Quantum-degenerate electron source

M. Zolotorev

AFRD LBNL

Working Group

E.D. Commins,
J. Oneill,
F. Sannibale,
M. Zolotorev

Enthusiast`s Group

P. Denes,
Z. Hussain,
G.V. Lebedev,
D.S. Robin,
R.W. Schoenlein,
W. Wan

Brightness & Degenerate Parameter

$$dN = \frac{dx dp_x \, dy dp_y \, dz dp_z}{h^3}$$

$$dN = \frac{dx d\beta_x \, dy d\beta_y \, dz d\beta_z}{(2\pi)^3 \lambda\!\!\!^{-}_C{}^3}$$

$$N = \frac{\epsilon_x \, \epsilon_y \, \epsilon_z}{\lambda\!\!\!^{-}_C{}^3}$$

$\lambda\!\!\!^{-}_C = 3.86 \; 10^{-11}$ cm

maximum number of particles per elementary volume at

$$\delta_F = \frac{N_e}{N} = \frac{N_e}{\epsilon_x \, \epsilon_y \, \epsilon_z} \lambda\!\!\!^{-}_C{}^3 = B \lambda\!\!\!^{-}_C{}^3$$

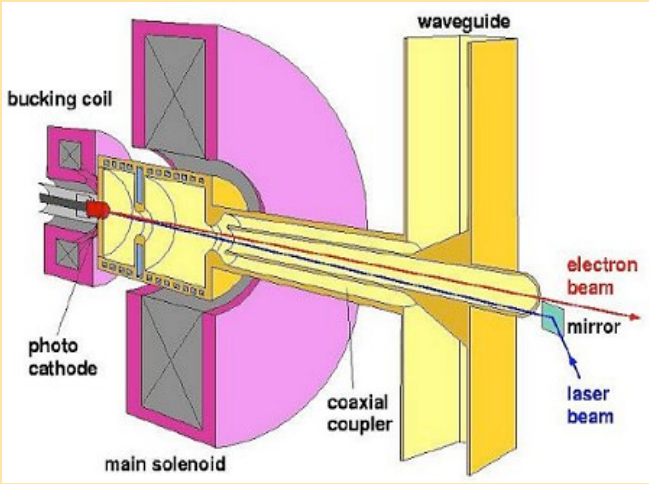
For fermions Pauli exclusion principle limit value of δ_F

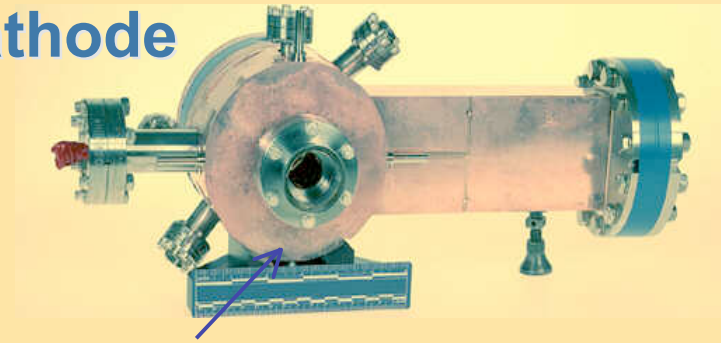
$\delta_F < 1$

In copper $\delta_F = .995$

High intensity sources (10-100 A)

RF Gun with Photocathode





ATF (BNL) Gun III (LINAC Injector):

- Energy ~ 2 MeV
- Normalized rms emittance of 2.6 mm mrad
- Charge of 1 nC
- Pulse length of 10 ps
- RF = 2856 MHz (1 MV/cm)

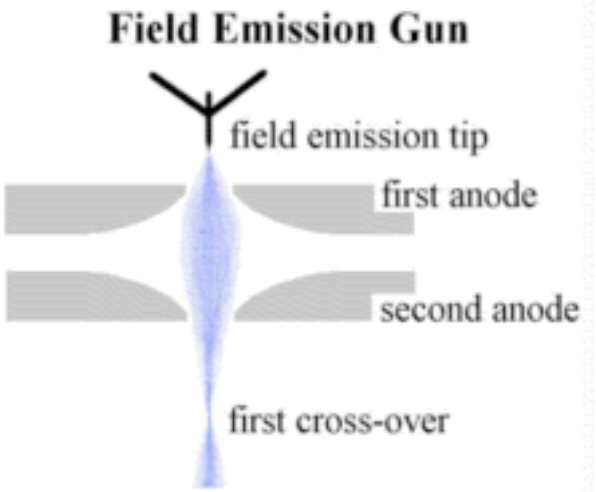
Charge densities up to 10^5 A/cm^2

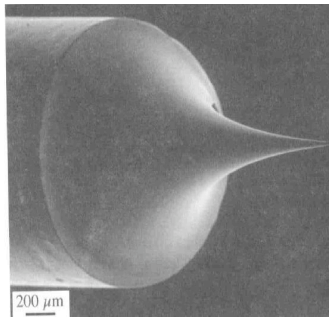
Gun for LCLS

$\delta_F \sim 10^{-11}$

Low intensity sources (1-10 nA)

Field Emission Electron Gun





THERMO Electro Corporation:
- Field at the cathode tip ~ 10 MV/cm
- 100 nm spot size at 5 nA sample current
- Current density ~ 50 A/cm^2
- Application: Electron microscope

Current densities on sample up to 10^5 A/cm^2

$\delta_F \sim 5\ 10^{-7}$

Electron effective temperature
$kT_e \sim .2$ eV

Modern developments including "nanotube"

$\delta_F \sim 10^{-5}$

Main killers - space charge and interbeam scattering (Borsch effect)

Liouville theorem

Space charge

nonlinear focusing effectively increase phase volume

Longitudinal: $T_f = T_i^2 / U$ Acceleration cooling

$T_{tr} >> T_{long}$ Rutherford scattering increase longitudinal phase volume

Transverse: $T \sim q^2 n^{1/3}$ fluctuation in potential energy

Main ideas

Cs^{133} $kT_{Cs} \sim 26$ meV

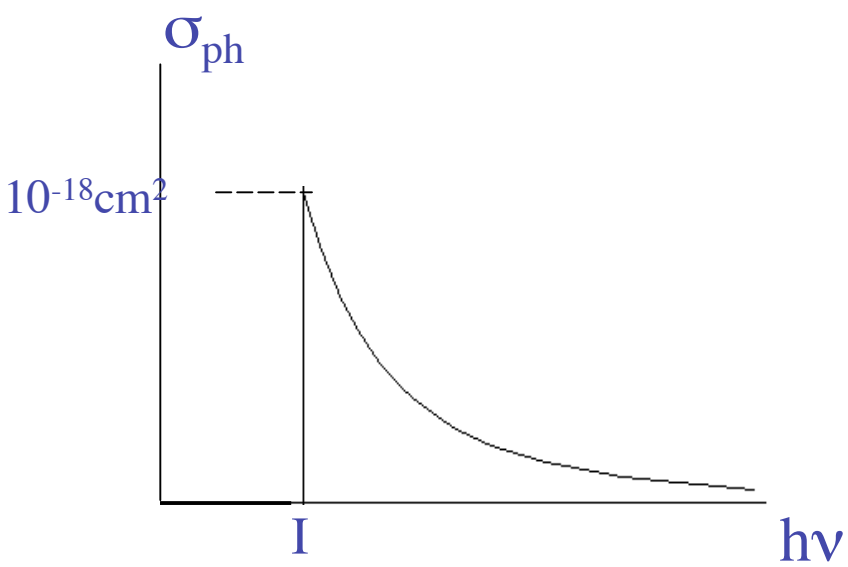
$kT_e \sim 26$ meV $m_e/M_{Cs} \sim 10^{-7}$ eV

