Fermi National Accelerator Lab

Electron Beam Profiler for Project-X

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Abstract

Large beam intensities in the Main Injector during the Project-X era will make the insertion of physical objects into the beam to measure transverse profiles virtually impossible. An alternative is to use a beam of low energy electrons as a probe of the transverse charge distribution of the proton beam. This paper discusses such a system and compares the Main Injector beam to the SNS beam.

1 Introduction

With the large intensities present in the Recycler and Main Injector for Project X, there needs to be a robust method for determining the transverse beam profiles that does not involve sticking something macroscopically rigid in the beam. Things such as the Flying Wires, Multi-wires, and OTR screens will not survive the large beam intensities. An alternative to these is either an Ionization Profile Monitor, or something like a gas jet or a beam of electrons. The latter has the advantage that it is essentially non-intrusive in that it doesn't require a hard scatter between the beam and the profiling substance. This idea has been around for a long time (see the References at the end) and in fact a working device has been installed at the Spallation Neutron Source (SNS) at Oak Ridge National Lab.

2 Theory

The principle of operation of an electron beam profiler is that you shoot an electron beam transversely to a beam of protons and measure the deflection angle (Fig. 1). This angle depends on the electron energy, the impact parameter of the electron beam with the proton beam, and the amount and transverse distribution of protons.

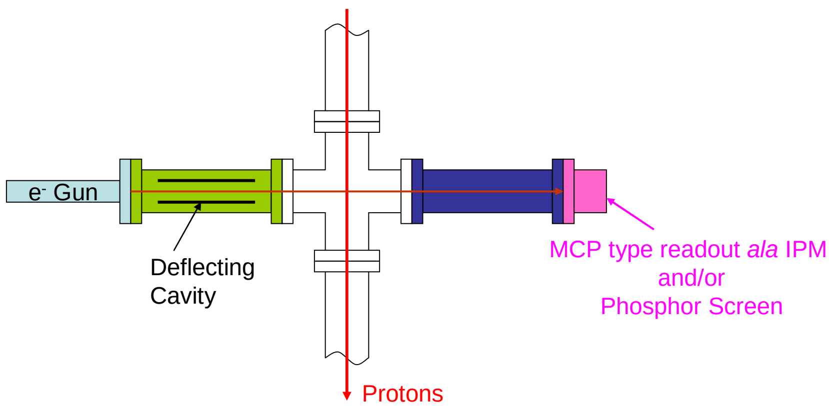
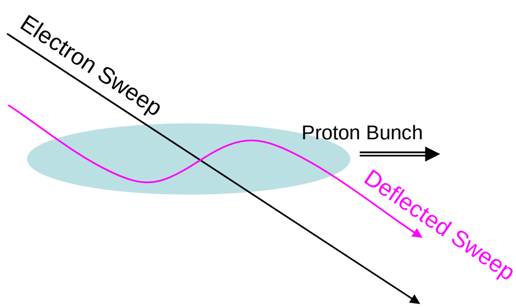


Figure 1: Schematic of an electron gun profile monitor.

To distinguish the deflection as a function of position, the electron beam must be swept not only across, but simultaneously along the proton beam, making a tilted sweep.



3 Simulation

We start with a coherent bunch (all particles have the same velocity, $\beta c\hat{\mathbf{z}}$). The electric and magnetic fields at location $\boldsymbol{r}$ from particles i at locations $\boldsymbol{x}_i$ are given in gaussian units by

$$\begin{aligned}\mathbf{E}(\mathbf{r},t) &= \gamma q\sum_i \frac{\left(\mathbf{r}-\left(\mathbf{x}_i+\beta ct\,\hat{\mathbf{z}}\right)\right)}{\left(b_i^2+\gamma^2 z_i^2\right)^{3/2}} \\ \mathbf{B}(\mathbf{r},t) &= \gamma q\beta c\sum_i \frac{\hat{\boldsymbol{\varphi}}_i}{\left(b_i^2+\gamma^2 z_i^2\right)^{3/2}}\end{aligned} \quad \textbf{(1)}$$

