PBAR NOTE 596
ATTENUATION OF WAVEGUIDE MODES WITH MULTI-LAYER ABSORBING WALLS

Dave McGinnis

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INTRODUCTION

Microwave modes in the beam pipe of the 4-8 GHz Debuncher Upgrade pickup and kicker arrays are attenuated with absorbing slabs placed on the sides of the beam pipe walls. Adding an additional layer of material with different microwave properties behind the absorbing slab can enhance the attenuation of the absorber. This note describes how to calculate the attenuation of rectangular waveguide modes with multi-layer absorbers placed on the top and side walls of the waveguide.

ABSORBER ON THE SIDE-WALLS OF THE WAVEGUIDE

The geometry of the problem is shown in Figure 1. Without the absorber, the dominant mode in the waveguide will be the Transverse Electric to Z 1,0 mode (TE^Z_{10}). With the introduction of the absorber, the waveguide modes can no longer be classified as transverse to Z (TE^Z, TM^Z) but can be classified as transverse to X (TE^X, TM^X).[1] Since the TE^X_{10} mode is the same as the TE^Z_{10} mode in the absence of absorber, and the incident mode on the absorbing section of waveguide will be TE^Z_{10} , we will consider TE^X modes which are even in X only. TE^X modes can be derived from a x-directed electric vector potential:

$$\vec{F}(x,y,z) = \hat{x}F_x(x,y)e^{-\gamma z} \tag{1}$$

The electric field is given as:

$$\vec{E} = -\frac{1}{\varepsilon}\vec{\nabla}\times\vec{F} \tag{2}$$

The magnetic field is given as:

$$\vec{H} = -\frac{1}{j\omega\mu\varepsilon}\vec{\nabla}\times\vec{E} \tag{3}$$

From Equations 1-3, electric field is:

$$\vec{E} = \left(0\hat{x} + \frac{\gamma}{\varepsilon}F_x\hat{y} + \frac{1}{\varepsilon}\frac{\partial F_x}{\partial y}\hat{z}\right)e^{-\gamma z} \tag{4}$$

The magnetic field is:

[1] Time Harmonic Electromagnetic Fields, R.F. Harrington, McGraw-Hill, Inc., 1961, pg. 158

$$\vec{H} = \frac{1}{j\omega\mu\varepsilon}\left(-\left(\frac{\partial^2 F_x}{\partial y^2} + \gamma^2 F_x\right)\hat{x} + \frac{\partial^2 F_x}{\partial x \partial y}\hat{y} - \gamma\frac{\partial F_x}{\partial x}\hat{z}\right)e^{-\gamma z} \tag{5}$$

To meet the boundary conditions in Region I of Figure 1, the electric vector potential must be of the form:

$$F_x(x, y) = F_1 \cos(f_1 x)\cos(k_y y) \tag{7}$$

The electric field in Region I is:

$$E_x(x, y) = 0 \tag{8}$$

$$E_y(x, y) = \frac{\gamma F_1}{\varepsilon_1}\cos(f_1 x)\cos(k_y y) \tag{9}$$

$$E_z(x, y) = -\frac{\gamma F_1}{\varepsilon_1}\frac{k_y}{\gamma}\cos(f_1 x)\sin(k_y y) \tag{10}$$

The magnetic field in Region I is:

$$H_x(x, y) = \frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{\gamma^2 - k_y{}^2}{\gamma^2}\cos(f_1 x)\cos(k_y y) \tag{11}$$

$$H_y(x, y) = -\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_1 k_y}{\gamma^2}\sin(f_1 x)\sin(k_y y) \tag{12}$$

$$H_z(x, y) = -\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_1}{\gamma}\sin(f_1 x)\cos(k_y y) \tag{13}$$

To meet the boundary conditions in Region II of Figure 1, the electric vector potential must be of the form:

$$F_x(x, y) = F_2 \cos(f_2 x + \delta_2)\cos(k_y y) \tag{14}$$

The electric field in Region II is:

$$E_x(x, y) = 0 \tag{15}$$

$$E_y(x, y) = \left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{\gamma F_1}{\varepsilon_1}\cos(f_2 x + \delta_2)\cos(k_y y) \tag{16}$$

$$E_z(x, y) = -\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{\gamma F_1}{\varepsilon_1}\frac{k_y}{\gamma}\cos(f_2 x + \delta_2)\sin(k_y y) \tag{17}$$

The magnetic field in Region II is:

$$H_x(x,y)=\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{\gamma^2-k_y{}^2}{\gamma^2}\cos(f_2 x+\delta_2)\cos(k_y y) \tag{18}$$

$$H_y(x,y)=-\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_2 k_y}{\gamma^2}\sin(f_2 x+\delta_2)\sin(k_y y) \tag{19}$$

$$H_z(x,y)=-\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_2}{\gamma}\sin(f_2 x+\delta_2)\cos(k_y y) \tag{20}$$

To meet the boundary conditions in Region III of Figure 1, the electric vector potential must be of the form:

$$F_x(x,y)=F_3\sin(f_3(a_3-x))\cos(k_y y) \tag{21}$$

The electric field in Region III is:

$$E_x(x,y)=0 \tag{22}$$

$$E_y(x,y)=\left(\frac{\varepsilon_1 F_3}{\varepsilon_3 F_1}\right)\frac{\gamma F_1}{\varepsilon_1}\sin(f_3(a_3-x))\cos(k_y y) \tag{23}$$

$$E_z(x,y)=-\left(\frac{\varepsilon_1 F_3}{\varepsilon_3 F_1}\right)\frac{\gamma F_1}{\varepsilon_1}\frac{k_y}{\gamma}\sin(f_3(a_3-x))\sin(k_y y) \tag{24}$$

The magnetic field in Region III is:

$$H_x(x,y)=\left(\frac{\mu_1}{\mu_3}\right)\left(\frac{\varepsilon_1 F_3}{\varepsilon_3 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{\gamma^2-k_y{}^2}{\gamma^2}\sin(f_3(a_3-x))\cos(k_y y) \tag{25}$$

$$H_y(x,y)=-\left(\frac{\mu_1}{\mu_3}\right)\left(\frac{\varepsilon_1 F_3}{\varepsilon_3 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_3 k_y}{\gamma^2}\cos(f_3(a_3-x))\sin(k_y y) \tag{26}$$

$$H_z(x,y)=-\left(\frac{\mu_1}{\mu_3}\right)\left(\frac{\varepsilon_1 F_3}{\varepsilon_3 F_1}\right)\frac{j\gamma}{\omega\mu_1}\frac{\gamma F_1}{\varepsilon_1}\frac{f_3}{\gamma}\cos(f_3(a_3-x))\cos(k_y y) \tag{27}$$