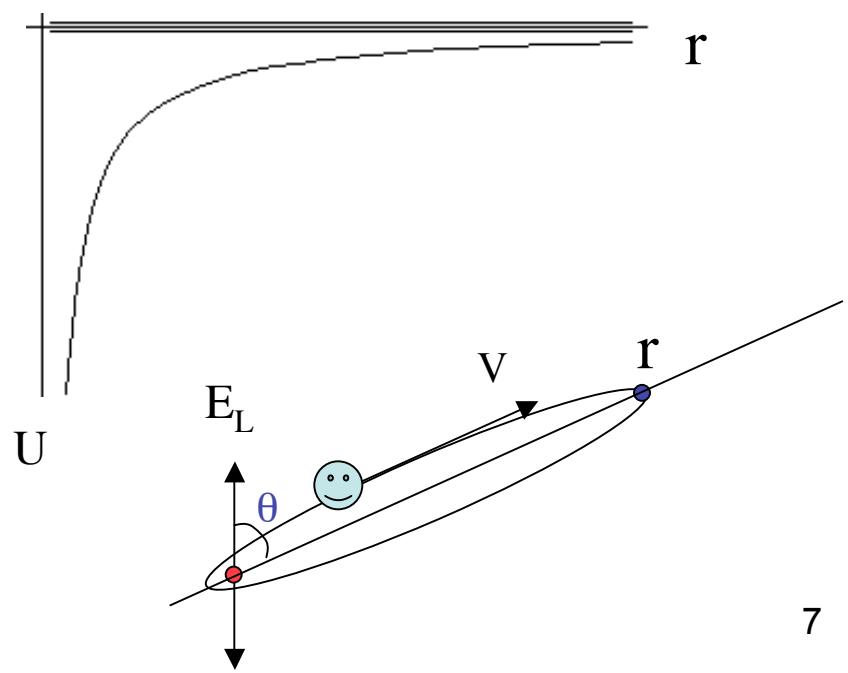
Compare with
$kT_e \sim .15 - .2$ eV

Plasma with hot ions and
extremely cool electrons

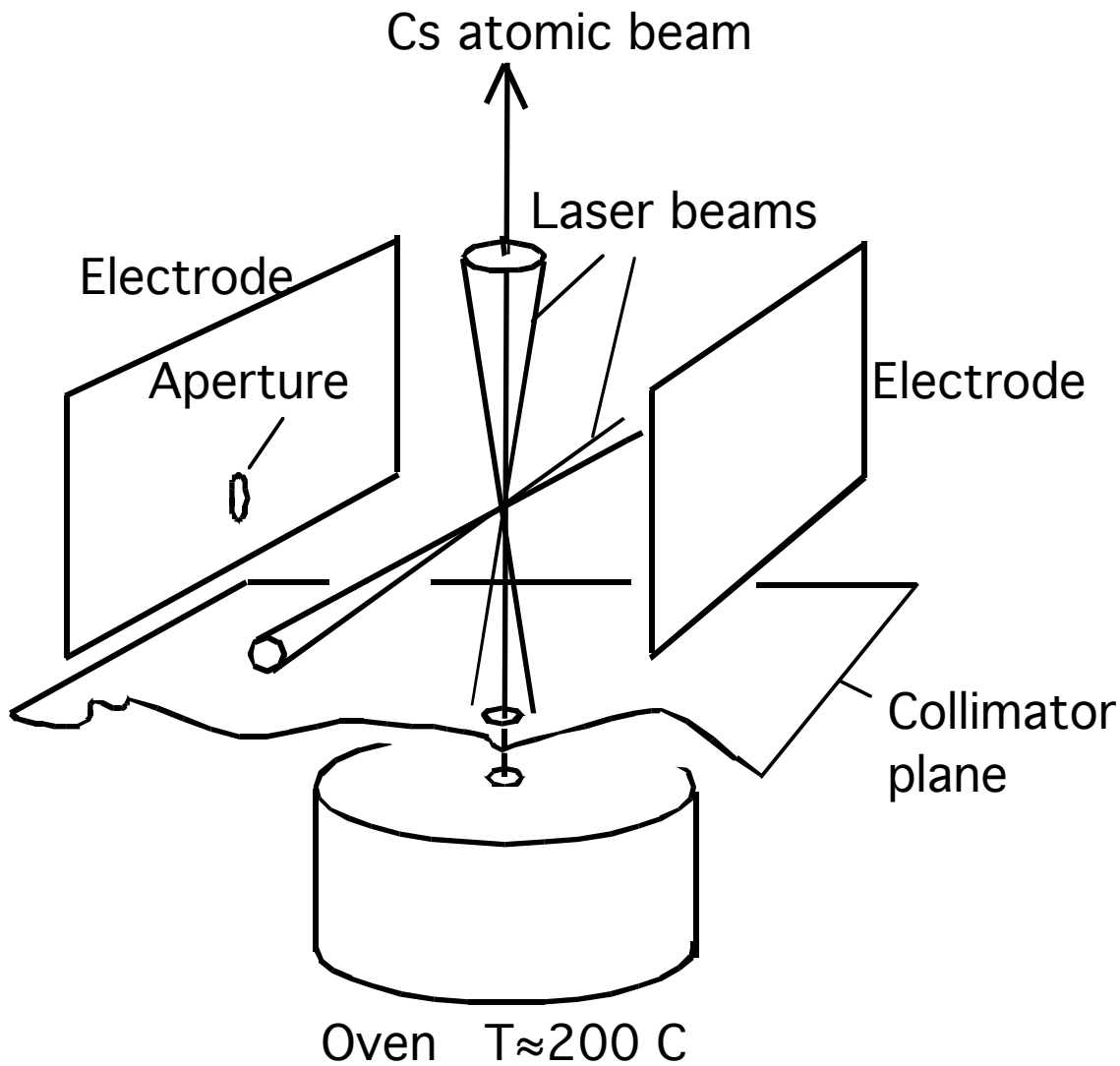
Problems: space charge when one tries to separate electrons and ions

Solution: make electrons one by one low intensity (< 1pA)
high brightness



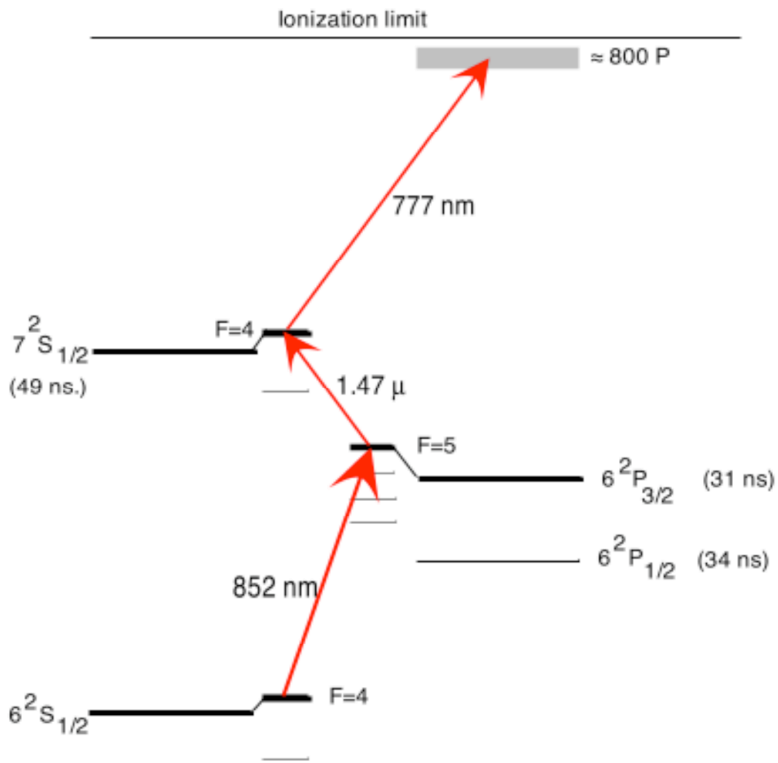


Apparatus schematic



Electron Excitation

In the interaction region,defined by the overlap of the lasers and atomic beam, we excite on average one Cs atom per laser pulse to a Rydberg state very close to vacuum (~-10^{-5} eV).The electron in the excited atom will have a large kinetic energy and will start to move from the ion.



Energy levels of atomic Cs
(not to scale)

n~1000 was populated in potassium [1]

T.Frey,X.Ling,B.G.Lindsay,K.A.Smith and F.B.Dunning

Waiting Period

After the laser pulse, we wait the time necessary (~40 ns) for the electron to go far from the nucleus (~65 μm) and reach the turning point of its classical orbit where its kinetic energy is negligible. At this point, a short pulsed voltage ionizes the atom, extracts the electron from the interaction region.

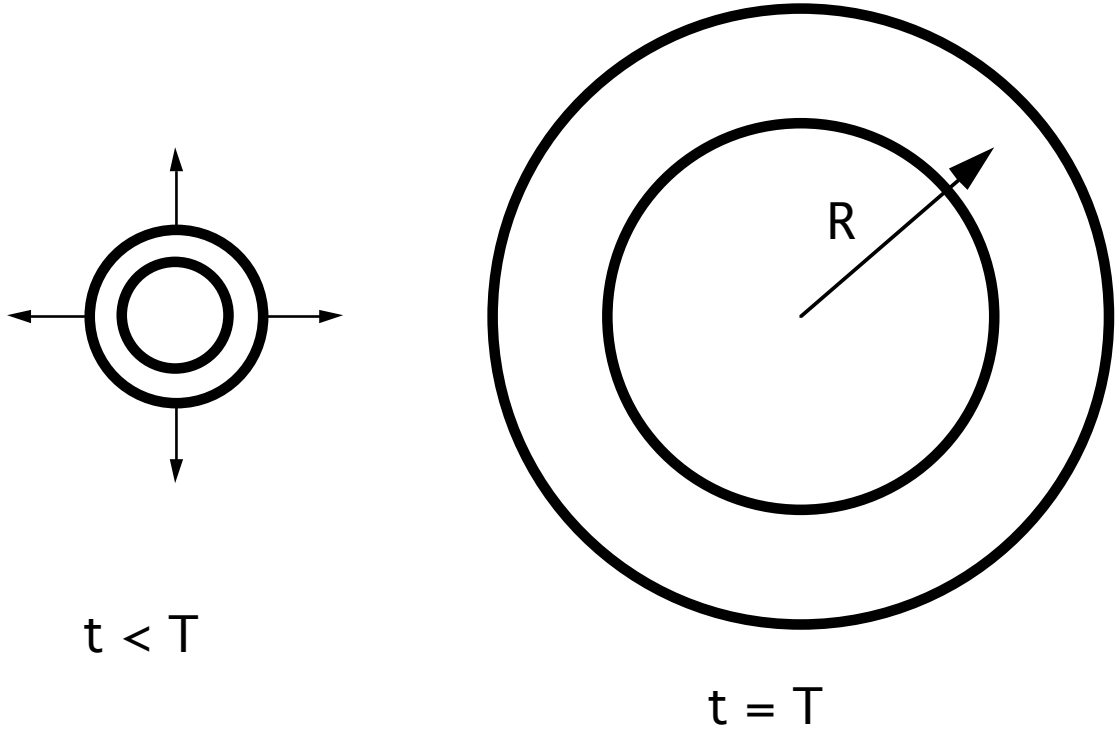
Electron Acceleration

DC voltage applied between next electrodes accelerates electron to desired energy.

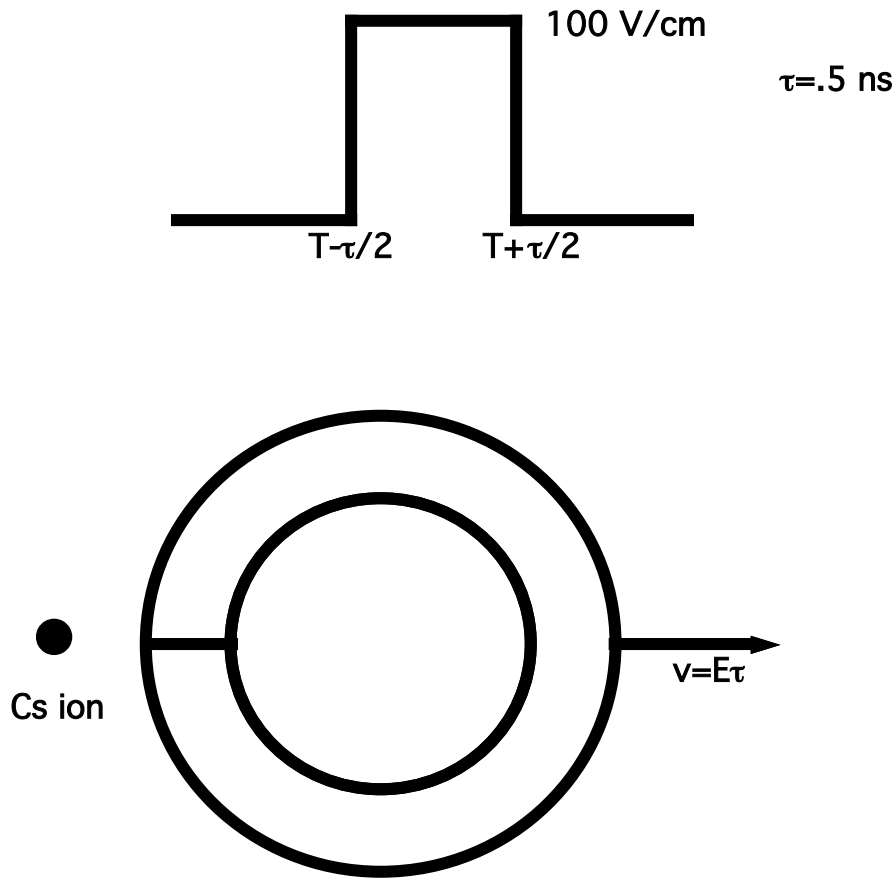
Ion Clearing

After the electron leaves the second region, a “clearing” pulsed field removes the residual ion before the beginning of the following cycle. In this way the electron produced in the next cycle does not interact with ions from previous pulses.

