Measurement and Correction of Beta Beating in the Fermilab Booster

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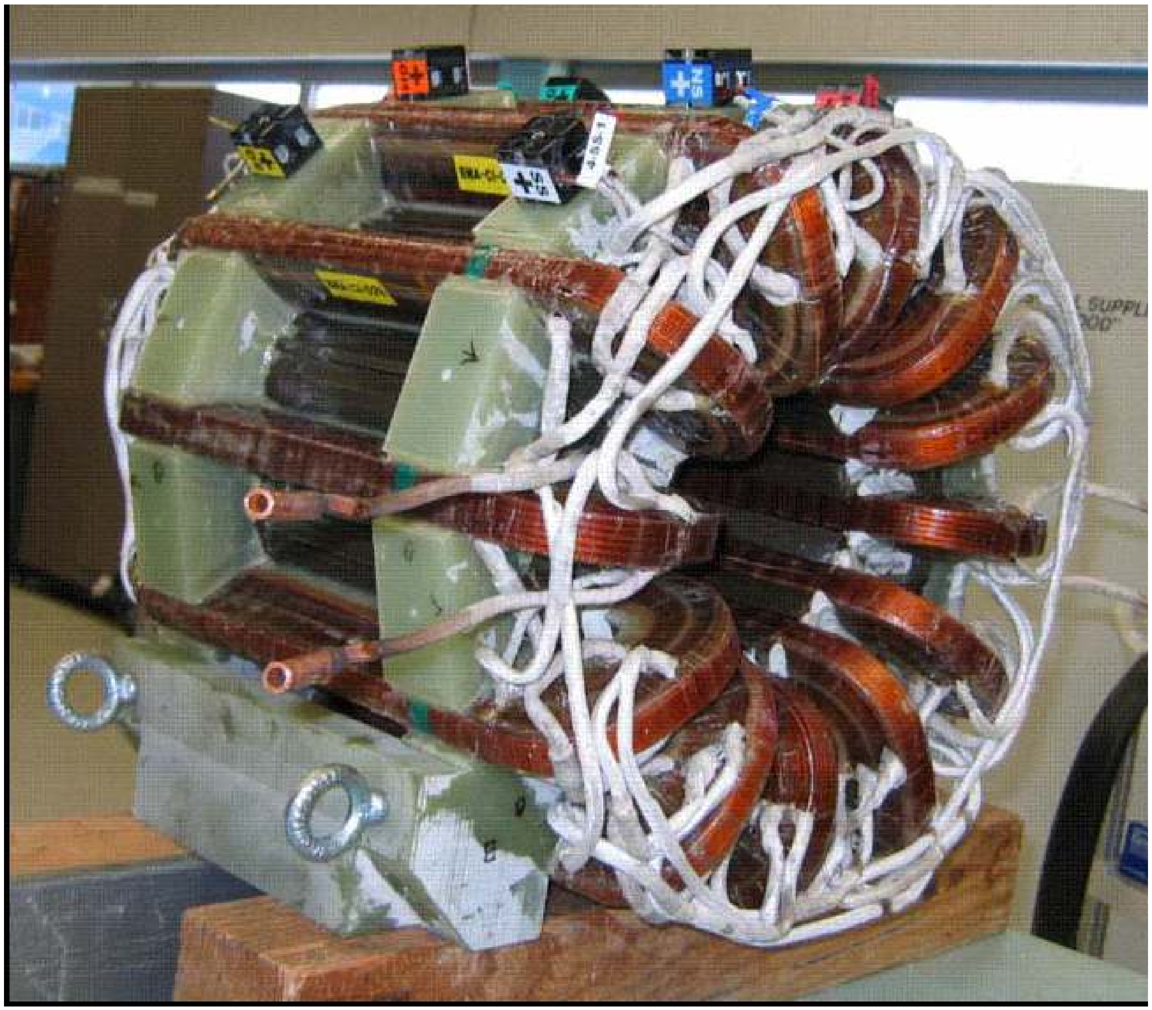
Motivation

New corrector magnet packages in the Booster each contain x and y dipole, normal and skew quad, and normal and skew sextupole elements.



•Each element in each of the 48 magnet packages is independently controllable

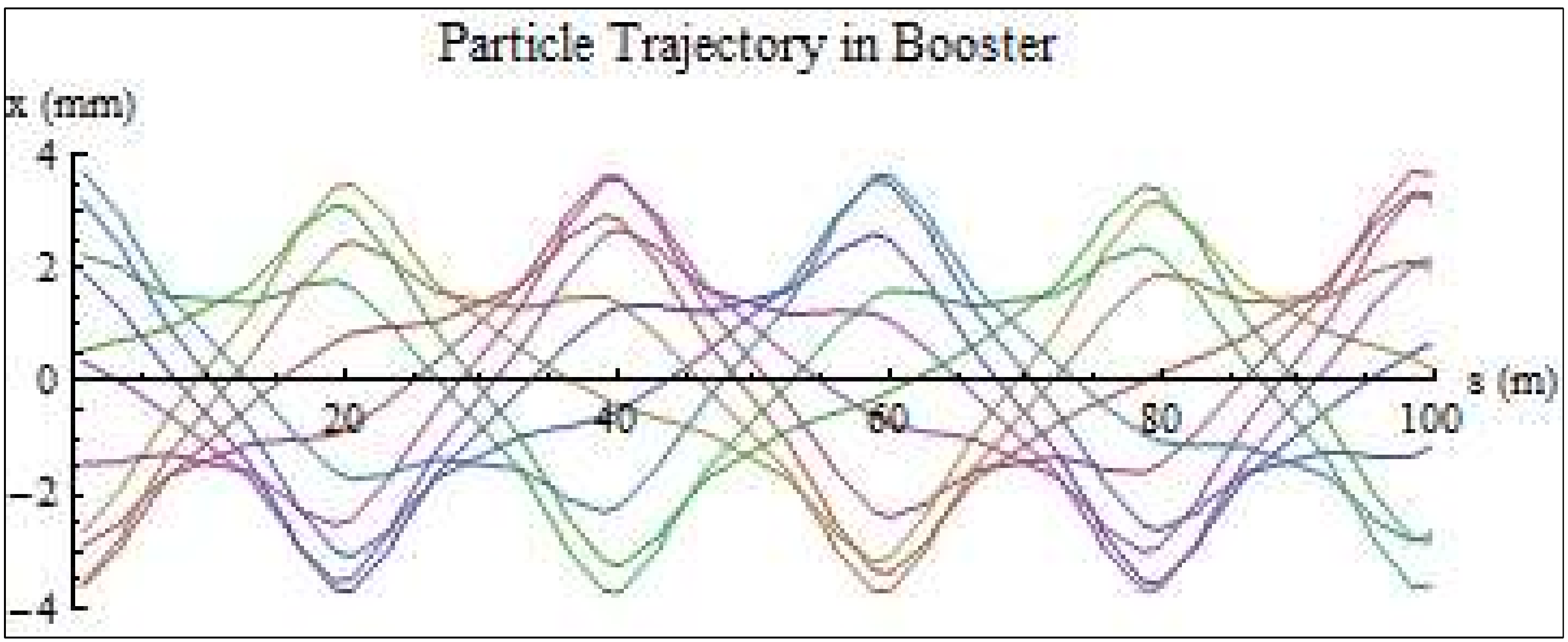
•Correctors now have ample strength to control the beam tune and position, but they're still being used much as the old correctors were

•One new use for the correctors is to measure and manipulate the beta function

Particle Dynamics Review:

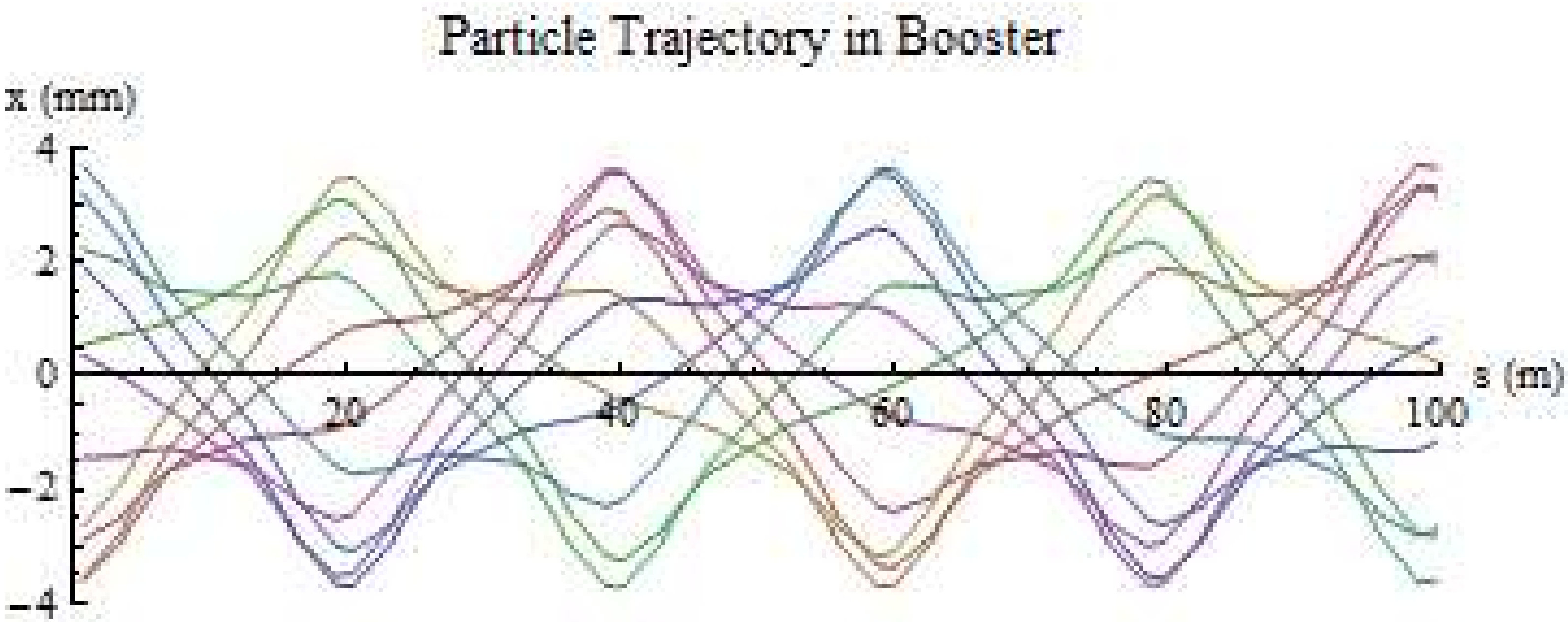
Motion of a Particle Through a Strong Focusing Lattice

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}}_{\text{Drift}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}}_{\text{Defocusing element}} \cdot \underbrace{\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}}_{\text{Drift}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}}_{\text{Focusing element}} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$



Particle position on multiple turns, modeled from the Booster's design values for quadrupole magnet strengths and magnet and drift lengths.

The Beta Function

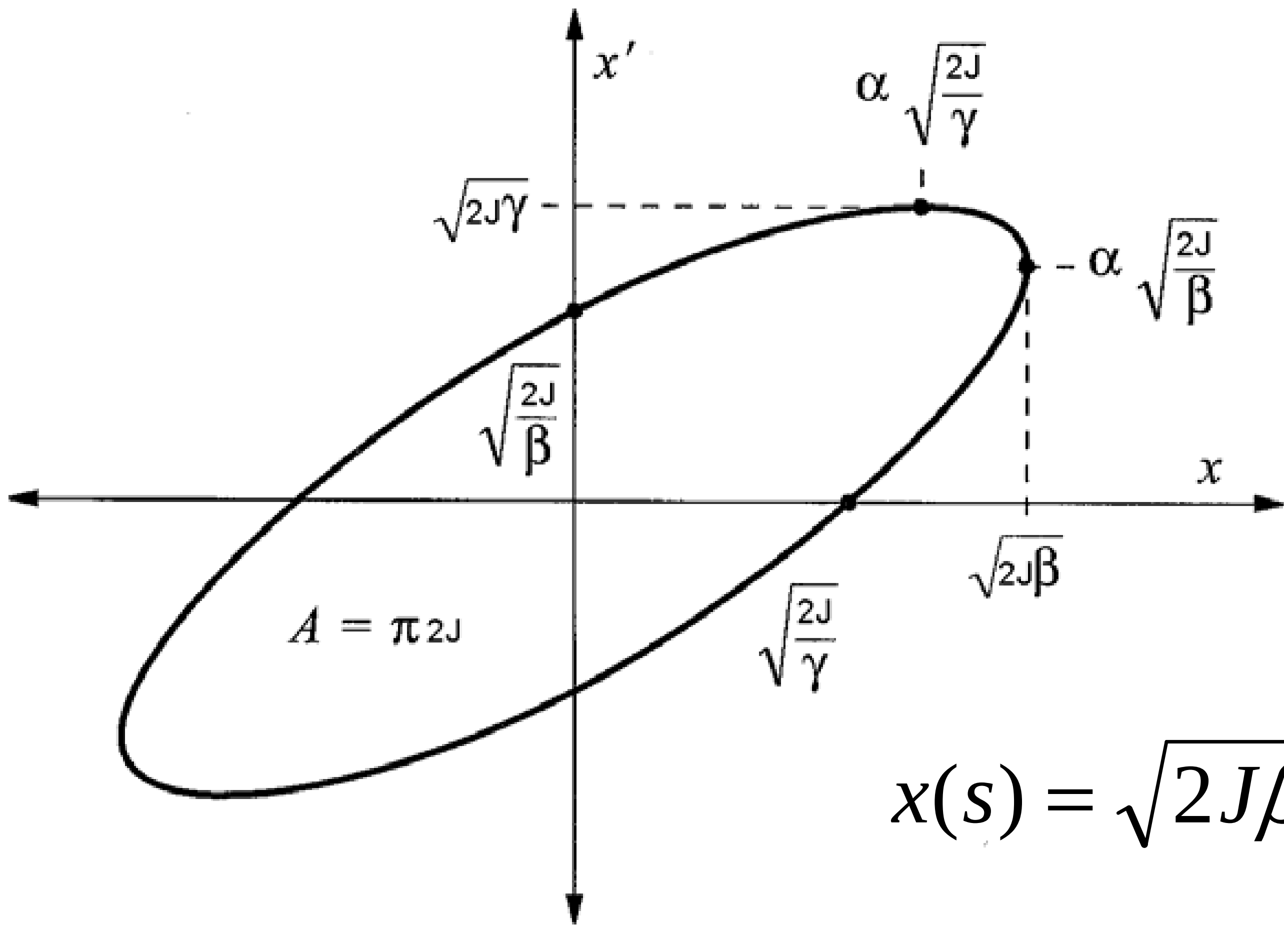


The beta function is proportional to the beam size envelope at a given point in the lattice. Beam size is determined by both the beta function and the emittance (average J for all particles).

Equation of motion for a single particle:

$$x(s) = \sqrt{2J\beta(s)}Sin(\psi(s) + \phi)$$

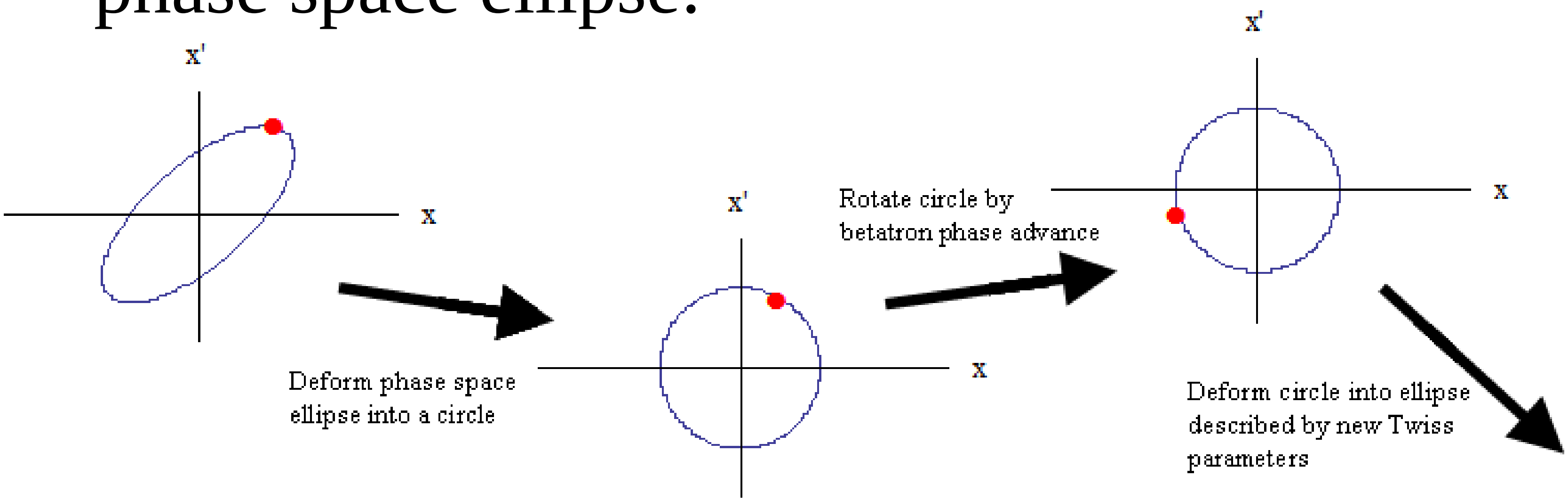
Particles with the same J and different φ lie on an ellipse in x x’ phase space.