For electric boundaries at both the top and bottom of the waveguide:

$$k_y=n\frac{\pi}{b} \tag{28}$$

where n is an integer. For an electric boundary on the bottom of the waveguide and a magnetic boundary on the top of the waveguide:

$$k_y=\left(n+\frac{1}{2}\right)\frac{\pi}{b} \tag{29}$$

At the interface of $x=a_1$ E_y, E_z must be continuous:

$$\cos(f_1 a_1) = \left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right)\cos(f_2 a_1 + \delta_2) \tag{30}$$

At the interface of $x= a_1$ H_y, H_z must be continuous:

$$f_1 \sin(f_1 a_1) = \frac{\mu_1}{\mu_2}\left(\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1}\right) f_2 \sin(f_2 a_1 + \delta_2) \tag{31}$$

Dividing Equation 31 by Equation 30 results in:

$$f_1 \tan(f_1 a_1) = \frac{\mu_1}{\mu_2} f_2 \tan(f_2 a_1 + \delta_2) \tag{32}$$

or:

$$\delta_2 = \tan^{-1}\left(\frac{\mu_2}{\mu_1}\frac{f_1}{f_2}\tan(f_1 a_1)\right) - f_2 a_1 \tag{33}$$

At the interface of $x=a_2$ E_y, E_z must be continuous:

$$\frac{F_3}{\varepsilon_3}\sin(f_3(a_3 - a_2)) = \frac{F_2}{\varepsilon_2}\cos(f_2 a_2 + \delta_2) \tag{34}$$

At the interface of $x= a_2$ H_y, H_z must be continuous:

$$\frac{1}{\mu_3}\frac{F_3}{\varepsilon_3} f_3 \cos(f_3(a_3 - a_2)) = \frac{1}{\mu_2}\frac{F_2}{\varepsilon_2} f_2 \sin(f_2 a_2 + \delta_2) \tag{35}$$

Dividing Equation 35 by Equation 34 results in:

$$\frac{f_3}{\tan(f_3(a_3 - a_2))} = \frac{\mu_3}{\mu_2} f_2 \tan(f_2 a_2 + \delta_2) \tag{36}$$

or:

$$\delta_2 = \tan^{-1}\left(\frac{\mu_2}{\mu_3}\frac{f_3}{f_2}\frac{1}{\tan(f_3(a_3 - a_2))}\right) - f_2 a_2 \tag{37}$$

The solution to the Helmholtz wave equation requires:

$$\gamma^2 - f_1{}^2 - k_y{}^2 + \omega^2 \mu_1 \varepsilon_1 = 0 \tag{38}$$

$$\gamma^2 - f_2{}^2 - k_y{}^2 + \omega^2 \mu_2 \varepsilon_2 = 0 \tag{39}$$

$$\gamma^2 - f_3{}^2 - k_y{}^2 + \omega^2 \mu_3 \varepsilon_3 = 0 \tag{40}$$

Subtracting Equation 38 from Equations 39 and 40 results in:

$$f_2^{\,2} = f_1^{\,2} + \omega^2\mu_1\varepsilon_1\left(\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1} - 1\right) = 0 \qquad (41)$$

$$f_3^{\,2} = f_1^{\,2} + \omega^2\mu_1\varepsilon_1\left(\frac{\mu_3\varepsilon_3}{\mu_1\varepsilon_1} - 1\right) = 0 \qquad (41)$$

Subtracting Equation 37 from Equation 33:

$$0 = \tan^{-1}\left(\frac{\mu_2}{\mu_3}\frac{f_3}{f_2}\frac{1}{\tan(f_3(a_3 - a_2))}\right) - \tan^{-1}\left(\frac{\mu_2}{\mu_1}\frac{f_1}{f_2}\tan(f_1 a_1)\right) - f_2(a_2 - a_1) \qquad (42)$$

The attenuation is:

$$\alpha = \left|\mathrm{IM}\left(\sqrt{\omega^2\mu_1\varepsilon_1 - k_y^{\,2} - f_1^{\,2}}\right)\right| \qquad (43)$$

The ratios of the vector potential amplitudes are:

$$\frac{\varepsilon_1 F_2}{\varepsilon_2 F_1} = \frac{\cos(f_1 a_1)}{\cos(f_2 a_1 + \delta_2)} \qquad (44)$$

$$\frac{\varepsilon_1 F_3}{\varepsilon_3 F_1} = \frac{\cos(f_1 a_1)}{\cos(f_2 a_1 + \delta_2)}\frac{\cos(f_2 a_2 + \delta_2)}{\sin(f_3(a_3 - a_2))} \qquad (45)$$

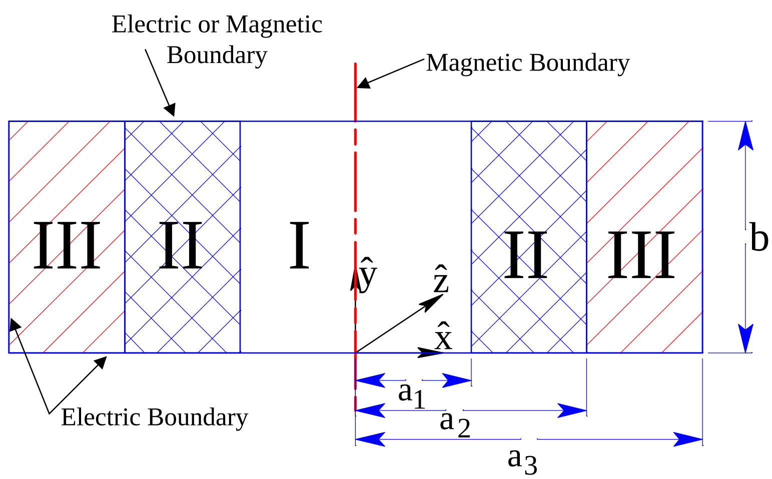

Figure 1. Geometry for sidebar absorbers.

