Back of the Envelope Calculation of the RF coupling β for the FNAL H- Injector RFQ

Cheng-Yang Tan

Accelerator Division/Tevatron

ABSTRACT: This is a quick back of the envelope calculation of the RF coupling β for the injector RFQ. The simple calculation shows that the minimum $\beta = 1.43$ for 60 mA beam.

COUPLING β

An RFQ is a resonant structure and not a travelling wave structure and so we can apply the usual RLC model and beam loading phasor diagrams in its analysis. Travelling wave structures cannot use the RLC model because the waves excited by earlier bunch passes also propagate down the structure.

Therefore, we will use the usual RLC model for the RFQ. We will assume that the beam going through the structure can be parametrized by a point bunch. Technically, we should follow a slice through the RFQ starting from DC and then to its bunched beam structure at the end, but this is too complicated to do. In other words, we are just doing a back of the envelope calculation to see what the coupling β should be.

The standard formula for the *optimum* coupling β is

$$\beta = 1 + \frac{I_0 R \cos\phi}{V_c} \tag{1}$$

where R is the shunt impedance, V_c is the cavity voltage which includes the beam induced voltage, ϕ is the synchronous phase angle † and I_0 is the beam current. The relationship between V_c and the generator voltage V_g is shown in Figure 1.

We cannot use β written in this form in (1) because the synchronous phase ϕ changes as the beam is accelerated and bunched in the RFQ. Therefore, we are going to rewrite (1) in terms of power

$$\left.\begin{aligned} \beta &= 1 + \frac{I_0 V_c \cos\phi}{\left(\frac{V_c^2}{R}\right)} \\ &= 1 + \frac{I_0 V_a}{\left(\frac{V_c^2}{R}\right)} \\ &= 1 + \frac{P_{\text{beam}}}{P_c} \end{aligned}\right\} \tag{2}$$

† maximum field is at $\phi = 0$, see PARMTEQM manual page 12 and 13

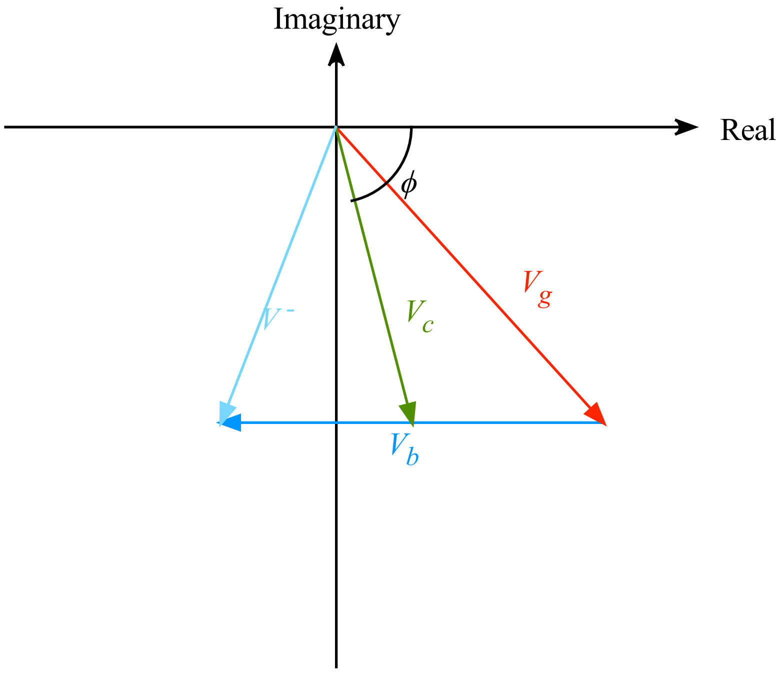

Figure 1 The usual phasor diagram for beam loading. The generator voltage V_g is pulled by the beam induced voltage $V_b/2$ and the accelerating voltage is $V_c \cos\phi$. Note that $-\pi/2 \leq \phi \leq 0$ for the RFQ.

where $V_a = V_c \cos\phi$ is the accelerating voltage, P_{beam} is the beam power and P_c is the power in the cavity which includes the beam induced power.

P_{beam} is trivial to calculate because

(i) $I_0 = 60$ mA from specifications.

(ii) $V_a = (750 - 35)$ kV $= 715$ kV because that is the equivalent voltage required to accelerate beam from 35 keV to 750 keV.

So

$$P_{\text{beam}} = I_0 V_a = (60 \times 10^{-3})[\text{A}] \times (715 \times 10^{3})[\text{V}] = 43 \text{ kW} \tag{3}$$

Now P_c is more complicated to calculate because it also depends on the induced voltage

V_b from the beam. However, we can make a good guess that because $|V_c| < |V_g|$[‡] then $P_c < P_g$ where P_g is the power of the generator. It is specified that $P_g < 100$ kW and so if the maximum generator power is used, we have a lower bound for β

$$\beta_{\text{min}} = 1 + \frac{43 \text{ [kW]}}{100 \text{ [kW]}} = 1.43 \tag{4}$$

For reference,[‡] the relationship between coupling β and s_{11} is

$$\beta = \text{VSWR} = \frac{1 + s_{11}}{1 - s_{11}} \tag{5}$$

‡ Only true if $-\pi/2 \leq \phi_s \leq 0$.

‡ R.G. Brown *et al*, "Lines, Waves, and Antennas. The Transmission of Electric Energy", pg. 284, 2nd Ed., J. Wiley and Sons, 1973.