or in integral form

$$\begin{aligned}\mathbf{E}(\mathbf{r},t) &= \gamma\iiint dp\ \rho(\mathbf{x})\frac{\left(\mathbf{r}-\left(\mathbf{x}+\beta ct\,\hat{\mathbf{z}}\right)\right)}{\left(b(\mathbf{x})^2+\gamma^2 z(\mathbf{x})^2\right)^{3/2}} \\ \mathbf{B}(\mathbf{r},t) &= \gamma\beta c\iiint dp\ \rho(\mathbf{x})\frac{\hat{\boldsymbol{\varphi}}(\mathbf{x})}{\left(b(\mathbf{x})^2+\gamma^2 z(\mathbf{x})^2\right)^{3/2}}\end{aligned} \quad \textbf{(2)}$$

where

$$\begin{aligned} z_i &= \left(\mathbf{r}-\mathbf{x}_i\right)\cdot\hat{\mathbf{z}}-\beta ct \\ \mathbf{b}_i &= \hat{\mathbf{z}}\times\left(\left(\mathbf{r}-\mathbf{x}_i\right)\times\hat{\mathbf{z}}\right) \\ b_i &= \left|\mathbf{b}_i\right| \\ \hat{\boldsymbol{\varphi}}_i &= \frac{\mathbf{b}_i}{b_i}\times\hat{\mathbf{z}}\end{aligned} \quad \textbf{(3)}$$

Working in general, a probe particle with mass *m* and charge $\tilde{q}$ will experience a force

$$\mathbf{F}(\mathbf{r},t) = \tilde{q}\left(\mathbf{E}(\mathbf{r},t)+\tilde{\boldsymbol{\beta}}\times\mathbf{B}(\mathbf{r},t)\right) \quad \textbf{(4)}$$

which is related to its acceleration via

$$\begin{aligned}\mathbf{F}(\mathbf{r},t) &= \frac{d\mathbf{p}}{dt} = m\frac{d\tilde{\gamma}\mathbf{v}}{dt} \\ \mathbf{F}(\mathbf{r},t) &= m\tilde{\gamma}\left(\mathbf{a}+\tilde{\gamma}^2\tilde{\boldsymbol{\beta}}\left(\tilde{\boldsymbol{\beta}}\cdot\mathbf{a}\right)\right)\end{aligned} \quad \textbf{(5)}$$

The tilde over the γ and β refer to the probe particle and the bold **β** means ***v****/c*.

This can be viewed as a matrix equation

$$\begin{pmatrix}F_x\\F_y\\F_z\end{pmatrix} = m\tilde{\gamma}\begin{pmatrix}1+\tilde{\gamma}^2\tilde{\beta}_x^2 & \tilde{\gamma}^2\tilde{\beta}_x\tilde{\beta}_y & \tilde{\gamma}^2\tilde{\beta}_x\tilde{\beta}_z \\ \tilde{\gamma}^2\tilde{\beta}_x\tilde{\beta}_y & 1+\tilde{\gamma}^2\tilde{\beta}_y^2 & \tilde{\gamma}^2\tilde{\beta}_y\tilde{\beta}_z \\ \tilde{\gamma}^2\tilde{\beta}_x\tilde{\beta}_z & \tilde{\gamma}^2\tilde{\beta}_y\tilde{\beta}_z & 1+\tilde{\gamma}^2\tilde{\beta}_z^2\end{pmatrix}\begin{pmatrix}a_x\\a_y\\a_z\end{pmatrix} \quad \textbf{(6)}$$

and inverted to find **a**

$$\begin{pmatrix}a_x\\a_y\\a_z\end{pmatrix} = \frac{1}{m\tilde{\gamma}\left(1+\tilde{\gamma}^2\tilde{\beta}^2\right)}\begin{pmatrix}1+\tilde{\gamma}^2\left(\tilde{\beta}_y^2+\tilde{\beta}_z^2\right) & -\tilde{\gamma}^2\tilde{\beta}_x\tilde{\beta}_y & -\tilde{\gamma}^2\tilde{\beta}_x\tilde{\beta}_z \\ -\tilde{\gamma}^2\tilde{\beta}_x\tilde{\beta}_y & 1+\tilde{\gamma}^2\left(\tilde{\beta}_x^2+\tilde{\beta}_z^2\right) & -\tilde{\gamma}^2\tilde{\beta}_y\tilde{\beta}_z \\ -\tilde{\gamma}^2\tilde{\beta}_x\tilde{\beta}_z & -\tilde{\gamma}^2\tilde{\beta}_y\tilde{\beta}_z & 1+\tilde{\gamma}^2\left(\tilde{\beta}_x^2+\tilde{\beta}_y^2\right)\end{pmatrix}\begin{pmatrix}F_x\\F_y\\F_z\end{pmatrix} \quad \textbf{(7)}$$