ABSORBER ON THE TOP AND BOTTOM WALLS OF THE WAVEGUIDE

The geometry of the problem is shown in Figure 2. Without the absorber, the dominant mode in the waveguide will be the Transverse Electric to Z 1,0 mode (TE^Z_{10}). With the introduction of the absorber, the waveguide modes can no longer be classified as transverse to Z (TE^Z, TM^Z) but can be classified as transverse to Y (TE^Y, TM^Y). Since the $TM^{Y}{}_{10}$ mode is the same as the $TE^{Z}{}_{10}$ mode in the absence of absorber, and the incident mode on the absorbing section of waveguide will be $TE^{Z}{}_{10}$, we will consider TM^{Y} modes which are even in Y only. TM^{Y} modes can be derived from a y-directed magnetic vector potential:

$$\vec{A}(x, y, z) = \hat{y} A_y(x, y) e^{-\gamma z} \tag{46}$$

The magnetic field is given as:

$$\vec{H} = \frac{1}{\mu} \vec{\nabla} \times \vec{A} \tag{47}$$

The electric field is given as:

$$\vec{E} = \frac{1}{j\omega\mu\varepsilon} \vec{\nabla} \times \vec{H} \tag{48}$$

From Equations 46-48, magnetic field is:

$$H = \left(\frac{\gamma}{} A_y x + 0y + \frac{1}{} \frac{\partial}{\partial x} \hat{} \right) e^{-\ z} \tag{49}$$

The electric field is:

$$\vec{E} = \frac{1}{j\omega\mu\varepsilon} \left(\frac{\partial^2 A_y}{\partial x \partial y} \hat{x} - \left(\frac{\partial^2 A_y}{\partial x^2} + \gamma^2 A_y \right) \hat{y} - \gamma \frac{\partial F_y}{\partial y} \hat{z} \right) e^{-\gamma z} \tag{50}$$

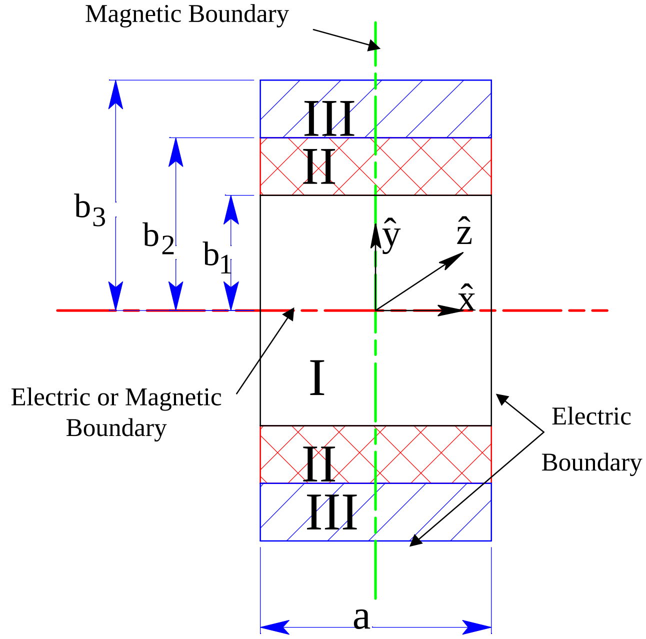

Figure 2. Geometry for absorber on the top and bottom of the waveguide.

Electric Boundary at y=0

If the array is operated in the difference mode for transverse cooling, the horizontal plane in the center of the aperture can be replaced with an electric conductor. To meet the boundary conditions in Region I of Figure 2 with an electric boundary for y=0, the magnetic vector potential must be of the form:

$$A_y(x,y) = A_1 \cos(k_x x)\cos(a_1 y) \tag{51}$$

The magnetic field in Region I is:

$$H_x(x,y) = \frac{\gamma A_1}{\mu_1}\cos(k_x x)\cos(a_1 y) \tag{52}$$

$$H_y(x,y) = 0 \tag{53}$$

$$H_z(x,y) = -\frac{\gamma A_1}{\mu_1}\frac{k_x}{\gamma}\sin(k_x x)\cos(a_1 y) \tag{54}$$

The electric field in Region I is:

$$E_x(x,y) = -\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{a_1 k_x}{\gamma^2}\sin(k_x x)\sin(a_1 y) \tag{55}$$

$$E_y(x,y) = \frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{\gamma^2 - k_x{}^2}{\gamma^2}\cos(k_x x)\cos(a_1 y) \tag{56}$$

$$E_z(x,y) = -\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{a_1}{\gamma}\cos(k_x x)\sin(a_1 y) \tag{57}$$

To meet the boundary conditions in Region II of Figure 2 with an electric boundary for y=0, the magnetic vector potential must be of the form:

$$A_y(x,y) = A_2 \cos(k_x x)\cos(a_2 y + \delta_2) \tag{58}$$

The magnetic field in Region II is:

$$H_x(x,y) = \left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)\frac{\gamma A_1}{\mu_1}\cos(k_x x)\cos(a_2 y + \delta_2) \tag{59}$$

$$H_y(x,y) = 0 \tag{60}$$

$$H_z(x,y) = -\left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)\frac{\gamma A_1}{\mu_1}\frac{k_x}{\gamma}\sin(k_x x)\cos(a_2 y + \delta_2) \tag{61}$$

The electric field in Region II is:

$$E_x(x,y) = -\left(\frac{\varepsilon_1}{\varepsilon_2}\right)\left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{a_2 k_x}{\gamma^2}\sin(k_x x)\sin(a_2 y + \delta_2) \tag{62}$$

$$E_y(x,y) = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)\left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{\gamma^2 - k_x^{\,2}}{\gamma^2}\cos(k_x x)\cos(a_2 y + \delta_2) \tag{63}$$

$$E_z(x,y) = -\left(\frac{\varepsilon_1}{\varepsilon_2}\right)\left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{a_2}{\gamma}\cos(k_x x)\sin(a_2 y + \delta_2) \tag{64}$$