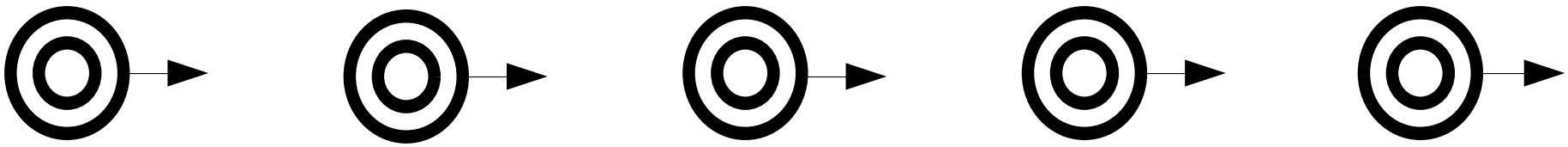
Expanding shell wave packet



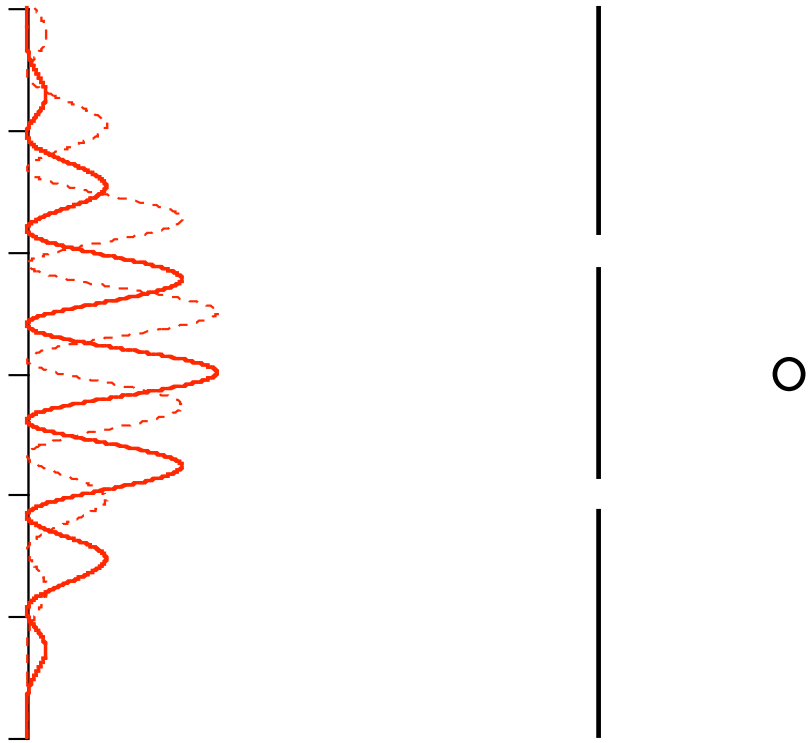
Acceleration pulse



TRAIN OF WAVE-PACKETS

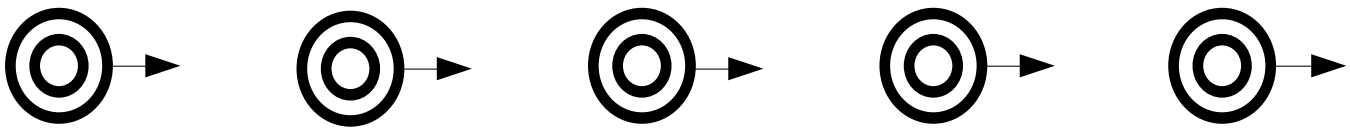


ANALOGY: 2- SLIT INTERFERENCE WITH **INDIVIDUAL** PHOTONS

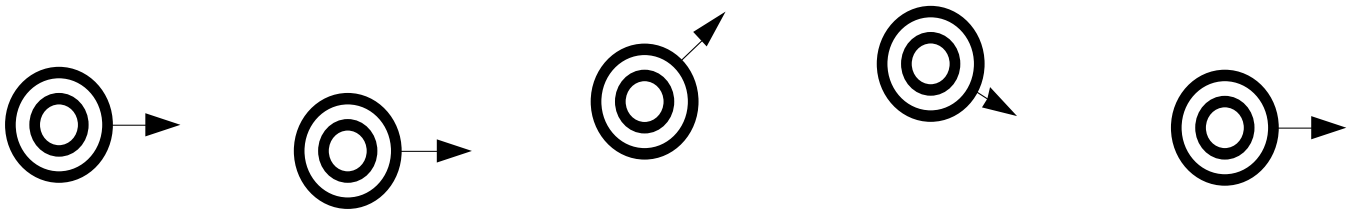


TRAIN OF WAVE PACKETS: IDEAL VS. ACTUAL

IDEAL



ACTUAL



FOR HIGH COHERENCE…

MINIMIZE LASER–ATOM INTERACTION VOLUME

MINIMIZE CESIUM ATOM VELOCITY SPREAD

WHY CESIUM?

- High vapor pressure at low temperatures: easy to make an atomic beam.
- Large mass: A=133

$$\therefore v \approx \sqrt{\frac{2kT}{M}} \approx 2 \bullet 10^4 \text{ cm/s}$$

- Cs atomic beam easily laser-cooled
- Cesium atomic structure well understood
- Cs laser wavelengths are convenient

WHY CHOOSE n = 800 ?

$$T = \frac{\hbar^3}{m_e e^4}\pi n^3 \qquad \left(= \pi n^3 \text{ in atomic units}\right)$$

$$R = 2n^2 a_0 \qquad \left(= 2n^2 \text{ in a.u. where } a_0 = \frac{\hbar^2}{m_e e^2} = 1\right)$$

When $n = 800$:

$$T = 38.9 \text{ ns}, \quad R = 1.28 \bullet 10^6 \ a_0 = .0068 \text{ cm}$$

QUANTUM MECHANICAL DESCRIPTION OF SOURCE BRIGHTNESS

When $\mathbf{b_F}$ approaches unity, the wave property of the electron becomes more and more important, and brightness, which in the classical case is invariant as a consequence of Liouville's theorem, is no longer conserved . The invariant in wave optics is no longer the brightness, but is instead the degeneracy parameter defined as the number of particles crossing the transverse coherence area in longitudinal coherence time. It follows that for any pure quantum state of polarized electrons, the degeneracy parameter is equal to unity. In all real situations, one does not have a pure quantum state, because one or more of the parameters describing the system fluctuate randomly. **In this case, one must employ the density matrix** (the quantum analog of the density in phase space for the classical physics of particles). When the parameter fluctuations become sufficiently strong, the off-diagonal elements of the density matrix become negligibly small, and one arrives at the limit where the classical physics of particles is appropriate, and the brightness recovers its meaning. However, for the source we are proposing, the parameter fluctuations are relatively small, and in order to give an adequate description of such a case, **we must express the degeneracy parameter in quantum mechanical terms**.

THE DENSITY OPERATOR (1)

DENSITY OPERATOR:

$$\rho = \sum_i p_i \left|\phi_i\right\rangle\left\langle\phi_i\right| \quad \text{with} \quad \sum_i p_i = 1$$

NORMALIZATION:

$$tr\left(\rho\right) = 1 \quad \text{if} \quad \left\langle\phi_i|\phi_i\right\rangle = 1 \ \text{ for all } i$$

THE DENSITY OPERATOR (2)

THE KEY QUANTITY IS $tr(\rho^2)$

$tr(\rho^2)=1$ FOR A PURE STATE, BUT…

$tr(\rho^2)<1$ FOR A STATISTICAL ENSEMBLE, AND…

$tr(\rho^2)$ DECREASES AS DISPERSION INCREASES

THE DENSITY OPERATOR (3)

$$\rho^2 = \left(\sum_i p_i \left|\phi_i\right\rangle\left\langle\phi_i\right|\right)\left(\sum_j p_j \left|\phi_j\right\rangle\left\langle\phi_j\right|\right)$$

$$tr\left(\rho^2\right) = \sum_{i,j} p_i p_j \int \left\langle x|\phi_i\right\rangle\left\langle\phi_i|\phi_j\right\rangle\left\langle\phi_j|x\right\rangle dx$$

$$= \sum_{i,j} p_i p_j \left|\int \left\langle x|\phi_i\right\rangle\left\langle\phi_j|x\right\rangle dx\right|^2$$

$$\therefore tr\left(\rho^2\right) = \sum_{i,j} p_i p_j \left|\int \psi_j^*(x)\psi_i(x)dx\right|^2$$

THE DEGENERACY PARAMETER (1)

$$\delta = tr(\rho^2)$$

$$= \sum_{i,k} p_i p_k \iint dxdy \left[\psi_i^*(x)\psi_k(x)\right]\left[\psi_k^*(y)\psi_i(y)\right]$$

$$\text{where} \quad \psi_i(x) = \langle x|\phi_i \rangle$$

$$\sum_i p_i \Rightarrow \int j(a)da \bullet \int J(q)dq$$

$$\sum_k p_k \Rightarrow \int j(b)db \bullet \int J(p)dp$$

$$\left(\hbar = m_e = e = 1\right)$$

$$\psi_i(x) \Rightarrow \psi(x-a) \bullet \exp\left[iq(x-a)\right]$$

$$\psi_j(x) \Rightarrow \psi(x-b) \bullet \exp\left[ip(x-b)\right]$$

THE DEGENERACY PARAMETER (2)

$$\delta = \iint j(a)j(b)dadb \iint J(q)J(p)dqdp \left| \int e^{i(q-p)x} \psi^*(x)\psi(x-(a-b))dx \right|^2$$

THE DEGENERACY PARAMETER (3)

TO MAKE THIS FORMULA USEFUL…

- $1 \Rightarrow 3$ SPATIAL DIMENSIONS

- $j(\vec{a}) = \frac{1}{\left(2\pi\sigma^2\right)^{3/2}} \exp\left[-\frac{\vec{a}^2}{2\sigma^2}\right]$

- $J(\vec{q}) = \frac{1}{\left(2\pi\eta^2\right)^{3/2}} \exp\left[-\frac{\vec{q}^2}{2\eta^2}\right]$

- and routine changes of variables, as well as algebra and calculus manipulations