and in short form, the solution for **a** can be written

$$\begin{aligned}\mathbf{a} &= \frac{1}{\tilde{\gamma}m\left(1+\tilde{\gamma}^2\tilde{\beta}^2\right)}\left[\mathbf{I}+\tilde{\gamma}^2\left(\tilde{\beta}^2\mathbf{I}-\tilde{\boldsymbol{\beta}}\tilde{\boldsymbol{\beta}}^T\right)\right]\mathbf{F} \\ &= \frac{1}{\tilde{\gamma}m}\left[\mathbf{I}-\frac{\tilde{\gamma}^2}{\left(1+\tilde{\gamma}^2\tilde{\beta}^2\right)}\tilde{\boldsymbol{\beta}}\tilde{\boldsymbol{\beta}}^T\right]\mathbf{F}\end{aligned} \tag{8}$$

where **I** is the identity matrix. Substituting from above leads to the full differential equation of motion for the probe particle.

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{\tilde{q}q\gamma}{\tilde{\gamma}m}\left[\mathbf{I}-\frac{\tilde{\gamma}^2}{\left(1+\tilde{\gamma}^2\tilde{\beta}^2\right)}\tilde{\boldsymbol{\beta}}\tilde{\boldsymbol{\beta}}^T\right]\sum_i \frac{\mathbf{r}-\left(\mathbf{p}_i+\beta ct\,\hat{\mathbf{z}}\right)+\beta c\left(\tilde{\boldsymbol{\beta}}\times\hat{\boldsymbol{\varphi}}_i\right)}{\left(\rho_i^2+\gamma^2 z_i^2\right)^{3/2}} \tag{9}$$

This of course is impossible to solve, so one must numerically do it using shortcuts and approximations. The simplest one of which is to assume that **a** is constant over short enough time durations. Then the solution is just

$$\mathbf{r}_{j+1} = \mathbf{r}_j + \mathbf{v}_j\Delta t + \frac{1}{2}\Delta t^2\left(\frac{\tilde{q}q\gamma}{\tilde{\gamma}m}\left[\mathbf{I}-\frac{\tilde{\gamma}^2}{\left(1+\tilde{\gamma}^2\tilde{\beta}^2\right)}\tilde{\boldsymbol{\beta}}\tilde{\boldsymbol{\beta}}^T\right]\sum_i \frac{\mathbf{r}-\left(\mathbf{p}_i+\beta ct\,\hat{\mathbf{z}}\right)+\beta c\left(\tilde{\boldsymbol{\beta}}\times\hat{\boldsymbol{\varphi}}_i\right)}{\left(\rho_i^2+\gamma^2 z_i^2\right)^{3/2}}\right)_j \tag{10}$$

where the subscript j on the parenthesis means evaluated at the j^{th} step.

This algorithm was implemented in MATLAB. The particles are represented by macro particles each carrying a portion of the charge. To avoid singularities as the electrons approach close to a macroparticle, the electric field of the macroparticle is linearly scaled to 0 at zero distance from it in an attempt to spread the macroparticle charge out into a uniform sphere of charge with a diameter similar to the average particle separation. To accomplish this, the following shielding term, S_i, is calculated for each macroparticle

$$\begin{aligned}\Delta_i &= \sqrt{\rho_i^2+\gamma^2 z_i^2} \\ S_i &= \begin{cases}\left(\Delta_i/\lambda_{\min}\right)^3 & : \quad \Delta_i < \lambda_{\min} \\ 1 & : \quad \Delta_i > \lambda_{\min}\end{cases}\end{aligned} \tag{11}$$

and multiplies the electric field from the macroparticle. The algorithm still suffers small fluctuations from the macroparticles, but the general behavior is more reliable with fewer macroparticles.