To meet the boundary conditions in Region III of Figure 2 with an electric boundary for y=0, the magnetic vector potential must be of the form:

$$A_y(x,y) = A_3 \cos(k_x x)\cos(a_3(y - b_3)) \tag{65}$$

The magnetic field in Region III is:

$$H_x(x,y) = \left(\frac{\mu_1 A_2}{\mu_3 A_1}\right)\frac{\gamma A_1}{\mu_1}\cos(k_x x)\cos(a_3(y - b_3)) \tag{66}$$

$$H_y(x,y) = 0 \tag{67}$$

$$H_z(x,y) = -\left(\frac{\mu_1 A_2}{\mu_3 A_1}\right)\frac{\gamma A_1}{\mu_1}\frac{k_x}{\gamma}\sin(k_x x)\cos(a_3(y - b_3)) \tag{68}$$

The electric field in Region III is:

$$E_x(x,y) = -\left(\frac{\varepsilon_1}{\varepsilon_3}\right)\left(\frac{\mu_1 A_2}{\mu_3 A_1}\right)\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{a_3 k_x}{\gamma^2}\sin(k_x x)\sin(a_3(y - b_3)) \tag{69}$$

$$E_y(x,y) = \left(\frac{\varepsilon_1}{\varepsilon_3}\right)\left(\frac{\mu_1 A_2}{\mu_3 A_1}\right)\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{\gamma^2 - k_x^{\,2}}{\gamma^2}\cos(k_x x)\cos(a_3(y - b_3)) \tag{70}$$

$$E_z(x,y) = -\left(\frac{\varepsilon_1}{\varepsilon_3}\right)\left(\frac{\mu_1 A_2}{\mu_3 A_1}\right)\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{a_3}{\gamma}\cos(k_x x)\sin(a_3(y - b_3)) \tag{71}$$

At the interface of y=b_1 H_x, H_z must be continuous:

$$\cos(a_1 b_1) = \left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)\cos(a_2 b_1 + \delta_2) \tag{72}$$

At the interface of y= b_1 E_x, E_z must be continuous:

$$a_1 \sin(a_1 b_1) = \frac{\varepsilon_1}{\varepsilon_2}\left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)a_2 \sin(a_2 b_1 + \delta_2) \tag{73}$$

Dividing Equation 73 by Equation 72 results in:

$$a_1 \tan(a_1 b_1) = \frac{\varepsilon_1}{\varepsilon_2} a_2 \tan(a_2 b_1 + \delta_2) \tag{74}$$

or:

$$\delta_2 = \tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\frac{a_1}{a_2}\tan(a_1 b_1)\right) - a_2 b_1 \tag{75}$$

At the interface of $y=b_2$ H_x, H_z must be continuous:

$$\frac{A_3}{\mu_3}\cos(a_3(b_2 - b_3)) = \frac{A_2}{\mu_2}\cos(a_2 b_2 + \delta_2) \tag{76}$$

At the interface of $y= b_2$ E_x, E_z must be continuous:

$$\frac{1}{\varepsilon_3}\frac{A_3}{\mu_3} a_3 \sin(a_3(b_2 - b_3)) = \frac{1}{\varepsilon_2}\frac{A_2}{\mu_2} a_2 \sin(a_2 b_2 + \delta_2) \tag{77}$$

Dividing Equation 77 by Equation 76 results in:

$$-a_3 \tan(a_3(b_3 - b_2)) = \frac{\varepsilon_3}{\varepsilon_2} a_2 \tan(a_2 b_2 + \delta_2) \tag{78}$$

or:

$$\delta_2 = \tan^{-1}\left(-\frac{\varepsilon_2}{\varepsilon_3}\frac{a_3}{a_2}\tan(a_3(b_3 - b_2))\right) - a_2 b_2 \tag{79}$$

The solution to the Helmholtz wave equation requires:

$$\gamma^2 - a_1{}^2 - k_x{}^2 + \omega^2\mu_1\varepsilon_1 = 0 \tag{80}$$

$$\gamma^2 - a_2{}^2 - k_x{}^2 + \omega^2\mu_2\varepsilon_2 = 0 \tag{81}$$

$$\gamma^2 - a_3{}^2 - k_x{}^2 + \omega^2\mu_3\varepsilon_3 = 0 \tag{82}$$

Subtracting Equation 80 from Equations 81 and 82 results in:

$$a_2{}^2 = a_1{}^2 + \omega^2\mu_1\varepsilon_1\left(\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1} - 1\right) = 0 \tag{83}$$

$$a_3{}^2 = a_1{}^2 + \omega^2\mu_1\varepsilon_1\left(\frac{\mu_3\varepsilon_3}{\mu_1\varepsilon_1} - 1\right) = 0 \tag{84}$$

Subtracting Equation 79 from Equation 75:

$$0=\tan^{-1}\left(-\frac{\varepsilon_2}{\varepsilon_3}\frac{a_3}{a_2}\tan\left(a_3\left(b_3-b_2\right)\right)\right)-\tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\frac{a_1}{a_2}\tan\left(a_1 b_1\right)\right)-a_2\left(b_2-b_1\right) \quad (85)$$

The attenuation is:

$$\alpha=\left|IM\left(\sqrt{\omega^2\mu_1\varepsilon_1-k_x{}^2-a_1{}^2}\right)\right| \quad (86)$$

To satisfy the boundary conditions that the E_y, E_z vanish at x= ±a/2 and H_y, H_z vanish at x= 0:

$$k_x=(2m+1)\frac{\pi}{a} \quad (87)$$

The ratios of the vector potential amplitudes are:

$$\frac{\mu_1 A_2}{\mu_2 A_1}=\frac{\cos\left(a_1 b_1\right)}{\cos\left(a_2 b_1+\delta_2\right)} \quad (88)$$

$$\frac{\mu_1 A_3}{\mu_3 A_1}=\frac{\cos\left(a_1 b_1\right)}{\cos\left(a_2 b_1+\delta_2\right)}\frac{\cos\left(a_2 b_2+\delta_2\right)}{\cos\left(a_3\left(b_3-b_2\right)\right)} \quad (89)$$