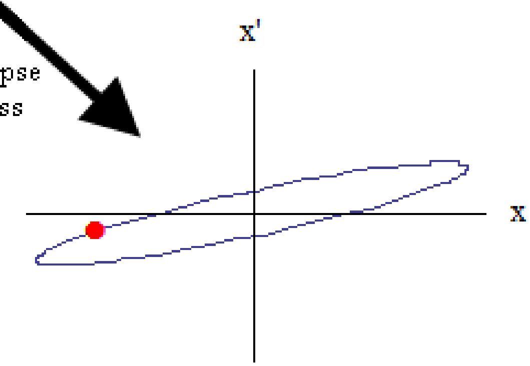


$$x(s) = \sqrt{2J\beta(s)}Sin(\psi(s)+\varphi)$$

$$x'(s) = \sqrt{2J}\,(-\frac{\alpha}{\sqrt{\beta(s)}}Sin(\psi(s)+\varphi)+\frac{1}{\sqrt{\beta(s)}}Cos(\psi(s)+\varphi))$$

Motion of a particle between two locations on a lattice can be described as deformations and a rotation of the phase space ellipse:



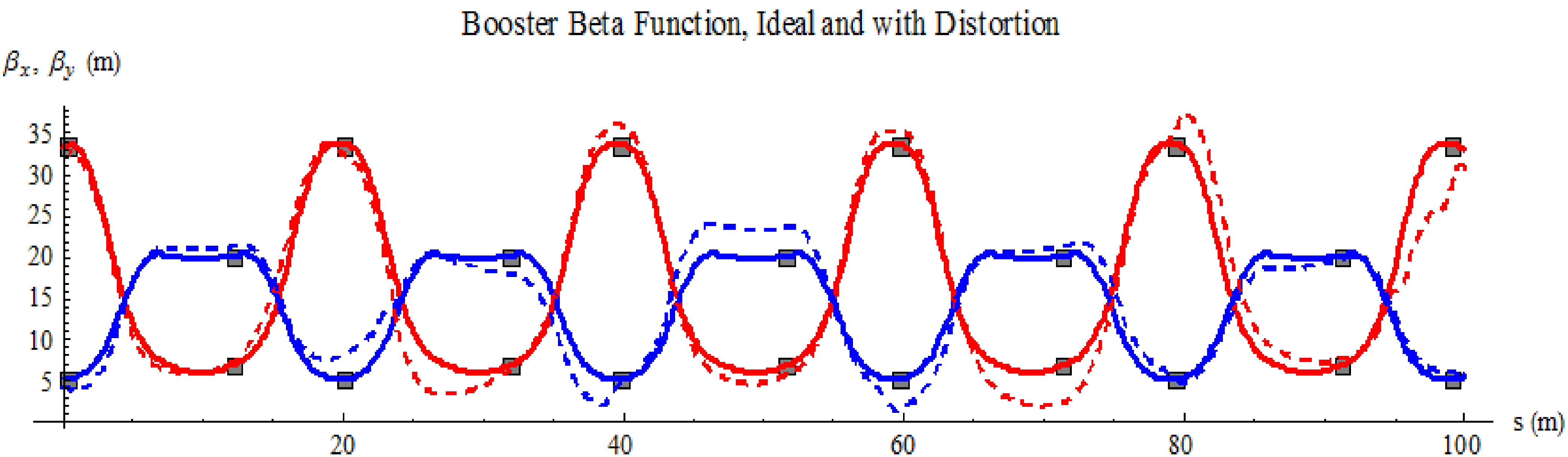


$$\underline{M} = \begin{pmatrix} \sqrt{\beta} & 0 \\ -\dfrac{\alpha}{\sqrt{\beta}} & \dfrac{1}{\sqrt{\beta}} \end{pmatrix} \cdot \begin{pmatrix} Cos\,\psi(s) & Sin\,\psi(s) \\ -Sin\,\psi(s) & Cos\,\psi(s) \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\beta_0} & 0 \\ -\dfrac{\alpha_0}{\sqrt{\beta_0}} & \dfrac{1}{\sqrt{\beta_0}} \end{pmatrix}$$

For one full turn around the machine: $$\boxed{\underline{M} = \begin{pmatrix} Cos(\mu)+\alpha Sin(\mu) & \beta Sin(\mu) \\ -\gamma Sin(\mu) & Cos(\mu)-\alpha Sin(\mu) \end{pmatrix}}$$

Betatron phase advance per turn: $$\boxed{Cos(\mu) = \frac{1}{2} Tr[M]}$$

Beta Function Distortion



Imperfections in the lattice cause distortions in the beta functions, which affects the physical size of the beam and may cause losses.

Deviation in the beta function caused by a quadrupole error δq at location s_0:

$$\frac{\Delta\beta(s)}{\beta(s)} = \frac{\delta q \cdot \beta(s_0)}{2 \cdot Sin 2\pi\nu} Cos\left(2\left|\psi(s_0) - \psi(s)\right| - 2\pi\nu\right)$$

Measuring the Beta Function

- If we add a weak thin quadrupole element to a ring, the change to the one-turn transfer matrix gives the change to the tune:

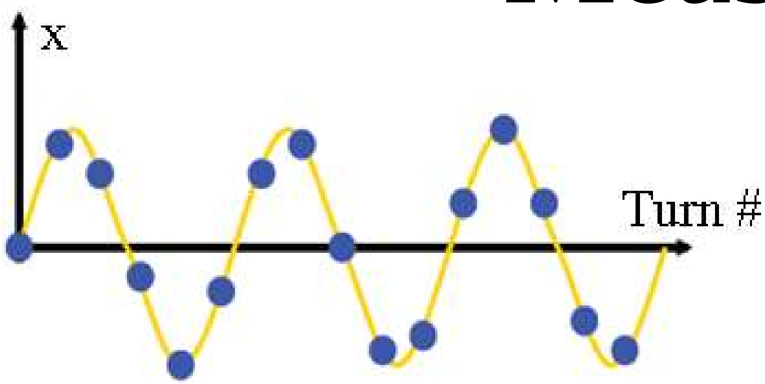
$$\underline{M} = \begin{pmatrix} Cos(\mu)+\alpha Sin(\mu) & \beta Sin(\mu) \\ -\gamma Sin(\mu) & Cos(\mu)-\alpha Sin(\mu) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$Cos(2\pi\nu) = \frac{1}{2} Tr[M] = Cos(2\pi\nu_0) - \frac{1}{2}\frac{\beta_0}{f} Sin(2\pi\nu_0)$$