THE DEGENERACY PARAMETER (4)

$$\delta = \frac{1}{\left(4\pi\sigma^2\right)^{3/2}} \int \exp\left(-\frac{\vec{u}^2}{4\sigma^2}\right) d^3\vec{u} \bullet \iint d^3\vec{r}_1 d^3\vec{r}_2$$

$$\left\{\exp\left[-\eta^2 \left|\vec{r}_1 - \vec{r}_2\right|^2\right] \psi^*(\vec{r}_1 - \frac{\vec{u}}{2}) \psi(\vec{r}_2 - \frac{\vec{u}}{2}) \psi^*(\vec{r}_2 + \frac{\vec{u}}{2}) \psi(\vec{r}_1 + \frac{\vec{u}}{2})\right\}$$

LET ψ BE CENTERED AT ORIGIN WITH SPREAD Δs IN EACH SPACE DIMENSION:

- IF $\sigma << \Delta s$ AND $\eta \Delta s << 1$:

$$\boxed{\delta \rightarrow 1 \quad \text{(Quantum limit)}}$$

- IF $\sigma >> \Delta s$ AND $\eta \Delta s >> 1$:

$$\boxed{\delta \rightarrow \prod_{i=xyz} \frac{1}{2\sigma_i \eta_i} \quad \text{(Classical limit)}}$$

What is the best shape for radial distribution at apogee ?

$$\psi(r) \sim e^{-\frac{(R-r)^2}{4\Delta^2}}$$

One wants the shell width **Δ** at apogee to be reasonably large because this relaxes restrictions on how small the interaction volume must be. Also, it is desirable that the radial part of the wave function describing the wave packet at apogee be real (apart from an arbitrary overall phase), so that the radial current density vanishes everywhere. These goals are achieved by suitable chirping of the 777 nm laser pulse

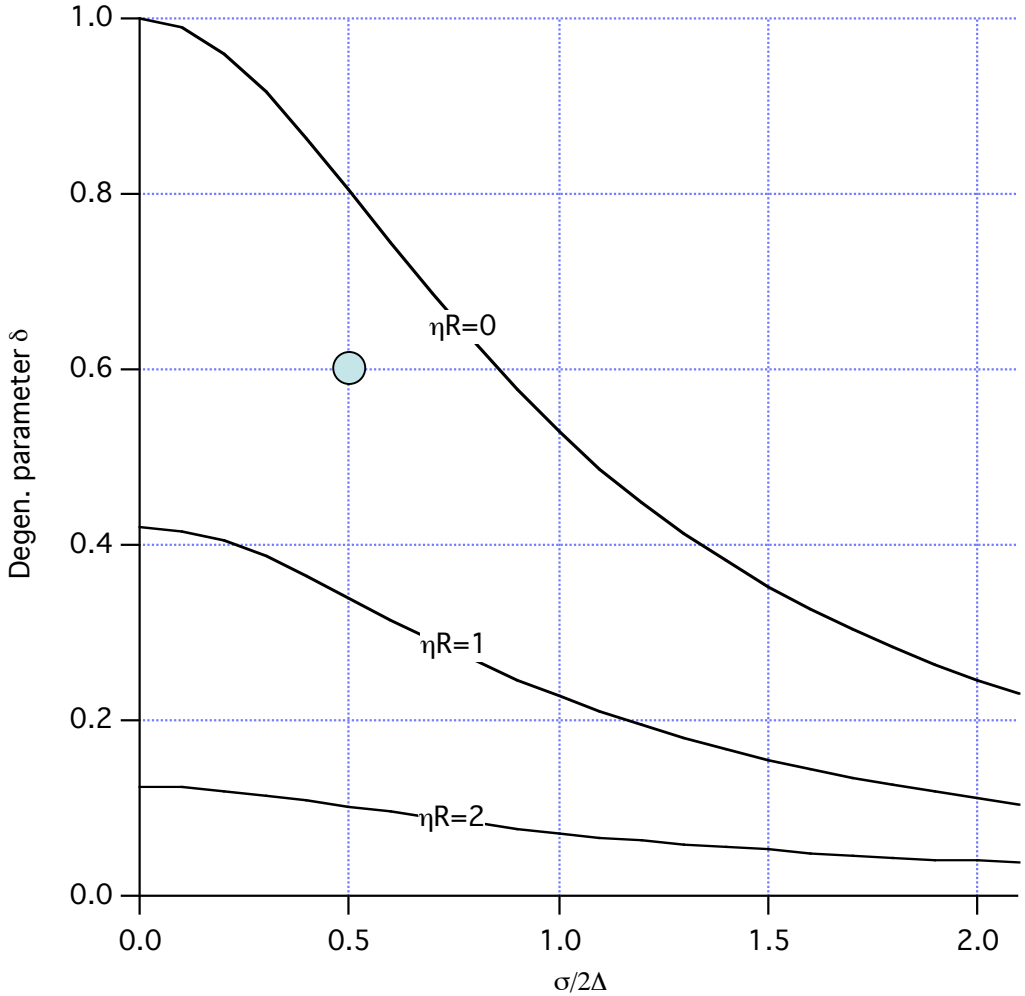
For Gaussian distributions Cs atoms in coordinate and momentum space and

$$\psi(\mathrm{r}) \sim e^{-\frac{(R-r)^2}{4\Delta^2}} \quad , \qquad \eta\Delta \ll 1 \ , \quad \sigma \ll R$$

12D integral for degeneracy parameter simplify to 1D

$$\delta \approx \int_0^1 \frac{\exp\left(-2\eta^2 R^2 x\right)}{\sqrt{1+\frac{\sigma^2}{2\Delta^2}+\left(\frac{\sigma^2}{2\Delta^2}\right)^2 x(1-x)}}\, dx$$

Degeneracy parameter for shell packet at apogee



$$\overline{n} = 800$$
$$R = 67\ \mu\text{m}$$
$$\Delta = 8\ \mu\text{m}$$
$$\eta = 1/R$$
$$V_{tr} = 173\ \text{cm}/s$$
$$\theta \simeq 10^{-2}\ \text{rad}$$

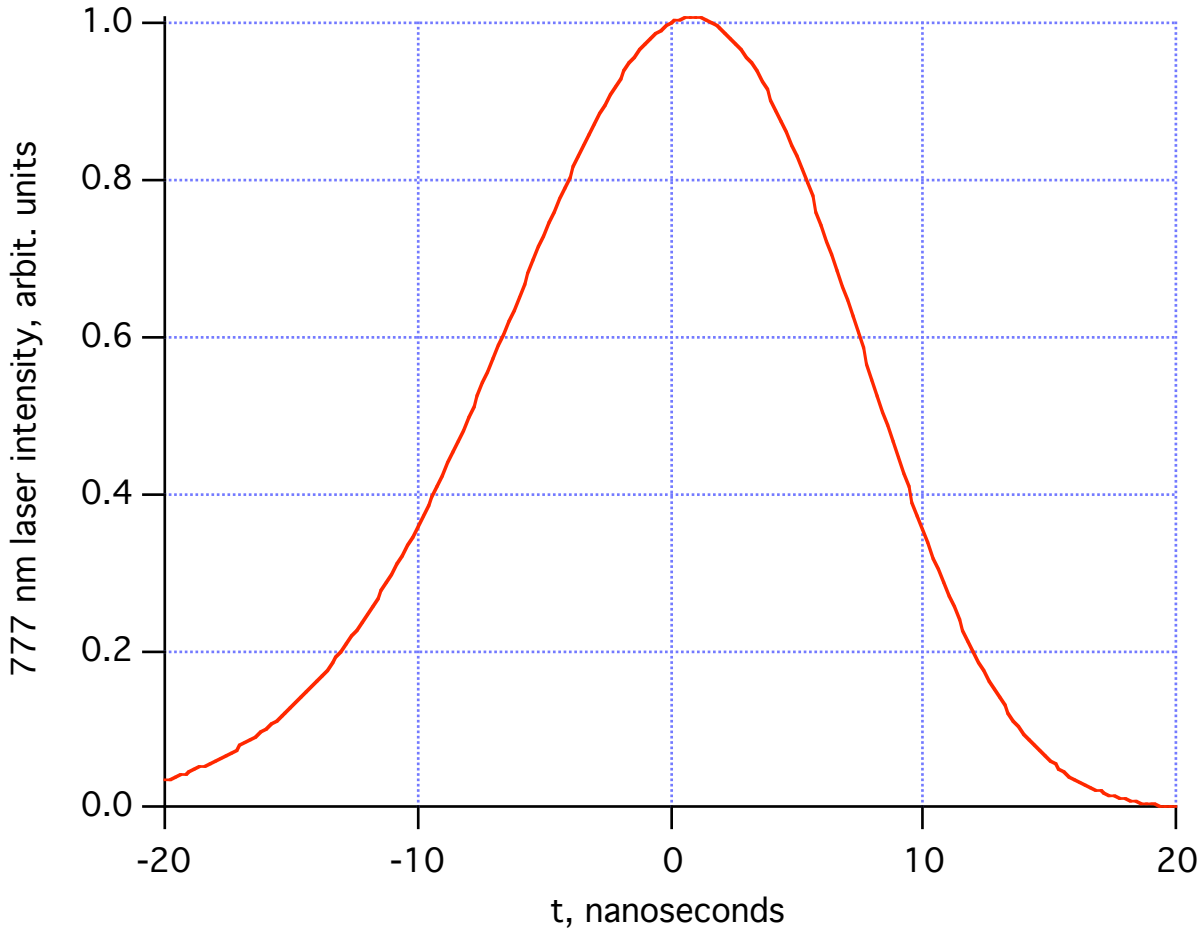
Kepler's laws and requirement that Ψ (t,r) is real in t=1 uniquely define:

$$\phi(t) = -\pi \overline{n} \left[\frac{3}{2} (1 - t)^{1/3} + \frac{t}{2} \right]$$

