Note on feedback systems for LLRF

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This notes shows a simple analysis of feedback systems applied to LLRF control.

1 Deriving the standard feedback system equations:

The figure below describes the basic feedback (FB) loop for a LLRF system. Set point (SP) and feed foward (FF) tables are indicated. H is the plant gain (i.e. gains associtated with RF up and down conversion, klystron, waveguides, cavity, etc...).

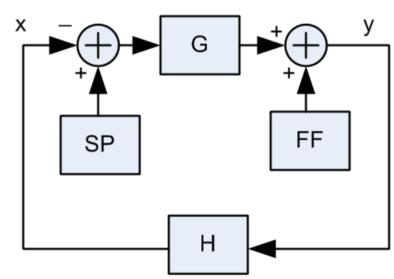

Figure 1: basic FB diagram for LLRF systems

Looking at the first summing junction in figure 1, the error signal $SP - x$ gets multiplied by the FB gain G and summed with the FF table at the second summing junction so that:

$$y = G(SP - x) + FF \tag{1}$$

Note that $x = Hy$, so replacing x by Hy in Eq. 1, we have:

$$y = G(SP - Hy) + FF$$
$$y(1 + GH) = G \times SP + FF$$

Finally,

$$y = \frac{G \times SP + FF}{1 + GH} \tag{2}$$

2 External system gain

2.1 Approach A: nulling the FF

In this approach, one sets the FF table to 0, the (normalized) SP to 1, and the FB gain to 1:

$FF = 0$

$SP = 1$

$G = 1$

Replacing these terms in Eq. 2, y becomes

$$y = \frac{1}{1+H} \tag{3}$$

When the system is calibrated, $H = 1$ corresponds to $y = 1/2$.

2.2 Approach B: nulling the error

In this approach, the FF table are set to 1, the FB gain is set to 1, and we are in the condition where the error is equal to 0. Note that if the error signal is 0, then $y = FF$. This approach is equivalent to replacing in Eq. 2:

$FF = 1$

$G = 1$

$y = FF$

so that Eq.2 simplifies to

$$y = \frac{SP+1}{1+H} = FF = 1 \tag{4}$$

which simplifies to $SP = H$. Here again, when the system is calibrated $H = 1$ corresponds to $SP = 1$ (i.e. the SP tables are scaled to the plant gain).

2.3 Adding a system calibration gain

In the previous section, we've looked at calibrated systems where the external system gain is 1. In this section, we complexify the LLRF system slightly by adding a calibration gain S, to compensate for an external system gain, which a priori is different from 1.

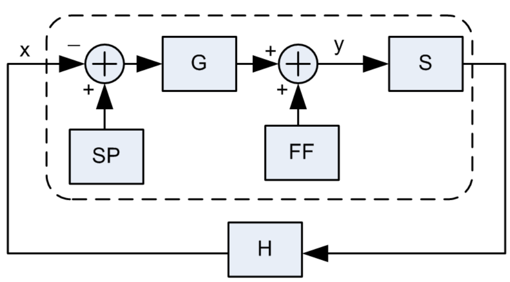

Figure 2: Adding a system calibration gain

Here again, y is the sum of the FF table and the error $(SP - x)$ scaled by G, so that $y = FF + G(SP - x)$. But now, $x = S \times Hy$. Replacing x into the y equation, one gets:

$$\begin{aligned} y &= FF + G(SP - S \times Hy) \\ y(1 + GSH) &= FF + G \times SP \end{aligned}$$

and finally, the system equation becomes:

$$y = \frac{FF + G \times SP}{1 + GSH} \tag{5}$$

Note that by setting $S = 1$ in the equation above, one gets the simpler system description of Eq. 2.
For the first calibration method, one applies $FF = 0$, $G = 1$ and $SP = 1$ in the equation above and solves for S when $y = 1/2$.

$$\begin{aligned} y = \frac{0 + 1 \times 1}{1 + SH} &= \frac{1}{2} \\ \frac{1}{1 + SH} &= \frac{1}{2} \\ 1 + SH &= 2 \end{aligned}$$

which simplifies to

$$S = \frac{1}{H} \tag{6}$$

This means that for an external system with a non-unity gain ($H \neq 1$), the LLRF system calibration is obtained by setting the external gain $S = H^{-1}$.

With the second calibration approach, one applies $FF = 1$, $G = 1$, and $SP = 1$ and solves for S when the error signal is null (i.e. $SP - x = 0$). Note that saying that the error is 0 is equivalent to saying that $y = FF$. Equation 5 now becomes:

$$\begin{aligned} y = \frac{1 + 1 \times 1}{1 + 1 \times SH} &= FF = 1 \\ \frac{2}{1 + SH} &= 1 \\ 1 + SH &= 2 \end{aligned}$$

which simplifies to

$$S = \frac{1}{H} \tag{7}$$

The two method are equivalent, when the system is calibrated $S = H^{-1}$, as found in Eq.6 and 7.

3 Including the external system gain in FB gains

In this section, a practical example is analyzed to illustrate the method explained above. It is inspired by the analysis carried while runnning the RFQ for HINS in November 2009 (logbook entry: `http://www-hins-crl.fnal.gov/hins/SingleEntry.jsp?entryPath=/Entries/2009/11month/11day/15hour/325_MHz_RF/LLRF/Log/Text_819`). The system corresponds to the one depicted in figure 1. At the time we ran the RFQ, we did not calibrated the system. This means our closed loop gain was not unity. So all the reported FB gain values (proportionnal and integral) were not meaningful (i.e. not calibrated). Running with a FF table of 1, one had to lower the SP table to minimize the error signal. This is equivalent to the second approach described above. We lowered the SP scale to 0.73 to zero out the error. As calculated in section 2.2, when the error is nulled, $H = SP$ is verified. In this example, this means that the external system gain H is equal to 0.73. So the gain settings should actually be scaled by 0.73 to correspond to calibrated gain readings. We ran with a proportional gain of 2.5, which corresponds to a calibrated gain of $2.5 \times 0.73 = 1.825$ and with an integral gain of 3200, which corresponds to a calibrated gain of $3200 \times 0.73 = 2336$.

3.1 scaling the integral gain in dB

The integral gain is set as an integer value, but can be scaled to dB. The scaling is given below (+20 dB every factor of 10):

$K_I = 16$	corresponds to -6 dB
$K_I = 160$	corresponds to 14 dB
$K_I = 1600$	corresponds to 34 dB

The equation to get a gain in dB from an integer value of K_I is

$$K_I([\mathrm{dB}]) = 20 \log \frac{K_I}{16} - 6 \tag{8}$$

So in our example, an integral gain $K_I = 2336$ corresponds to +37 dB.