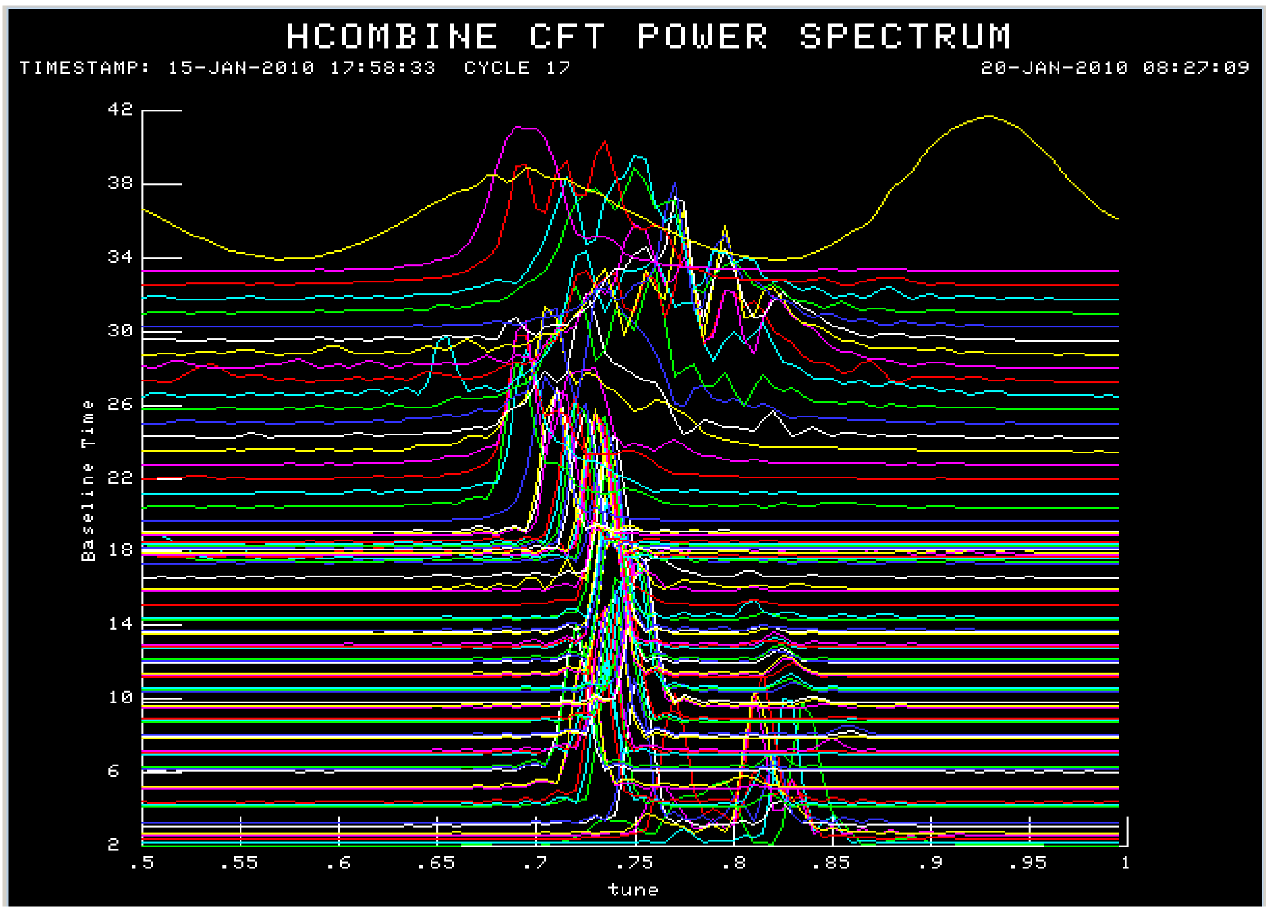
- If δq=1/f is small enough, the above expression can be simplified and the resulting tune shift is proportional to the beta function at the location of the quad error:

$$\boxed{\begin{aligned} \delta\nu_x &= \frac{1}{4\pi}\beta_x \cdot \delta q \\ \delta\nu_y &= -\frac{1}{4\pi}\beta_y \cdot \delta q \end{aligned}}$$

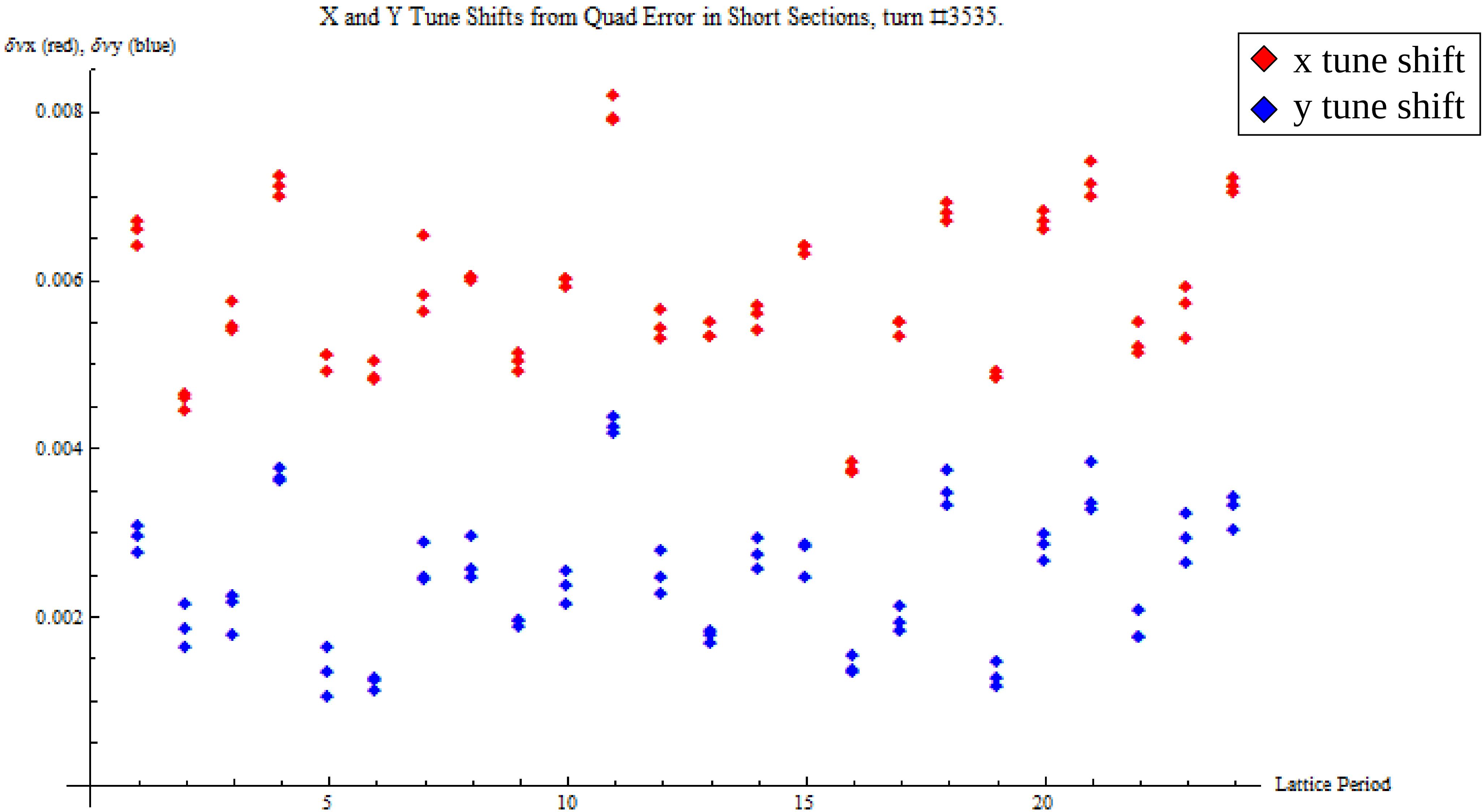
Measuring the Tune



When the beam is kicked transversely, it performs betatron oscillations around the closed orbit. A Fourier transform of the beam position at a given location on successive turns shows the betatron tune.



Tune Shift Measurement Results



$$\delta\nu_x = \frac{1}{4\pi}\beta_x \cdot \delta q$$

$$\delta\nu_y = -\frac{1}{4\pi}\beta_y \cdot \delta q$$

Figure shows the change in x and y tunes due to a single quad error in a particular short section drift period. For an uncoupled machine, the tune shift *should* have a different sign in each plane; coupling in the Booster is large enough to cause the low-beta-plane tune to move in the wrong direction.