<u>Magnetic Boundary at y=0</u>

If the array is operated in the sum mode for longitudinal cooling, the horizontal plane in the center of the aperture can be replaced with a magnetic conductor. To meet the boundary conditions in Region I of Figure 2 with an magnetic boundary for y=0, the magnetic vector potential must be of the form:

$$A_y(x,y)=A_1\cos\left(k_x x\right)\sin\left(a_1 y\right) \quad (90)$$

The magnetic field in Region I is:

$$H_x(x,y)=\frac{\gamma A_1}{\mu_1}\cos\left(k_x x\right)\sin\left(a_1 y\right) \quad (91)$$

$$H_y(x,y)=0 \quad (92)$$

$$H_z(x,y)=-\frac{\gamma A_1}{\mu_1}\frac{k_x}{\gamma}\sin\left(k_x x\right)\sin\left(a_1 y\right) \quad (93)$$

The electric field in Region I is:

$$E_x(x,y)=\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{a_1 k_x}{\gamma^2}\sin\left(k_x x\right)\cos\left(a_1 y\right) \quad (94)$$

$$E_y(x,y)=\frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{\gamma^2-k_x{}^2}{\gamma^2}\cos\left(k_x x\right)\sin\left(a_1 y\right) \quad (95)$$

$$E_z(x,y) = \frac{j\gamma}{\omega\varepsilon_1}\frac{\gamma A_1}{\mu_1}\frac{a_1}{\gamma}\cos(k_x x)\cos(a_1 y) \tag{96}$$

At the interface of y=b_1 H_x, H_z must be continuous:

$$\sin(a_1 b_1) = \left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)\cos(a_2 b_1 + \delta_2) \tag{97}$$

At the interface of y= b_1 E_x, E_z must be continuous:

$$-a_1\cos(a_1 b_1) = \frac{\varepsilon_1}{\varepsilon_2}\left(\frac{\mu_1 A_2}{\mu_2 A_1}\right)a_2\sin(a_2 b_1 + \delta_2) \tag{98}$$

Dividing Equation 98 by Equation 97 results in:

$$-\frac{a_1}{\tan(a_1 b_1)} = \frac{\varepsilon_1}{\varepsilon_2}a_2\tan(a_2 b_1 + \delta_2) \tag{99}$$

or:

$$\delta_2 = \tan^{-1}\left(-\frac{\varepsilon_2}{\varepsilon_1}\frac{a_1}{a_2}\frac{1}{\tan(a_1 b_1)}\right) - a_2 b_1 \tag{100}$$

Subtracting Equation 100 from Equation 75:

$$0 = \tan^{-1}\left(-\frac{\varepsilon_2}{\varepsilon_3}\frac{a_3}{a_2}\tan(a_3(b_3 - b_2))\right) - \tan^{-1}\left(-\frac{\varepsilon_2}{\varepsilon_1}\frac{a_1}{a_2}\frac{1}{\tan(a_1 b_1)}\right) - a_2(b_2 - b_1) \tag{101}$$

The ratios of the vector potential amplitudes are:

$$\frac{\mu_1 A_2}{\mu_2 A_1} = \frac{\sin(a_1 b_1)}{\cos(a_2 b_1 + \delta_2)} \tag{102}$$

$$\frac{\mu_1 A_3}{\mu_3 A_1} = \frac{\sin(a_1 b_1)}{\cos(a_2 b_1 + \delta_2)}\frac{\cos(a_2 b_2 + \delta_2)}{\cos(a_3(b_3 - b_2))} \tag{103}$$

A SAMPLE DESIGN

The design parameters of a sidebar waveguide absorber (as shown in Figure 1) operating between 5-7 GHz in the difference mode will be explored in this section. The width of Region I is 40 mm. The material in Region II is a ferrite from Trans-Tech called TT2-111R[2]. The material in Region III is alumina with a dielectric constant of 10. Figures 3-5 are contour plots of the attenuation as a function of the thickness of Regions II and III at 5, 6, and 7 GHz, respectively. Figure 6 shows the product at 5, 6, and 7 GHz of the attenuation as a function of absorber thickness.

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[2] PBAR Note 594, "Analysis of Microwave Properties for Various Absorbing Materials", Dave McGinnis, June 30,1998

The average attenuation in the band is defined as:

$$\langle \mathrm{L} \rangle = \frac{1}{3}\left(\mathrm{loss}_{5\mathrm{GHz}} + \mathrm{loss}_{6\mathrm{GHz}} + \mathrm{loss}_{7\mathrm{GHz}}\right) \tag{104}$$

To design an absorber that has flat attenuation over a wide bandwidth, the following function is defined:

$$\chi = \frac{3\langle \mathrm{L} \rangle}{\sqrt{\left(\mathrm{loss}_{5\mathrm{GHz}} - \langle \mathrm{L} \rangle\right)^2 + \left(\mathrm{loss}_{6\mathrm{GHz}} - \langle \mathrm{L} \rangle\right)^2 + \left(\mathrm{loss}_{7\mathrm{GHz}} - \langle \mathrm{L} \rangle\right)^2}} \tag{105}$$

This function is plotted in Figure 7.

- Comparing Figure 6 and Figure 7, the best set of parameters for large attenuation and wide bandwidth is to have the thickness in Region II to be 1.9 mm and Region III to be 1.6 mm.
- If Region II and Region III both consist of TT2-111R, the optimum combined thickness for maximum attenuation at 6 GHz is 2.8 mm.
- To have the maximum attenuation at 6 GHz, Region II should be 1.86 mm and Region III should be 2.3 mm.

Figure 8 is the attenuation as a function of frequency for the three different absorber designs. Figures 9 through 11 are the electromagnetic fields at 6 GHz as a function of position in the waveguide.

