Half-Integer Resonant Extraction from the Debuncher

(the early years: vol.1)

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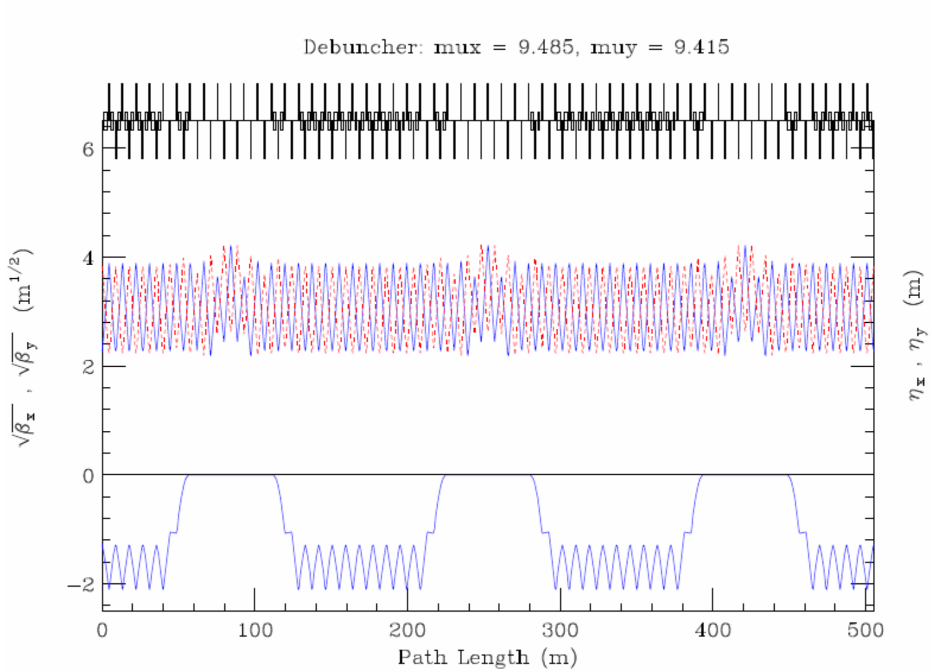
Philosophy

- The non-linear nature of the half-integer resonance guarantees that the extraction characteristics are inter-dependent functions of the extraction parameters: these being; the strengths of the harmonic quadrupole and octupole circuits, the effective betatron phase difference between the harmonic quadrupoles and septum, the septum offset, and the initial tune separation from the half-integer.
- It is precisely because extraction is a non-linear phenomenon that numerical tracking codes are **not** the appropriate tools for initial exploration of the parameter space. The degree of insight into the system dynamics by such an approach is severely limited.

- In this (very!) preliminary work the intention is to produce analytic expressions that can be used to explore the system dynamics and narrow the range of acceptable extraction parameters. Expressions for the step-size at the septum, maximum circulating beam amplitude & extraction inefficiency are derived under the conditions that exist at the end of extraction; that is, when there is zero stable phase-space. This simplification does not qualitatively alter conclusions one obtains from the general case, but the resulting equations are more readily transparent to interpretation.

- off we go then ……

Debuncher Lattice



- This particular 'missing magnet' dispersion suppressing scheme **requires** that the arc & suppressor cells have 60° of horizontal phase advance – this must not (or should not) be changed.

- With the phase of each straight section cell also adjusted to 60° the overall tune of the machine is exactly 9.50.

Idealized Equations of Motion Near the Half-Integer

- With only 19th harmonic quadrupole & 0th harmonic octupole perturbations, an idealized Hamiltonian, which is an approximate constant of the motion, can be written as: $$\widehat{H} = [\delta - \widehat{q}\cos(2\phi - \chi)]R^2 - 3\widehat{\lambda}R^4$$
- The linear betatron phase advance $\widehat{\theta} \equiv \int ds/\nu\beta$ is conjugate to $\widehat{H}$.
- $\delta = (19/2 - \nu)$ is the fractional tune separation, and action-angle variables are:

$$\beta R^2 = x^2 + (\beta p + \alpha x)^2; \phi = \frac{19}{2}\widehat{\theta} + \tan^{-1}[(\beta p + \alpha x)/x]$$

- The quadrupole driving strength is defined as $\widehat{q} \equiv [q_s^2 + q_c^2]^{1/2}$, where qs & qc are the orthogonal sine and cosine-like contributions;

$$q_s = \frac{1}{2\pi}\oint ds \frac{B'\beta}{2B_0\rho} \cdot \sin(19\widehat{\theta}) \qquad q_c = \frac{1}{2\pi}\oint ds \frac{B'\beta}{2B_0\rho} \cdot \cos(19\widehat{\theta})$$