Eigentunes When Error Skew Quad Fields are Present

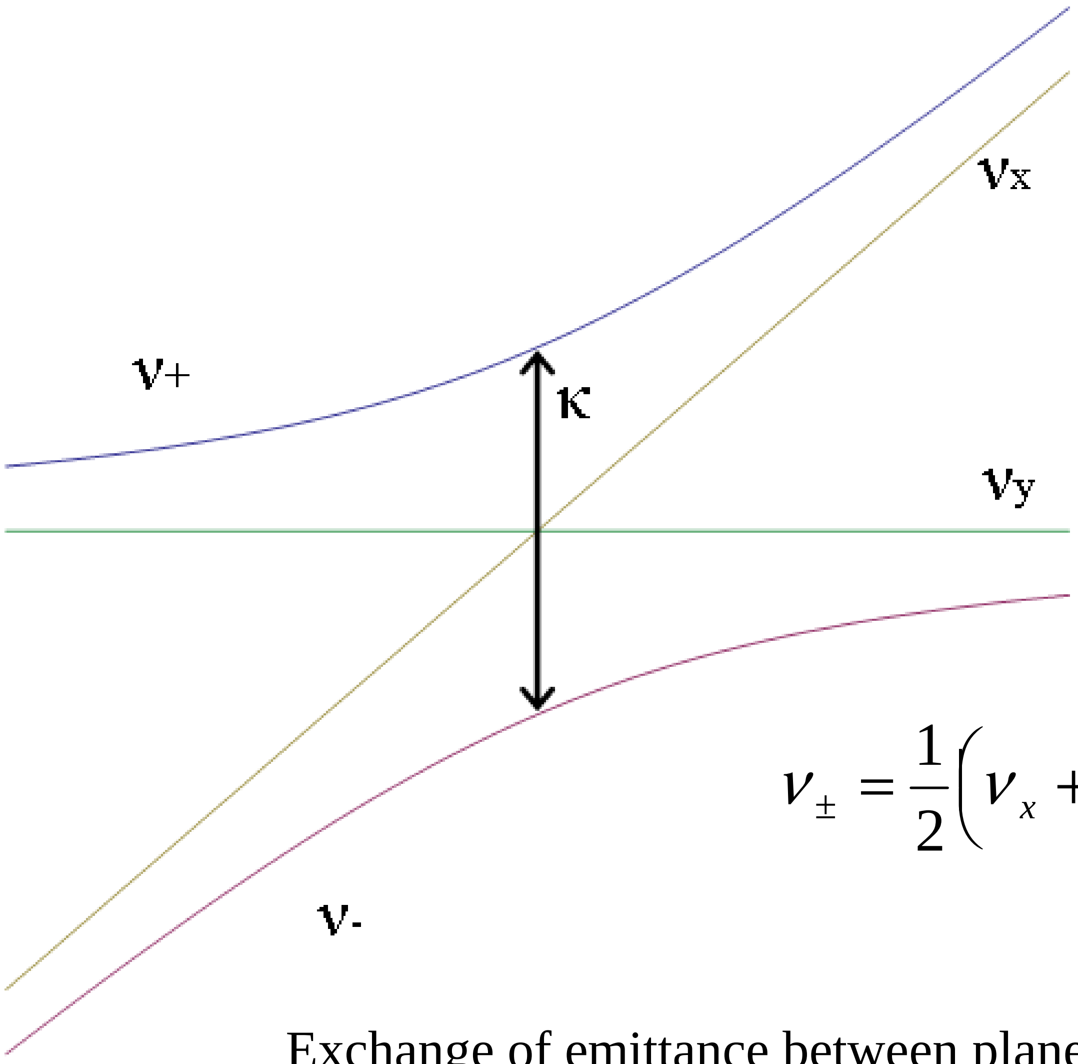
- Willeke and Ripken (*Methods of Beam Optics*, 2000) treat unintentional skew quad fields as a perturbation on the uncoupled motion
- The constants of integration are allowed to vary:

$$x(s) = \sqrt{2J(s)\beta(s)}\,Sin(\psi(s) + \phi(s))$$

- They derive an expression for observable eigentunes in terms of unperturbed tunes and resonance width κ:

$$\nu_{\pm} = \frac{1}{2}\left(\nu_x + \nu_y \pm \sqrt{\kappa^2 + \left(\nu_x - \nu_y\right)^2}\right)$$

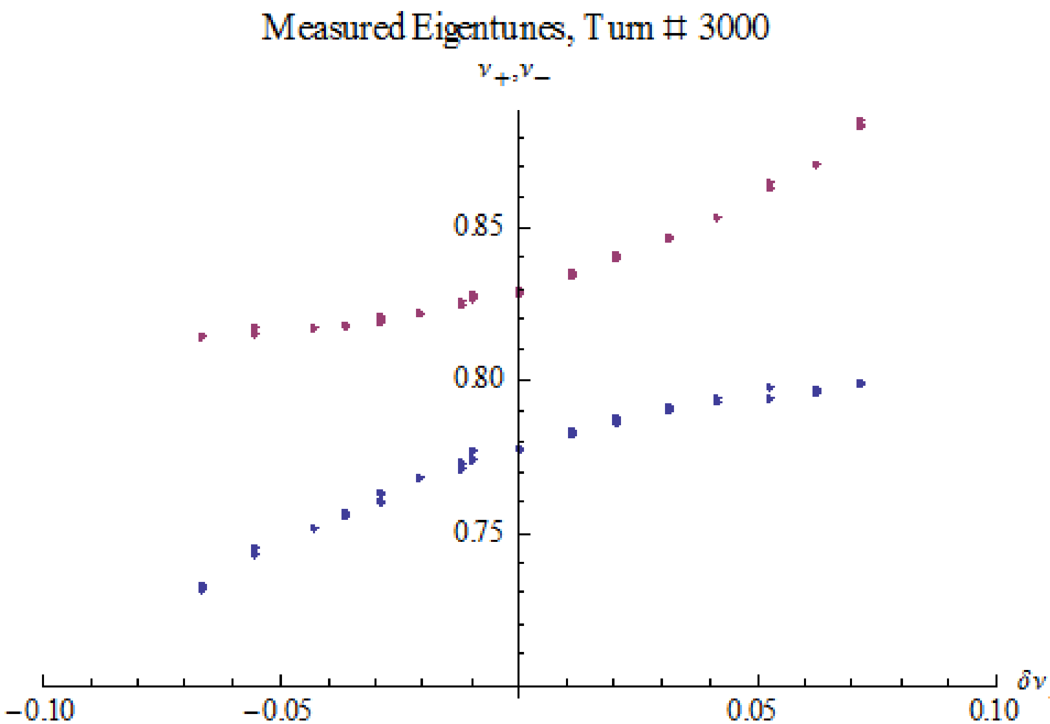
Change to eigentunes as horizontal tune is varied and vertical tune is held constant:



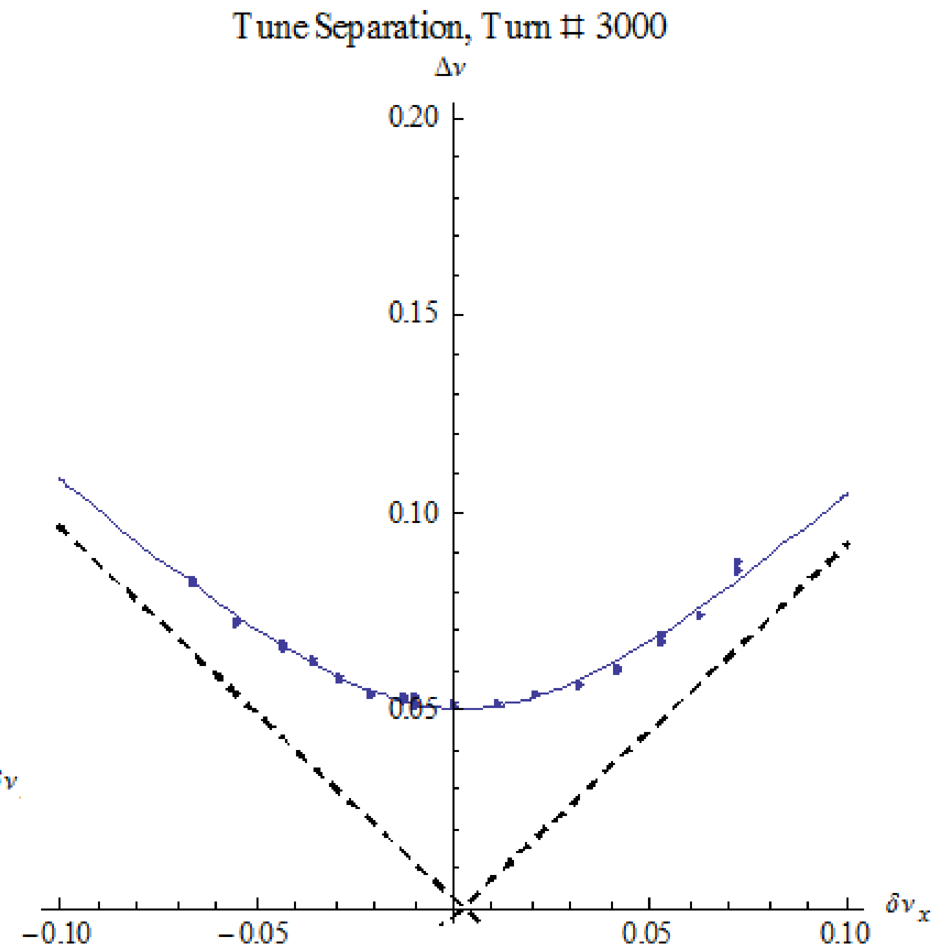
$$\nu_\pm = \frac{1}{2}\left(\nu_x + \nu_y \pm \sqrt{\kappa^2 + \left(\nu_x - \nu_y\right)^2}\right)$$