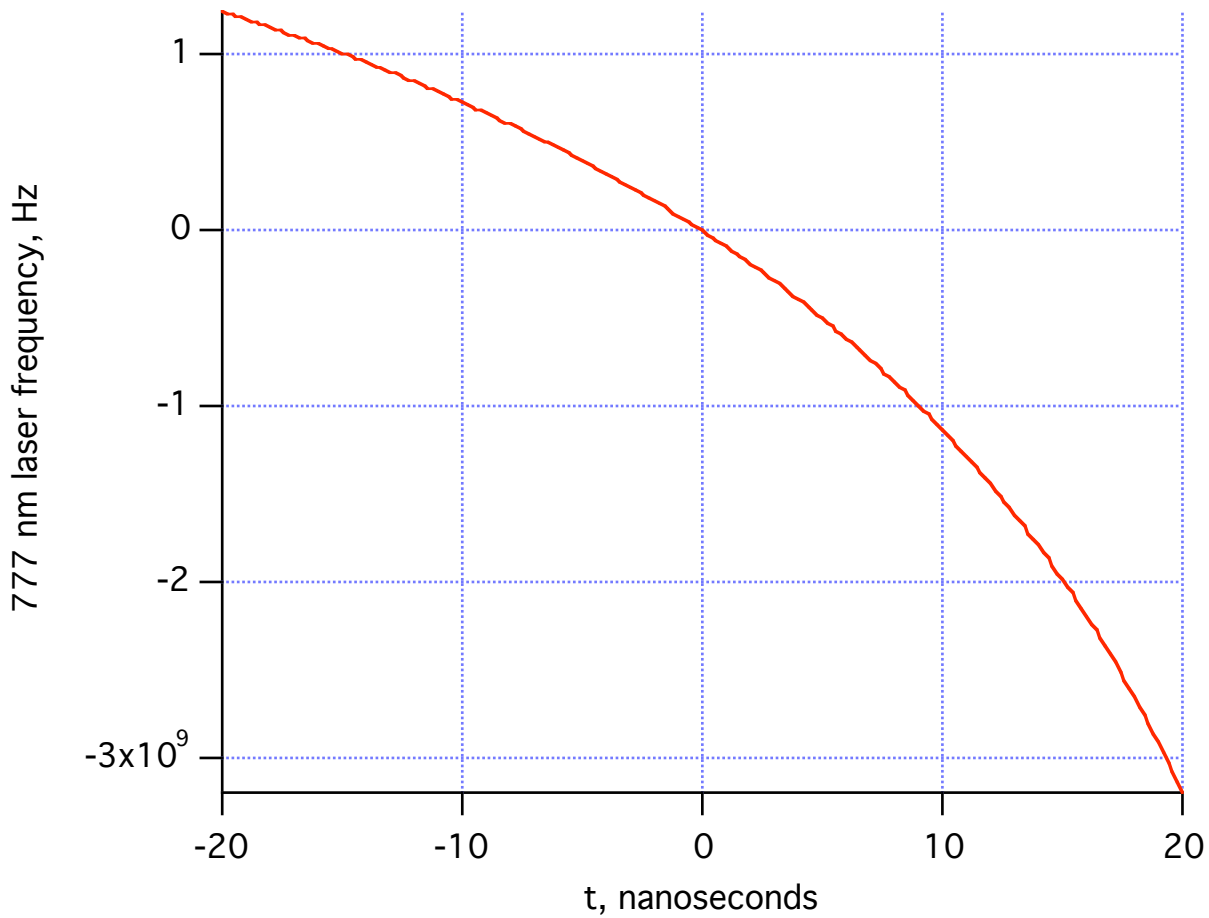
$$A(t) \sim (1 - t)^{-1/3} \Psi\left[(1 - t)^{2/3}\right]$$

$$\Psi(r) \sim r^{1/2} A\left(1 - r^{3/2}\right)$$

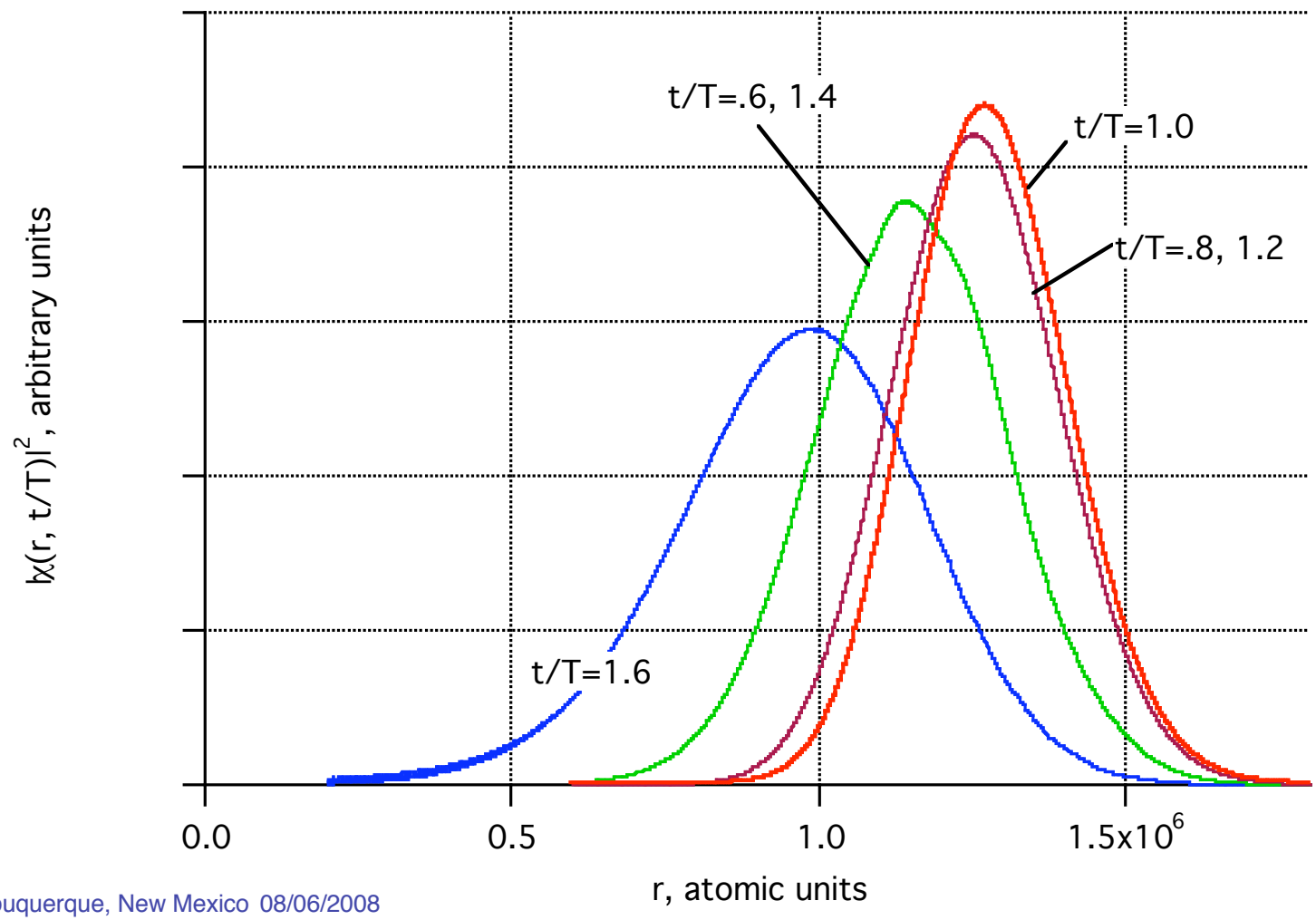
Chirped 777 nm laser intensity



Chirped 777 nm laser frequency



Radial probability density versus t / T



HOW A REAL DEVICE CAN FAIL

- Stray magnetic fields (> .002 G)
- Stray electrons (e.g. laser generated photo-electrons)
- Scattering of Rydberg atoms by background gas
- **Stray electric fields**

  (must be << .001 V/cm)

WHY THIS CONSTRAINT ON STRAY ELECTRIC FIELDS?

ELECTRIC FIELD AT e^- DUE TO CESIUM ION, AT APOGEE:

$$E = \frac{e}{R^2} = \frac{1}{\left(1.28 \bullet 10^6\right)^2} \frac{e}{a_0^2} = .0031 \text{ V/cm}$$

Experimental Problems to be solved (1)

- Atomic beam

  Vacuum system and oven designed, constructed.

  Oven not yet operated.

  No provision yet for laser cooling of atomic beam.

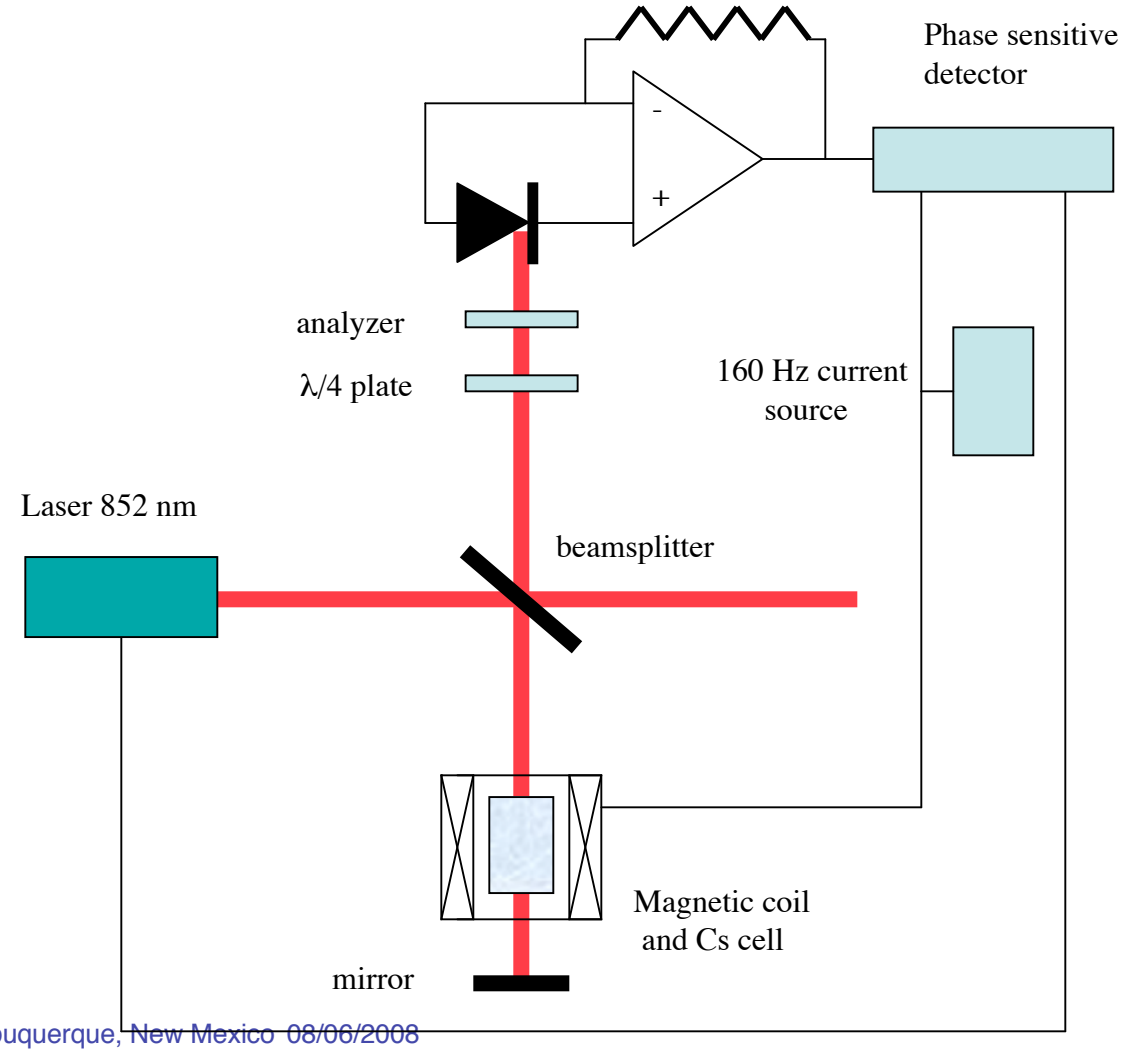
- Laser System

  Active stabilization of 852 nm and 1.47μ lasers achieved.