- χ determines the phase-space orientation at the septum $\chi \equiv \tan^{-1}[qs/qc]$.
- The 0th harmonic octupole term is the integral around the ring:

$$\widehat{\lambda} = \frac{1}{2\pi}\oint ds \frac{B'''\beta^2}{96B_0\rho}$$

Hamiltonian (cont'd)

- For brevity of notation it is convenient to change the normalization of the Hamiltonian: $H \equiv \hat{H}/\delta$; $\theta \equiv \delta\hat{\theta}$ is conjugate to H, advancing by $2\pi\delta$ per turn, and the explicit dependence on the octupole field can be removed by rescaling R^2 to $r^2 \equiv (6\lambda/\delta)\cdot R^2$.
- The approximate constant of the motion is now $\kappa \equiv (6\lambda/\delta)\cdot H$:

$$\kappa = [1 - q\cos(2\phi - \chi)]r^2 - r^4/2$$

- In this normalization $q \equiv \hat{q}/\delta \to 1$ in the limit of zero stable phase space.
- Hamilton's eqn's of motion are determined from:

$$\frac{dr^2}{d\theta} = -\frac{\partial\kappa}{\partial\phi} = -2qr^2\sin(2\phi - \chi)$$

$$\frac{d\phi}{d\theta} = +\frac{\partial\kappa}{\partial r^2} = [1 - q\cos(2\phi - \chi)] - r^2$$

- At the unstable fixed points:

$$r_F^2 = [1 - q\cos(2\phi - \chi)] \to (1 - q)$$

Hamiltonian (cont'd)

- The separatrices in general satisfy:

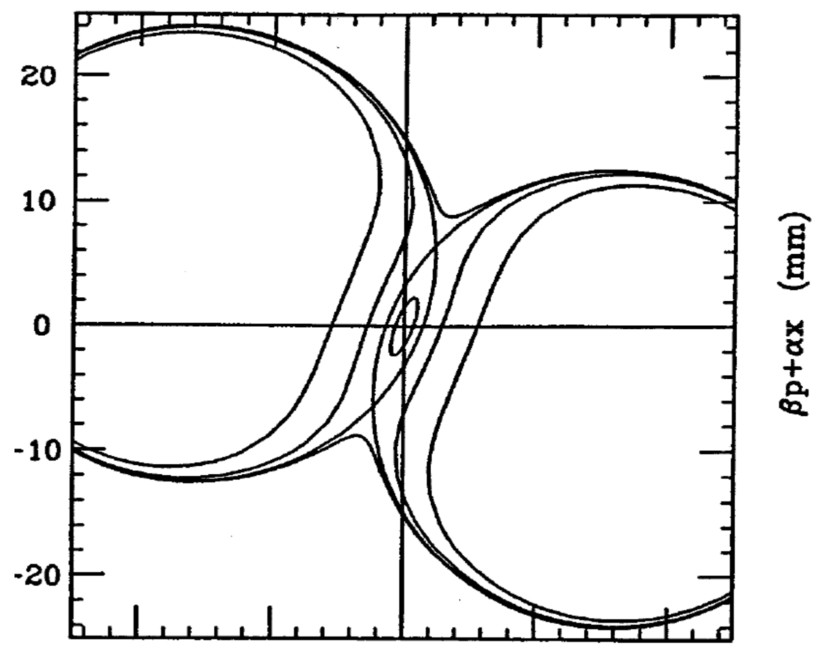
$$r^4 - 2[1 - q\cos(2\phi - \chi)]r^2 - (1-q)^2 = 0$$

- This is the description of 2 overlapping circles of unit radii, which is most easily seen by switching to co-ordinates at the septum $\hat{x}[\hat{x}'] \equiv r\cos(\phi)[r\sin(\phi)]$. The curves describing the separatrices, become:

$$[\hat{x} \pm \sqrt{q}\sin(\chi/2)]^2 + [\hat{x}' \mp \sqrt{q}\cos(\chi/2)]^2 = 1$$