Exchange of emittance between planes is $\propto \frac{\kappa^2}{\kappa^2 + (\nu_x - \nu_y)^2}$
so coupling effects are weaker when tunes are further apart

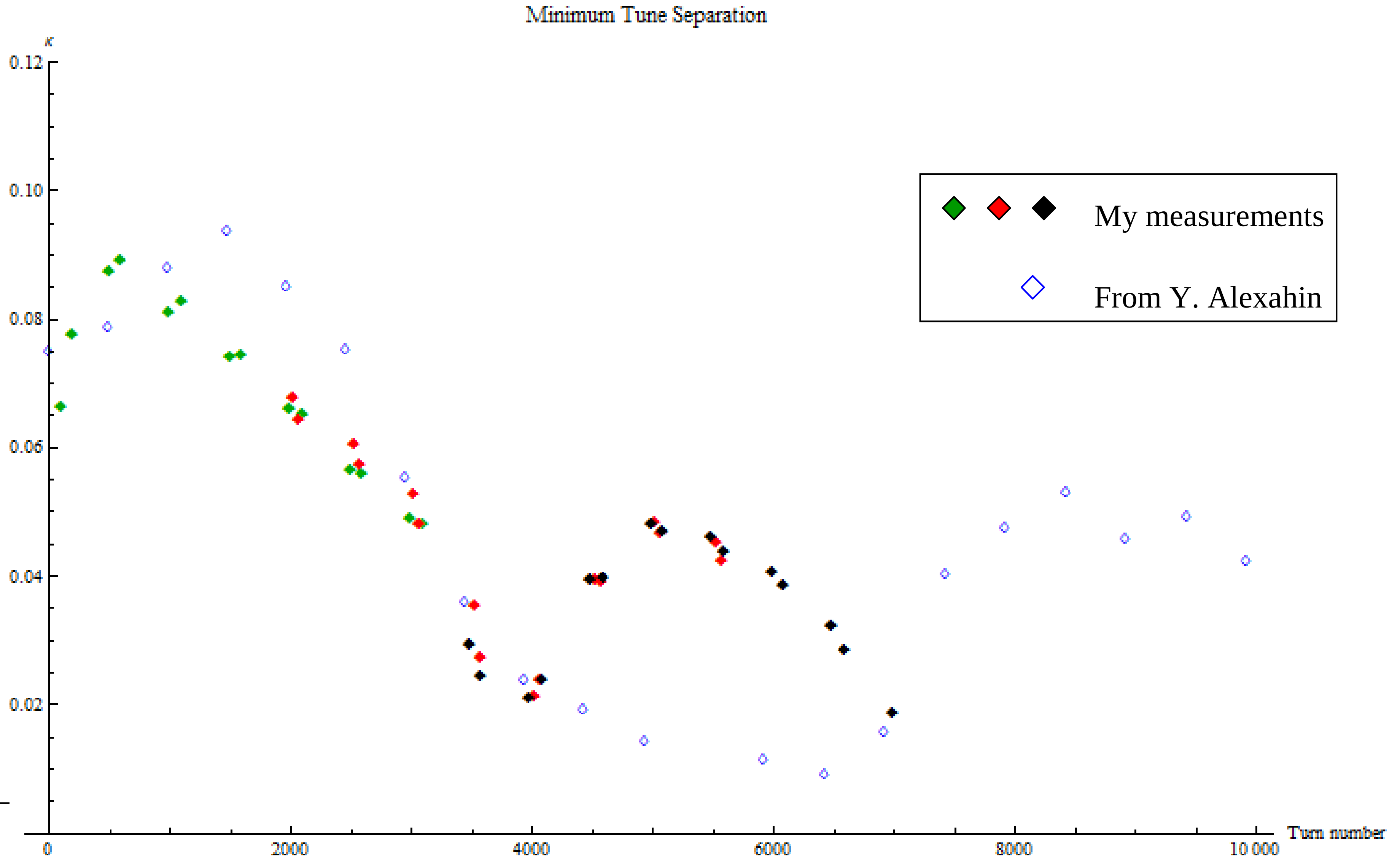
Measurements of resonance width/ minimum tune separation

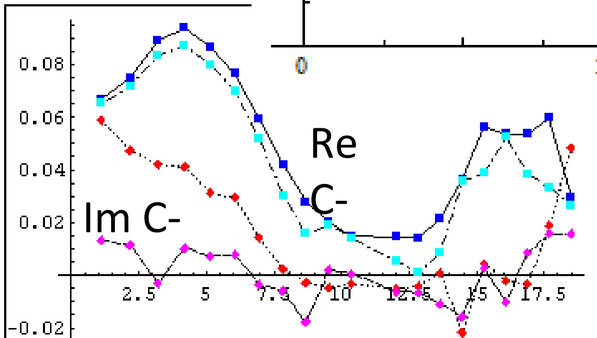


Measured eigentunes as x tune is varied and y tune is held constant. I made these measurements every 500 turns for the first 7000 turns of the acceleration cycle.



Hyperbolic fit to the difference between the measured eigentunes, which is used to determine minimum tune separation.





Result of similar study by Y. Alexahin; |C-| ~ κ

Solid markers show minimum tune separation through the early part of the acceleration cycle, found by fitting a hyperbola to the eigentune separation as a function of x tune. (Each color is from a data set with a tune bump put in for a few thousand turns.) Outlined markers are from Y. Alexahin.

Calculating Betas using coupled expression

We know how the eigentunes relate to the unperturbed tunes, so we can find how the eigentunes change when a small quad bump is introduced:

$$\nu_{\pm} = \frac{1}{2}\left(\nu_x + \nu_y \pm \sqrt{\kappa^2 + \left(\nu_x - \nu_y\right)^2}\right)$$