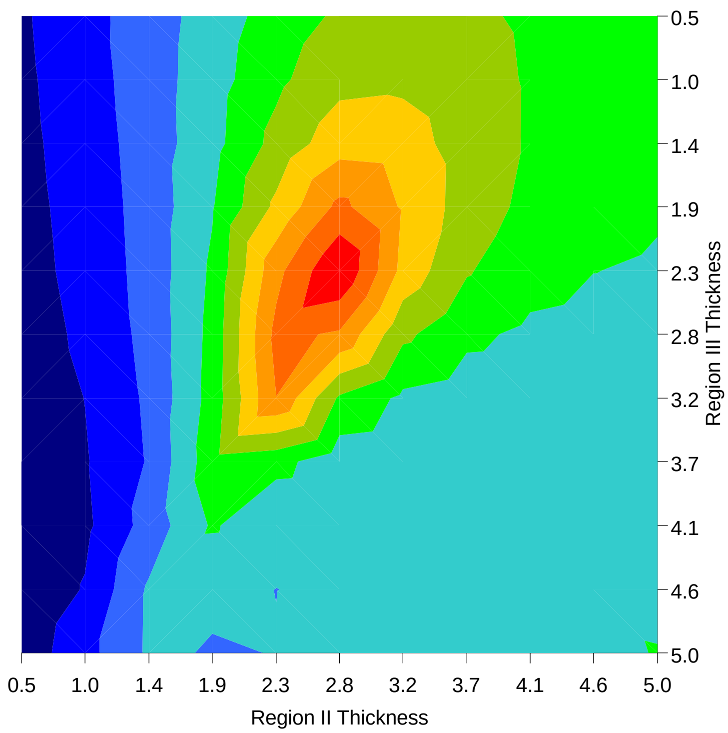

Figure 3. Attenuation at 5 GHz for a TT2-111R/Alumina sidebar absorber as a function of material thickness. Dimensions are in mm.

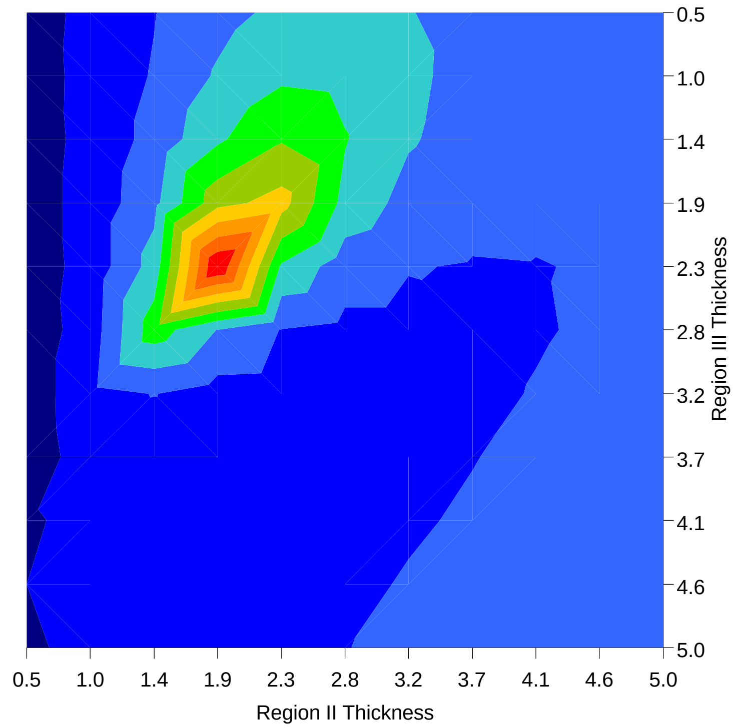

Figure 4. Attenuation at 6 GHz for a TT2-111R/Alumina sidebar absorber as a function of material thickness. Dimensions are in mm.

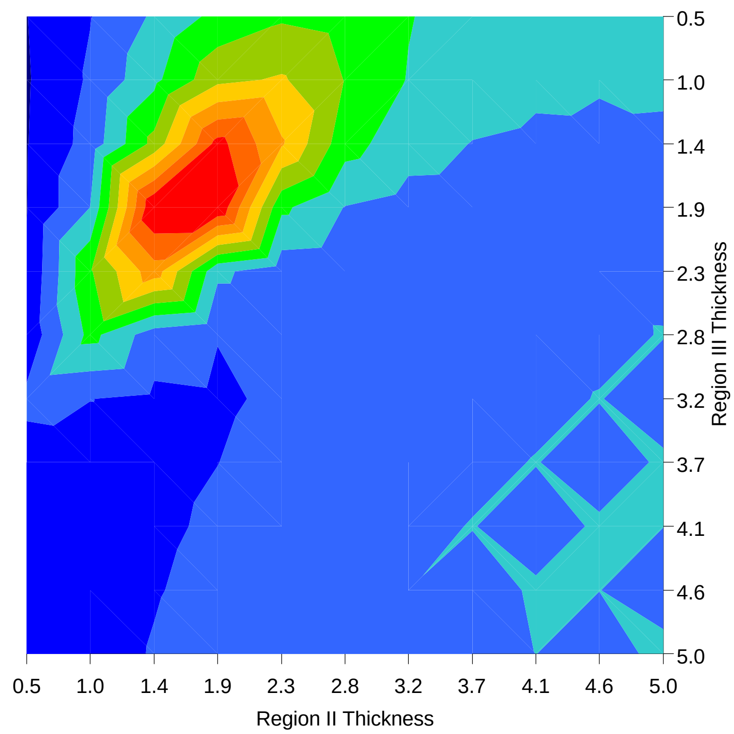

Figure 5. Attenuation at 7 GHz for a TT2-111R/Alumina sidebar absorber as a function of material thickness. Dimensions are in mm.

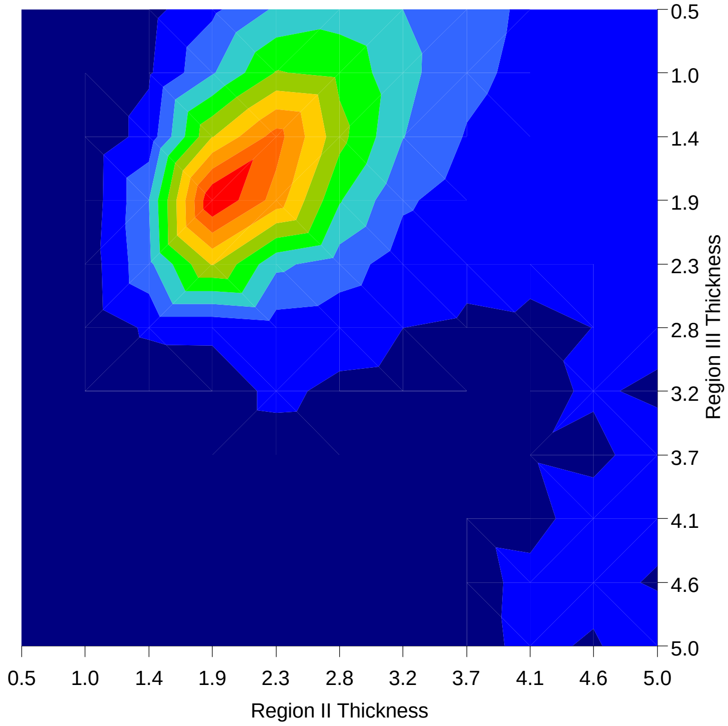

Figure 6. Product of the attenuation at 5,6, & 7 GHz for a TT2-111R/Alumina sidebar absorber as a function of material thickness. Dimensions are in mm.