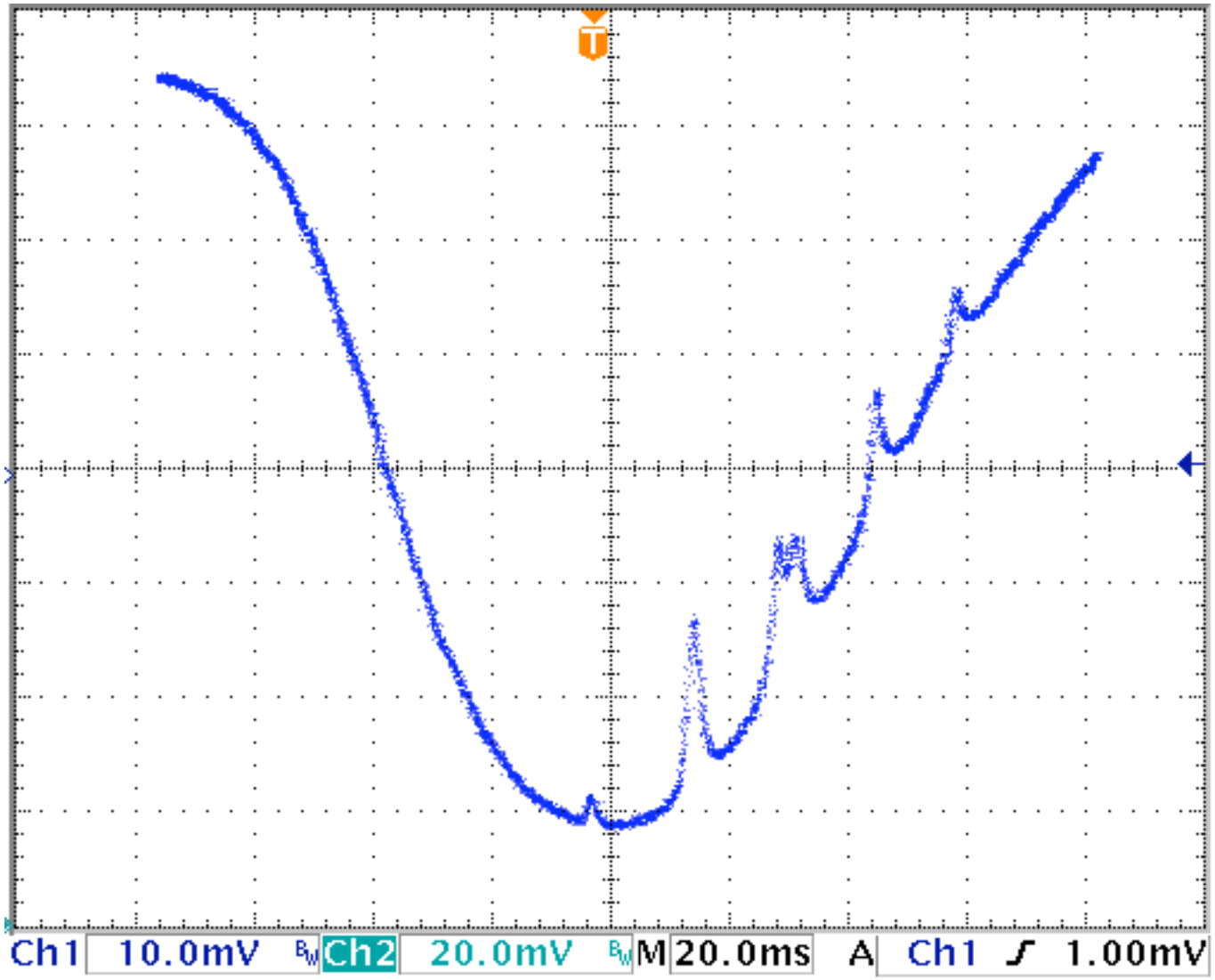
  Present 777 nm laser is inadequate for real experiment;

  no satisfactory procedure for stabilization yet.