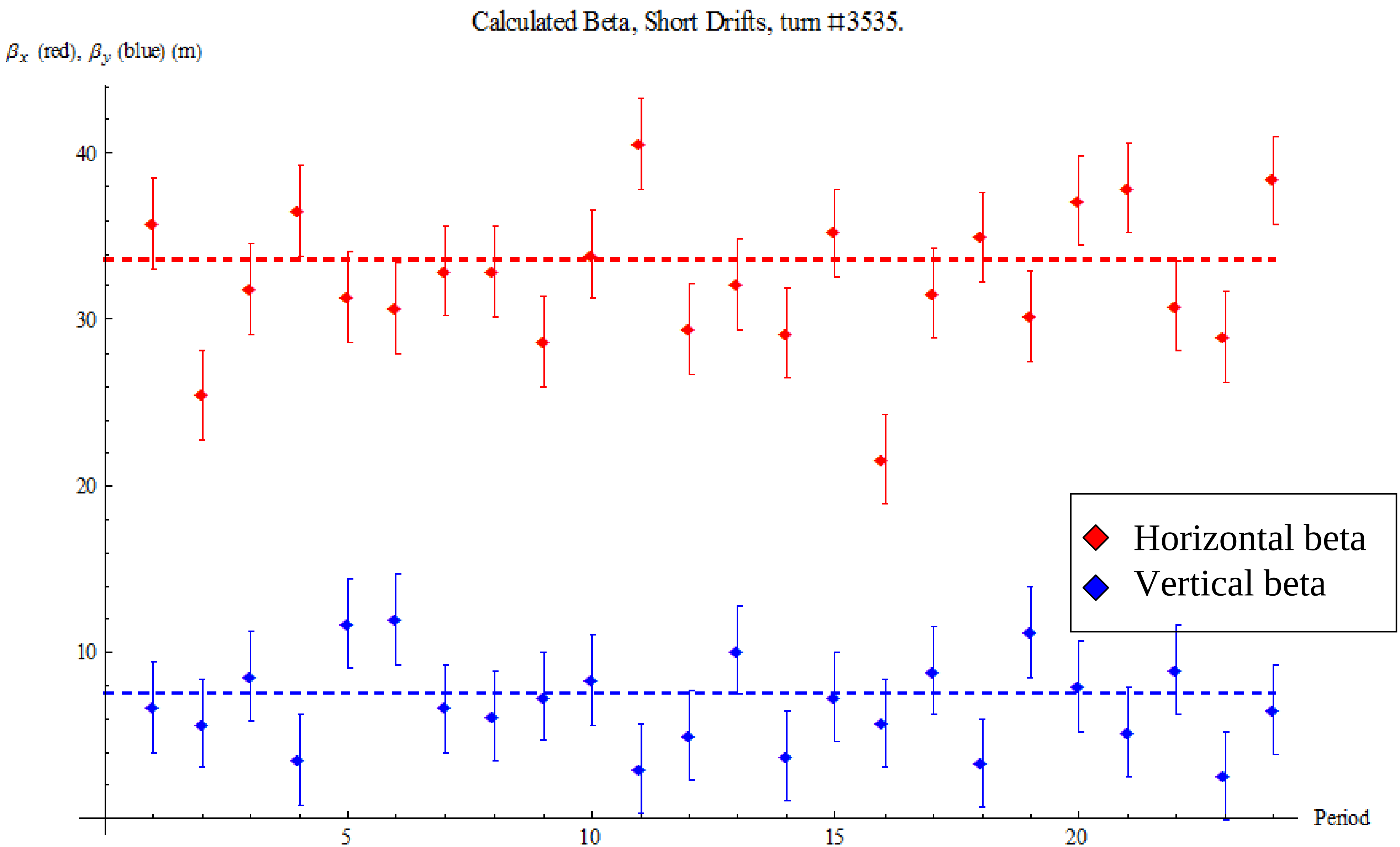
We know how the unperturbed, uncoupled tunes should change, so set $\nu_x \to \nu_x + \frac{1}{4\pi} q\beta_x, \quad \nu_y \to \nu_y - \frac{1}{4\pi} q\beta_y$

Expand the resulting expression to first order in q:

$$\boxed{\delta\nu_{\pm} = \frac{q}{8\pi}\left(\beta_x\left(1 \mp \frac{\nu_x - \nu_y}{\sqrt{\kappa^2 + \left(\nu_x - \nu_y\right)^2}}\right) + \beta_y\left(1 \pm \frac{\nu_x - \nu_y}{\sqrt{\kappa^2 + \left(\nu_x - \nu_y\right)^2}}\right)\right)}$$

(This reduces to the familiar uncoupled expressions when κ=0.)

Calculated Beta Functions at Each Short Section Corrector Location



Beta functions calculated using the measured minimum tune separation and the relationship between eigentune shifts, uncoupled tunes, and beta functions given above. The dashed lines show the design value of the beta functions at the location of the short section corrector magnets.

Uncertainty in the Beta Function Measurements

- Sources of error:
  - Determination of “average” (unmodified) tune (significant error source, since tune wanders over the course of hours or days)
  - Measurement of shifted tune
  - Measurement of minimum tune separation κ
  - Small fluctuations in quad magnet strength
- Uncertainty in beta measurements is ~2.7 m
- Much of the apparent variation that I see in the beta function is really due to random fluctuations in the tune; I’ll need to make many measurements of each tune shift to get a better statistical average value for beta.

Correcting the Beta Function

The change to the beta function at location s, caused by small quad errors δq_i at locations s_i, is approximately linear in δq_i:

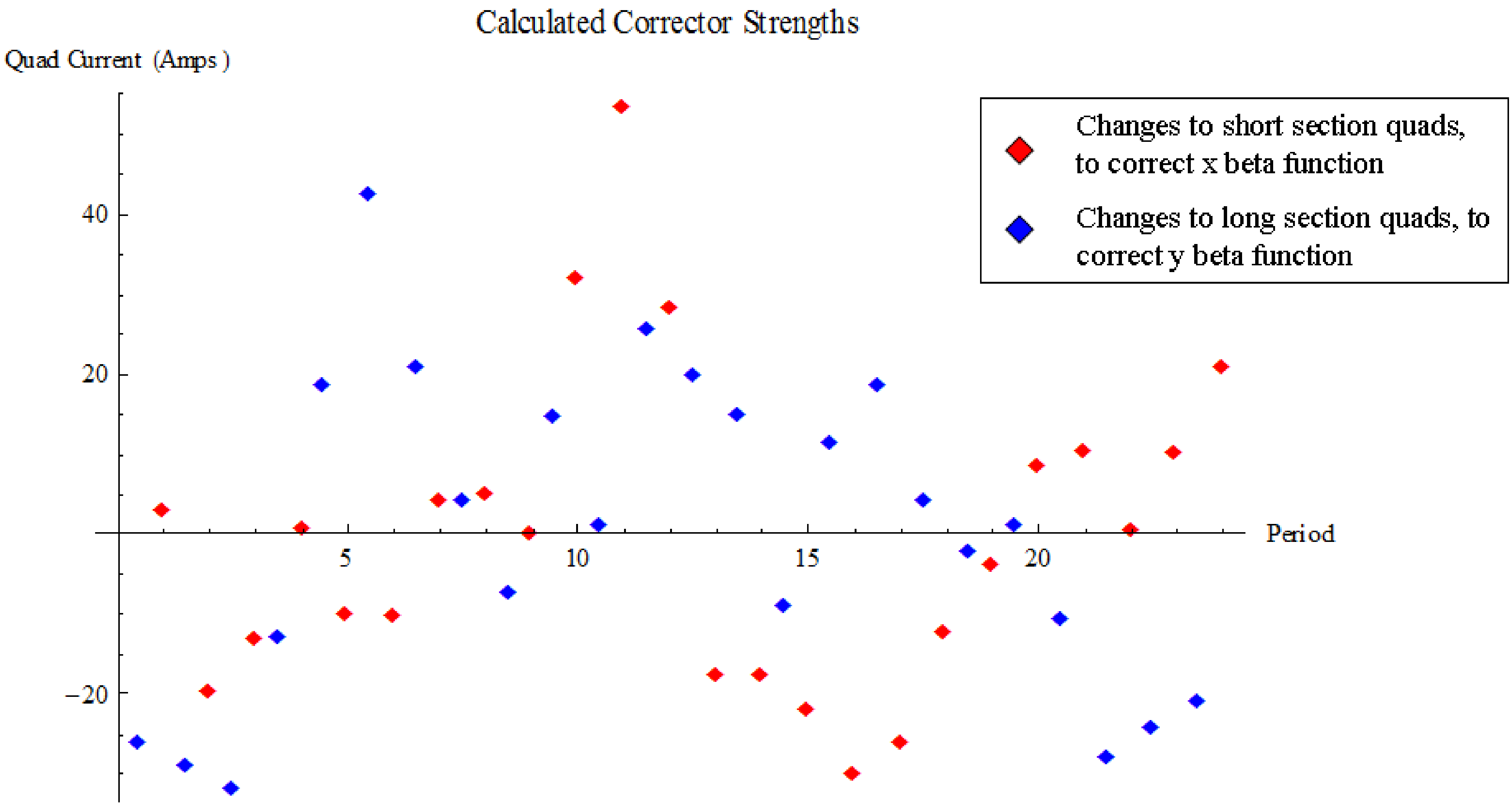
$$\Delta\beta(s) = \sum_i \frac{\delta q_i \cdot \beta(s_i)\beta(s)}{2 \cdot Sin 2\pi\nu} Cos\left(2\left|\psi(s_i) - \psi(s)\right| - 2\pi\nu\right)$$

The change in beta at multiple locations caused by quad errors at those locations can be expressed as a matrix equation:

$$\Delta\vec{\beta} = \overline{M} \cdot \Delta\vec{q}$$

$$M_{ij} = \frac{\beta(s_i) \cdot \beta(s_j)}{2 \cdot Sin 2\pi\nu} Cos\left(2\left|\psi(s_i) - \psi(s_j)\right| - 2\pi\nu\right)$$

This expression can be inverted to solve for a set of quadrupole strengths that will cancel out the beta errors. We decided to only use the 24 high-beta correctors in each plane in this calculation (ie solve for the settings that would correct the vertical beta function in the 24 long sections using the long section quads, and separately solve for the settings that would correct the horizontal beta function in the short sections using the 24 short section quads).