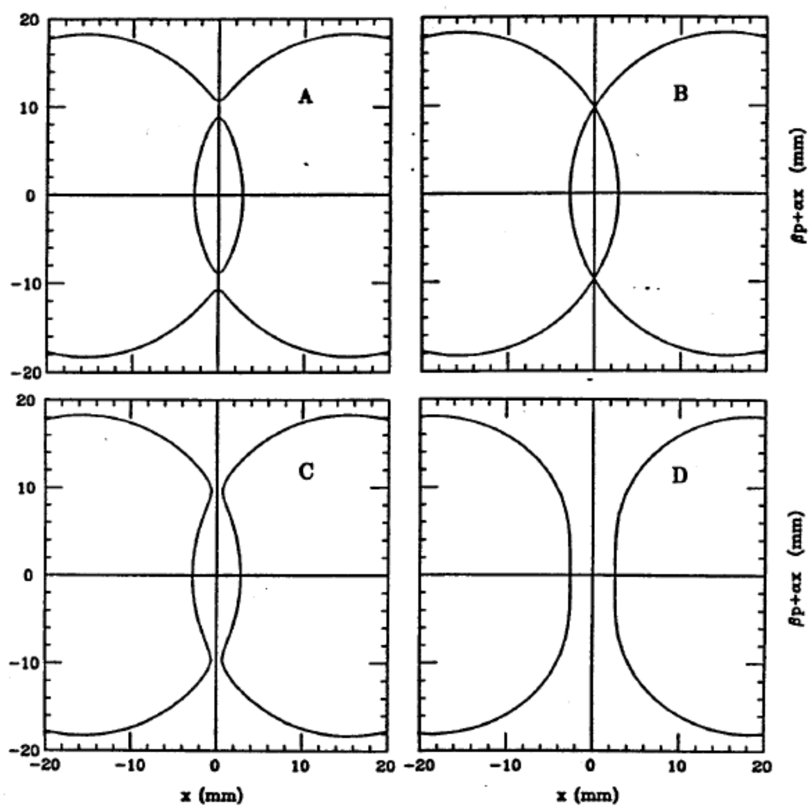
- The stable region is the area enclosed by the overlap of the circles. Extraction begins when q and λ are such that the overlap has shrunk to an area equal to the emittance of the circulating beam;

$$\boxed{\pi\varepsilon = \frac{\delta}{3\hat{\lambda}}\left\{\sin^{-1}[\sqrt{1-q}] - \sqrt{q}\sqrt{1-q}\right\}}$$



Representative contours of constant H. The innermost contour is stable. Next is the stability limit with fixed points at the intersection of the 2 circles. The remaining contours are unstable.

Contour Evolution During Extraction



a) central stable & outer unstable contours are unconnected;

b) stability limit, with the stable region defined by the overlap of the circles;

c) both contyours are unstable, and;

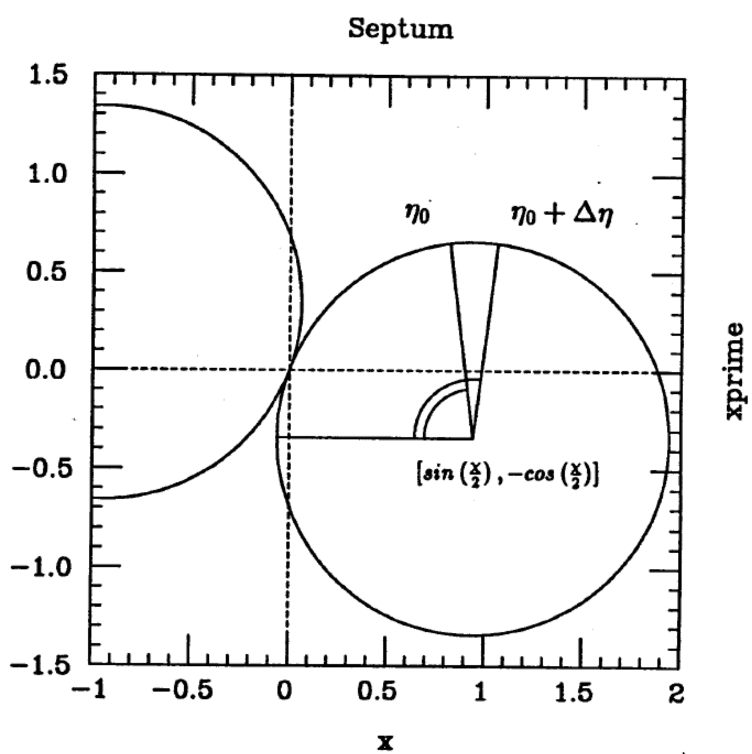
d) the end of extraction ($q = \delta$)

From this point forward only results derived at the end of extraction ($q=\delta$) are presented!