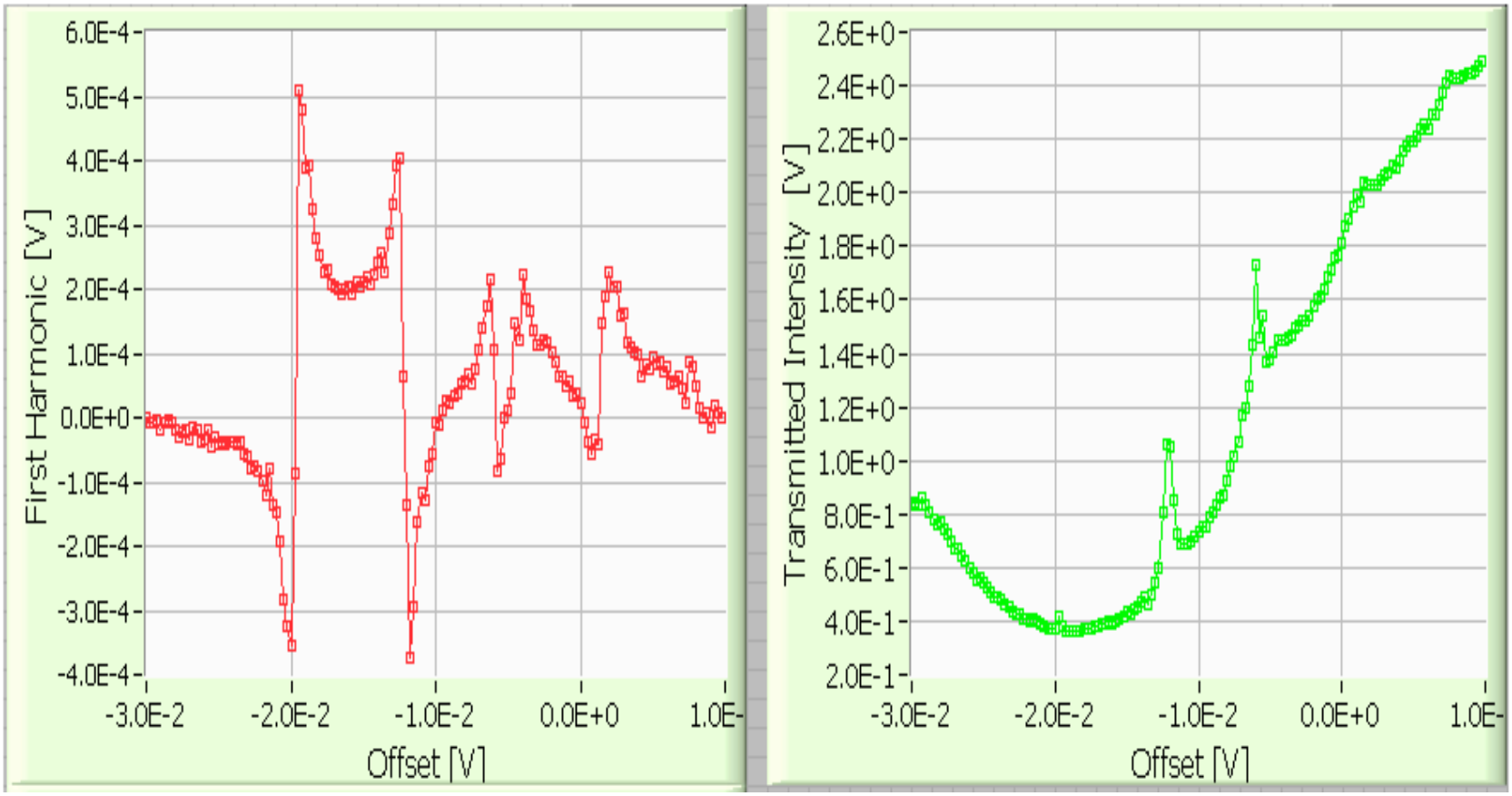
Doppler-free Faraday rotation 852nm



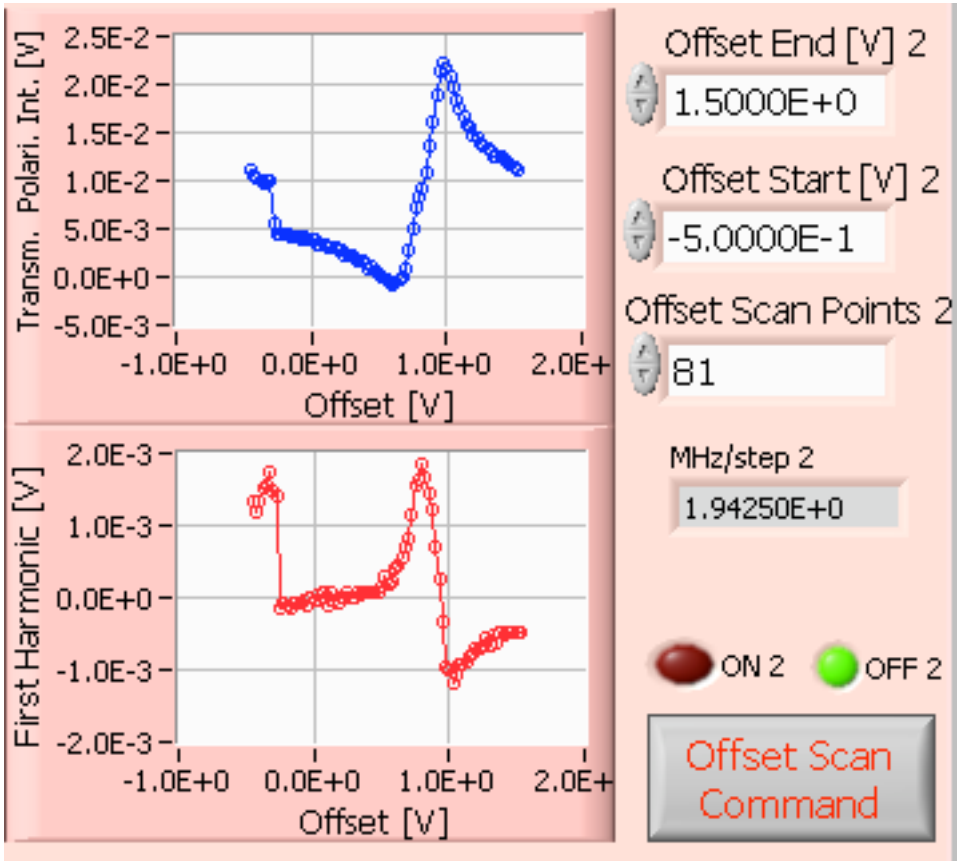
Doppler-free spectroscopy 852nm data



Doppler-free Faraday rotation 852nm data



Faraday rotation 1.47 mkm



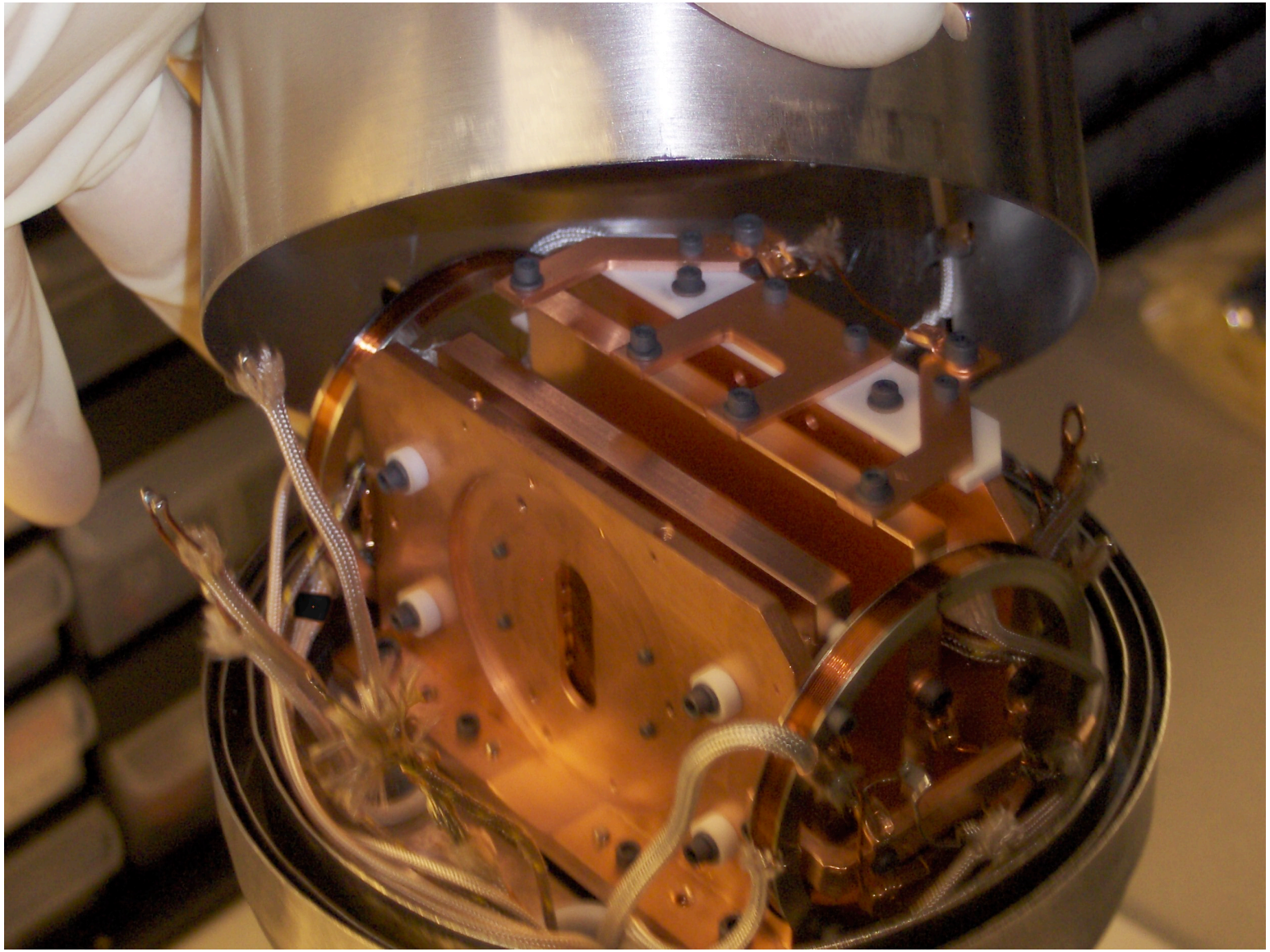
Experimental Problems to be solved (2)

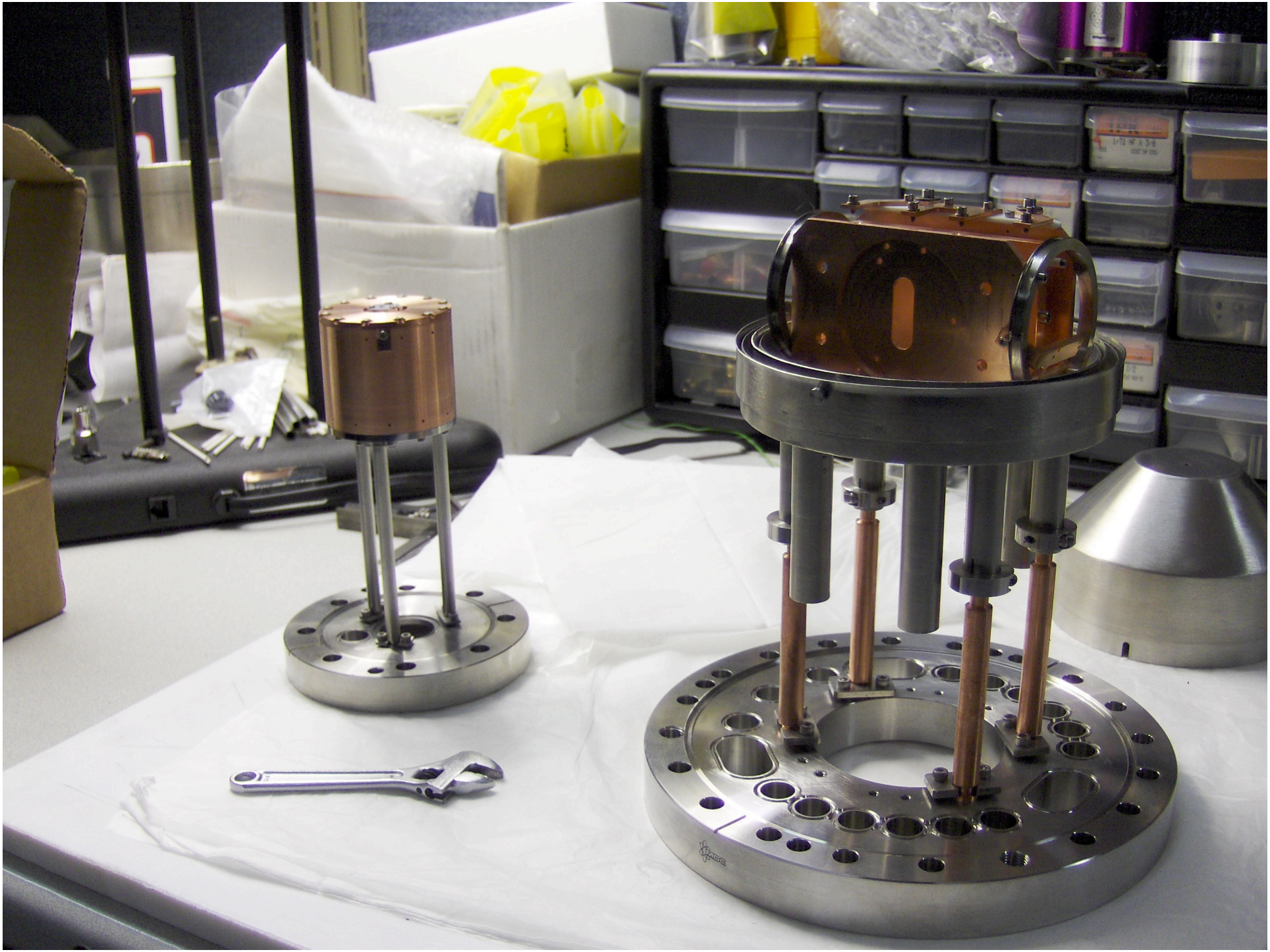
- Stray electric field measurements

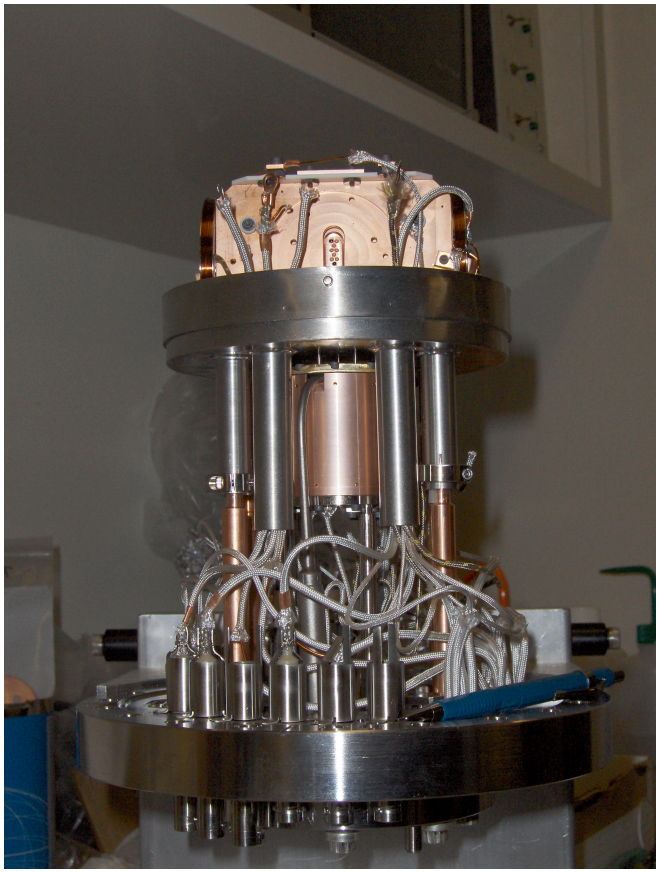
  Time-of-flight system designed and constructed, now being tested.