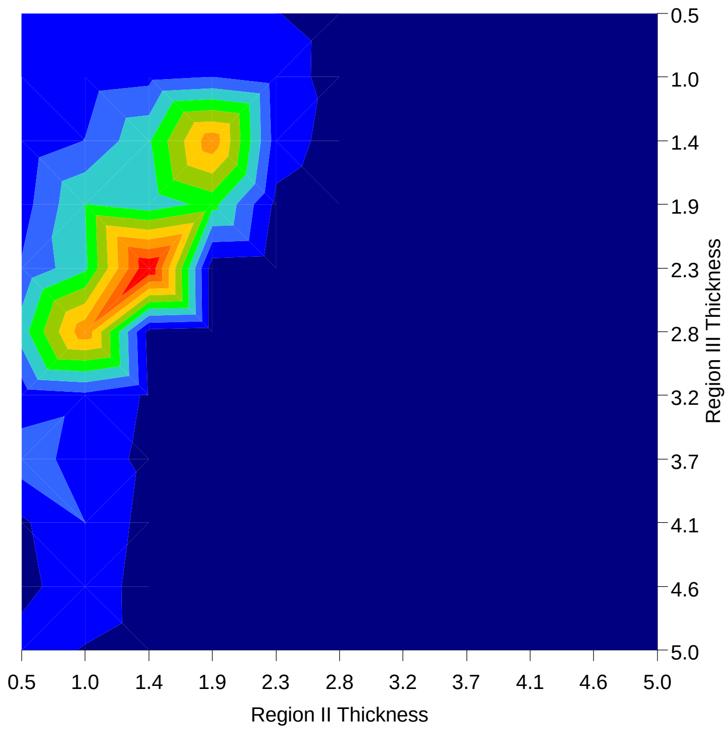

Figure 7. Equation 105 as a function of material thickness. Dimensions are in mm.

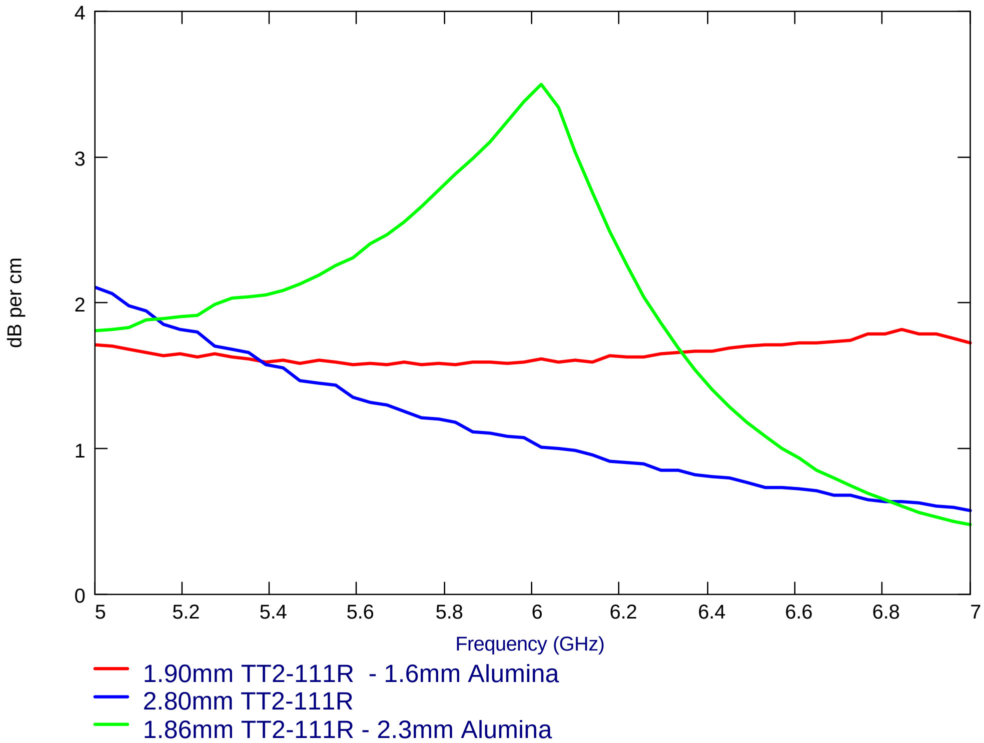

Figure 8. Attenuation as a function of frequency for three different sidebar absorber designs.