Step Size at the Septum

Given δ, λ & septum offset, the extraction inefficiency and maximum amplitude of the circulating beam are determined by the step size at the septum. The motion of particles in $\hat{x} - \hat{x}'$ space can be described by an angle of rotation about the circle center, where, at the end of extraction (q=1) the angle η is:

$$\begin{aligned} \hat{x} - sin(\chi/2) &= -cos(\eta) \\ \hat{x}' + cos(\chi/2) &= +sin(\eta) \end{aligned}$$



Step Size (cont'd)

The two-turn increase $\Delta\eta$ in η is found by first differentiating the preceding definitions of x, x':

$$\begin{aligned} sin(\eta)\frac{d\eta}{d\theta} &= \frac{d\hat{x}}{d\theta} = -r sin(\phi) \cdot \frac{d\phi}{d\theta} + cos(\phi) \cdot \frac{dr}{d\theta} \\ &= 2\left[\hat{x}' + cos(\chi/2)\right]\left[\hat{x} sin(\chi/2) - \hat{x}' cos(\chi/2)\right] \\ \Rightarrow \frac{d\eta}{d\theta} &= 2\left[1 - sin(\eta + \chi/2)\right] \end{aligned}$$

The increase $\Delta\eta$ in η_0 is solved from the integral equation:

$$\int_{\eta_0}^{\eta_0+\Delta\eta} \frac{d\eta}{2\left[1 - sin(\eta + \chi/2)\right]} = 4\pi\delta$$

Where the right hand side follows from θ advancing by $4\pi\delta$ in two turns. The solution is straightforward:

$$\boxed{\Delta\eta = 2 \cdot tan^{-1}\left\{\frac{4\pi\delta \cdot \left[1 - sin(\eta_0 + \chi/2)\right]}{1 + 4\pi\delta \cdot cos(\eta_0 + \chi/2)}\right\} \approx \mathbf{8\pi\delta \cdot [1 - sin(\eta_0 + \chi/2)]}}$$

The final approximation for $\Delta\eta$ follows from the tune separation δ << 1.

Maximum Amplitude of Circulating Beam

Aperture restrictions (physical or dynamic) limit the maximum amplitude permissible for the circulating beam. Referring to the diagram showing the separatrices at the septum, the amplitude at the septum is the distance from the origin ($\eta = \pi/2 - \chi/2$) to the point $\eta = \eta_0 + \Delta\eta$.

If β_{max} is the largest betatron function in the machine, the maximum amplitude is simply:

$$\boxed{a_{max} = \sqrt{\frac{\delta\beta_{max}}{6\lambda}} \cdot 2sin\left[\frac{1}{2}\left(\eta_0 + \Delta\eta - \frac{\pi}{2} + \frac{\chi}{2}\right)\right]}$$

Extraction Inefficiency

- Extraction inefficiency is the fraction of particles lost hitting the septum wires relative to the number extracted. Assuming that all particles hitting the wire are lost then inefficiency can be expressed as:

$$f = \frac{\int_{\hat{x}_0}^{\hat{x}_0+\hat{\omega}} d\hat{x}/(d\hat{x}/d\theta)}{\int_{\hat{x}_0}^{\hat{x}_0+\Delta} d\hat{x}/(d\hat{x}/d\theta)}$$

- The denominator is the total number extracted. The finite upper limit reflects that the maximum value of x is limited by the step size. The numerator is the relative number hitting the wire of width ω. For a thin wire dx/dθ does not change appreciably across the width & can be removed from under the integral sign. From which follows:

$$\boxed{f = \sqrt{\frac{6\lambda}{\delta\beta}} \cdot \omega \cdot \{8\pi\delta \cdot \sin(\eta_0)[1 - \sin(\eta_0 + \chi/2)]\}^{-1}}$$

- Using the small-δ approximation for $\Delta\eta$ & noting that $\sin(\eta_0) \approx 1$ always, it follows that:

$$f \approx \sqrt{\frac{6\lambda}{\delta\beta}} \cdot \frac{\omega}{\Delta\eta}$$

Septum Offset

It is desirable to minimize the slow-extracted beam size at the extraction Lambertson. For a Lambertson located 90° in phase D/S of the septum it is clear that this will be accomplished if $\hat{x}'(\eta_0) = \hat{x}'(\eta_0 + \Delta\eta)$; i.e;

$$\begin{aligned} sin(\eta_0) &= sin(\eta_0 + \Delta\eta) \\ \Rightarrow \eta_0 &= \frac{\pi}{2} - \frac{\Delta\eta}{2} \end{aligned}$$