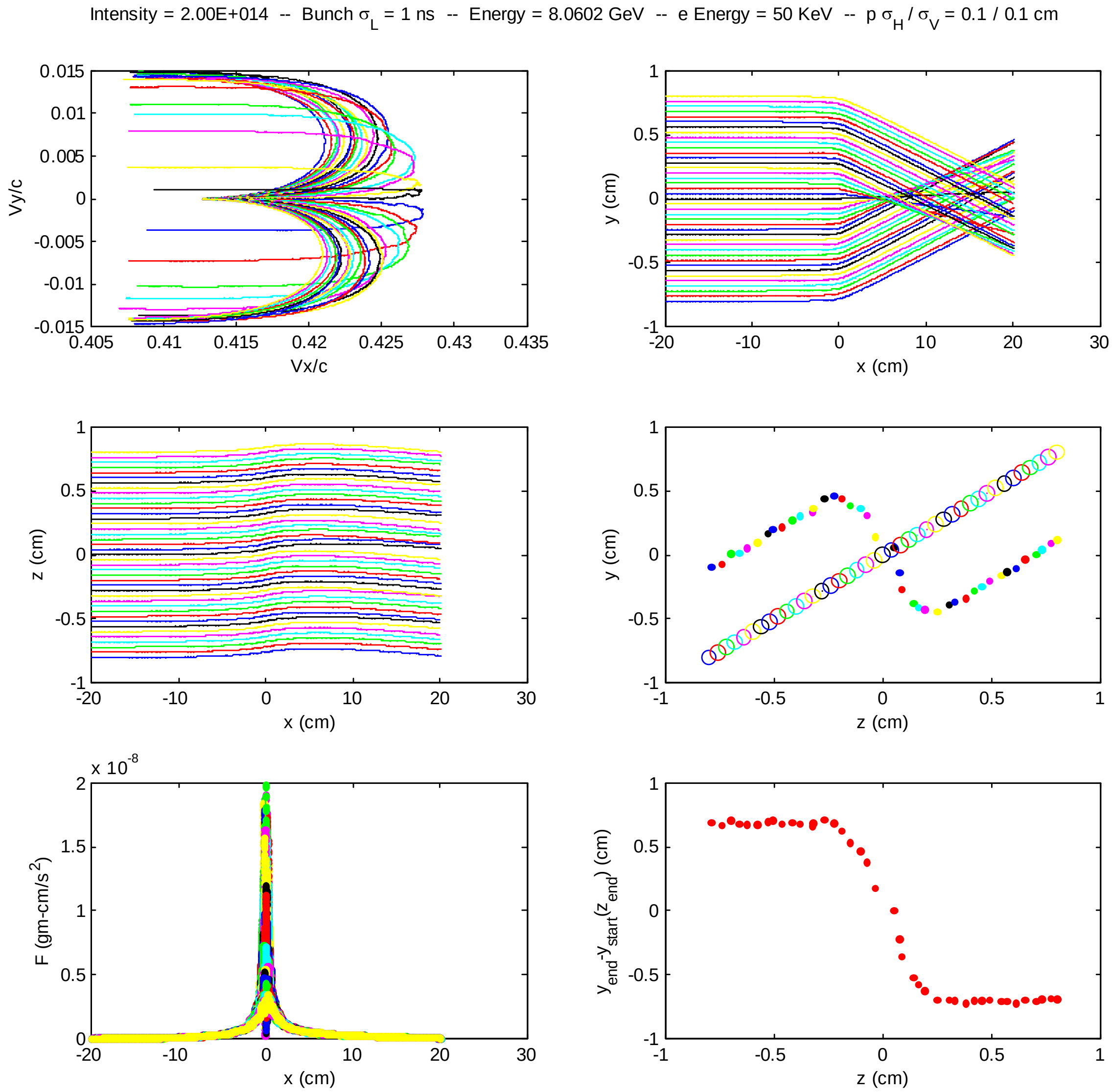


Figure 2: Results from a 50 KeV electron beam passing through an 8 GeV proton beam with intensity 2E14 protons and an RMS beam size of 1 mm in both dimensions. The middle right plot shows the starting positions (circles) and the ending positions (points) of the electrons.