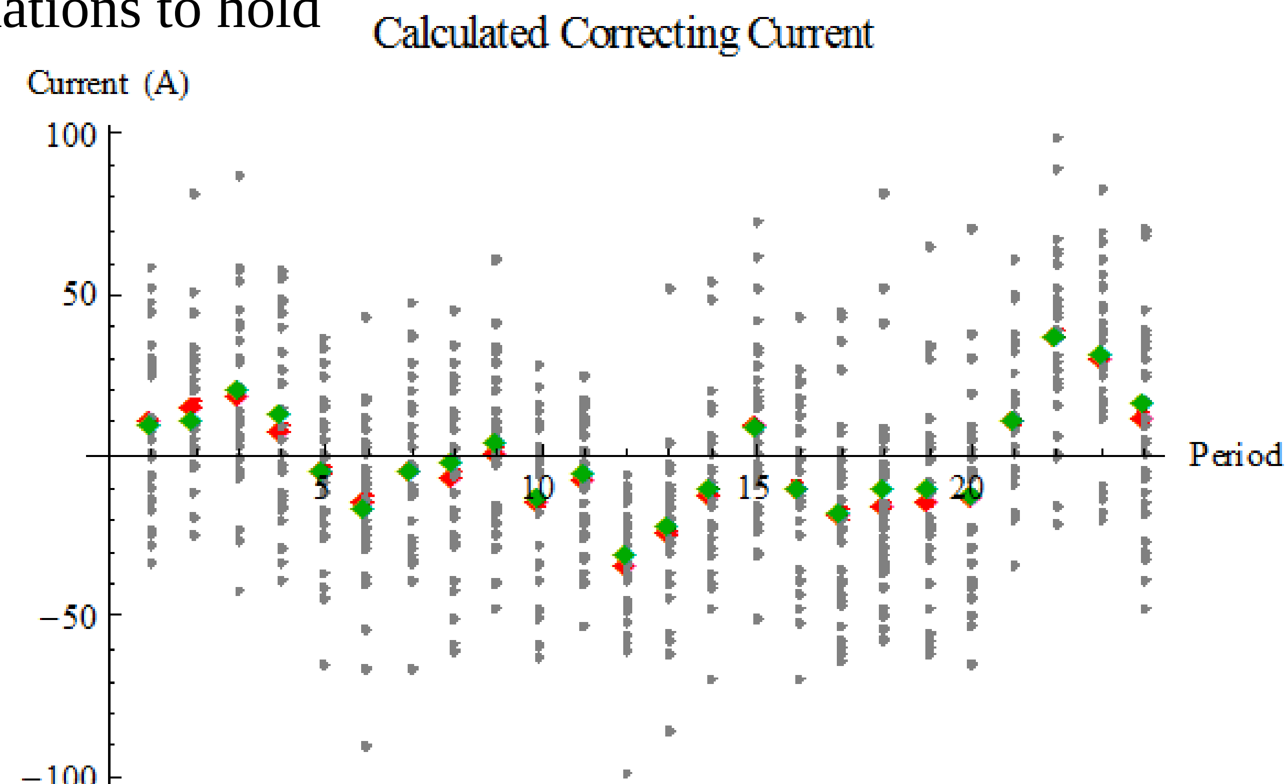


Calculated quadrupole strength changes needed to correct the beta function errors, based on three measurements of beta at each point. For reasons discussed on the next slide, we decided to scale these currents by 1/5 to partially correct the beta function, then repeat the process.

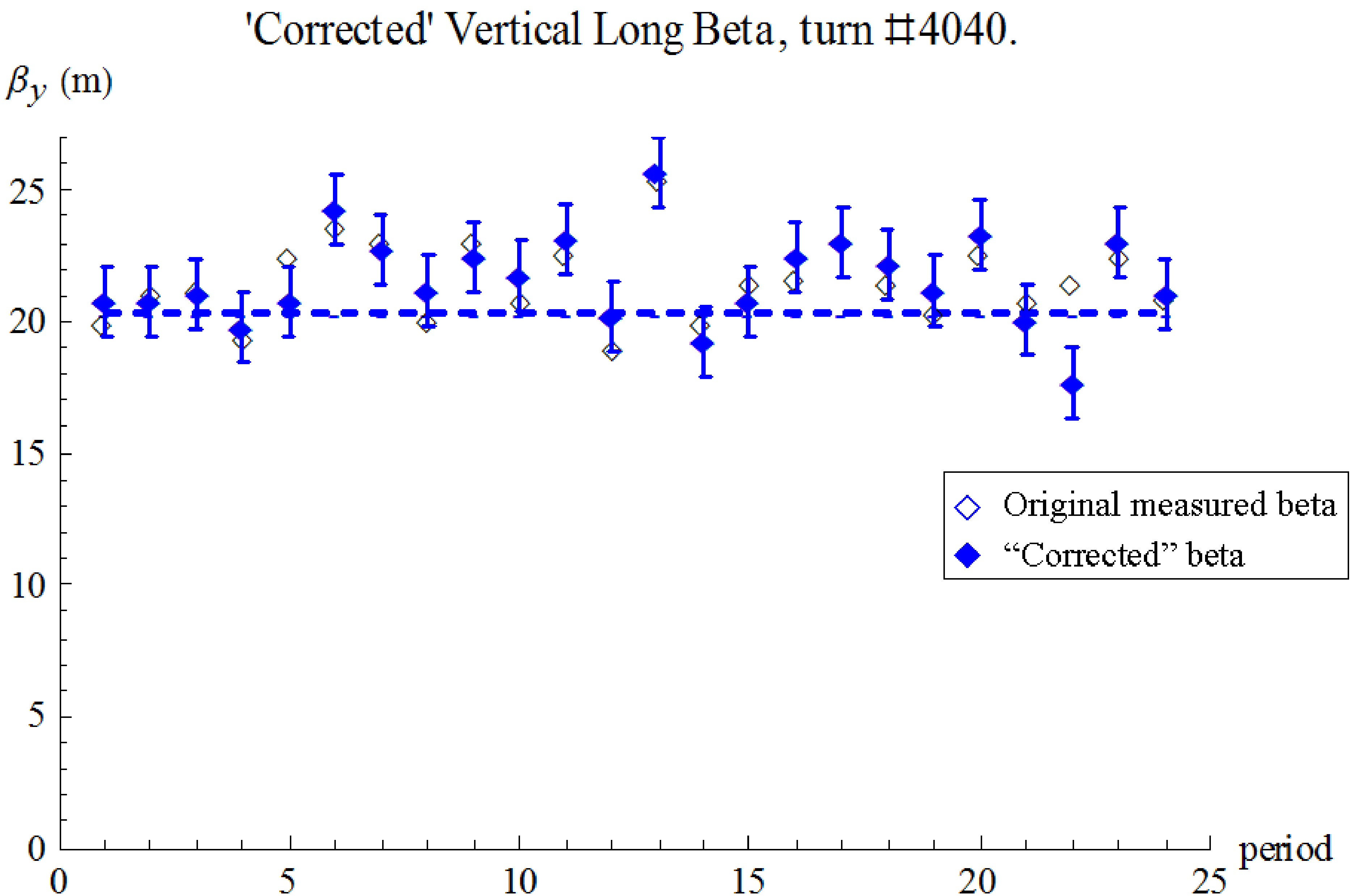
Correcting the Beta Function

Problems with this method:

1. Uncertainty in beta leads to large uncertainty in the correction
2. The corrections found are generally too large for “small δq” linearizing approximations to hold



To get an idea of the uncertainty in the correcting current, I calculated corrections 20 times as I added 20 different sets of random errors to the measured beta functions. The uncertainty in current is large; I’d get better results if I measured the beta function many times and averaged the results



Beta function measured after "correcting" quad current offset was added; no significant reduction in beta beat is apparent. This is not too surprising considering that I didn't take enough measurements of the original beta function to get an accurate average value, and the correcting quad currents were only ~1/5 of what would be required to fully correct the beta function.

Future Plans

- Automate the beta function measurement process using ACL so that more data can be collected, giving a more accurate value for beta
- Correct beta beating using an iterative process of partial corrections
- Measure beam losses when the beta function irregularities have been corrected to see if efficiency is improved
- Try to reduce losses in sensitive areas of the ring and steer lossed towards collimators using local beta bumps

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