The step-size $\Delta\eta$ can rewritten exclusively as a function of the angle χ and tune separation δ:

$$\boxed{\Delta\eta = 2 \cdot sin^{-1}\left\{\frac{4\pi\delta}{1+(4\pi\delta)^2}\left[-cos(\chi/2) + \sqrt{1+[4\pi\delta sin(\chi/2)]^2}\right]\right\}}$$

Exploring Parameter Space

With step-size related to septum offset η_0 by the previous expressions, the other relevant extraction parameters can be simplified to:

$$x_{sep} = \sqrt{\frac{\delta\beta_{sep}}{6\lambda}} \cdot [sin(\chi/2) - sin(\Delta\eta/2)]$$

$$a_{max} = \sqrt{\frac{\delta\beta_{max}}{3\lambda}} \cdot [1 - cos(\Delta\eta/2 + \chi/2)]^{1/2}$$

$$f = \sqrt{\frac{6\lambda}{\delta\beta_{sep}}} \cdot \frac{\omega}{8\pi\delta \cdot cos(\Delta\eta/2)[1 - cos(\Delta\eta/2 - \chi/2)]}$$

One possible procedure for exploring variation of maximum beam amplitude & inefficiency with the extraction parameters can proceed as follows: given a tune separation δ and harmonic phase $\chi/2$, step-size at the septum is defined, and; given an offset x_{sep}, the octupole strength λ required is fixed. There are no free parameters at this point and a_{max} and f are completely defined. Variation of δ, x_{sep} and $\chi/2$ then spans the parameter space.

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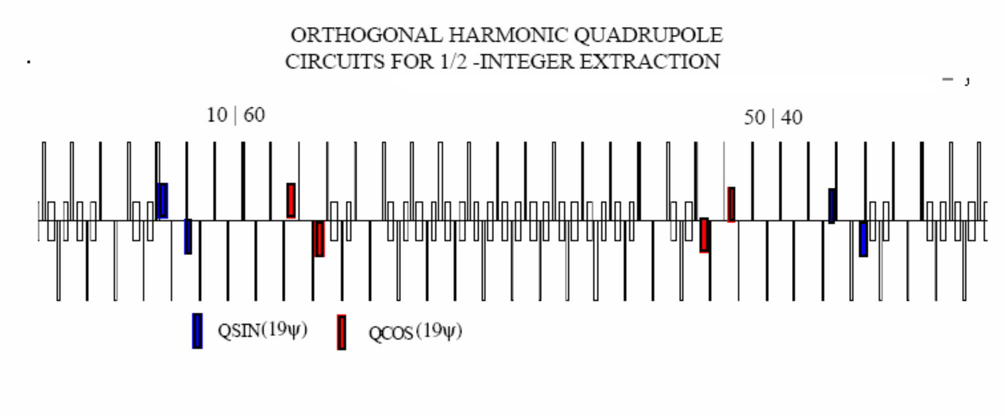
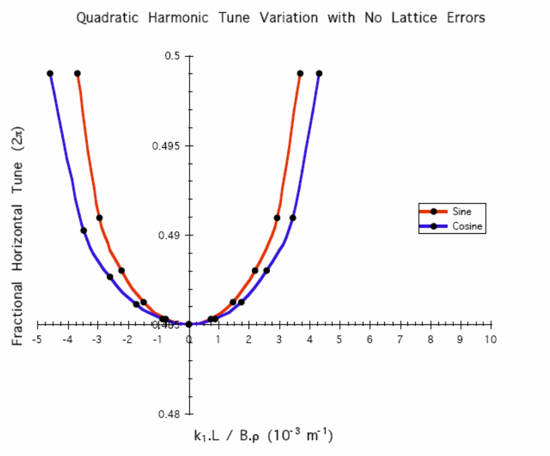
Layout of Extraction Circuits

- The harmonic quad circuits are comprised of a total of 8 magnets split into 2 orthogonal families. The sin(19ψ) & cos(19ψ) families are interleaved across the 10/60 & 54/40 straights, as shown below.
- The harmonic circuits perform 3 functions:
  1) Determine the phase-space orientation at the septum;
  2) Control the extraction ramp, and;
  3) Correct the intrinsic half-integer stopband of the machine.