4 Comparison with SNS

Since SNS already has a working electron beam profiler, we should compare our simulations with theirs. Wim Blokland at SNS has a simulation that assumes an infinitely long cylindrically symmetric proton beam. This approach avoids the previously mentioned macroparticle instabilities and is faster since much of it can be calculated analytically. Since the beam is assumed to be transversely Gaussian and longitudinally infinite, there is no dependence on the energy of the protons[1]. The main difference between SNS and us is that we have 53 MHz bunched beam whereas SNS has essentially no bunching (the entire ring is one bucket).

4.1 Bunched / Unbunched Intensities

Start with a beam with the following parameters

I	Beam intensity (# of particles)
ρ_0	Maximum beam density in center of bucket (# of particles / cm)
h	Number of RF buckets in ring
n	Number of filled RF buckets
τ	RF bucket period (ns)
σ	Sigma of Gaussian distribution in each RF bucket (ns)

Assuming a roughly Gaussian time distribution in the filled buckets, the total charge is

$$\begin{aligned} I &\cong n\int \rho_0 e^{-\frac{1}{2}\frac{t^2}{\sigma^2}}dt \\ I &\cong n\rho_0\sigma\sqrt{2\pi} \end{aligned} \qquad \textbf{(12)}$$

So the charge density at the center of the bucket is

$$\rho_0 = \frac{I}{n\sigma\sqrt{2\pi}} \qquad \textbf{(13)}$$

A uniform time distribution has a charge density of

$$\rho_0' = \frac{I'}{n\tau} \qquad \textbf{(14)}$$

Some other important facts:

- The velocity of an electron with kinetic energy E_k is

$$\begin{aligned} &v = \beta c = c\sqrt{1-\frac{1}{\gamma^2}} = c\sqrt{1-\frac{m^2}{\left(E_k+m\right)^2}} \\ &\text{where} \quad \gamma = \frac{E}{m} = \frac{E_k+m}{m} \end{aligned} \qquad \textbf{(15)}$$

  - A 1 KeV electron has a velocity of ~1.9×10^9 cm/s giving it a 10 cm beampipe crossing time of ~5 ns, which is well within one bucket. A 60 KeV electron has a crossing time of ~0.7 ns.

- One can also ask the question of how far down the beam the electron can ‘see’. Meaning how far away can the electron still feel the field of a proton taking into consideration the

[1] An infinite, radially-symmetric beam is calculable from Gauss’ Law which is one of Maxwell’s equations which in turn are invariant under Lorentz transformations. As such, the energy of the protons cannot affect the calculation of the electric field.

Lorentz pancake effect. For a proton with energy *E*, the transverse E field seen by the electron a distance *b* away the axis of motion of the proton drops by half when the longitudinal distance between the proton and electron is b/γ. So for a 8 GeV proton, this distance at the edge of a 10 cm beampipe is ~5/8 cm which is ~20 ps away. So being conservative and saying that $\pm 4b/\gamma$ is the window of visibility, then, at the edge of the beampipe, the electron can see 80 ps in either direction.

These two facts imply that as far as the electron is concerned, it can't tell the difference between a multi-nsec bunch and a continuous beam. Thus we can make the approximation that the beam is continuous with a charge density corresponding to the maximum charge density in a bunch.

Now referring to the equations above, the beam intensity of a continuous beam, *I'*, needed for a charge density corresponding to the maximum in a bunched beam is

$$\rho_0 = \rho_0' \quad \Rightarrow \quad \frac{I'}{n\tau} = \frac{I}{n\sigma\sqrt{2\pi}}$$
$$I' = \frac{I\tau}{\sigma\sqrt{2\pi}} \tag{16}$$

