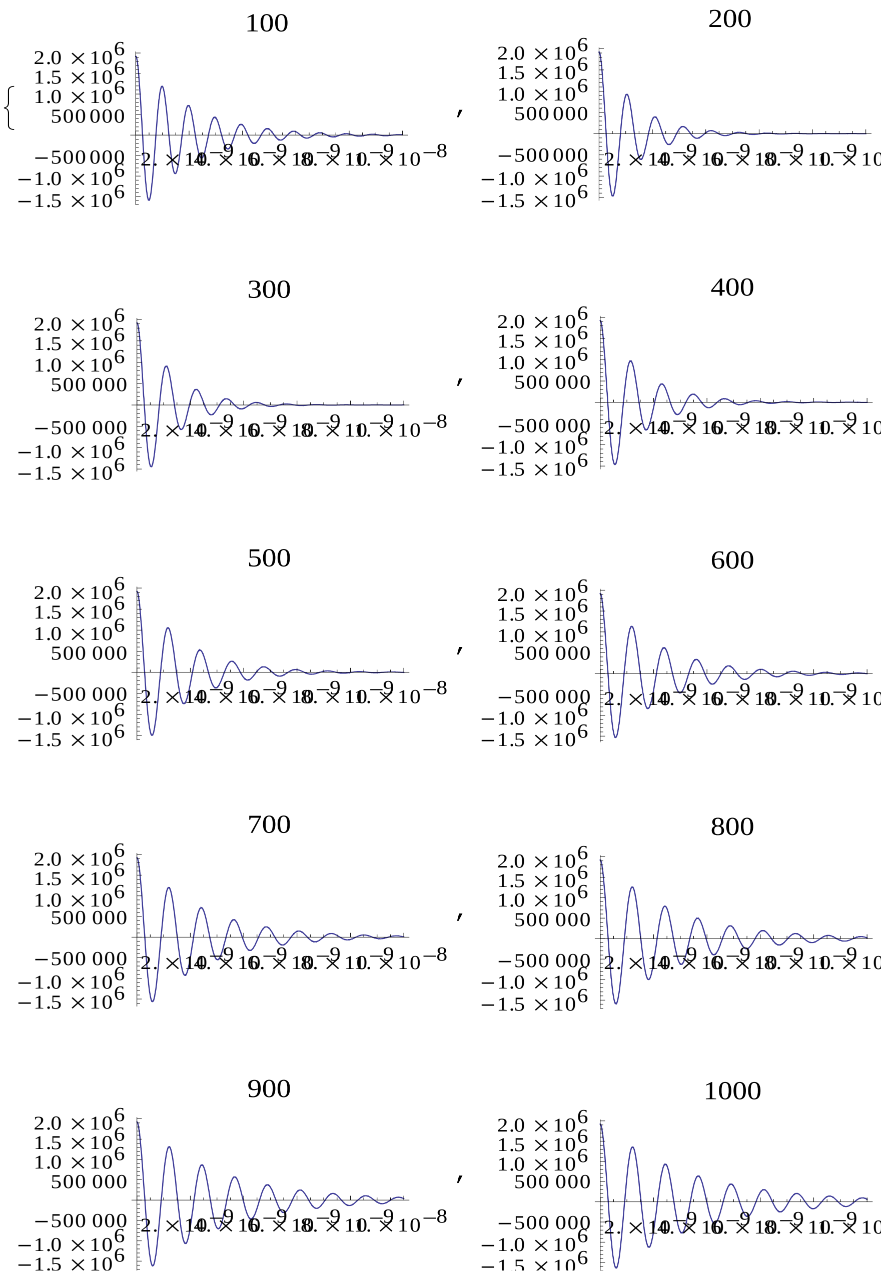


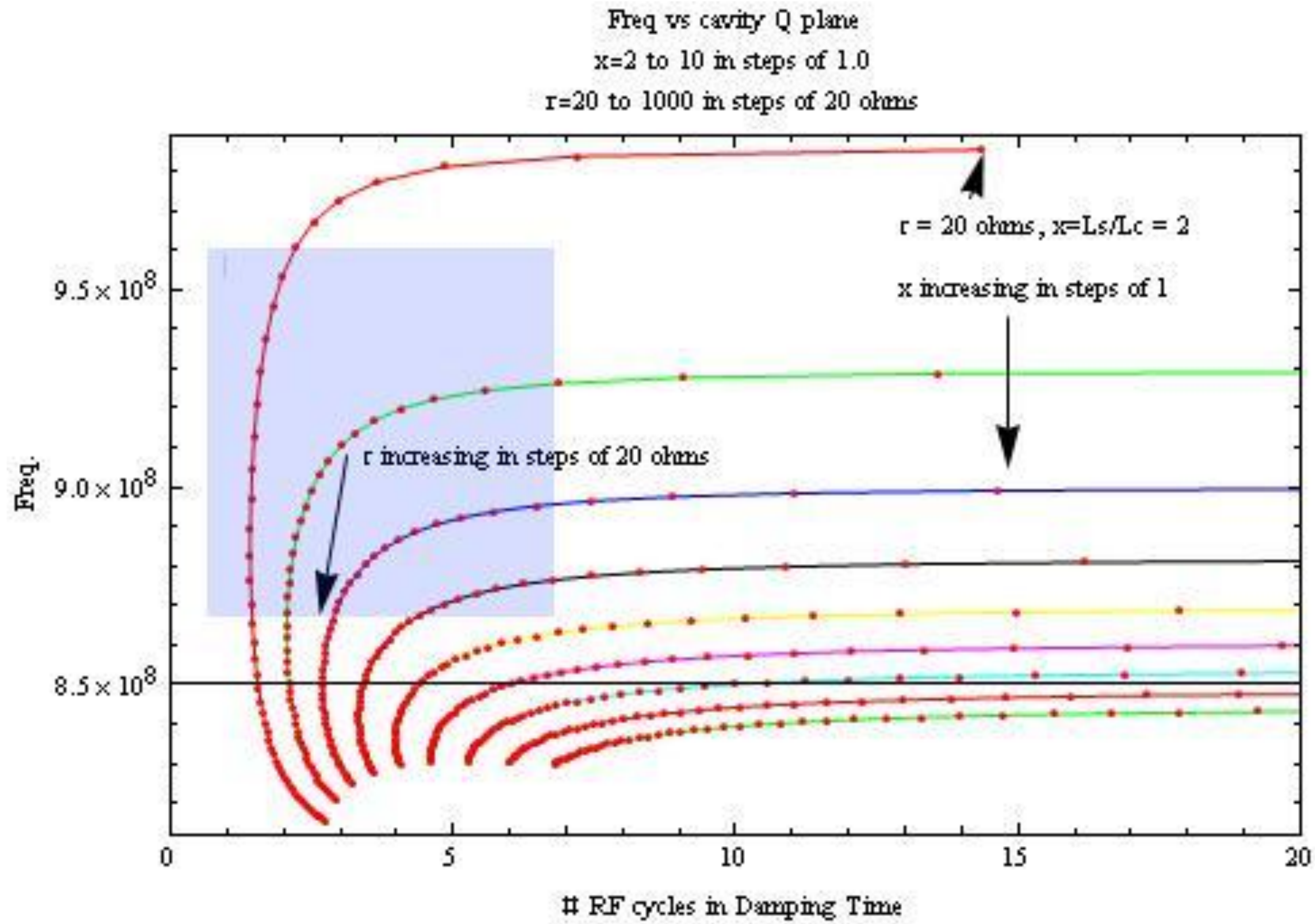


$$z[p,x,r] = \frac{1}{\frac{1}{pL} + pC + \frac{1}{(xpL + r)}}$$

L_s Fixed at 1.5 the cavity inductance

Out[24]=





Freq vs cavity Q plane
x=2 to 10 in steps of 1.0
r=20 to 1000 in steps of 20 ohms
r = 20 ohms, x=Ls/Lc = 2
x increasing in steps of 1
r increasing in steps of 20 ohms
Freq.
9.5 × 10^8
9.0 × 10^8
8.5 × 10^8
0
5
10
15
20
RF cycles in Damping Time

Questions

For a better model

1. For E/p > 14.1 and some electrons in the center of the cavity, model the growth of the plasma as a function of time. How many cycles does it take to reach the electrodes. Why does Paschen Slope seem to depend on electrode material? Why does H_2 follow the DC breakdown curve but N_2 doesn't?
2. For the above case, predict the light out in the visible region. What is the molecular spectrum that is excited. Can we follow the initiation of a discharge? Model the effects of SF_6
3. Make a model of an emitter and follow the electrons in both the break down and non-break down mode. Does light come out? Can it give us information similar to what is observed with dark current for the vacuum case?
4. Model a break down in the plateau region. If field emission starts it, what is the subsequent history?
5. Get a firm handle on the hydrogen atom and ion chemistry so we know all of the reaction rates.
6. Make a good model for the history of the beam induced plasma.

For a functional cavity

1. Verify the loading calculations. The predictions indicate serious trouble at 10^{11} and a disaster if used in the initial capture region where there are many more pi mesons and protons than muons.
2. Is there something less toxic than SF_6 that will eat the electrons?
3. How does one remove the ions between pulses or is it even necessary. It is necessary to get rid of the SF_6 debris as it is very toxic but maybe this could be done at a much slower rate if the ionization isn't a problem.