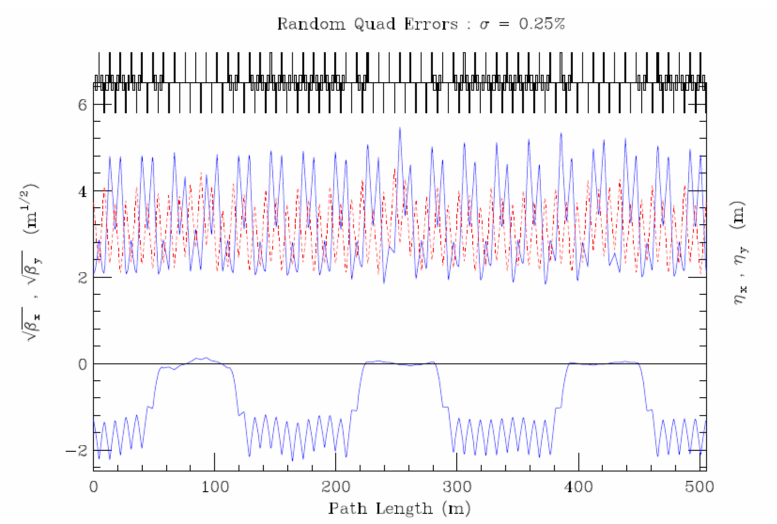
- With an even number of opposite polarity magnets in each family, there is no linear tune shift. As the circuits are ramped, there **is** a quadratic tune variation, which always drags the machine tune towards the half-integer.

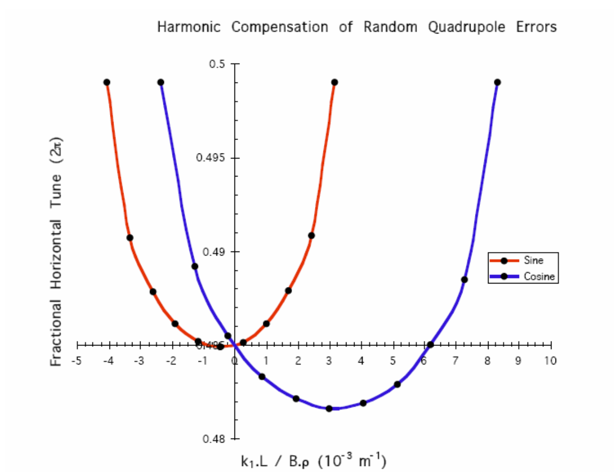




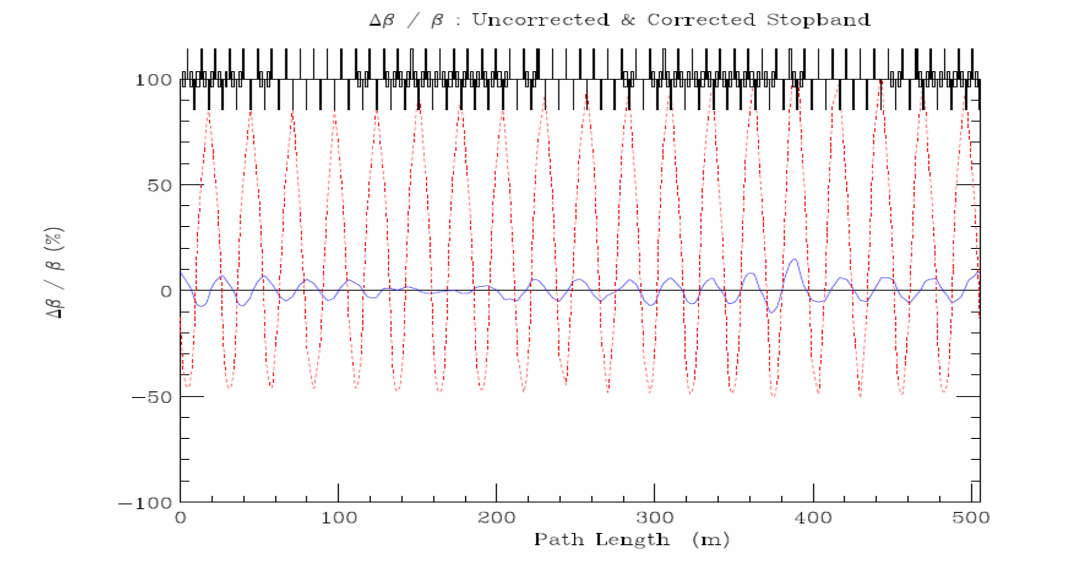
Compensation of Intrinsic Machine Quad Errors ($\Delta\beta/\beta$)

- Random quad errors generate an additional (unwanted) half-integer driving term.
- The procedure for cancelling these errors to 1st order relies on the orthogonality of the harmonic quad circuits:
  - The sin & cosine-like circuits are ramped independently both positive & negative to determine the minimum machine tune;
  - These gradients then serve as revised 'zero', or 'baseline' values for the extraction quad circuits.
- The philosophy of this approach is to first inject quadrupole errors of the correct strength & phase to cancel the machine errors, and then the desired $\Delta\beta/\beta$ errors appropriate for extraction can be applied.