Comparing with SNS,

Parameter	**SNS (1 GeV)**	**Project X (MI)**
Design Beam Intensity	1.4×10^{14} (1.4 MW 60 Hz)	2×10^{14}
Ring Length (μs)	1	11
Charge density (particles / cm)	5.4×10^{9}	0.6×10^{9}
Bunch Length (ns)	---	1-2
Peak Charge Density (particles / cm)	5.4×10^{9}	4×10^{9}
I'	1.4×10^{14}	1×10^{15}
For comparison, a 300E9 proton bunch Peak charge density		2×10^{9}

Figure 3 shows calculations with an infinite beam of 1 GeV, the goal of which is to validate my calculation in a regime that the SNS calculation was designed to do. The magnitudes of the deflections are similar for both the SNS calculation and my calculation. Figure 4 shows calculations for 2×10^{14} protons at 8 GeV with 1 ns bunch RMS. The difference between the two calculations can probably be attributed to the fact that the SNS beam is still continuous and the intensities were set such that the peak proton intensity was the same. So the electrons actually see slightly fewer protons in the bunched beam than in the continuous beam.

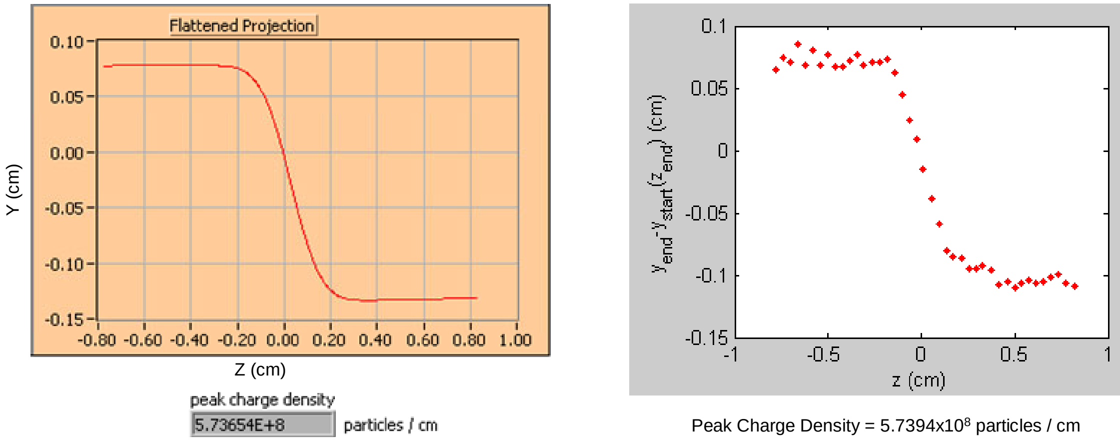


Figure 3: Comparison off SNS (left) and RMTK (right) calculations of electron beam deflection. The electrons are 50 KeV, and the protons are 1 GeV and a continuous, infinite beam.