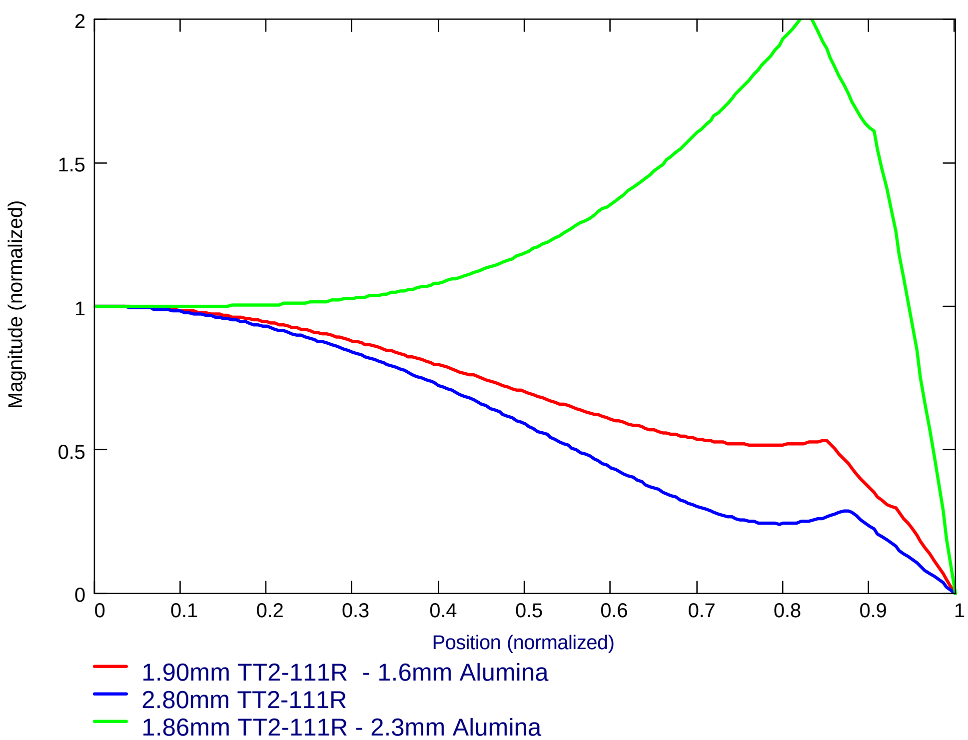

Figure 9. Magnitude of the electric field in the y direction at 6 GHz as a function of position in the waveguide. X=0 is the center of Region I.

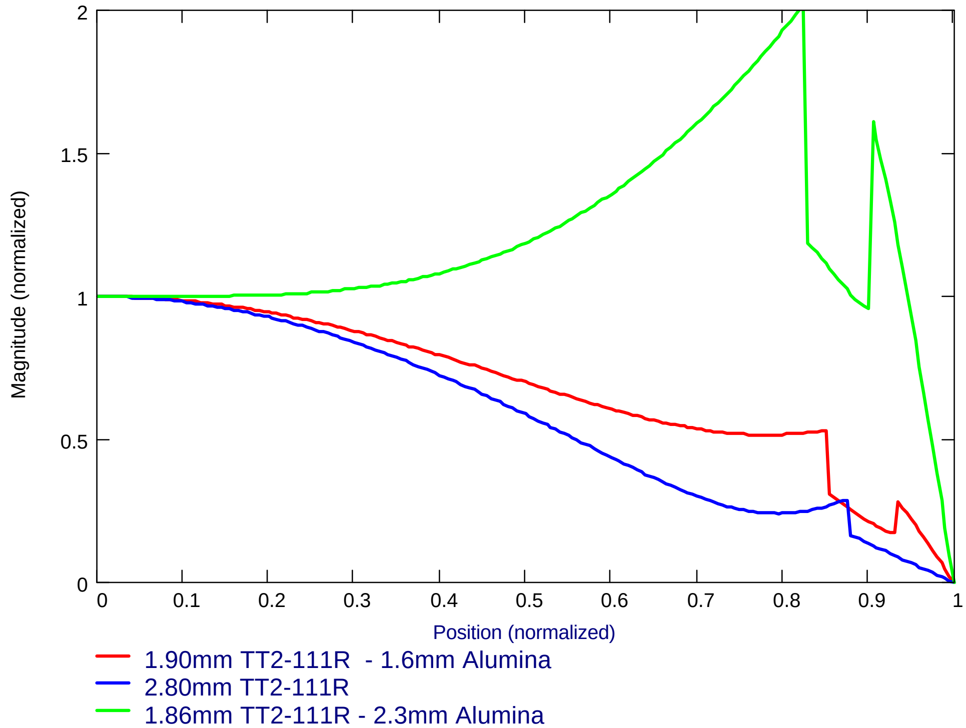

Figure 10. Magnitude of the magnetic field in the x direction at 6 GHz as a function of position in the waveguide. X=0 is the center of Region I.

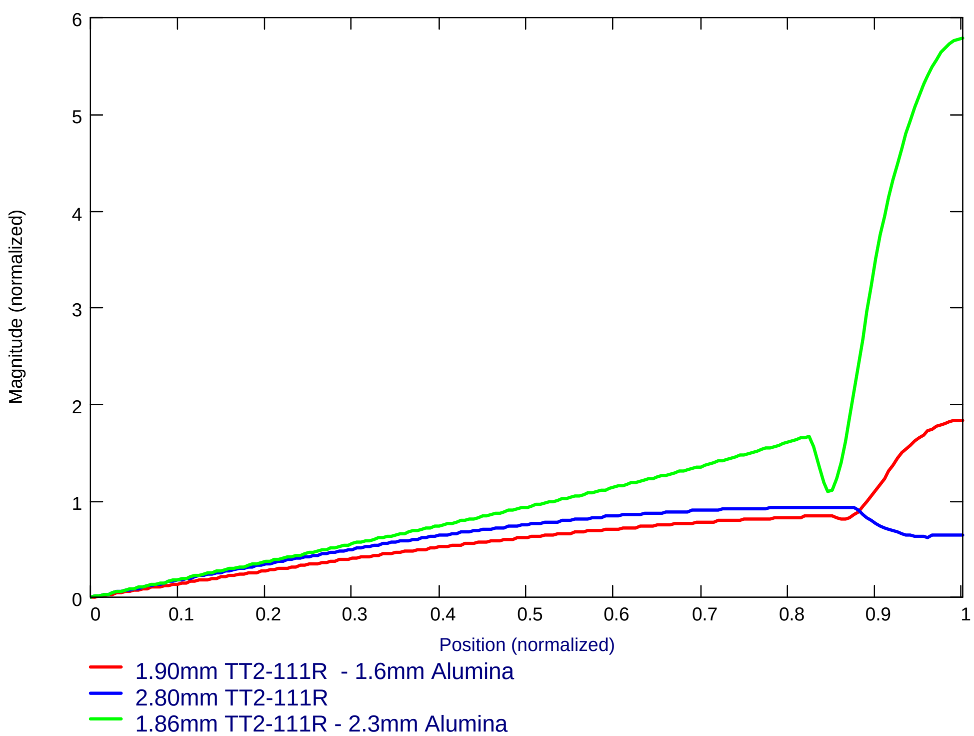

Figure 11. Magnitude of the magnetic field in the z direction at 6 GHz as a function of position in the waveguide. X=0 is the center of Region I.