Alignment Tolerances for Booster Correctors

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Abstract

This note examines the effect of misalignment of the new correctors in the Fermilab Booster. Both lateral offset and angle (pitch, yaw) are considered. It is found that incoherent alignment errors on the order of 3 mm will result in closed orbit distortions of less than 1 mm and manageable tune shifts. Systematic alignment errors should be kept below a millimeter. Correctors are insensitive to angular errors well beyond any that could conceivably occur during installation.

Roll tolerances are discussed briefly. The Booster is a highly coupled machine by nature, so arguments involving tune coupling are difficult. An alignment of better than 10 milliradians will keep the dipole coupling below 1%.

1 Introduction

Type	**Field/gradient**	**Effective Length (m)**	**Integrated Field/gradient**
Horizontal Dipole	.0357 T	.44	.0157 T-m
Vertical Dipole	.0357 T	.44	.0157 T-m
Normal Quadrupole	.49 T/m	.36	.176 T
Skew Quadrupole	.031 T/m	.37	.0115 T
Normal Sextupole	5.87 T/m^2 T	.34	2.0 T/m
Skew Sextupole	5.87 T/m^2 T	.34	2.0 T/m

Table 1: Specifications for the Booster corrector packages.

A lateral offset of the correctors can affect the beam in two ways:

- A closed orbit distortion due to the anomalous dipole moment caused by quadrupole and sextupole terms.
- A tuneshift caused by the anomalous quadrupole of the offset sextupole.

There should be relatively little sensitivity to rotational errors. To first order the anomalous dipole of the quadrupole term will be zero, as will the anomalous quadrupole of the sextupole, leaving only a slight anomalous dipole of the sextupole.

Corrector specifications are discussed in [1] and are summarized in Table 1.

2 Lateral Offsets

The closed orbit distortion caused by a single dipole is given by

$$\Delta x(s) = \frac{\theta\sqrt{\beta_0\beta(s)}}{2\sin\pi\nu}\cos\left[\psi(s) - \pi\nu\right]$$

We can approximate the scale of the closed orbit distortions δ caused by incoherent anomalous integrated dipole moments $\delta(Bl)$ at the N(=24) maximum beta β_{max} points as

$$\delta = \frac{\sqrt{N}}{2}\frac{\delta(Bl)}{(B\rho)}\frac{\beta_{max}}{2\sin\pi\nu}$$

where the additional factor of 1/2 comes from the incoherent sum of the phase advance angles. The overall scaling factor is slightly larger for the horizontal plane, where

$$\frac{\beta_{max}}{2\sin\pi\nu} = \frac{33}{2\sin(\pi 6.7)} = 20.4 \text{ m}$$

so we will concentrate on that. If we set the maximum allowable closed orbit distortion to 1 mm, this gives a maximum anomalous dipole moment δBl_{max} .64 and 6.0 Gm per magnet at injection and extraction, respectively.

If the magnet is offset by an amount Δ, the anomalous dipole due to the quadrupole and sextupole terms will be

$$\Delta B = B'\Delta + B''(\Delta)^2$$

We see that the quadrupole term will dominate until we get several *centimeters* from the center, so we will just consider that.

The quadrupole strength was established by considering the maximum possible tune shift at extraction, so we can assume that the maximum common quad setting scales down with $B\rho$ and simply use the extraction value, given an allowable lateral offset error of

$$\Delta = \frac{\delta Bl_{max}}{B'l_{max}} = \frac{.0006}{.176} = .0034 \text{ m}$$

or a 3.4 mm incoherent misalignment. Note that if there is a *systematic* offset, then the effects will add coherently ($\sqrt{24} \rightarrow 24$), adding about another factor of 5 sensitivity, for a systematic limit of about .7 mm.

Lateral offsets of the sextupole can also introduce anomalous quadrupole terms, with the effective focal length given by

$$f_{eff} = \frac{(B\rho)}{2B''l\Delta}$$

The tuneshift due to a single quadrupole error is

$$\Delta\nu = \frac{1}{4\pi}\frac{\beta_0}{f}$$

so the tuneshift associated with random misalignment of the sextupoles is

$$\Delta\nu = \frac{\sqrt{N}}{4\pi}\frac{2\beta_0 B''\Delta}{(B\rho)}$$

If the sextupole correctors are at full strength, this would be a tuneshift of .006 at extraction for Δ = 3.4 mm. It would be a somewhat worrisome tuneshift of .055 at injection, but it is unlikely the sextupoles would be run full strength at injection.

3 Angular Errors

To first order, the anomalous dipole due to the quadrupole is zero for a purely angular misalignment, as is the anomalous quadrupole due to the sextupole. The only remaining effect is the anomalous dipole from the sextupole, given by

$$\delta Bl = \int_{-l/2}^{1/2} B''(\Delta\theta s)^2 ds = \frac{B''\Delta\theta^2 l^3}{12}$$

or

$$\Delta\theta_{max} = \sqrt{\frac{12\delta Bl_{max}}{B''l^3}}$$

Even if we plug in the maximum sextupole strength and the .6 Gm anomalous dipole allowed at injection, this gives an allowable angular error of 57 milliradians, well outside of any conceivable misalignment of the magnet.

4 Roll Errors

The Booster is a highly coupled machine by nature, so arguments involving tune coupling are problematic. We note in passing that a roll error of less than 10 milliradians would keep the dipole coupling of 1%. This should be well within the standard alignment tolerances, and a significant improvement over the previous corrector system.

5 Conclusion

Using a very conservative argument, in which the beam remains in the center of the beam pipe and we try to avoid all significant effects of misalignment, we find that an incoherent lateral misalignment on the order of 3 mm will still result in acceptable closed orbit distortions and tune shifts. Under these assumptions, systematic misalignments should be kept below 1 mm.

The system is insensitive to angular and roll misalignments at any level which could conceivably occur during installation.

References

[1] E. Prebys, *et al*, FNAL-BEAMS-DOC 1881-v5 (2006)