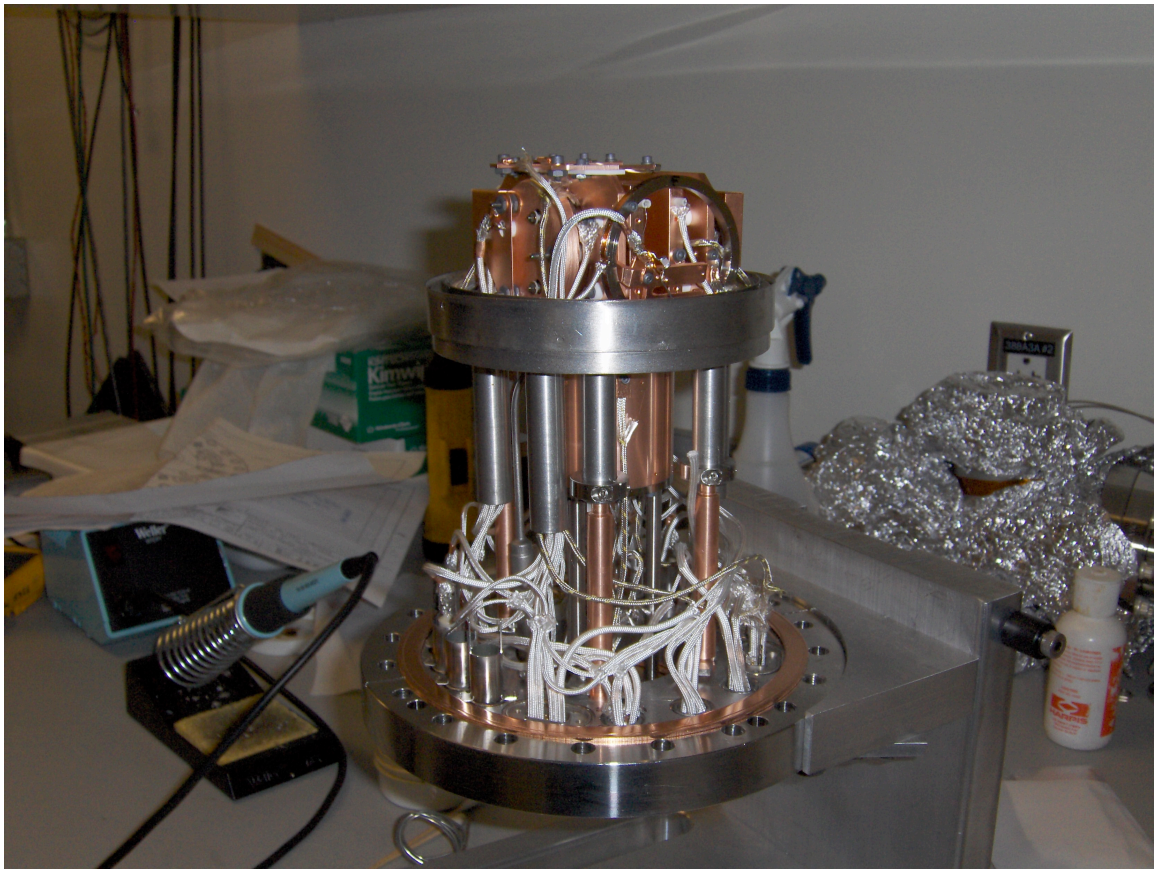
- Stray magnetic fields

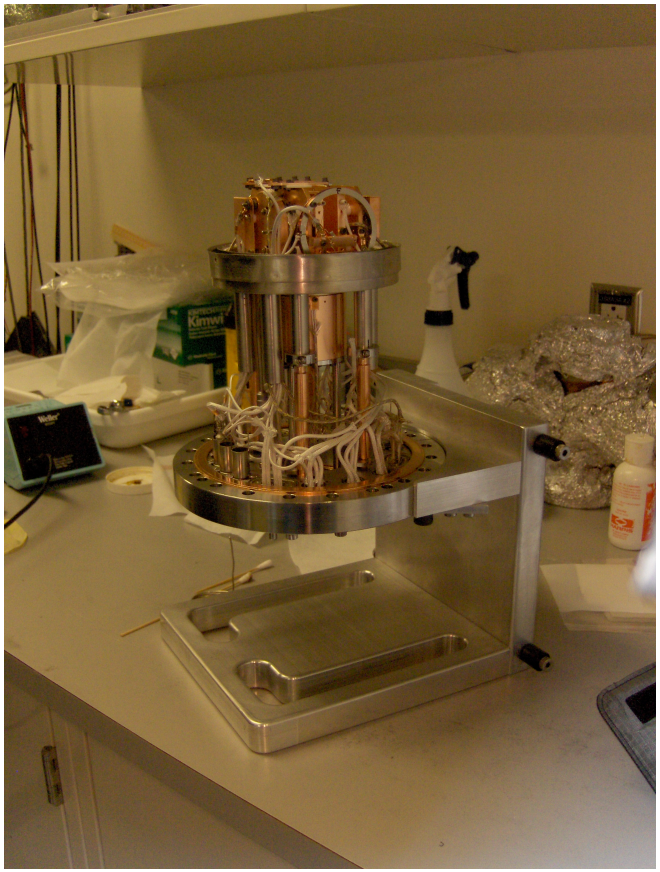
  Existing shields should be adequate.







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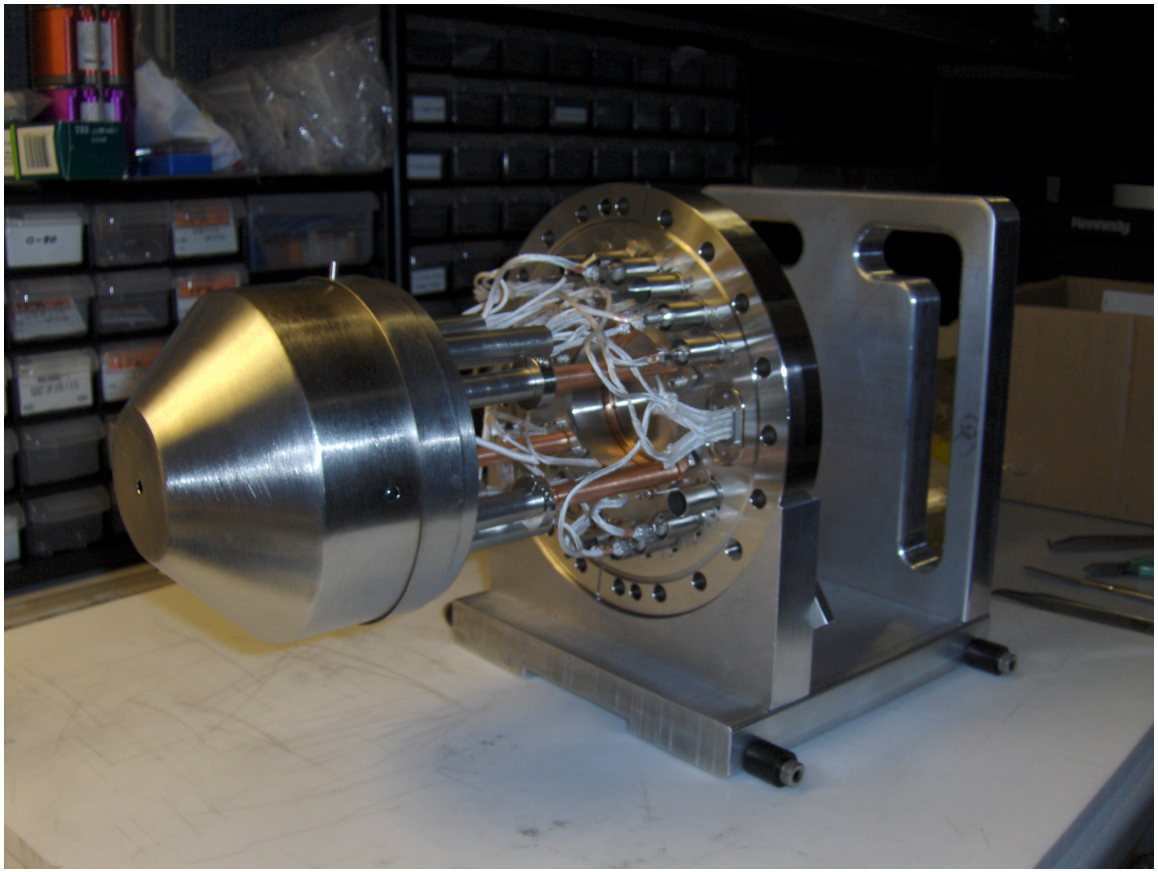

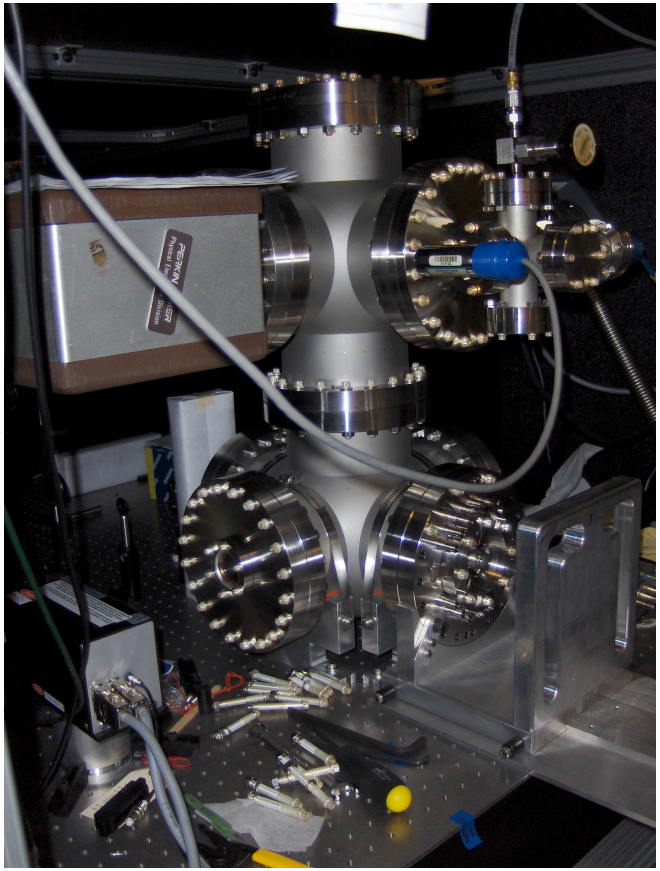
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We are still working

Thank You !