Horizontal Δβ/β Correction (contd)



Uncorrected Stopband Δβ/β		Corrected Stopband Δβ/β	
Max	RMS	Max	RMS
110%	50%	15%	<5%

Summary

- Simple analytic expressions for extraction characteristics have been developed that will be used to initially narrow the range of acceptable extraction parameters.

- Modeling the Debuncher including orthogonal harmonic quadrupole circuits has been completed & shown to be appropriate for canceling random quadrupole field errors in the machine.

- the saga continues ……

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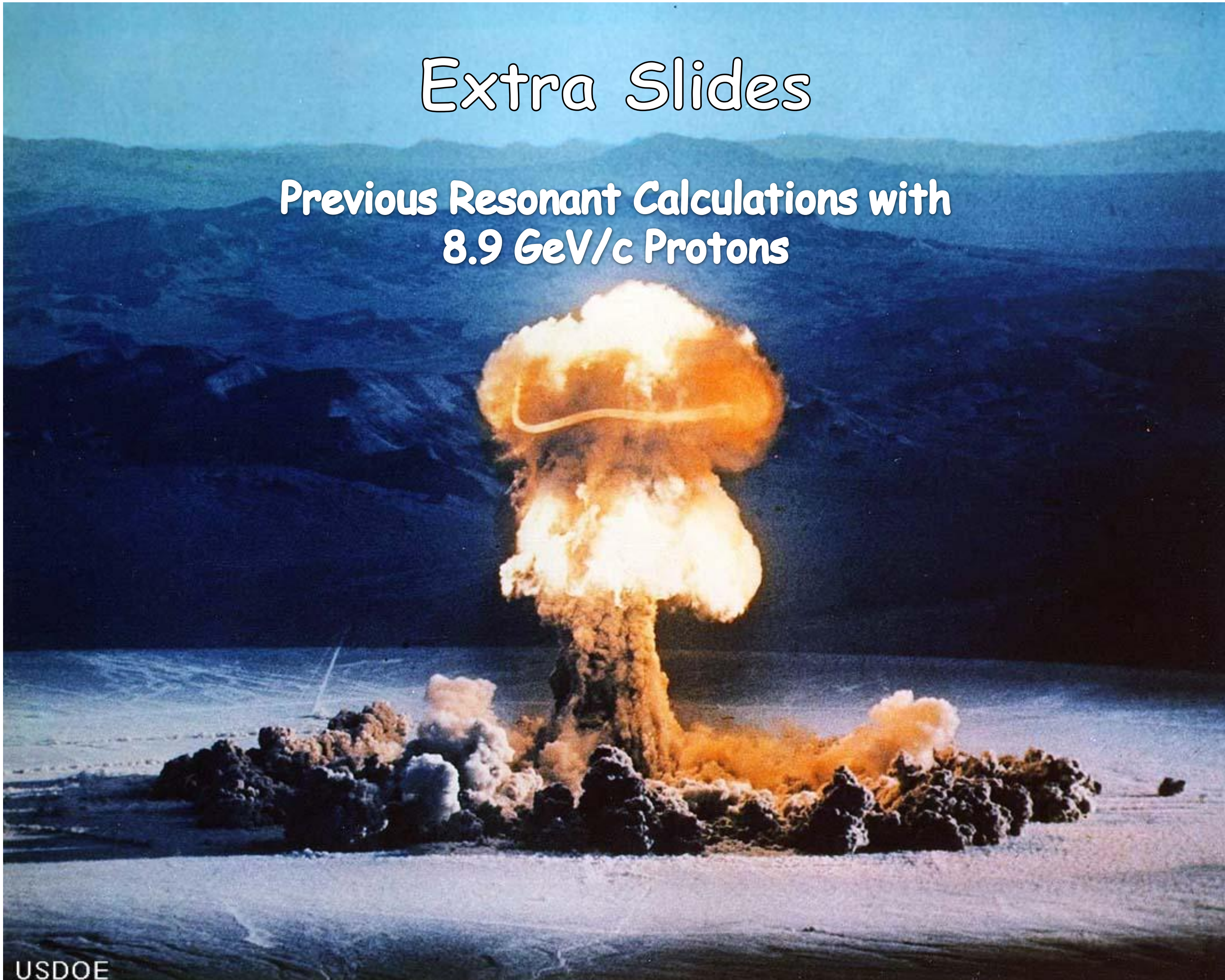
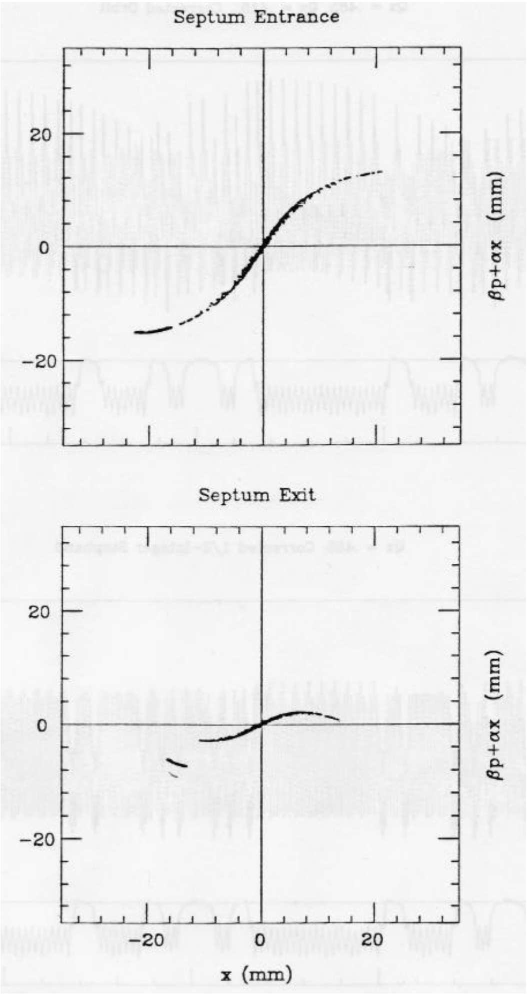
Extra Slides