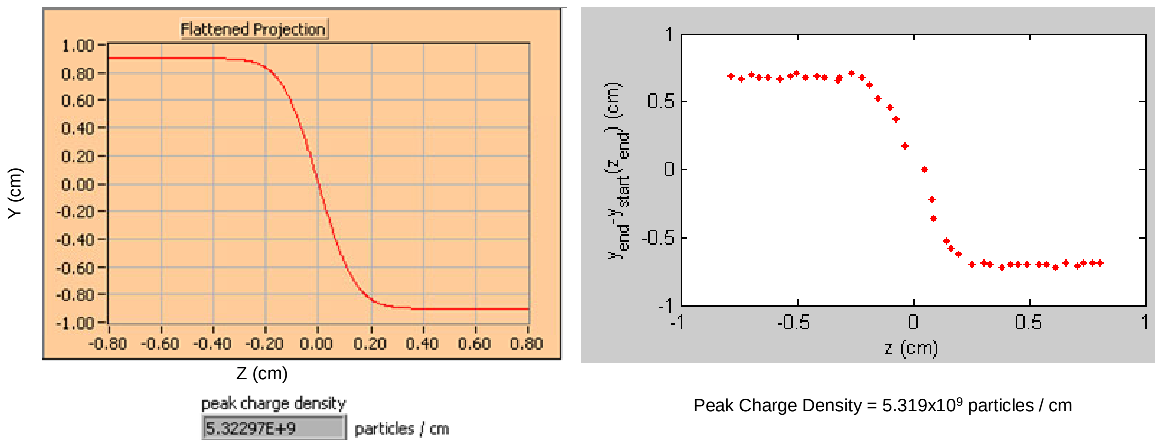


Figure 4: Comparison of SNS (left) and RMTK (right) calculations of electron beam deflection. The electrons are 50 KeV and the protons for the RMTK model are 8 GeV with 2E14 protons in 500 buckets around the Main Injector, each with RMS bunch length of 1 ns. The proton intensity in the SNS model is adjusted to give the same peak lineal density as the Main Injector bunched beam. Since the bunch length is only 1 ns, the SNS electron beam actually sees more protons than the MI beam resulting in a slightly larger deflection.

5 Bunch Slicing

To measure the transverse profile of a single bunch it is necessary to either gate the electron beam, or to sweep the electron beam very rapidly to effectively slice the proton beam. Slicing the electron beam requires a deflecting field ***E*** that produces the correct magnitude of position swing at the proton beam, and that has the correct ***dE****/dt* to slice the beam in the required time interval. Let's assume the following facts:

- We wish to slice through τ ns in the middle of a proton bunch
- The proton beam is a distance *D* cm from the deflecting field and we want the deflected electron beam to scan through a range of $\pm R$ cm to either side of the proton beam.

  $\beta_T = \beta \tan(\theta)$

  $\tan(\theta) = R / D$

  where β_T is the transverse velocity

The required transverse kinetic energy is then

$$E_{KT} = mc^2\left(\frac{1}{\sqrt{1-\beta_T^2}} - 1\right)$$
$$E_{KT} = mc^2\left(\frac{D}{\sqrt{D^2 - \left(1-\frac{m^2c^4}{\left(E_K+mc^2\right)^2}\right)R^2}} - 1\right) \tag{17}$$

For a D of 20 cm and an R of 7 mm, the transverse kinetic energy is ~53 eV.

For a deflecting field length of λ, the electric field, **E**, required to achieve the desired transverse kinetic energy, E_{KT}, is (using $v = at$),

$$\beta_T = \frac{\mathbf{E}q\lambda}{mc^2\,\beta_Z} = \frac{\mathbf{E}q\lambda\left(E_k+mc^2\right)}{mc^2\sqrt{E_K^2+2E_Kmc^2}} \quad \text{...and...} \quad \beta_T = \sqrt{1-\frac{m^2c^4}{\left(E_{KT}+mc^2\right)^2}}$$
$$q\mathbf{E}\lambda = \frac{mc^2}{\left(E_k+mc^2\right)}\sqrt{E_K^2+2E_Kmc^2}\sqrt{1-\frac{m^2c^4}{\left(E_{KT}+mc^2\right)^2}} \tag{18}$$

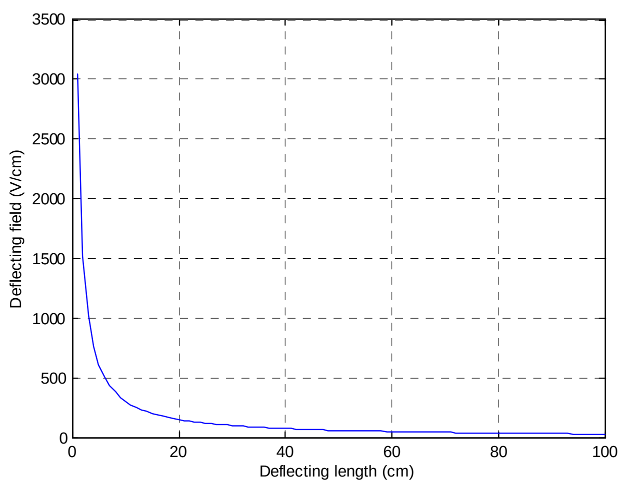


Figure 5: Deflecting field required for a given deflecting length to achieve 53 eV transverse kinetic energy.

For an electron beam current of 2 mA, the number of electrons per unit length along the scan path, Q, is

$$Q = \frac{I_e\tau}{2R} \tag{19}$$

which for a current of 2 mA and a scan time of 1 ns is ~900 electrons/μm.

6 References

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