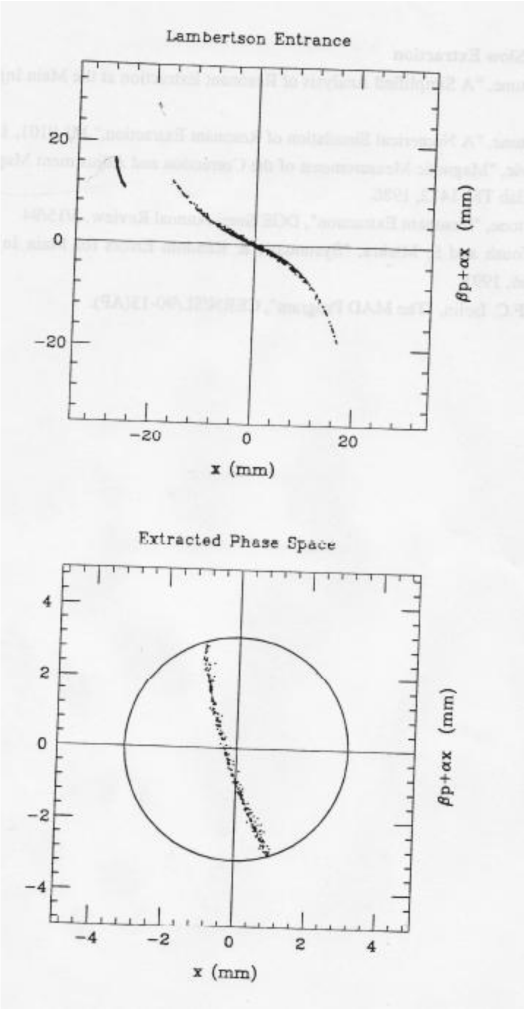
Previous Resonant Calculations with 8.9 GeV/c Protons



MI Extraction

120 GeV/c, Cx = Cy = +5, $\Delta p/p_{95\%}$ = ±0.04% ⇒ **12mm** separation at Lambertson





Recycler Extraction

8.9 GeV/c, Cx = Cy = -10, $\Delta p/p_{95\%}$ = ±0.13% ⇒ **ZERO** separation at Lambertson!